Determination of the Aqueous Sublayer Thicknesses at an Air-Water Interface

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ABSTRACT

The thicknesses of the viscous and thermal sublayers in the water beneath an air-water interface are obtained by an application of the theory of rough-wall flows to results obtained in a laboratory wind, water-wave research facility. For fully rough flow the dimensionless viscous sublayer thickness δ_{v_+} is proportional to the square root of the roughness Reynolds number h_+ based on mean roughness height, i.e., $\delta_{v_+}=0.37h_+^{\frac{1}{2}}$. In addition, if Pr is the (molecular) Prandtl number, the dimensionless thermal sublayer thickness $\delta_{t_+}=0.37h_+^{\frac{1}{2}}\,\mathrm{Pr}^{-\frac{1}{2}}$.

1. Introduction

It is well established now that the heat transfer in the surface layer just beneath an air-water (the ocean-atmosphere) interface is controlled by a molecular process (Saunders, 1967; McAlister and McLeish, 1969; Hasse, 1971; Paulson and Parker, 1972; Hill, 1972; and Miller *et al.*, 1975). Wu (1971) postulated that the molecular (thermal and viscous) sublayer thicknesses at the air-sea interface are the same as those developed along a smooth solid surface and that the thermal sublayer is about one-half of the thickness of the viscous sublayer. He presented no experimental data to substantiate his hypothesis. Thus, to date no experimentally verified quantitative estimates have been obtained for the thickness of the molecular-dominated aqueous sublayer. Furthermore, the analytical and experimental results for solid rough walls of Owen and Thomson (1963) and Yaglom and Kader (1974) and the experimental results of Wu (1975) for air-water interfaces leave little doubt that, at wind speeds above, say, 3 m s^{-1} the air-sea interface is no longer smooth, and its aqueous sublayers must behave quite differently from those on a smooth solid surface.

Accordingly, we have used the results of Miller *et al.* (1975) and have extended the theory of Hasse (1971) to obtain quantitative estimates of the aqueous sublayer thicknesses.

2. Previous work

Saunders (1967) presented a simple theory which related the surface-water/bulk-water temperature difference at the ocean-air interface to the heat transfer and the stress on the interface. His result was given in the form

$$\lambda=\frac{\rho_w c_{p_w} U_{*_w}\Delta T_t}{Q_T}\left(\frac{\kappa}{\nu}\right), \tag{1}$$

where λ is a dimensionless heat transfer coefficient, ρ_w is the water density, c_{p_w} is the specific heat of water, U_{*_w} is the friction velocity in the water, ΔT_t is the temperature difference across the surface layer, Q_T is the total heat flux through the surface layer, κ is the thermal diffusivity through water and ν is the kinematic viscosity of water. He hypothesized that $\lambda=7$ from existing field data and the assumption that ΔT_t occurs across the conduction region of the surface layer.

Hasse (1971) determined ΔT_t as a function of Q_T through use of the fundamental equation for heat transfer in the water and wind tunnel determinations of the effective thermal diffusivity in the water boundary layer beneath the air-water interface. His diffusivity equation did not require determination of the thermal or viscous sublayer thicknesses but employed an empirical diffusivity relationship derived for solid boundaries and assumed to be correct through the entire water surface layer.

Paulson and Parker (1972) sought to test the hypotheses of Saunders (1967) in a laboratory facility. They found that λ was, indeed, a constant; but the value (=15) that they obtained was not in agreement with the results of others (see Miller *et al.*, 1975, Table 2.2).

Using experimental data and Owen and Thomson's (1963) theory, Kondo (1975) estimated the bulk transfer coefficients for heat and mass between a rough sea surface and the air flow above it. He discussed the apparent thickness of the viscous layer in air in relation to his determination of the roughness of the sea surface and formulated expressions for these layer thicknesses as functions of surface state. He found that the effective thickness of the viscous sublayer is thinner than that

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predicted by the relation ($\delta_{v_+}=11.6$) for a smooth flat solid surface. Interestingly, he noted the experimental evidence of Hill (1972) which shows that the rate of decrease of the sublayer thickness with increasing friction velocity is larger than that predicted from the theory for flow over an aerodynamically smooth surface.

Finally, Miller *et al.* (1975) presented an investigation of the energy exchange and temperature structure within the water, thermal layer at an air-water interface under the action of a turbulent wind. The effects of wind, water waves, air-water temperature differences, and fetch on the surface temperature and the thermal-layer heat transport were evaluated. Measurements were made in the Stanford Wind, Water-Wave Channel. A subset of the Miller *et al.* (1975) data is used here to establish the sublayer thicknesses at the air-water interface.

3. Theoretical foundations

There are two parts to the formulation, the first being a heat flux formulation and the second a formulation of the flow characteristics.

Hasse (1971) suggests treating the flux in the surface layer in differential form, *viz.* (see Fig. 1)

$$Q_T=-\rho_w c_{p_w} K\frac{\partial T}{\partial z}, \tag{2}$$

where $K=K(z)$ is the *effective* thermal diffusivity and T is the water temperature. If $\partial T/\partial x$ is essentially zero, and we speak of a steady mean flow so $\partial T/\partial t\equiv 0$, then

$$Q_T\approx -\rho_w c_{p_w} K\frac{dT}{dz}. \tag{3}$$

In the surface layer Q_T is constant below the level of back radiation and in the absence of solar (incoming) radiation. We ignore the temperature effects on ρ_w and c_{p_w} across the thermal layers. Then,

$$\frac{dT}{dz}=-\left(\frac{Q_T}{\rho_w c_{p_w}}\right)\frac{1}{K(z)}. \tag{4}$$

Owen and Thomson (1963) and Yaglom and Kader (1974) discussed the flow over and heat transfer from rough surfaces. They envision a region of molecularly dominated flow around and between the roughness protuberances. Dimensional arguments allow definition of a viscous sublayer in this flow with a thickness of the order of the square root of the roughness Reynolds number based on the mean height h of the protuberances. We can define then the following layers (see Fig. 1):

1) A thermal sublayer of thickness δ_t in which the molecular diffusivity κ dominates the eddy diffusivity.

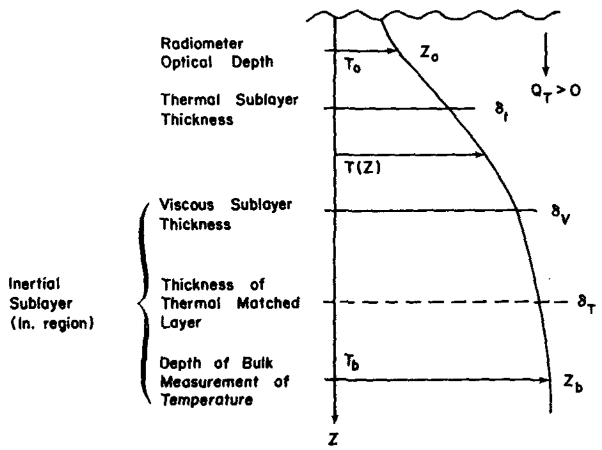

FIG. 1. Schematic of aqueous surface layer.

2) A viscous sublayer of thickness δ_v in which the molecular viscosity ν dominates the eddy viscosity.
3) A thermal matched layer at $z=\delta_T$, where the eddy diffusivity becomes equal to a value appropriate to a logarithmic variation of temperature in a turbulent boundary layer.

Yaglom and Kader (1974) defined the values of the necessary eddy viscosities and eddy diffusivities in the surface layers; we employ their formulations here. Owen and Thomson (1963) defined a transfer coefficient

$$B=\frac{Q_T}{\rho_w c_{p_w} U_{*w}\Delta T_t},$$

where $\Delta T_t=T_0-T_b$. Normally we write

$$\frac{1}{B}=\frac{\rho_w c_{p_w} U_{*w}\Delta T_t}{Q_T}. \tag{5}$$

The immediate objective of our work was to obtain an explicit relationship between $1/B$ and the parameters of the flow. Our goals then were *first* to use the results of Miller *et al.* (1975) to find the relation between δ_v and the boundary roughness h for rough flows and *second* to employ this relation to predict $1/B$ for any rough flow.

The key parameters of the problem are summarized as follows (see Fig. 1):

- B heat transfer coefficient [see Eq. (5)]
- c_{p_w} heat capacity of water
- H mean surface roughness height
- k' Kármán constant=0.40 [*or* we could use $1/\alpha=1/2.12=0.47$ according to Yaglom and Kader (1974) for $Pr_t=0.85$; this change has a small effect and is not used here]
- $K(z)$ effective thermal diffusivity in water
- Pr molecular Prandtl number$=\nu/\kappa$
- Pr_t turbulent Prandtl number
- Q_T total heat transport (positive downward)

T	temperature in water
T_a	air temperature
T_b	water temperature at $z=z_b$
T_0	water temperature at $z=z_0$
U_{*w}	$(\tau_0/\rho_w)^{\frac{1}{2}}$ = friction velocity
z	vertical coordinate (measured positive downward from mean water level)
z_b	depth of "bulk" water temperature measurement [z_b = 100 mm for the Miller *et al.* (1975) data]
z_0	radiometer optical depth [$z_0 \approx 140\ \mu$m for the Miller *et al.* (1975) data]
δ_v	viscous sublayer thickness
δ_t	thermal sublayer thickness
δ_T	thermal matching layer thickness
η	deviation of water surface from the mean water level
κ	thermal diffusivity of water
ν	kinematic viscosity of water
ρ_a	density of air
ρ_w	density of water
τ_0	surface shear stress

We also define the following Reynolds numbers:

$$h_+ = U_{*w}h/\nu;\quad z_+ = U_{*w}z/\nu;$$
$$z_{b_+} = U_{*w}z_b/\nu;\quad z_{0_+} = U_{*w}z_0/\nu;$$
$$\delta_{t_+} = U_{*w}\delta_t/\nu;\quad \delta_{T_+} = U_{*w}\delta_T/\nu;\quad \delta_{v_+} = U_{*w}\delta_v/\nu.$$

The essence of the analysis is prescription of $K(z)$. Following Yaglom and Kader (1974) we have, for fully rough flows where $h_+>100$,

$$\epsilon_H = a'_H\nu h_+^{-\frac{3}{2}} z_+^3, \tag{6}$$

$$\epsilon_M = a'_M\nu h_+^{-\frac{3}{2}} z_+^3, \tag{7}$$

near the rough wall, where ϵ_H and ϵ_M are the eddy diffusivities for heat and momentum, respectively, and a'_H and a'_M are supposed constant for any given boundary geometry. Eqs. (6) and (7) arise directly from continuity arguments and the requirement that ϵ_H and ϵ_M be of order ν at $z=\delta_v$.

For the layers shown in Fig. 1, using the definitions of δ_v, δ_t and δ_T and Eqs. (6) and (7) leads to the following conditions:

$$\text{at } z_+=\delta_{t_+},\quad \epsilon_H = a'_H\nu h_+^{-\frac{3}{2}}\delta_{t_+}^3 = \kappa, \tag{8}$$

$$\text{at } z_+=\delta_{v_+},\quad \epsilon_M = a'_M\nu h_+^{-\frac{3}{2}}\delta_{v_+}^3 = \nu, \tag{9}$$

$$\text{at } z_+=\delta_{T_+},\quad \epsilon_H = a'_H\nu h_+^{-\frac{3}{2}}\delta_{T_+}^3 = k'\nu\delta_{T_+}. \tag{10}$$

Thus we require $\epsilon_H=\kappa$ at the edge of the thermal sublayer, $\epsilon_M=\nu$ at the edge of the viscous sublayer, and $\epsilon_H = k'U_{*w}z$ as a matching condition so the eddy diffusivity given by the cubic estimate matches that of the logarithmic layer (cf. Kader and Yaglom, 1972, Section 3.2). Note $\delta_{v_+}\neq\delta_{T_+}$; indeed, for water we show below that $\delta_{v_+}\approx\frac{1}{2}\delta_{T_+}$.

Yaglom and Kader (1974) argue that $\delta_{v_+}=O(h_+^{\frac{1}{2}})$, while the constants a'_H and a'_M are not functions of h_+ or Pr but may be dependent on the surface roughness shapes. From Eq. (8)

$$\delta_{t_+}^3 = \left(\frac{1}{a'_H\,\mathrm{Pr}}\right)h_+^{\frac{3}{2}}$$

or

$$\delta_{t_+} = \left(\frac{1}{a'_H\,\mathrm{Pr}}\right)^{\frac{1}{3}}h_+^{\frac{1}{2}}, \tag{11}$$

i.e.,

$$\delta_{t_+} = O(h_+^{\frac{1}{2}})$$

as a consequence of the form chosen for ϵ_H, which was based on the previously determined result that $\delta_{v_+}=O(h_+^{\frac{1}{2}})$. From (9), we similarly obtain

$$\delta_{v_+}^3 = \frac{1}{a'_M}h_+^{\frac{3}{2}}$$

or

$$\delta_{v_+} = \left(\frac{1}{a'_M}\right)h_+^{\frac{1}{2}}. \tag{12}$$

It follows immediately, as Yaglom and Kader (1974) showed, that

$$\delta_{t_+}/\delta_{v_+} = O(\mathrm{Pr}^{-\frac{1}{3}}).$$

It is useful to go further here and, using (11) and (12), to find

$$\delta_{t_+}/\delta_{v_+} = \left(\frac{a'_M}{a'_H\,\mathrm{Pr}}\right)^{\frac{1}{3}}. \tag{13}$$

Now as $\mathrm{Pr}\to 1$, $\delta_{t_+}\to\delta_{v_+}$; therefore, we conclude that $a'_M=a'_H$. Because this is true, on returning to the general relation (13) we have *for all* Pr

$$\delta_{t_+}/\delta_{v_+} = \mathrm{Pr}^{-\frac{1}{3}}. \tag{14}$$

From (10)

$$\delta_{T_+}^2 = k'h_+^{\frac{3}{2}}\left(\frac{1}{a'_H}\right)$$

or

$$\delta_{T_+} = \left(\frac{k'}{a'_H}\right)^{\frac{1}{2}}h_+^{\frac{3}{4}}.$$

Thus

$$\delta_{t_+} = \left(\frac{1}{k'\,\mathrm{Pr}}\right)^{\frac{1}{3}}\delta_{T_+}^{\frac{2}{3}}. \tag{15}$$

Combining (14) and (15) yields

$$\delta v_+ = k'^{-\frac{1}{3}}\delta_{T_+}^{\frac{2}{3}}$$

or

$$\frac{\delta_{v_+}}{\delta_{T_+}} \approx \frac{1.4}{\delta_{T_+}^{\frac{1}{3}}}. \tag{16}$$

For $\delta_{T_+}\approx 10$, $\delta_{v_+}/\delta_{T_+}\approx 0.6$.

Using the above we have

$$z_0\leqslant z\leqslant\delta_T,\quad K(z)=\kappa + a'_H\nu h_+^{-\frac{3}{2}}z_+^3,$$
$$\delta_T\leqslant z\leqslant z_b,\quad K(z)=\kappa + k'U_{*w}z.$$

From (11)

$$a_H' \nu h_+^{-\frac{1}{3}} = \kappa \delta_{t_+}^{-3},$$

so

$$z_0 \leqslant z \leqslant \delta_T : K(z) = \kappa[1 + (z_+/\delta_{t_+})^3], \tag{17}$$

$$\delta_T \leqslant z \leqslant z_b : K(z) = \kappa(1 + k' \,\mathrm{Pr} z_+). \tag{18}$$

Therefore, (4) can be integrated to give

$$\int_{T_0}^{T_b} dT = \left(\frac{-Q_T}{\rho_w c_{p_w}}\right) \int_{z_0}^{z_b} \frac{dz}{K(z)}$$

or

$$\frac{T_b - T_0}{\left(\dfrac{-Q_T}{\rho_w c_{p_w}}\right)} = \int_{z_0}^{\delta_T} \frac{dz}{\kappa\left[1 + \left(\dfrac{z_+}{\delta_{t_+}}\right)^3\right]} + \int_{\delta_T}^{z_b} \frac{dz}{\kappa(1 + k' \,\mathrm{Pr} z_+)} = I_1 + I_2.$$

If $\Delta T_t = T_0 - T_b$ and $Q_T < 0$ for heat flow upward,

$$\frac{T_b - T_0}{\left(\dfrac{-Q_T}{\rho_w c_{p_w}}\right)} = \frac{-\Delta T_t}{\left(\dfrac{-Q_T}{\rho_w c_{p_w}}\right)} = \frac{\rho_w c_{p_w} \Delta T_t}{Q_T} = \frac{1}{B U_{*w}}.$$

Thus

$$\frac{1}{B} = U_{*w}(I_1 + I_2). \tag{19}$$

Now

$$I_1 = \int_{z_0}^{\delta_T} \frac{dz}{\kappa\left(1 + \dfrac{z_+^3}{\delta_{t_+}^3}\right)} = \frac{\nu}{\kappa U_{*w}} \int_{z_0}^{\delta_{T_+}} \frac{dz_+}{1 + \delta_{t_+}^{-3} z_+^3}$$

$$= \frac{\mathrm{Pr}}{U_{*w}} \int_{z_{0_+}}^{\delta_{T_+}} \frac{dz_+}{1 + \delta_{t_+}^{-3} z_+^3},$$

$$I_1 = \frac{\mathrm{Pr}}{U_{*w}} \left[\frac{\delta_{t_+}}{3} \left\{ \frac{1}{2} \ln \frac{(\delta_{t_+}^2 - \delta_{t_+} z_{0_+} + z_{0_+}^2)(\delta_{t_+} + \delta_{T_+})^2}{(\delta_{t_+}^2 - \delta_{t_+}\delta_{T_+} + \delta_{T_+}^2)(\delta_{t_+} + z_{0_+})^2} + (3)^{\frac{1}{2}} \left(\tan^{-1} \frac{2\delta_{T_+} - \delta_{t_+}}{(3)^{\frac{1}{2}} \delta_{t_+}} - \tan^{-1} \frac{2 z_{0_+} - \delta_{t_+}}{(3)^{\frac{1}{2}} \delta_{t_+}} \right) \right\} \right]. \tag{20}$$

$$I_2 = \int_{\delta_T}^{z_b} \frac{dz}{\kappa(1 + k' \,\mathrm{Pr} z_+)} = \frac{\nu}{\kappa U_{*w}} \int_{\delta_{T_+}}^{z_{b_+}} \frac{dz_+}{1 + k' \,\mathrm{Pr} z_+}$$

$$= \frac{\mathrm{Pr}}{U_{*w}} \times \frac{1}{k' \,\mathrm{Pr}} \ln \frac{1 + k' \,\mathrm{Pr} z_{b_+}}{1 + k' \,\mathrm{Pr} \delta_{T_+}}$$

$$= \frac{1}{k' U_*} \ln \frac{1 + k' \,\mathrm{Pr} z_{b_+}}{1 + k' \,\mathrm{Pr} \delta_{T_+}}. \tag{21}$$

Combining (19)–(21) produces

$$\frac{1}{B} = \frac{\mathrm{Pr}\delta_{t_+}}{3} \left[\frac{1}{2} \ln \left\{ \frac{(\delta_{t_+}^2 - \delta_{t_+} z_{0_+} + z_{0_+}^2)(\delta_{t_+} + \delta_{T_+})^2}{(\delta_{t_+}^2 - \delta_{t_+}\delta_{T_+} + \delta_{T_+}^2)(\delta_{t_+} + z_{0_+})^2} \right\} + (3)^{\frac{1}{2}} \left\{ \tan^{-1}\left(\frac{2\delta_{T_+} - \delta_{t_+}}{(3)^{\frac{1}{2}} \delta_{t_+}} \right) - \tan^{-1}\left(\frac{2 z_{0_+} - \delta_{t_+}}{(3)^{\frac{1}{2}} \delta_{t_+}} \right) \right\} \right] + \frac{1}{k'} \ln \left\{ \frac{1 + k' \,\mathrm{Pr} z_{b_+}}{1 + k' \,\mathrm{Pr} \delta_{T_+}} \right\}. \tag{22}$$

In view of (14) and (15), relation (22) gives $1/B$ explicitly as a function of δ_{v_+} and Pr for any measured or selected values of z_{0_+} and z_{b_+}.

4. Relevant experimental results

The data of Miller *et al.* (1975) can be used together with Eq. (22) to obtain an estimate of δ_{t_+} and δ_{v_+}. We seek specifically, for $h_+ > 100$, the constant a_+ in the relation

$$\delta_{v_+} = a_+ h_+^{\frac{1}{4}}, \quad h_+ > 100 \tag{23}$$

according to the hypothesis of Yaglom and Kader (1974).

Measurements were made in the Stanford Wind, Water-Wave Research Facility (Fig. 2) and reported in Miller *et al.* (1975). To establish a_+ we used 24 cases of wind-generated waves [a subset of the Miller *et al.* (1975) data] with $T_b - T_a \approx 2.5$, 5.0 and 7.5°C at fetches of 9.5 and 14.5 m. The basic data are given in Table 1.

The facility is about 35 m long; the test section is approximately 20 m long, 0.9 m wide and 1.93 m high. The channel is filled with water to a depth of about 1 m, leaving a 1 m deep air flow section. Air flow is produced by drawing air through the test section with an airfoil bladed fan at the downstream end of the channel. A honeycomb is situated at the end of the test section to suppress secondary flows caused by the centrifugal action of the fan. The air inlet is a curved section with a series of turning vanes, surmounted by a set of filters. An additional honeycomb and several small mesh screens further straighten and condition the air flow after passage through the inlet.

In the water a beach composed of a slanting solid surface and baskets of stainless steel lathe shavings is used to minimize wave reflections into the test section. The water is heated from below by six 9.5 mm diameter Chromolox electric heating cables which are located 50 mm above the channel floor, extend from near the wave-generating plate to the channel end, and supply up to 90 kW through a temperature controller giving relatively constant air-water temperature differences (±0.2°C) over an experimental run.

Water surface temperature T_0 was measured at an effective depth $z_0 = 140$ μm with an infrared radiometer employing an indium antimonide detector. Bulk water

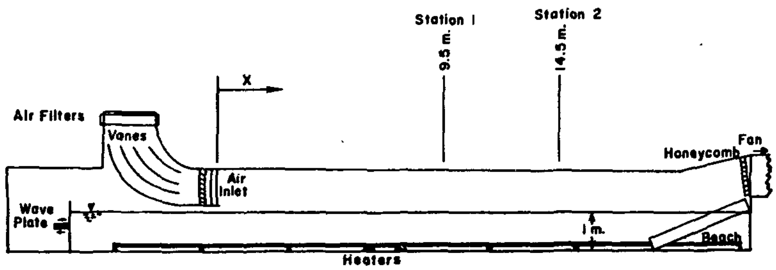

FIG. 2. Schematic of the Stanford Wind, Water-Wave Research Facility.

temperature was measured at a depth $z_b=100$ mm with a bead-in-glass thermistor. Extracting the surface-temperature/bulk-temperature difference involved special calibration and computational techniques and consideration of the wavelength dependent, water and air, optical properties within the detector bandwidth (1.5–5.5 μm); details are given in Miller *et al.* (1975). Calibrations showed the temperature difference ΔT_t to be accurate within ± 0.01°C [yielding a possible error (at the 90% confidence level) which does not exceed 18% for the smallest ΔT_t in Table 1 or 2.5% for the largest ΔT_t].

Other parameters, such as the mean free-stream windspeed, temperature and humidity and water surface elevation, were obtained as well. They formed the basis for calculating the total heat transfer Q_T and surface roughness Reynolds numbers h_+ through the use of data collected by previous investigators in the Stanford Facility (see, i.e., Mangarella *et al.*, 1973; Bole and Hsu, 1969). The latent and sensible transfers were computed from previous data; radiative transfer was estimated from the measured temperatures at the surface. The possible errors in Q_T and h_+ are estimated not to exceed 14 and 10%, respectively.

Three specific points need discussion before we use the experimental data.

a. The value of U_{*w}

Hill (1972), Hasse (1971), Saunders (1967) and we follow the concept of stress continuity ennunciated, e.g., by Shemdin (1972), *viz.*

$$U_{*w} = (\rho_a/\rho_w)^{\frac{1}{2}} U_{*a}, \tag{24}$$

TABLE 1. Experimental data.

Station	Fetch (distance from air inlet) (m)	Wind-speed U_∞ (m s^{-1})	Bulkwater temp. T_b (°C)	Water-air temp. diff. T_b-T_a (°C)	Prandtl no. Pr	Shear velocity U_{*w} (m s^{-1})	Total heat transfer $-Q_T$ (mW cm^{-2})	$-\Delta T_t = T_b - T_0$ (°C)	Transfer coefficient $1/B$	Roughness Reynolds number h_+
1	9.5	10.0	22.92	2.43	6.6	0.0201	80.2	0.11	11.2	455
1	9.5	7.6	22.52	2.37	6.6	0.0134	63.1	0.13	11.1	204
1	9.5	5.1	22.80	2.32	6.6	0.0073	44.0	0.16	11.1	58
1	9.5	2.6	23.12	2.11	6.6	0.0026	25.4	0.33	14.4	2.0
1	9.5	1.0	24.01	2.77	6.5	0.0006	16.5	0.46	7.3	0.1
1	9.5	10.0	25.78	4.54	6.2	0.0202	108.8	0.14	10.5	481
1	9.5	7.6	25.68	4.94	6.2	0.0133	97.6	0.17	9.8	216
1	9.5	5.0	26.35	5.64	6.1	0.0071	70.5	0.29	12.1	60.6
1	9.5	2.5	26.37	5.11	6.2	0.0026	37.9	0.45	12.8	2.0
1	9.5	0.8	26.57	4.90	6.2	0.0005	19.5	0.67	7.2	0.1
1	9.5	9.9	30.43	9.39	5.6	0.0201	160.8	0.28	14.5	522
1	9.5	7.5	31.61	8.88	5.4	0.0132	142.0	0.29	11.2	238
1	9.5	5.0	32.43	8.24	5.4	0.0071	99.2	0.41	12.2	68
1	9.5	2.5	33.18	8.09	5.3	0.0026	53.6	0.73	14.5	2.4
1	9.5	0.9	33.61	6.67	5.3	0.0006	26.7	0.80	7.1	0.1
2	14.5	10.1	21.65	2.48	6.6	0.0206	60.6	0.12	17.4	585
2	14.5	7.5	21.44	1.98	6.8	0.0133	*	*	*	*
2	14.5	5.1	21.90	2.37	6.7	0.0074	31.1	0.17	17.4	75
2	14.5	10.1	25.19	5.81	6.3	0.0206	84.4	0.17	17.7	620
2	14.5	7.5	24.60	5.67	6.4	0.0132	57.4	0.15	14.0	272
2	14.5	5.1	24.44	5.49	6.4	0.0074	43.3	0.19	13.6	78
2	14.5	9.9	28.12	7.33	5.9	0.0200	104.7	0.15	11.9	636
2	14.5	7.5	27.58	7.37	6.0	0.0132	73.0	0.17	12.7	286
2	14.5	5.0	27.64	7.78	6.0	0.0073	56.1	0.22	11.9	80

* Value not used because of errors in ΔT_t and in radiation and latent heat transfer calculations or data.

where the shear velocity U_{*_a} in the air was determined from wind velocity data. Wu (1975) points out that for his laboratory facility the fraction of the momentum flux $\rho_a U_{*a}^2$ from the air which goes directly to drift currents ranges from about 0.6 to 0.8 in the rough flow regime, the remaining flux going to wave generation. In a rough boundary case for heat transfer where eddying is driven by the drift current boundary layer flow and the random water wave motion together, it is not entirely clear whether one should use U_{*_w} from (24) or some fraction thereof as a scaling velocity [the fraction ranges from $(0.6)^{\frac{1}{2}} \approx 0.8$ to $(0.8)^{\frac{1}{2}} \approx 0.9$, apparently]. Pending further evidence we have used (24).

b. The value of h

Miller *et al.* (1975) determined the variance of the water surface displacement $(\overline{\eta^2})^{\frac{1}{2}}$. Colonell (1966) ran tests in the Stanford Facility and, using field results as well, showed that wave heights in the laboratory and the field (in the absence of swell) follow a Rayleigh probability distribution to a good approximation [cf. Kinsman (1965) for support of this point]. From Colonell (1966) then we have the relationship between the mean wave height h and the variance

$$h = (2\pi)^{\frac{1}{2}} (\overline{\eta^2})^{\frac{1}{2}} = 2.5 (\overline{\eta^2})^{\frac{1}{2}}. \tag{25}$$

The mean wave height h corresponds in our cases to the mean height of roughness elements used by Yaglom and Kader (1974) for solid walls. The fact that the wave roughness distribution is essentially the same in laboratory and field (barring the presence of swell) suggests strongly that our results will hold for the field.

Miller *et al.* (1975) tabulated a roughness Reynolds number $\eta_+ = U_{*_w} (\overline{\eta^2})^{\frac{1}{2}} / \nu$. Using (25) leads to

$$h_+ = 2.5 \eta_+$$

which is given here in Table 1.

c. The nature of the sublayer

Calculation of $R = U_d x / \nu$ [where x is the fetch and, according to Wu (1975), U_d = drift velocity $\approx 0.55 U_{*_a}$ and so $U_d \approx 15.7 U_{*_w}$] shows that for a free-stream windspeed $U_\infty > 5.0$ m s^{-1} the aqueous boundary layer in the Stanford facility should be turbulent if it acts as the boundary layer on a smooth plate. Similarly, estimates of the boundary layer thickness on a flat plate show that the thickness expected is greater than $z_b = 100$ mm for all flows; hence, our measuring points are well within the boundary layer [Wu (1975) shows log profiles for all windspeeds to depths greatly exceeding z_b for unheated cases.].

In the Stanford facility the water is heated from below by immersion heaters as noted above. The resulting unstable conditions act to destroy any mean temperature variations in the vertical except where the molecular-dominated and intermediate layers develop properly under the forced convection of the shear- and wave-induced drift current beneath the interface. A series of rough measurements of the vertical temperature profile made in our channel under the conditions of Table 1 (except $T_b - T_a = 10°C$) confirms that there is no significant gradient in the bulk water zone when the heaters are on. [Arya (1975) shows a similar trend for his unstable wall flows (cf., his Fig. 1 for $R_i < 0$ cases).] On the other hand, there is a measurable gradient at low wind speeds when the heaters are off.

Now our theory assumes the existence of several flow regions including (for $z > \delta_T$) an inertial subrange of velocity and the accompanying logarthmic temperature region (cf. Tennekes and Lumley, 1972). In the Stanford facility it is clear that the logarithmic zone of temperature variation (for $z \geqslant \delta_T$) does not exist. Thus, in our analysis of the Miller *et al.* (1975) data we delete the zone $\delta_T \leqslant z \leqslant z_b$ from the theory to accommodate the experimental conditions; the result is that T_b is measured effectively at $z = \delta_T$, *not* at $z = z_b$. This leads to a consistent result and, because we are seeking to determine δ_v and δ_t which are less than δ_T, this deletion of the zone $z > \delta_T$ has no effect on the analysis for δ_v and δ_t.

5. Calibration of the theory

We began with the 24 cases listed in Table 1. The indicated case was subsequently discarded as being obviously in error on several counts not related to the present analysis. The range of data was chosen to include runs clearly not within the province of the theory so that we could clearly establish that the behavior for $h_+ > 100$ was not continued for $h_+ < 100$. Using $z_{b_+} = \delta_{T_+}$ in (22) we obtained estimates of δ_{T_+}, δ_{v_+} and δ_{t_+} for experimentally determined $1/B$, h_+ and Pr.

Fig. 3 presents our results. Several points are clear:

1) For $h_+ > 100$ the aqueous sublayer is fully rough and $\delta_{v_+} \propto h_+^{\frac{1}{4}}$. Indeed, $a_+ = 0.37$ so

$$\delta_{v_+} \approx 0.37 h_+^{\frac{1}{4}}. \tag{26}$$

The very small downward drift of data points in the range $h_+ > 100$ in Fig. 3 is probably due to the change in surface shapes and roughness density with windspeed [see the remarks following Eqs. (6) and (7)].

2) If we use (26) and the given data (for cases where $h_+ \geqslant 100$) to predict $1/B = \lambda$ Pr under the assumption that the logarithmic layer exists, we find $1/B$ values in the range 20 to 30 and λ values in the range 3.4 to 4.8 which are in reasonable agreement with the work of Hill (1972) and McAlister and McLeish (1969). These results are also *not* inconsistent with the values obtained by Hasse (1971) and Saunders (1967). However, Saunders (1967) hypothesis is inconsistent with the analysis of Owen and Thomson (1963) in that λ ought to vary with h_+ and Pr. Indeed, from (22) and

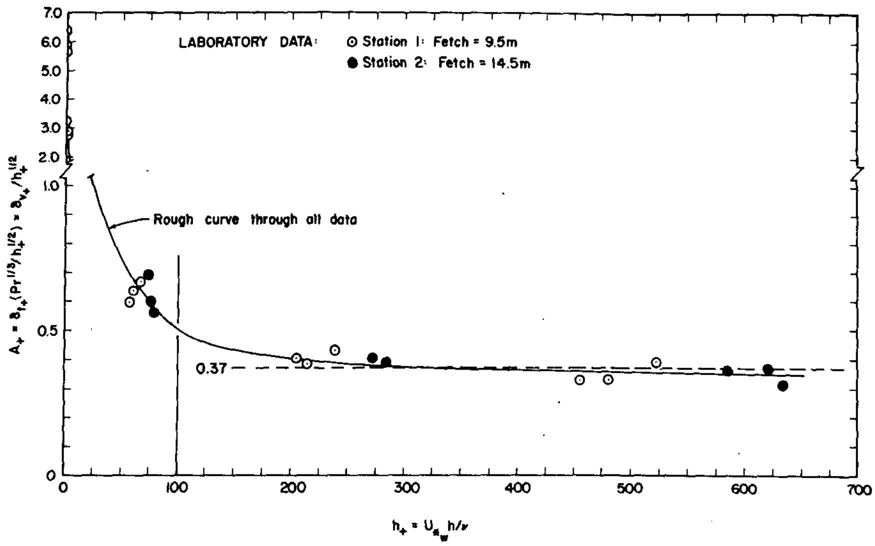

FIG. 3. Determination of the coefficient a_+ for $h_+ \geq 100$.

(26) we deduce that, *roughly*, with regard to Pr,

$$1/B \propto Pr^{2/3} \text{ so } \lambda \propto Pr^{-2/3}.$$

6. Conclusions

The above theory provides a rational means for evaluating the aqueous sublayer thickness at an air-water interface. The hypothesis of Yaglom and Kader (1974) for the viscous sublayer thickness for fully rough solid wall flows is found to be valid for fully rough air-water interface flows as well. Pending further experimental verification, the thicknesses δ_{v_+} and δ_{t_+} can be given by

$$\delta_{v_+} = 0.37h_+^{1/2}$$

$$\delta_{t_+} = 0.37h_+^{1/2}\,Pr^{-2/3}$$

for $h_+ \geq 100$. Eq. (22) then yields an estimate for the sublayer Stanton number B.

Wu's (1971) conjecture that the viscous sublayer thickness at the air-sea interface can be assumed to be the same as for a solid surface is incorrect for fully rough flows as illustrated in Fig. 4. While δ_{v_+} is less than the solid-wall value of 11.6 for the laboratory experiments, δ_{v_+} clearly can exceed 11.6 for field conditions.

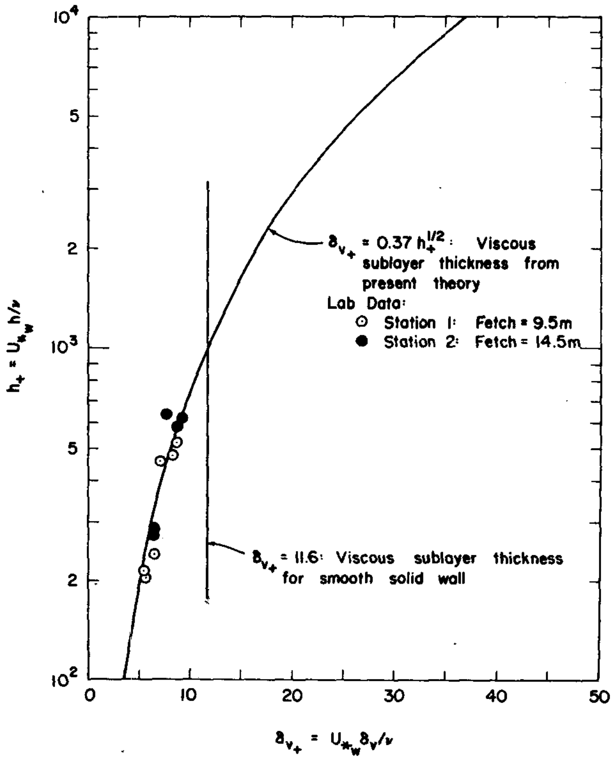

FIG. 4. Variation of nondimensional viscous sublayer thickness with surface roughness Reynolds number.

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