Is scattering just fallibilism in the wave-ice fandango?

Vernon Squire
with help from Fabien Montiel and Johannes Mosig

University of Otago, New Zealand

Presentation
September 2016, Washington DC

What I will talk briefly about

1. Montiel, F., V. A. Squire, and L. G. Bennetts. Evolution of directional wave spectra through finite regular and randomly perturbed arrays of scatterers. *SIAM J. Appl. Math.*, 75(2), 630–651, doi:10.1137/1409739062015, 2015.
2. Montiel, F., V. A. Squire, and L. G. Bennetts. Attenuation and directional spreading of ocean wave spectra in the marginal ice zone. *J. Fluid Mech.*, 790, 492–522, doi:10.1017/jfm.2016.21, 2016.
3. Mosig, J. E. M., F. Montiel, and V. A. Squire. Comparison of viscoelastic-type models for ocean wave attenuation in ice-covered seas. *J. Geophys. Res.*, 120, doi:10.1002/2015JC010881, 2015.
4. Mosig, J. E. M., F. Montiel, and V. A. Squire. Water wave scattering from a mass loading ice floe of random length using generalised polynomial chaos. *Wave Motion* (to appear).
5. Squire, V. A., and F. Montiel. Evolution of directional wave spectra in the marginal ice zone: a new model tested with legacy data. *J. Phys. Oceanogr.* (to appear 2016, doi:10.1175/JPO-D-16-0118.1).
6. Montiel, F., and V. A. Squire. Breakup of ice floes in marginal ice zones and its implication for floe size distribution. *Underway* 2016/17.

JFM and JPO papers

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Attenuation and directional spreading of ocean wave spectra in the marginal ice zone

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A theoretical model is used to study wave energy attenuation and directional spreading of ocean wave spectra in the marginal ice zone (MIZ). The MIZ is constructed as an array of tens of thousands of compliant circular ice floes, with randomly selected positions and radii determined by an empirical floe size distribution. Linear potential flow and thin elastic plate theories model the coupled water–ice system. A new method is proposed to solve the time-harmonic multiple scattering problem under a multidirectional incident wave forcing with random phases. It provides a natural framework for tracking the evolution of the directional properties of a wave field through the MIZ. The attenuation and directional spreading are extracted from ensembles of the wave field with respect to realizations of the MIZ and incident forcing randomly generated from prescribed distributions. The averaging procedure is shown to converge rapidly so that only a small number of simulations need to be performed. Far-field approximations are investigated, allowing efficiency improvements with negligible loss of accuracy. A case study is conducted for a particular MIZ configuration. The observed exponential attenuation of wave energy through the MIZ is reproduced by the model, while the directional spread is found to grow linearly with distance. The directional spreading is shown to weaken when the wavelength becomes larger than the maximum floe size.

Key words: sea ice, wave scattering, wave–structure interactions

1. Introduction

There is now growing evidence that ocean surface waves have a significant impact on the seasonal advance and retreat of sea ice in the Arctic and Southern Oceans. Satellite observations have shown that the energy content of wave spectra in the polar oceans has been trending upwards in the last three decades, more significantly than at lower latitudes (Young, Zieger & Babanin 2011). Recent *in situ* observations and hindcasts of energetic wave fields at high latitudes (Kohout *et al.* 2014; Thomson & Rogers 2014; Collins *et al.* 2015) support these long-term trends and suggest an increasing impact of waves on the morphology of ice-covered oceans. In particular, waves contribute to the rapid decline of sea ice extent and thickness observed in the

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Evolution of Directional Wave Spectra in the Marginal Ice Zone: A New Model Tested with Legacy Data

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ABSTRACT

Field experimental data from a 1980s program in the Greenland Sea investigating the evolution of directional wave spectra in the marginal ice zone are reanalyzed and compared with the predictions of a new, phase-resolving, three-dimensional model describing the two-dimensional scattering of the waves by the vast number of ice floes that are normally present. The model is augmented with a dissipative term to account for the nonconservative processes affecting wave propagation. Observations reported in the experimental study are used to reproduce the ice conditions and wave forcing during the experiments. It is found that scattering alone underestimates the attenuation experienced by the waves during their passage through the ice field. With dissipation, however, the model can replicate the observed attenuation for most frequencies in the swell regime. Model predictions and observations of directional spreading are in agreement for short to midrange wave periods, where the wave field quickly becomes isotropic. For larger wave periods, little spreading can be seen in the model predictions, in contrast to the isotropic or near-isotropic seas reported in the experimental study. The discrepancy is conjectured to be a consequence of the inaccurate characterization of the ice conditions in the model and experimental errors.

1. Introduction

Over 30 years of passive microwave radiometry data collected by several satellites demonstrate that sea ice morphology is changing. In the Arctic, we have experienced more than a 55% decrease in summer sea ice extent during that period (Meier et al. 2013; Jeffries et al. 2013), and there have been reductions in the thickness too (Kwok and Rothrock 2009; Wadhams et al. 2011). In contrast, the Southern Ocean has experienced a modest increase in maximum sea ice extent along with greater variability in its spatial distribution around the Antarctic continent (Simpkins et al. 2013). Compelling evidence suggests that global climate change is responsible in both cases, noting that the physical processes causing the observed effects are different between the two polar oceans.

The primary agent causing metamorphosis of the sea ice during the Arctic summer is the positive ice–albedo feedback effect. An accompanying trending upward of wind and wave intensity (Young et al. 2011; Thomson and Rogers 2014; Thomson et al. 2016) further assists sea ice attrition, as ocean waves propagating through fields of sea ice can fracture the ice floes (see, e.g., Squire et al. 1995; Squire 2007, 2011), enhancing their melting in the summer and aiding freezing in the winter. These latter effects have been identified as potential contributors to the observed sea ice extent trends in the Arctic (Thomson and Rogers 2014) and Southern Oceans (Kohout et al. 2014). Granting that the deleterious contribution from penetrating ocean waves is most pronounced within, say, 100 km of the ice edge, the transformation of the summer Arctic to being more like a marginal ice zone (MIZ; usually defined as being the region of ice receptive to open-ocean processes) signifies an increasingly pivotal role for harmful ocean waves that is fueling contemporary polar oceanographic research such as the U.S. Office of Naval Research (ONR) initiative acknowledged later.

Ocean waves entering an MIZ reduce in amplitude as they propagate farther into the ice field, with the rate of attenuation being directly related to the wave frequency

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MIZ 101

Revision – what we all know

Ocean waves in the range $T = 5$–$20\,\mathrm{s}$ ...

- reduce in amplitude as they travel through sea ice fields due to
  - *dissipative* energy loss $-\alpha_{\mathrm{dis}}$
  - *conservative* wave scattering $-\alpha_{\mathrm{scat}} \Rightarrow$ redistribution of wave energy $\Rightarrow$ localization $\Rightarrow$ exponential decay
- are redistributed directionally, depending on their period
- break up and move ice floes around if sufficiently energetic
- are governed by the energy flux equation, here with both scattering and dissipation ice source terms included, viz.

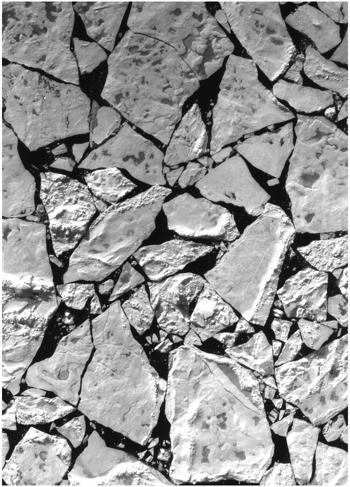
$$\mathcal{D}_t E = \sum S = S_{\mathrm{in}} + S_{\mathrm{nl}} + S_{\mathrm{ice}},\ E = E(\mathbf{x}, t; \omega, \theta),\ S_{\mathrm{ice}} = S_{\mathrm{ice}}(\mathbf{x}, t; \omega, \theta)$$

$$\frac{1}{c_{\mathrm{g}}} S_{\mathrm{ice}} = -(\alpha_{\mathrm{scat}} + \alpha_{\mathrm{dis}}) E + \int_0^{2\pi} K(\theta - \theta') E \,\mathrm{d}\theta$$

$$\text{such that } \int_0^{2\pi} S_{\mathrm{ice}} \,\mathrm{d}\theta' = 0,\ -\alpha_{\mathrm{scat}} + \int_0^{2\pi} K(\theta - \theta') \,\mathrm{d}\theta = 0$$

(note that the columns of matrix $\alpha_{\mathrm{scat}}\mathbf{I} - \mathbf{K}$ will add to 0 in discrete version $\Rightarrow$ rows are linearly dependent, so at least 1 zero eigenvalue).

MIZ genesis and maintenance due to waves



MIZ morphology

- created by wave-induced ice breaking
- random distribution of small floes; $a \sim \mathrm{O}(10\text{–}100)\,\mathrm{m}$
- wave activity in swell regime; $T = 5\text{–}20\,\mathrm{s}$, i.e. $ka = \mathrm{O}(1)$ where scattering dominates
- waves experience attenuation and directional spreading.

Where we want to be

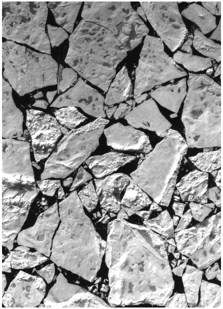
Properly modelling waves in the MIZ, allowing for large amplitudes and energy dissipation

But the primary goal here is . . .

To model attenuation *and directional spreading* of ocean waves in the MIZ due entirely to conservative wave scattering by random arrays of ice floes and to embed parametrizations of their effects in ice/ocean models and OGCMs.

The model, which is described is nauseous detail in 2 and is based upon reference 1, has the components:

- MIZ generated using a floe size distribution (FSD) extracted from a specified probability distribution e.g. the bounded power law function[1], $f(a) = (a/a_{\min})^{-\kappa}$ with $a_{\min}$, $a_{\max}$, κ and concentration, drawn from observations ⟸
- forcing defined by an incoherent directional wave spectrum with prescribed angular spreading $A(\tau) = (2/\pi)\cos^2(\tau)$ for $-\pi/2 \le \tau \le \pi/2$, and a random phase φ.
- the ability to produce outputs (attenuation, directional spreading, breakup, refraction, etc.) using ensemble averaging techniques.

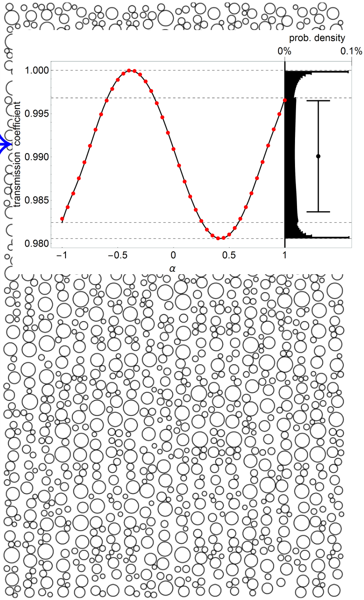


[1] Toyota et al. (2011)

The model 2

Physics and assumptions

- Finite number of circular floes with prescribed randomized radius and thickness to be checked with gPC 4
- Linear water wave theory
- Each floe is represented as a thin viscoelastic plate, i.e. dissipative effects such as floe collisions, ridge-building and rafting, overwash, viscous damping, inelasticity, turbulence, vortex shedding, etc., are *parameterized*
- Periodic motion
- Multiple scattering in deterministic framework.

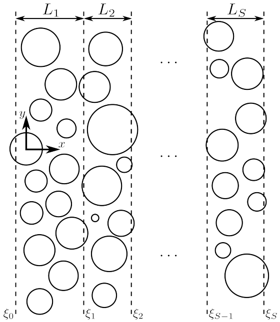


The strip-clustering method

Existing 2(+1)D multiple scattering techniques are usually limited to a few hundred scatterers (floes). To remedy this shortcoming, we have developed the strip-clustering method.

The MIZ is subdivided into contiguous strips of designated finite width running parallel to the ice edge. The solution procedure is then decomposed into 3 scales; floe, strip and MIZ.

Dissipation is now included in a linear paradigm.



Method computes attenuation and directional spreading of wave spectra, providing . . .

- An efficient means to solve 2(+1)D multiple scattering problems by large arrays, i.e. O(10^3–10^4) of scatterers (e.g. ice floes)
- A representation of the wave field at each strip boundary, i.e. we can extract forward wave energy $E^+(\xi_j)=\int_{-\pi/2}^{\pi/2} S_j^+(\chi)\,\mathrm{d}\chi$ and directional spread $\sigma_1(\xi_j)=\sigma_1\left(S_j^+(\chi)\right)$ at each strip interface $x=\xi_j$, where $S_j^+(\chi)=\left|A_j^+(\chi)\right|^2$ is the forward wave energy spectrum
- A framework to study scattering by random arrays and fully directional wave fields.

Attenuation and directional spreading

Exponential attenuation

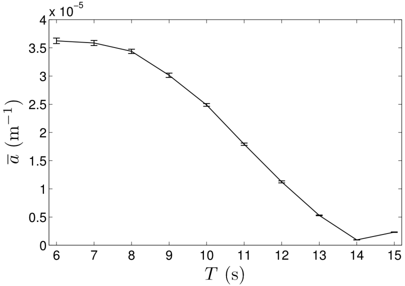
$$\overline{E^+}(x) = E_0\, \mathrm{e}^{-\bar{a}x}$$

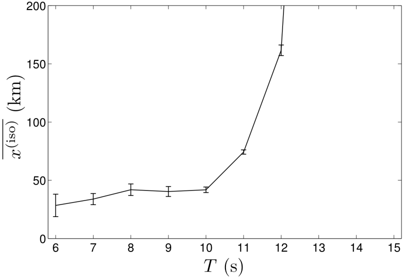
Linear spreading

$$\overline{\sigma_1}(x) = \bar{s}x + \sigma_1^0$$

$$\downarrow$$

$$x^{\mathrm{iso}} = \left(\sigma_1^{\mathrm{iso}} - \sigma_1^0\right) / \bar{s}$$



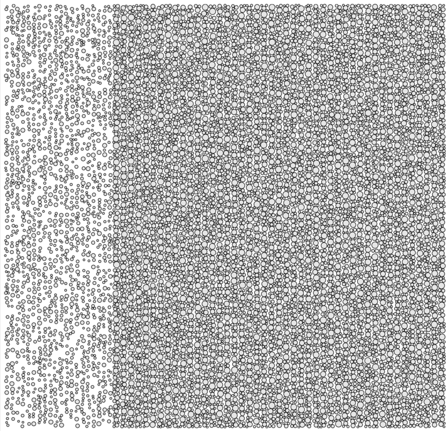


Some early field observations in Greenland Sea[2]

Tentative 'parameterization'

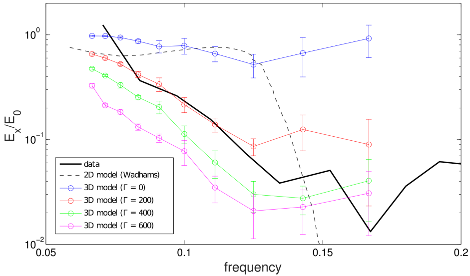
that the "edge" appears to change shape. Station 0904 lay 4.1 km inside a loose pack composed mainly of first-year floes of typical thickness 2.5 m and typical diameter 250 m. At this station the IOS buoy was

heave–tilt sensor was deployed on an ice floe. The final station, 0905, was accomplished in close pack in which the IOS buoy could not be put out, so only the SPRI heave–tilt sensor was deployed. As can be seen from

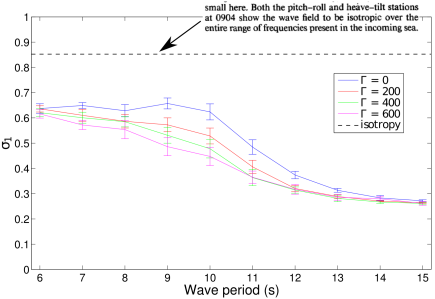


[2] Wadhams et al. (1986)

Attenuation and directional spreading



A reasonable fit to data as long as dissipation is included, but note the poor fit of 2(+1)D scattering model at higher frequencies.

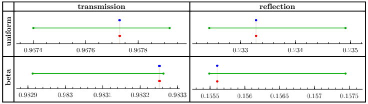


Observations are suspicious, as it is unlikely that the incoming ocean waves would become isotropic at all frequencies, i.e. at long periods, so near the ice edge.

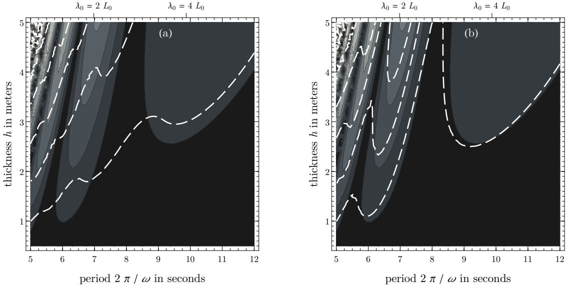
Generalised polynomial chaos (gPC)

Goal: to find an alternative to Monte Carlo (MC) averaging

MC requires a vast number N of simulations to achieve good accuracy, as the standard deviation of the mean goes as $\sqrt{N}$. The basic idea of gPC is that the averaging is done up front, i.e. pre-simulation. Any ice model can be used, e.g. mass loading, [3], finite elastic plate.



Suppose floe length is $L_0 + \alpha L_1$ where $0 < L_1 < L_0$ and α is a random variable drawn from a probability distribution on $[-1, 1]$. The expectations of the transmission and reflection coefficients are computed using three different methods, i.e. Monte Carlo (green) and two gPC methods (blue and red), for $\alpha \in \mathcal{U}, \mathcal{B}$. The displayed MC interval is defined as mean $\pm$ standard deviation, estimated from 10 individual MC runs of 10^4 values with different random seeds, each of which takes about 60 s. Six digit accuracy is obtained for the two gPC methods, but Monte Carlo produces significantly larger intervals despite its runtime being more than 20 times greater than the runtime of any of the gPC methods.

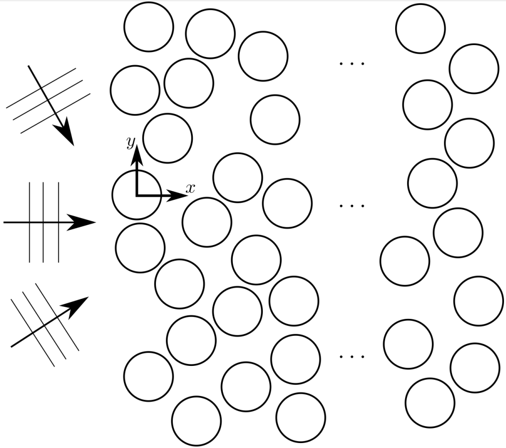


Contours of the deterministic reflection coefficient (shaded) overlayed with contours of its expectation (white dashed lines), for $\alpha \in \mathcal{U}, \mathcal{B}$. The uniform distribution $\mathcal{U}$ smears out the contours of expectation compared to the beta distribution $\mathcal{B}$ because $\mathcal{B}$ is narrower, which is intuitive.

But the floes sometimes break too

Goal

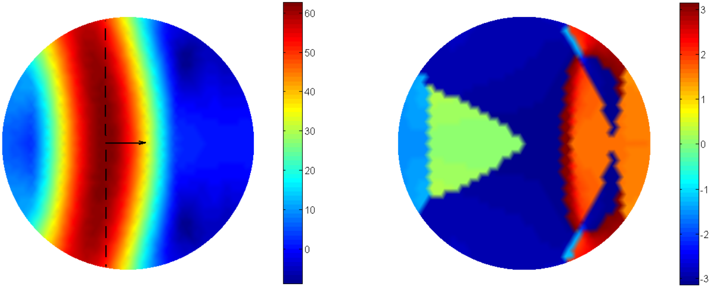
To use linear hydroelasticity and multiple scattering theory to devise a model of floe breakup in a randomized 2(+1)D MIZ which can ultimately be used to create a FSD.



Analysis of the breakup of an ice floe

Single floe breaking criterion

Maximum principal strain $\varepsilon_m >$ critical breaking strain ε_c, where ε_c and the sea ice rigidity can potentially accommodate the brine volume gradient that always exists through the ice thickness because of the temperature and salinity gradients. Floe breaks perpendicular to ε_m.

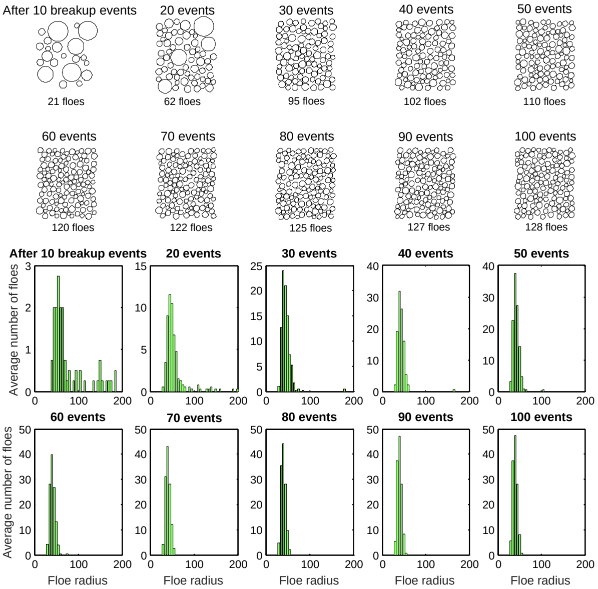


LH plot shows absolute value of the larger of the two principal strains, with dashed line showing where the floe breaks for a nearly long-crested wave travelling in the direction of the arrow.

RH plot shows the direction of the largest principal strain, noting that the angle is $\pm\pi$ commensurate so that the axes are parallel for dark blue and light green, and that the larger and smaller principal strains will switch precedence as the right floe edge is approached.

The asymptotic FSD

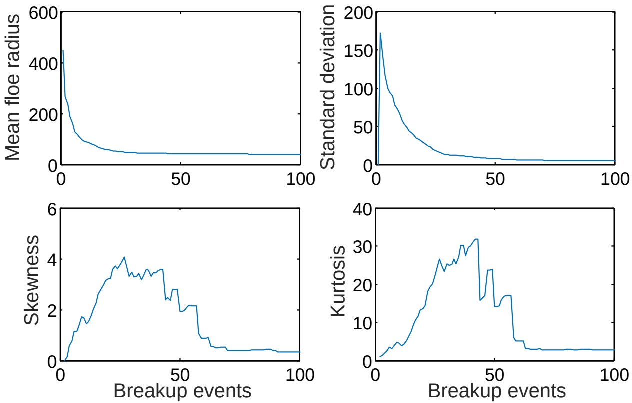
Gradual evolution of a single floe during breakup, starting with a single vast ice floe



How the FSD changes as the MIZ breaks up. The schematics are just one realisation, while the FSDs are ensemble averages.

The asymptotic FSD

Evolution of the moments of the distribution



Mean, standard deviation, skewness and kurtosis of FSD as a function of breaking events. Blue denotes breakup along a diameter, while red denotes breakup perpendicular to principal direction.

Thanks. Any comments or questions?