Wind-Induced Drift in Contained Bodies of Water

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ABSTRACT

From a review of experimental evidence it is concluded that in confined bodies of water the ratio of surface drift to surface friction velocity, u_s/u_*, is a function of water depth that precludes the application of laboratory-derived relationships between these velocities under conditions prevailing in the open sea. A simple empirical model is proposed for contained bodies of water that requires

$$u_s = (w_{*,s}/k)[\ln(H/Z_{0,s}) - 1],$$

$$U(\hat{Z}) = [1 + (\rho_w/\rho_a)^{1/2}]u_s + (u_*/k)\ln(\hat{Z}/0.368H),$$

where u_s is the surface current, $U(\hat{Z})$ the wind speed at height $\hat{Z}$ above water surface, $w_{*,s}$ the surface friction velocity in the water, u_* the surface friction velocity in the air, $Z_{0,s}$ the roughness length in the water at the surface, H the depth of water, ρ_w the density of water, ρ_a the density of air and k the von Kármán constant. These relationships have some support in the experimental evidence presently available.

1. Introduction

When air blows over water in a closed laboratory channel, a wind-induced drift current is produced in the water. Early work, studying the relation between the surface wind-induced current u_s and a reference wind speed U_r, may be summarized as (Keulegan, 1951; Van Dorn, 1953; Francis, 1951; Vines, 1962; Masch, 1963; Fitzgerald, 1964)

$$u_s/U_r = F(u_sH/\nu_w), \tag{1.1}$$

where H is the water depth and ν_w the kinematic viscosity of water. For values of the Reynolds number $u_sH/\nu_w > 10^4$, the function F becomes only weakly dependent on the Reynolds number. Under these conditions Wu (1973, 1975), for example, recommends replacing (1.1) by

$$u_s/u_* = \text{constant}, \tag{1.2}$$

where the reference velocity is taken to be the surface friction velocity u_* in the air.

On the basis of (1.2) Wu then presents arguments relating his data to the geophysical situation of the wind-induced drift current at sea. The purpose of this note is to reexamine the conclusion u_s/u_* = constant and its implications.

For our purposes it will be convenient to also write (1.2) in terms of the surface friction velocity $w_{*,s}$ in the water. In the absence of wave drag due to growing waves, the tangential stress τ_0 must be continuous at the air-water interface, i.e.,

$$\tau_0 = \rho_a u_*^2 = \rho_w w_{*,s}^2, \tag{1.3}$$

where ρ_a and ρ_w are the densities of air and water, respectively. Then

$$u_s/w_* = \text{constant}\,(\rho_w/\rho_a)^{1/2} \approx \text{constant}. \tag{1.4}$$

2. Review of experimental data (u_s and u_*)

Kondo (1976) has recently summarized evidence of the relationship between u_s and u_* and finds a strong depth dependence. Fig. 1 shows the data summarized by Kondo for contained bodies of water, plus several additional studies. Grouping the studies into two classes, $H \leq 30$ cm and 30 cm $< H \leq 200$ cm, the dependence on depth is apparent.

To understand why depth becomes influential we now examine a typical steady-state water current profile in a closed channel (see Fig. 2). The current follows a logarithmic profile from the surface and merges with another near the bottom. The fitted profile crosses the zero line at $z/H \approx 0.66$.

3. Estimate of u_s

Fitzgerald and Mansfield (1965) present a simple argument which allows an estimate of u_s. In our discussion we shall generalize their argument to

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$\frac{w}{u_s}$

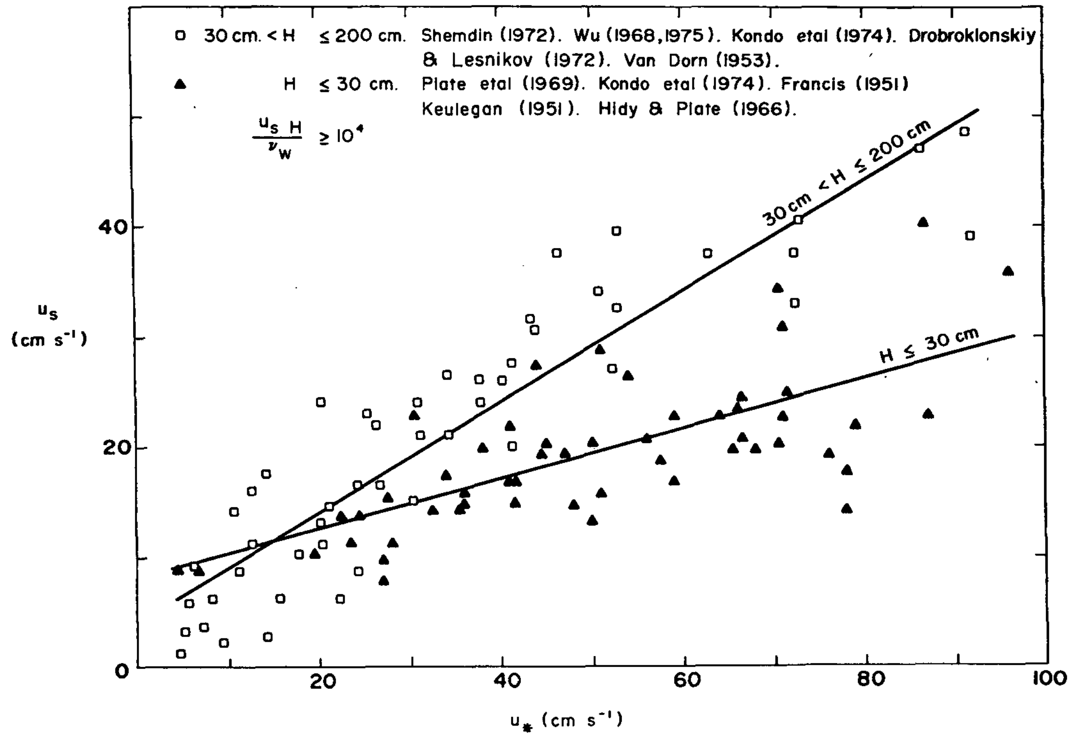

FIG. 1. Experimental results of surface drift current (u_s) as a function of surface friction velocity (u_*) in the air, for contained bodies of water, when $(u_sH/\nu_w) \geq 10^4$. The solid lines are linear regression lines for the two classes of data $H \leq 30$ cm, 30 cm $< H \leq 200$ cm.

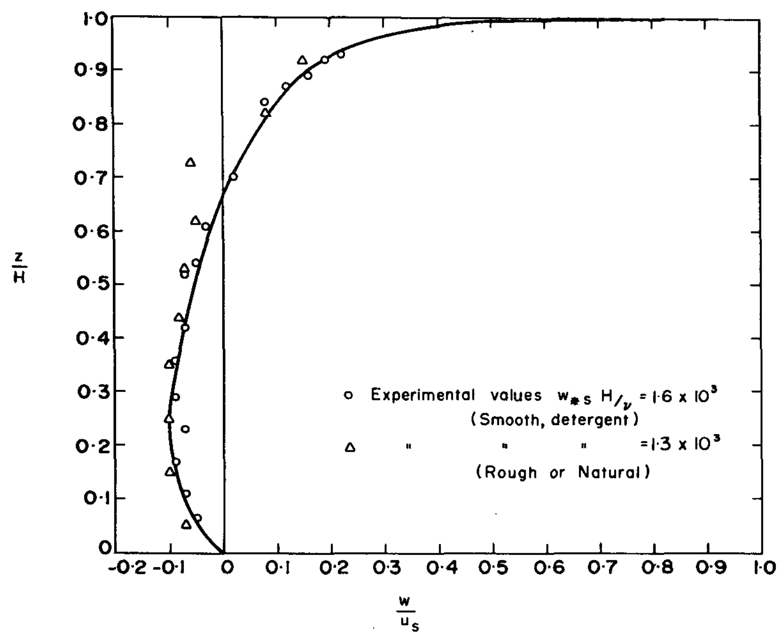

FIG. 2. The water current profile. The data are taken from Fitzgerald and Mansfield (Fig. 3, 1965). The solid lines indicate logarithmic fits of Eqs. (3.1) and (3.2) to the set of circled values. The lower part described by (3.2) was fitted so that $n = 1.05$.

TABLE 1. The dependence of X on the ratio of friction velocities.

n	1.1	1.08	1.06	1.05	1.04	1.03	1.02	1.01
λ	0.316	0.283	0.245	0.224	0.200	0.173	0.141	0.100
$H/Z_{0,b}$	214	384	880	1558	3308	9610	53000	2×10^6
X	1.033	1.024	1.015	1.012	1.008	1.005	1.003	1.001

include transitional and fully rough flow, in addition to smooth flow in the water. Empirically Fig. 2 shows that the current profile is well approximated by fitting two logarithmic curves, one for the upper region (i.e., the wind-induced current and upper part of the return flow) and one for the lower region (i.e., the bottom part of the return flow). These curves can be represented by

$$u_s - \bar{w} = (w_{*,s}/k) \ln[(H - Z)/Z_{0,s}], \quad (3.1)$$

$$-\bar{w} = (w_{*,b}/k) \ln(Z/Z_{0,b}), \quad (3.2)$$

where $\bar{w}$ is the current, k the von Kármán's constant ($k \approx 0.4$), $w_{*,s}$ and $w_{*,b}$ the friction velocities in the water at the surface and the bottom, respectively, and $Z_{0,s}$ and $Z_{0,b}$ the roughness lengths in the water at the surface and the bottom, respectively.

Although the tangential stress gradient $\partial\tau/\partial z = (\rho/g)(\partial H/\partial x) =$ constant, in a long open channel, many workers (cf. Table 2) have reported a logarithmic profile in the upper region.

As noted by Goldstein (1938, p. 336) in commenting on flow in a smooth pipe, "It is a matter of good fortune rather than sound reasoning" that logarithmic laws hold to a good approximation throughout the entire flow.

We now require that the value of the current determined by (3.1) be equal to that determined by (3.2) when $Z = Z_c$, i.e.,

$$u_s = (w_{*,s}/k) \ln[(H - Z_c)/Z_{0,s}] - (w_{*,b}/k) \ln(Z_c/Z_{0,b}). \quad (3.3)$$

We also require that the net horizontal transport of mass is zero, i.e., for uniform density

$$\int_{Z_{0,b}}^{H-Z_{0,s}} \bar{w}dZ = \int_{Z_{0,b}}^{Z_c} \bar{w}dZ + \int_{Z_c}^{H-Z_{0,s}} \bar{w}dZ = 0. \quad (3.4)$$

Eqs. (3.4), (3.1) and (3.2) then give the relation

$$u_s(H - Z_c) = (w_{*,s}/k)\{Z_{0,s} + (H - Z_c) \times \ln[(H - Z_c)/Z_{0,s}] - H + Z_c\} + (w_{*,b}/k) \times \{Z_{0,b} + Z_c \ln(Z_c/Z_{0,b}) - Z_c\}. \quad (3.5)$$

We further require that the stress profile (in unaccelerated portions of steady-state flow) be linear with height, i.e.,

$$\frac{\partial\tau}{\partial z} = \frac{w_{*,s}^2}{H - Z_c} = \frac{w_{*,b}^2}{Z_c}. \quad (3.6)$$

Provided $Z_{0,s} \ll H\ln[H/(nZ_{0,s})]$ and $Z_{0,b} \ll H(n - 1/n)$, where $n \equiv (\tau_0 + \tau_b)/\tau_0 = (w_{*,s}^2 + w_{*,b}^2)/w_{*,s}^2$ and τ_b is the stress at the bottom, from (3.3), (3.5) and (3.6) we obtain, after some algebraic manipulation,

$$\ln(H/Z_{0,s}) = \ln(1 + \lambda^2) + (\lambda^3 + 1)/(\lambda^2 + 1) + k(u_s/w_{*,s}), \quad (3.7)$$

$$\ln(H/Z_{0,b}) = \ln(1 + \lambda^2) + (\lambda^3 + 1)/(\lambda^3 + \lambda) - \ln\lambda^2, \quad (3.8)$$

where $\lambda \equiv w_{*,b}/w_{*,s} = (n - 1)^{1/2}$.

It is convenient now to consider (3.1) evaluated at the level Z_F where $\bar{w} = 0$, i.e.,

$$u_s = (w_{*,s}/k) \ln(H - Z_F)/(Z_{0,s}). \quad (3.9)$$

From (3.7) and (3.9) we obtain

$$\ln[H/(H - Z_F)] = \ln(1 + \lambda^2) + (\lambda^3 + 1)/(\lambda^2 + 1) = X. \quad (3.10)$$

The values of X from (3.10) and $H/Z_{0,b}$ from (3.8) are given in Table 1 for various values of n and λ. From Table 1 it is seen that for $n < 1.1$, which covers the range of values of interest (Francis, 1953; Van Dorn, 1953; Reid[1], 1957; Fitzgerald and Mansfield, 1965), we may write (3.10) and (3.9) as

$$Z_F = H[1 - \exp(-1)] \approx 0.632H, \quad (3.11)$$

$$u_s = (w_{*,s}/k)[\ln(H/Z_{0,s}) - 1], \quad (3.12)$$

to within 3%.

Eq. (3.12) explicitly shows that the ratio $u_s/w_{*,s}$ is not constant but should depend on the depth of the water, and offers an explanation of the behavior shown in Fig. 1. It follows conversely, that if $u_s/w_{*,s}$ were constant then the surface roughness length would be determined by the water depth as pointed out by Reid (1957).

Eq. (3.12) has another interesting implication.

[1] We wish to acknowledge that a reviewer has drawn our attention to Reid (1957) who has given a much more general treatment of steady-state channel flows employing Montgomery's (1943) generalized mixing-length hypothesis and the presumed relationship between stress and velocity shear due to Prandtl and von Kármán. For the special case of zero net flow and $H/Z_{0,s} > 100$ Reid deduces that n is dependent only on $H/Z_{0,b}$ and (from his Fig. 10) is <1.1 for $H/Z_{0,b} > 900$. The reviewer points out that for $n < 1.1$ only minor errors result in neglecting the log profile near the bottom and if the log-deficit profile is assumed from the surface Eq. (3.12) follows.

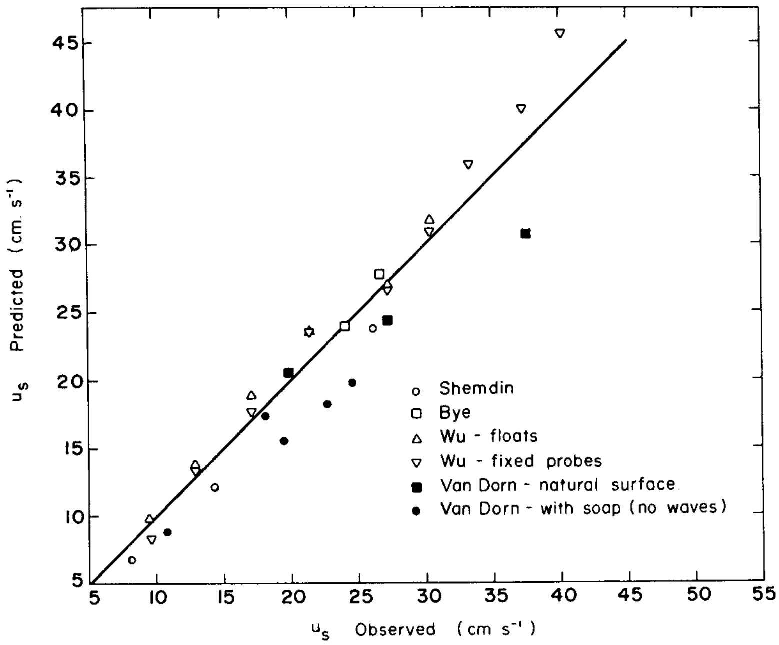

FIG. 3a. Observed values of u_s and predicted values from Eq. (3.12) from those data sources listed in Table 2. The solid line indicates perfect agreement.

The equation for air flow in neutral conditions is

$$U - u_s = (u_*/k)\ln(\hat{Z}/Z_{0,a}), \qquad (3.13)$$

where $\hat{Z}$ is the height above the water surface ($\hat{Z} = Z - H$) and $Z_{0,a}$ is the surface roughness length for the air flow. Preliminary results by Kondo (1976) show that for $hw_*/\nu_w > 67$ (where h is the rms height of water surface irregularity associated with the high-frequency waves; $h \approx 0.1 + 0.009\,u_*$ for h in centimeters and u_* in centimeters per second), the roughness lengths $Z_{0,a}$ and $Z_{0,s}$ are equal. For the case $Z_{0,a} \approx Z_{0,s}$ Eqs. (3.12), (3.13) and (1.3) give

$$U(\hat{Z}) = u_s[1 + (\rho_w/\rho_a)^{1/2}] + (u_*/k)\ln[\hat{Z}\exp(1)/H] \approx 30.3\,u_s + (u_*/k)\ln(\hat{Z}/0.368H). \qquad (3.14)$$

For $\hat{Z} \approx 0.368H$ then

$$u_s/U \approx 3.3\% \qquad (3.15)$$

which is independent of u_* and thus is independent of water surface roughness, i.e., whether or not waves are present, provided only that 1.3 is closely approximated.

Several workers such as Keulegan (1951), Van Dorn (1953) and Fitzgerald (1964), for film surface pressure <33 dyn cm^{-1}, have noted this striking result that u_s/U was independent of the presence of waves. Van Dorn's experiment best approximates the conditions of the model and will be considered further in the next section. Fig. 3 shows that within the scatter of the experimental data (3.12) provides a good estimate of u_s, although there is a suggestion that Wu's data systematically deviates from the relationship at high wind speed.

4. Test of the model

In Figs. 3a and 3b we present a test of the model based on laboratory, pond and lake data. The

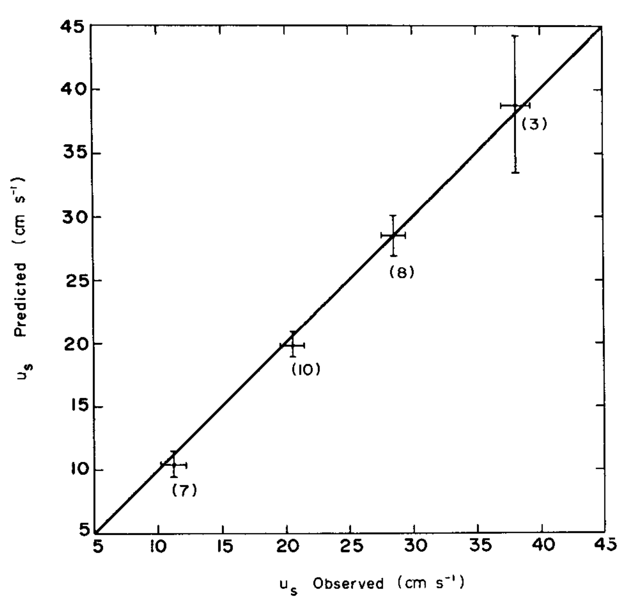

FIG. 3b. Observed and predicted values of u_s from Fig. 3a averaged by 10 cm s^{-1} class intervals of u_s observed, with standard errors of means. The number of observations in each range is in parentheses. The solid line indicates perfect agreement.

TABLE 2. Data for testing u_s estimate model.

Experiments	Location	Comments
Van Dorn (1953)	Rectangular model yacht pond, ~220 m × 60 m × 1.85 m	Stress measured by water surface tilt (set-up). $Z_{0,a}$ derived from wind speed at $\hat{Z}$ = 25 cm from (3.13) for all points in his Fig 9; u_* determined from his empirical relation (27). We have assumed $Z_{0,s} = Z_{0,a}$. Experiments with waves present (natural) and also when waves are damped by application of soap (detergent).
Bye (1965)	Lake Lough Neagh, depth 10 m	u_*, w_*, $Z_{0,a}$, $Z_{0,s}$ obtained from (1.3) and simultaneous air speed, water current logarithmic profiles with the assumption $Z_{0,a} = Z_{0,s}$.
Shemdin (1972)	Laboratory channel, 45.7 m × 1.83 m × 0.915 m	u_*, w_*, $Z_{0,s}$ from measured logarithmic profiles of wind speed and current (but not at same wind speed). Mechanically produced waves also present. Finds (1.3) satisfied.
Dobroklonskiy and Lesnikov (1972)	Laboratory channel, 25 m × 1.23 m × 0.80 m	Measured logarithmic profiles of wind speed and current. Finds (1.3) satisfied. Does not give u_s values and data does not allow a linear extrapolation to the surface (cf. Wu). Not used in Fig 3.
Kondo *et al.* (1974)	Laboratory channel, lake, sea.	Depths not specified. Finds $Z_{0,a} = Z_{0,s}$ for $hw_*/\nu_w \geq 67$ (see text). Not used in Fig 3.
Wu (1975)	Laboratory channel, ~22 m × 1.5 m × 1.24 m	u_*, w_*, $Z_{0,a}$ and $Z_{0,s}$ from measured wind speed and current profiles; both fixed probes and floats gave current profiles. Used linear extrapolation to obtain u_s (cf. D & L above); finds $Z_{0,a} \neq Z_{0,s}$ and (1.3) not satisfied; part of channel uncovered.

measurements, especially those of u_s, present experimental difficulties and some comments about the different data sets are given in Table 2.

Finally in this section we compare Eq. (3.14) with Van Dorn's (1953) observation of u_s and $U(\hat{Z})$ for $\hat{Z}$ = 25 and 100 cm at the downwind edge of the pond. From Van Dorn's empirical stress equation (27), and the parameters in his Tables III and IV we obtain the expected and observed drift-wind factors $(u_s/U_{\hat{z}})$ in Table 3. This agreement between the observations and the predictions [Eq. (3.14)] is very encouraging and suggests this model may have practical applications in relating the drift current to the wind over confined bodies of water.

5. Summary

This review of experiments providing observations of u_s and permitting evaluation of u_* (or w_*) confirm the evidence of Kondo (1976) that the relationship between u_s and u_* is depth dependent.

An explanation of this depth dependence has been offered through a simple empirical model with the following characteristics: 1) logarithmic profiles of current flow in the upper and lower arms of the circulation; 2) uniform gradient of stress; and 3) zero net horizontal transport of mass. Such a model indicates that the expression

$$u_s = (w_{*,s}/k)[\ln(H/Z_{0,s}) - 1] \quad (3.12)$$

should be true to better than 3% for all situations with pond Reynolds number $Hu_s/\nu > 10^4$ and $n < 1.1$.

Eq. (3.12) is shown in Fig. 3 to be in agreement with observations where $Z_{0,s}$ is available and or when (1.3) holds and $Z_{0,s} \approx Z_{0,a}$; for these latter two conditions it follows that

$$U(\hat{Z}) = 30.3u_s + (u_*/k)[\ln(\hat{Z}/0.368H)]. \quad (3.14)$$

Eq. (3.14) is shown in Table 3 to be in agreement with the relevant but limited observations of Van Dorn, and predicts

$$u_s/U(\hat{Z}) = 0.033 \text{ for } \hat{Z} \text{ near } 0.368H,$$

irrespective of surface state, i.e., natural or affected by capillary-wave damping detergent.

Further experimental evidence is required to confirm the predictions of (3.12) and (3.14) and their relevant geophysical applications to confined bodies of water.

TABLE 3. Expected and observed drift-wind factors $[u_s/U(\hat{Z})]$ for wind $U(\hat{Z})$ at 25 and 100 cm, with surface-wind stress relationships also affected by detergent (after Van Dorn, 1953).

$\hat{Z}$ Height of wind observation (cm)	Value of $u_s/U(\hat{Z})$ expected by Eq. (3.14)* (a) Clear surface	(b) Detergent added	Observed average of (a) and (b)
25	0.042	0.038	0.041
100	0.030	0.031	0.030

* (a) is at 8–10 m s^{-1} wind speed; (b) is average over all wind observations.

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