Similarity Scaling of Turbulence Spectra in Marine and Atmospheric Boundary Layers

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ABSTRACT

Original turbulence spectra from the marine bottom surface layer, together with spectra derived from other authors, are compared with atmospheric surface layer spectra. The spectra from the two boundary layers are found to coincide when scaled by variance and measuring height.

1. Introduction

The general Monin-Obukhov similarity theory of turbulence in a thermally stratified surface layer postulates that over a horizontally homogeneous surface the turbulence structure is determined only by the height z from the surface, the surface shear velocity u_*, the surface heat flux and a buoyancy parameter (Arya and Sundararajan, 1976; Monin and Obukhov, 1954). For the particular case of neutral stability this indicates that when velocities are scaled by u_*, and lengths are scaled by z, the shapes of velocity spectra outside the dissipation range should reduce to universal forms. There is ample evidence that this is valid in the surface region of the atmospheric boundary layer (see, e.g., Monin and Yaglom, 1975; Kaimal *et al.*, 1972) and if the theory is of general validity it might be expected that spectra from the marine bottom boundary layer will coincide with the atmospheric spectra. Recent measurements in a neutrally stable marine bottom boundary layer have allowed spectra to be calculated covering a wide range of wavenumbers with reasonably tight confidence limits. These, together with spectra derived from other authors, are compared here with the atmospheric spectra, and are shown to coincide with these when scaled by variance and measuring height.

2. Measurement of spectra

The new observations reported here were made as part of a study of the movement of sediment by turbulent tidal streams. They were taken in 14 m depth of water flowing over a sandy bed in Start Bay, southwest England (50°14.30′N, 3°37.90′W), and consisted of measurements of horizontal and vertical velocities made with electromagnetic current meters (Tucker, 1972; Heathershaw, 1975). The sensors were mounted at heights of 30 and 140 cm on a frame sitting on the sea bed and aligned by a fin with the mean flow. Sediment (175 μm sand) was intermittently in suspension during the periods of strongest flow. Profiles of temperature, salinity and suspended sediment concentration were measured to determine the stability of the water column. All the gradients were small and combined to give a gradient Richardson number, mainly due to the sediment concentration gradient, of 0.020 at 30 cm height and 0.027 and 140 cm height, indicating that the stability can be classified as near neutral (Lumley and Panofsky, 1964). This will frequently be the case in shallow seas with strong tidal mixing.

Conditions were not ideal for obtaining definitive spectra, as the measurements were made in an area where the sand was both rippled and formed into sandwaves (50–100 cm high, 7–10 m wavelength). However, this disadvantage was offset by the long stationary data set obtainable at the site, since the asymmetry of the tidal cycle there gave a long period of effectively steady flow during the ebb tide.

An uninterrupted data set of 252 min duration recorded in this period was chosen for analysis, and digitized at intervals of 0.2 s. The data set was divided

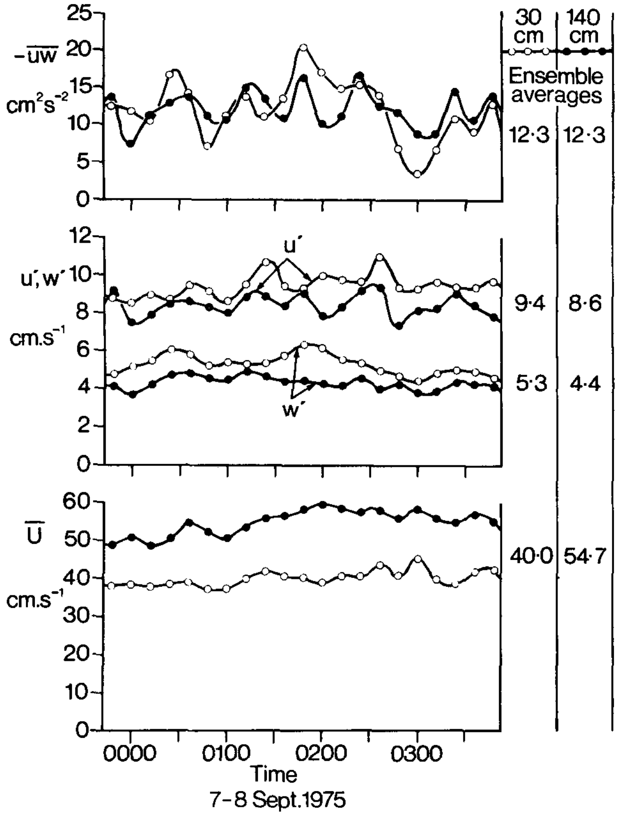

FIG. 1. Time series and mean values of $\bar{U}$, u', w' and $\overline{uw}$.

into a sequence of 21 records of 12 min duration, each of which was rotated into coordinates aligned with the mean flow, and had a linear trend removed. The mean value of the horizontal current $\bar{U}$, the rms value u' of the horizontal velocity fluctuation u, the rms value w' of the vertical velocity fluctuation w and the mean product $\overline{uw}$ were calculated for each record. The resulting sequences of values of u', w' and $\overline{uw}$ at both heights (Fig. 1) were shown to be statistically stationary at the 95% confidence level by application of the run test (Bendat and Piersol, 1971). In addition the sequences were checked for autocorrelation of successive 12 min samples. It was found that $\overline{uw}$ and w' at $z=30$ cm were both significantly autocorrelated at the 95% level for one lag (12 min) only, presumably due to short-term trends, but this should not significantly affect the results. The other parameters were not autocorrelated.

Energy spectra of u and w, and the cospectrum of uw, were obtained for each record using an FFT technique (Rayment, 1970), followed by block-averaging of the raw estimates into 20 blocks equally spaced on a logarithmic frequency scale.

We consider first the energy spectrum of u. Comparison of the spectral estimates with the values of $\bar{U}^2$ showed that the criteria discussed by Lumley (1965) concerning the application of Taylor's hypothesis were satisfied. This justified forming a one-dimensional wavenumber spectrum, $\hat{E}_{uj}(k)$, where j indicates the jth record, and $k=2\pi f\bar{U}^{-1}$ for a frequency f. As the time series is stationary the 21 spectra can be ensemble-averaged to give a mean spectrum $E_u(k)$ at each of the two measuring heights. The spectra are plotted in Fig. 2 as a function of the dimensionless wavenumber $k^*=kz$ in accordance with similarity scaling. Instead of scaling the ordinate by u_*^2, however, a frequently used alternative scaling by total variance has been applied, as will be discussed later, and the spectra presented in equal-area, equal-energy form as

$$kE_u(k)\Big/\int_0^\infty E_u(k)dk.$$

Confidence limits for the mean spectrum $E_u(k)$ were determined directly from the variability of the individual spectra $\hat{E}_{uj}(k)$ at each value of k. Error bars around the spectrum at 30 cm height indicate one standard error either side of the mean. The confidence limits at 140 cm are similar. A correction for spatial averaging across the sensor was applied for $k>0.1$ cm^{-1}, although because the averaging volume of the current meters is not known accurately, the correction may cause an overestimation by up to 20% at the highest wavenumbers.

The energy spectrum $E_w(k)$ of the vertical velocity

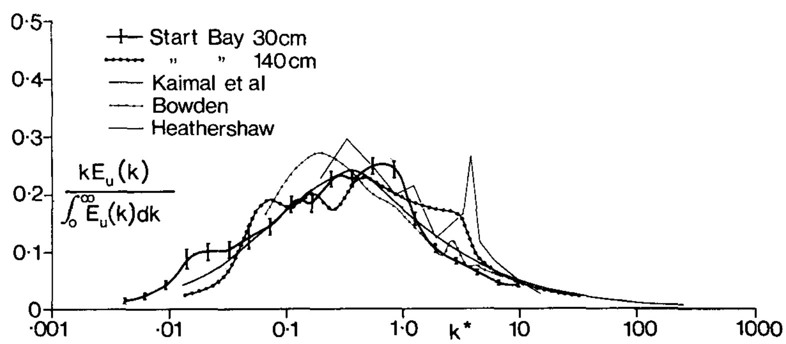

FIG. 2. Energy spectra of horizontal velocity. Standard errors are shown on the Start Bay 30 cm spectrum.

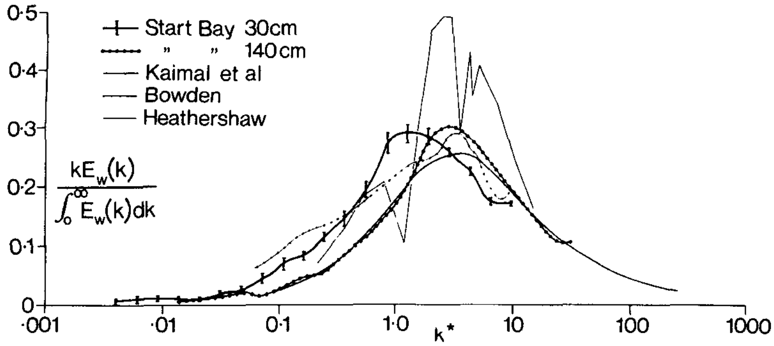

FIG. 3. Energy spectra of vertical velocity. Standard errors are shown on the Start Bay 30 cm spectrum.

and the cospectrum $C_{uw}(k)$ were obtained in the same way and are shown in Figs. 3 and 4, respectively.

3. Discussion

Comparisons are made in Figs. 2, 3 and 4 with spectra obtained under neutral conditions in the atmospheric surface layer, and with other spectra from the marine bottom surface layer. The atmospheric spectra chosen are those of Kaimal *et al.* (1972), who present definitive spectra from their own measurements and also show that these agree well with those of other workers. The work of Bowden (1962) and Heathershaw (1976a,b), both in the Irish Sea, and both using electromagnetic current meters, afford further examples of measurements in a near-neutrally stable marine bottom boundary layer. The spectra from these two sources have been smoothed at large k in this presentation. Those of Heathershaw were calculated for a single 10 min record, and hence show considerable scatter.

A number of points should be noted when comparing the spectra:

1) True similarity scaling nondimensionalized the spectral values $E_u(k)$ and $E_w(k)$ by the square of the shear velocity u_*, but this scaling was found to give poor agreement between the magnitudes of the spectra from different authors. Some measure of the disagreement between the spectra when scaled by u_*^2 can be obtained by examining the ratios u'^2/u_*^2 and w'^2/u_*^2, shown in Table 1. There is known to be considerable fluctuation in the ratio u'^2/u_*^2 as measured in the atmosphere; Busch (1973) quotes values ranging from 3.2 to 5.3, though the marine values in Table 1 are higher again than this. The ratio w'^2/u_*^2 is generally thought to be quite well-defined, although Busch (1973) again quotes a considerable range of values from 1.4 to 2.6. All the values in Table 1 fall within this range.

To better demonstrate that the length scales of the turbulence do exhibit similarity, normalization to give unit area under the spectrum was chosen instead. The cospectrum is non-dimensionalized by

$$u_*^2 = \int_0^\infty C_{uw}(k)dk$$

with either scaling.

2) The atmospheric spectra show a well-defined inertial subrange at large k^* with a $k^{-5/3}$ dependence

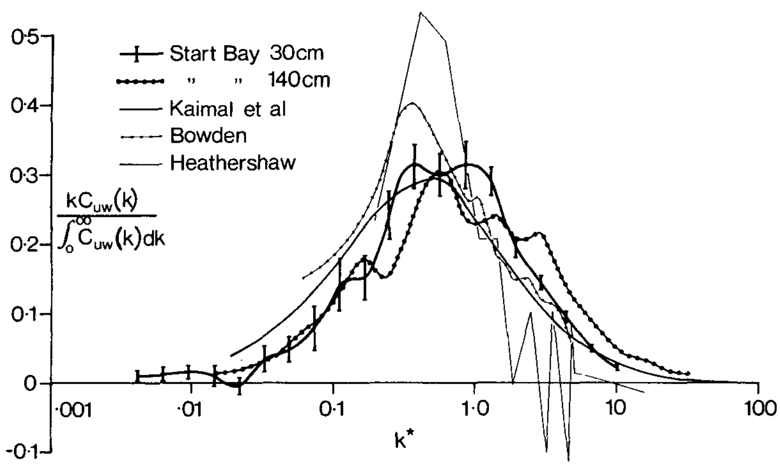

FIG. 4. Cospectra of horizontal and vertical velocities. Standard errors are shown on the Start Bay 30 cm cospectrum.

TABLE 1. The ratios u'^2/u_*^2 and w'^2/u_*^2 for the for the spectra of Figs. 2–4.

	u'^2/u_*^2	w'^2/u_*^2
Start Bay, $Z=30$ cm	7.3	2.3
Start Bay, $Z=140$ cm	6.0	1.6
Kaimal *et al.* (1972)	4.4	1.4
Bowden (1962)	6.3	1.4
Heathershaw (1976)	12.6	2.5

in the energy spectra and a $k^{-7/3}$ dependence in the spectrum. The Start Bay spectra exhibit the same behavior when plotted on log-log scales (not shown), although the Reynolds numbers based on Eulerian integral length scale and rms velocity lie between 1.3×10^4 and 1.3×10^5, which is less than the minimum value of 1.4×10^5 which Tennekes and Lumley (1972) show to be necessary for the existence of an inertial subrange.

3) The spectra $E_w(k)$ (Fig. 3) show poorer correspondence than do either $E_u(k)$ or $C_{uw}(k)$. This is surprising, as similarity scaling is generally thought to be most appropriate for vertical velocities (Lumley and Panofsky, 1964). In particular the spectrum for Start Bay at $z=30$ cm is shifted toward small wavenumber. This anomaly is the subject of further investigation.

4) An additional comparison has been made (but not shown here) with observations by McPhee and Smith (1976) of $E_w(k)$ and $E_u(k)$ in the boundary layer under Arctic pack ice. These spectra show reasonable correspondence with those presented here, except that $E_u(k)$ does not have a peak, but continues to increase toward small wavenumber.

Although there are differences in detail, it is clear that the spectra from all these surface layers have the same general form, giving confirmation that during periods of steady flow similarity scaling can be applied to the length scales of turbulence in the surface region of a neutrally stable marine bottom boundary layer.

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