Waveform Aliasing in Satellite Radar Altimetry

Walter H. F. Smith and Remko Scharroo

Abstract—The full deramp pulse compression scheme employed by satellite radar altimeters digitizes each radar echo at a sampling rate matched to the chirp bandwidth. Echo power is undersampled by a factor of 2 when the power samples are simply obtained by squaring the magnitude of the echo samples, without first resampling, as is done in all altimeters to date. This results in inadequate sampling of the leading edge of the waveform if the significant wave height (SWH) is low. For a typical Ku-band altimeter with a chirp bandwidth of 320 MHz, simple squaring should be inadequate over ocean surfaces with SWH less than 2 m, that is, half of the ocean. Simply zero-padding the digital samples prior to range Fourier transform alleviates the problem introduced by magnitude squaring. Data from the CryoSat altimeter are used to demonstrate this remedy, and it is found that this reduces the variance in estimated range by 10% and in SWH by 20%. These improvements are confined to a range of SWH values between 1 and 4 m. Zero-padding also seems to have some small impact on values estimated over very flat surfaces (SWH $\ll 1 \text{ m}$), although theory suggests that better resolution of such surfaces would require a bandwidth exceeding 320 MHz. The 500-MHz bandwidth of the Ka-band altimeter on the Satellite with ARGOS and AltiKa mission should encounter these difficulties at smaller SWH values. Onboard range tracking and automatic gain control loops in future altimeters might be improved if zero-padding were employed during onboard waveform processing.

Index Terms—Geophysical sea measurements, ocean surface, pulse compression methods, pulse measurements, pulse modulation, radar altimetry, radar remote sensing, signal sampling, spaceborne radar, synthetic aperture radar (SAR).

I. INTRODUCTION

C ONVENTIONAL pulse-limited satellite radar altimeters produce a digital "waveform" measuring the mean of backscattered power as a function of two-way travel time, averaged over a radar cycle, approximately one twentieth of a second. The waveform's amplitude, the time when the leading edge rises, and the duration of the rise determine the surface backscatter coefficient, range, and surface roughness, respectively. Feedback control loops within the instrument react to changes in the waveform for automatic gain control (AGC) and for the target tracking scheme that maintains the position of the leading edge of the waveform near a "track point" within the

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"range window" spanned by the waveform sample sequence (see Fig. 1).

Jensen [1] remarked that the process by which altimeters obtain their waveforms should be prone to distortion due to undersampling and aliasing. He suggested that the problem would be eliminated if each echo's complex amplitude sequence were interpolated to double its sampling rate prior to the step that obtains the echo power as the squared magnitude of the complex amplitude. Until recently, it was not possible to investigate these ideas directly, because the telemetry from most altimeters furnishes only the averaged waveform power samples and not the complex amplitude samples of individual radar echoes. Envisat (2002-2012) did yield individual echo samples collected over 1 s of flight, but only one such collection per minute. Cryosat-2 (launched April 2010) telemeters all individual radar echo samples collected from flight segments in its "SAR" or "SARIn" modes [2], and these data are publicly available in the "full bit rate" (FBR) data product. CryoSat's SAR mode performs target tracking and echo sampling in a manner like that of conventional altimeters, and thus, we believe these data are most suited to investigating the possibility of waveform aliasing.

This paper tests both the waveform aliasing claim and the resampling cure proposed in [1]. Section II details how we produce waveforms, both conventional and resampled (see Fig. 1), from CryoSat SAR mode FBR data. Section III describes the retrieval of geophysical parameters continuously along profiles over ocean surfaces, as well as the estimation of their running means and standard deviations as is customary in the ocean altimeter practice. Section IV analyzes all such data from one mapping subcycle (29 days) of data, finds significant differences in the retrieved values and their variances, and demonstrates that the differences and variance reductions are most pronounced when the sea surface roughness is low. Section V explores the aliasing question theoretically, derives the bandwidth of a hypothetical idealized echo arising from a Dirac delta function pulse meeting a Gaussian rough surface, and suggests that this hypothetical bandwidth offers some insight into the range of roughness conditions under which the aliasing claimed in [1] may arise and how this range is related to the instrument's chirp bandwidth. Section VI discusses findings and presents conclusions.

II. FROM INDIVIDUAL ECHO SAMPLES TO AVERAGED WAVEFORMS

A. Raw Samples of Individual Echoes

Beginning with Seasat (launched in 1978) and continuing through all currently planned missions (Sentinel-3 and Jason-3, to launch in 2015, and Jason-CS, to launch in 2019), all



pulse-limited satellite radar altimeters have employed a pulse compression [3], [4] and time stretching [5] technique known as "full deramp" of a linear FM chirp [2], [6]–[9]. A chirp generator sweeps through a range of frequencies B over a time duration T, such that the sweep rate, a = B/T, remains constant. Each pulse transmitted by the instrument consists of one chirped signal, with transmitted bandwidth $B_{TX} =$ 350 MHz and duration $T_{TX} = 49 \ \mu$ s (numerical values here are those of CryoSat [2], but all altimeters operate in a similar manner).

A target tracking loop forecasts the expected round-trip travel time of the pulse, t_E , and this is split into coarse, t_C , and fine, t_F , components, $t_E = t_C + t_F$, such that t_C is an integer number of ticks of the clock that triggers events in the instrument. CryoSat's 80-MHz clock quantizes t_C in steps of 12.5 ns. At time t_C , another chirp is generated with the same sweep rate, a, and mixed with the output from the receiver, and the mixed signal is filtered and digitized. To avoid transient edge effects at the beginning and end of the mixed signal, only the intermediate $T_{\rm RX} = 44.8 \ \mu s$ is digitized, resulting in an effective chirp bandwidth at reception B of 320 MHz; B = $aT_{\rm RX}$. The digitizing collects N = 128 digital samples of the in-phase, I, and quadrature, Q, components in equally spaced time increments $\Delta T = T_{\rm RX}/N = 350$ ns spanning $T_{\rm RX}$. The raw echo samples furnished in the Cryosat FBR SAR data product are these 128 I and Q samples.

B. Range Delay Sampling by DFT

A point reflector at two-way travel time t is advanced or retarded with respect to the chirp deramp time by a range delay, $d = t - t_C$. This generates in the deramped and filtered signal a pure tone [continuous wave (CW) signal] of frequency $f_{\rm CW} = ad$ [9], [10, Sec. 7.5.3], [11, Sec. 8.2]. Surface scattering generates a linear combination of many point reflections, and hence, the digitized echo signal is a linear combination of many such CW tones, FM chirp time compression being a linear process [5].

Let the subscript k label the sequence of I and Q samples of an echo, form the complex sample sequence $Z_k = I_k + jQ_k$, with $j = \sqrt{-1}$, and let z_t indicate the complex sample sequence obtained by a discrete Fourier transform (DFT) applied to Z_k . Because Z_k is a time series spanning T_{RX} with sampling rate ΔT , z_t is a frequency series spanning $|f| \leq (2\Delta T)^{-1}$ with sampling rate $\Delta f_{\rm CW} = 1/T_{\rm RX}$. The sequence z_t can be viewed as a set of complex amplitude coefficients of CW tones corresponding to discrete frequencies. However, through the chirp slope, these CW frequencies correspond to delay times, because $f_{\rm CW} = ad$. Thus, z_t also represents a time series spanning a range window of delay times $|d| = |f_{\rm CW}/a| \le N/2B$ and sampled at a rate $\Delta t = \Delta f_{\rm CW}/a = B^{-1}$. Thus, applying a DFT to the I, Q sample sequence completes the process of pulse compression and yields a time series of complex echo amplitudes with sample index representing range delay referred to the chirp deramp time, t_C .

C. Very Rough Surface Scattering and Incoherent Averaging of Echo Power

Under typical conditions, the surface scattering process is that of a "very rough" surface [12], so that each z_t is a circularly symmetric complex Gaussian random variable. The phase of z_t is random, and thus, in a conventional altimeter, it is not used, and only its squared magnitude, the power $p_t = |z_t|^2$, is of interest. The statistical distribution for the power in an individual echo has a variance equal to the square of the



mean, and thus, individual echoes are too noisy to yield useful geophysical information. Therefore, a conventional instrument averages the power over a radar cycle to obtain an averaged power waveform, $\langle p_t \rangle$.

D. Time Alignment During Averaging

As the spacecraft follows an elliptical orbit around an ellipsoidal Earth, the range to the surface changes at a rate, \dot{r} , as high as 35 m/s, or as much as 1.6 m during one radar cycle (47.17185 ms for CryoSat). This change in range is equivalent to a change in delay time of nearly $4\Delta t$, that is, 4/B or 12.5 ns, necessitating a target tracking scheme to keep successive echoes aligned at the same sample points within the z_t series. In order that averaging should not blur the leading edge, degrading range precision and making the surface roughness appear higher than it is, a fine height adjustment is made to each echo by applying a linear phase ramp to its digitized echo sequence

$$Z'_{k} = Z_{k} \exp\left[-j2\pi k \left(\frac{t_{F}}{D}\right)\right] \tag{1}$$

prior to the DFT. This causes z_t to be aligned to $t_C + t_F$, rather than t_C . Here, $D = N \Delta t$ is the total range delay time spanned by the range window, that is, 400 ns.

When the power in successive echoes is averaged over a radar cycle, t_C is fixed during the averaging cycle, and t_F is changed from echo to echo. In a conventional altimeter, this occurs on board, autonomously, with successive values of t_F obtained from the \dot{r} forecast by the target tracking loop. In the CryoSat FBR SAR data analyzed here, we applied (1) prior to the DFT and subsequent averaging. When this is done (either on board in a conventional instrument or by us in our analysis here), the time sampling points in the averaged waveform then are referenced to a "window delay time," t_W , which is the average of the $t_C + t_F$ values used in the averaging cycle: $t_W = \langle t_C + t_F \rangle$.

E. Closed Burst Effects—The Tracking Echo and Tracker Noise

CryoSat has a limited capacity to store and downlink individual complex echoes, and thus, its SAR and SARIn modes are deployed mainly over sea ice and continental ice margin areas; over ice-free ocean, it mainly operates in its "LRM" mode (for low-rate mode or low-resolution mode). In LRM mode, it functions like other conventional altimeters, continuously emitting pulses with a pulse repetition frequency (PRF) around 1.95 kHz, a rate intended to maximize the number of independent "looks" per radar cycle [13], [14] to maximize the variance reduction available through incoherent averaging of $\langle p_t \rangle$. The waveforms so obtained are conventional pulselimited waveforms, but are sometimes called LRM waveforms in the CryoSat context to distinguish them from waveforms obtained in other modes. They average 91 pulses and have approximately 91 statistically independent looks. The onboard target tracking and AGC loops operate on these waveforms when the instrument is in LRM mode.

CryoSat's SAR mode transmits four "bursts" of pulses per radar cycle. Each burst contains 64 pulses emitted at a PRF around 18.18 kHz, and thus, burst transmission lasts for 3.52 ms. However, the burst repetition interval is 11.8 ms, so that the instrument is actively measuring only about 30% of the available time, in contrast to a conventional instrument or CryoSat's LRM mode, which measures continuously. This design may be suboptimal for an ocean altimeter [15] and will be changed in the future Jason-CS instrument. It was a good choice for CryoSat's ice objectives and budget constraints, as it allowed the instrument to achieve pulse-to-pulse coherency to support SAR and SARIn mode aperture synthesis while minimizing cost and risk by utilizing the then-existing altimeter designs with extensive flight heritage.

When the instrument is in SAR mode, it creates a pulselimited waveform for use by the target tracking and AGC loops. This "tracking echo" is formed by incoherently averaging the power in every ninth echo in each burst, numbering the echoes from 0 to 63 and taking those echoes whose number modulo 9 is zero. This is done to approximate the PRF of the LRM mode. In this scheme, only 32 echoes are averaged per radar cycle, in contrast to the 91 averaged in LRM mode, so that the tracking echo has a higher speckle noise level than the noise level on the LRM waveform.

F. Smoothing of the Tracker Noise

Because the input to the onboard range tracking loop is noisier in SAR mode than in LRM mode, the tracker's forecast of the arrival time of an echo t_E is likewise noisier. We computed the tracker's forecast range as $r_E = c(t_C + t_F)/2$, taking the propagation speed c to be that of light in a vacuum, and then estimated the tracker's performance by forming a quasi-height as orbit height minus r_E minus geoid height. The low-frequency components of this quasi-height are not due to tracker errors but instead contain true ocean height signals, calibration constants, and propagation delays. We ascribed the high-frequency components to tracker error, high and low being with respect to 0.4 Hz, the natural resonance of the alpha-beta tracker loop. We found high-frequency quasi-height power levels about three times higher in SAR mode than in LRM mode, commensurate with the 91:32 ratio of the number of echoes averaged in the tracked waveforms.

In order that our waveform averaging from SAR data should imitate a conventional instrument as closely as possible, we derived our own t_F values under the following constraints: 1) the average over the cycle of the sum $t_C + t_F$ equaled that of the values given in the data product, and 2) the rate of change of the sum $t_C + t_F$ was constant over a cycle and implied that $\dot{r} = \dot{h}$, where \dot{h} is the rate of change of the orbit ephemeris height above the Earth ellipsoid, a variable supplied in the FBR data product.

The first constraint ensures that the net window delay t_W obtained in our process is the same as that which would have been used on board if the instrument had been functioning as a conventional altimeter. This constraint has an important effect on the root mean square (RMS) of nominal 1-Hz averages of range retrievals made at a nominal 20-Hz rate (radar cycle

rate). Random jumps in the averaged waveform's leading edge position within the window occur in our processing just as they would have occurred on board in a conventional instrument. After "retracking" (see Section III), we then expect our range retrievals to have the same random noise level that they would have had if the instrument had been functioning conventionally. Indeed, the range uncertainties found below are similar, after accounting for the 91:32 ratio of effective number of looks, to those found by retracking of conventional LRM mode data [16].

The second constraint is an approximation that ignores the rate of change of range due to the slope of the sea surface, which produces negligible change in range over the averaging time. We made this approximation because it allowed us to obtain more correct echo alignment than we would have obtained using the SAR mode range tracker's forecast value. We also allowed t_F to vary in a linear way with each echo in a burst, rather than merely changing it once per burst. These ensure correct alignment of the leading edge of the waveform during echo averaging over a radar cycle, which ensures that the leading edge rise will not be smeared out, causing an apparent increase in the surface roughness or the SWH H.

G. Calibration Mode 1

Because pulse transmission and reception are not continuous in CryoSat's SAR and SARIn modes, there are transient thermal effects in the instrument that occur within each burst and cause small variations in echo amplitude and phase from one pulse to the next. Pulse-to-pulse phase coherence is necessary for aperture synthesis, and thus, the instrument has a calibration mode known as "Cal 1" that supplies small corrections to the amplitude and phase of each pulse echo. We applied these to the echoes prior to the application of (1), although our application requires only incoherent averaging and does not exploit the phase information. Employing the Cal 1 amplitude correction ensures that each echo gets the correct weight in the average. Using the Cal 1 phase correction ensures that each echo gets the intended time alignment in (1). The corrections are very small and unlikely to have an appreciable effect on our results, but we included them in our processing for completeness.

H. Calibration Mode 2

After forming z_t , p_t , and $\langle p_t \rangle$ by the above steps, a slight ripple is visible in the plateau of the resulting waveform. This is due to the filter applied in the receiver after the echo is mixed with the receive chirp and before it is digitized. The instrument's calibration mode 2 supplies a correction known as "Cal 2" that can be applied to a p_t or $\langle p_t \rangle$ sequence to remove this ripple. We applied this correction.

I. Conventional and Resampled Waveforms

According to the Nyquist–Shannon sampling theorem [17], a time series sampled at Δt correctly reconstructs signal frequencies in the frequency band $\pm 1/(2\Delta t)$, that is, $\pm B/2$; thus, the sampling rate of z_t is correctly matched to the chirp bandwidth B. The frequency content of p_t should be double that of z_t , however, due to the magnitude squaring operation $p_t = |z_t|^2$, and therefore, to avoid aliasing p_t , it ought to be sampled at a sampling rate twice that of z_t [1], that is, $\Delta t/2$. In a conventional instrument, it is not; the p_t sequence is formed by squaring the z_t sequence without any resampling. Reference [1] suggested that all conventional altimeter instruments are therefore producing aliased waveforms.

Jensen [1] suggested that this waveform aliasing could be avoided if the Z'_k sequence obtained from (1) were simply extended with an equal number of zeros prior to application of the DFT. This would effect a sinc-function interpolation of the z_t sequence, doubling its sampling rate while leaving its frequency content unchanged, after which the magnitude squaring operation should yield a p_t series sampled at a rate that would produce no aliasing.

In this paper, we produce both 128-element conventional waveforms and 256-element resampled waveforms from the same FBR I and Q data by applying Cal 1, smoothing the t_F values as above, applying (1) with the smoothed t_F values, then either applying a 128-element DFT or zero-padding and applying a 256-element DFT, then magnitude squaring the resulting 128-element and 256-element complex amplitude sequences, and then incoherently averaging the power for a radar cycle. To the resulting averages, we then apply the Cal 2 correction. The Cal 2 correction data are supplied as 128-element arrays and can be directly applied to the 128-element conventional waveforms. For the 256-element resampled waveforms, we interpolated Cal 2 with a sinc-function interpolant before applying it. Fig. 1 shows the result of this processing.

The official Level-1b waveform data products generated by the CryoSat mission for times when the instrument is in SAR or SARIn mode include coherent processing for aperture synthesis, followed by "multilooking" [2], and thus are very different from the waveforms shown here. The intent here is to simulate conventional pulse-limited waveforms, that is, employing incoherent averaging without aperture synthesis. When waveforms such as shown here are derived from CryoSat SAR data, they are sometimes called "pseudo-LRM" or "PLRM" in the CryoSat community. This term is ambiguous. Some mean by it waveforms that utilize only every ninth echo, while others mean an incoherent average of all echoes. Our waveforms include all the echoes in each radar cycle.

J. Paradox? Something for Nothing?

Fig. 1 illustrates that the same raw I and Q data can yield waveforms with different leading edge shapes, hence differences in the retrieved geophysical parameters, depending on whether the processing is conventional or includes the resampling step. At first, this may seem paradoxical, as zero-padding cannot add new information. When two mathematical operations are performed in sequence, the order in which they are performed is irrelevant only in the case that both are linear operations. The operation that forms power from the squared magnitude of complex amplitude is nonlinear, and therefore, interpolation followed by squaring does not produce the same result as squaring followed by interpolation. Thus, the 256-element waveform cannot be obtained by interpolating the 128-element waveform. Zero-padding does not add



Fig. 2. Profiles of (top) sea surface height anomaly ("ssha"), (middle) H ("swh"), and (bottom) backscatter ("sig0") estimates obtained from radar cycle averages of received power at (red curves) nominal 20-Hz rhythm, (yellow curves) 1-Hz averages of these, and (blue curves) 1-Hz standard deviation of the 20-Hz data around the 1-Hz average. In the blue zone, the instrument is in SAR mode, and the estimates are obtained from echo data processed as in a conventional instrument (N = 128, left panels) and with zero-padding (N = 256, right panels). In the white zones, the instrument is in LRM mode, and the estimates are obtained from waveforms averaged on board in the conventional manner. Data were acquired on March 11, 2012, in the South Atlantic from a descending pass that enters the SAR mode at 07:25:15 UTC time.

information, but it may be preventing the loss of information. The next two sections investigate whether this loss has practical consequences, how large these consequences are, and under what conditions they occur.

III. EFFECT OF RESAMPLING ON GEOPHYSICAL PARAMETER RETRIEVAL

We processed long arcs of CryoSat SAR data, producing 128- and 256-element waveforms $\langle p_t \rangle$ at a nominal "20-Hz" rhythm, that is, one such average per radar cycle. We then put the resulting waveforms through an "MLE-3"-like "retracker" [18], which does a least squares fit to each waveform to recover a correction to range delay, an estimate of the wave height parameter H, and an estimate of the backscattered power, σ^0 [16]. The N = 128 and N = 256 versions are compared in Fig. 2. Due to the DFT used to convert echo samples to range delay times, the waveforms exhibit a "wrap around" as they obey a periodic boundary condition at the edges of the range window (see Fig. 1). Therefore, in estimating geophysical parameters by least squares fitting, we chose to exclude the first and last 12 samples in the 128-element waveforms.

At the 20-Hz radar cycle rhythm, there are fluctuations in the fitted geophysical parameters, and thus, oceanographers commonly average 20 consecutive values to obtain averaged parameters at a nominal "1-Hz" rhythm. Because H and σ^0 are not expected to change significantly during 1 s of flight, a simple average is used for these parameters, and the standard deviation of the 20 values around the mean is used to assess the precision of the 20-Hz values that went into the average. Since range will change significantly during 1 s, it is customary to form a raw sea height as orbit height minus range at the 20-Hz rhythm and then average the raw sea height to a 1-Hz rhythm. Because the raw height contains the geoid variations, we have removed the Danish Technical University 2010 mean sea surface model "DTU10" [19] from the raw height, and thus, we plot a height anomaly in Fig. 2. The simple average of the height anomaly and the standard deviation of the 20 height anomaly values around the mean are also shown.

In the blue box area in Fig. 2, CryoSat is in SAR mode, and we have performed the analysis described above. Outside the blue box, it is in LRM mode and functions as a conventional altimeter, supplying one 128-element mean power waveform per radar cycle. We have retracked these LRM waveforms as well, in order to compare the values retrieved by our process applied to the SAR mode data against the values we retrieve from the 128-element waveforms processed on board in the LRM mode.

Comparing the 1-Hz averages of the geophysical retrievals from either the 128-element or the 256-element waveform we have generated with the averaged retrievals from the LRM waveforms, we do not see any apparent shift in the levels of sea surface height (SSH) anomaly H or σ^0 in Fig. 2, suggesting that there is no bias between our retrievals from SAR data and the retrievals from LRM data. It also appears that the 1-Hz average level of the geophysical parameters is essentially the same in the 128- and 256-element versions; there is no obvious shift in the mean levels between these two versions.

The backscatter standard deviations, which are labeled " $\sigma \operatorname{sig0}$ " in Fig. 2, are about the same between LRM and SAR modes, as the AGC loop maintains the digitized receiver output at a nearly constant level regardless of variations in backscatter; thus, most of the information about σ^0 comes from the AGC and not the fitting of geophysical models to the averaged waveforms. There are clear differences in the standard deviations, which are labeled " σ ssha" and " σ swh," in Fig. 2. These are higher in the SAR data than in the LRM data by a factor of roughly $\sqrt{91/32}$, as expected from the differences in the number of statistically independent looks at the ocean surface during a radar cycle [15], [20].

Visible also in Fig. 2 is a small decrease in the σ values for SSH anomaly and H from the N = 256 version compared with the N = 128 version. For the sea height anomaly, the σ decrease is about 1 cm, or about 20% reduction in height variance. For the H values, the σ decrease is about 0.15 m, or about 30% reduction in variance.

IV. CYCLE AVERAGES OF 1-HZ VALUES

We carried out an analysis such as that shown in Fig. 2 for all available CryoSat SAR mode data over open and ice-free ocean areas collected between June 11 and July 8, 2012. This time period corresponds to "cycle 29" in the Radar Altimeter Database System [21]. CryoSat's orbit has an "exact repeat" (meaning ground tracks repeat within ± 1 km at the equator) every 369 days, with nearly repeating subcycles approximately every 29 days, and we have examined one such subcycle. We made 128- and 256-element waveforms at a 20-Hz rhythm, retracked these to obtain geophysical retrievals at 20 Hz, and averaged the retrieved parameters to a 1-Hz rhythm, thus obtaining mean and standard deviation values at 1 Hz throughout the 29-day period. This yielded 156 267 sample points, restricted to the ocean areas where CryoSat is in SAR mode (see Fig. 3).

Fig. 1 shows that 128- and 256-element waveforms made from the same raw echo samples may display leading edges



Fig. 3. (Top) Map of ocean locations sampled by CryoSat while its instrument was in SAR mode, during the period June 11 to July 8, 2012, for which the pulse-limited waveform appears to show valid ocean and the Wave Watch 3 (WW3) model indicates valid data. After averaging along-track to 1-s averages, as in Fig. 2, there are 143 388 such valid averaged points. Color indicates the SWH H value at each point, according to WW3, as in the histogram below the map. (Bottom) Histogram of WW3 H values at the points shown in the map above. Counts are thousands of points; histogram bin width is 0.2 m.

that will yield different H estimates. We also expect that this phenomenon is more likely when the true H is small, so that the waveform's leading edge is steep. In order to explore this possibility, we wanted to assign to each of our 1-Hz estimates an H value determined independently of our fitted parameters.

We used H values from the Wave Watch 3 Climate Forecast System Reanalysis hindcast model (WW3) [22], which is available from the NOAA National Centers for Environmental Prediction web site. This model does not assimilate altimeter estimates of H, but estimates H independently, considering wind forcing and swell propagation. We interpolated these model H values to the space and time sample points of the altimeter data. At some of our retrieval points, the WW3 model grids indicated unavailable or invalid data; thus, these points were not analyzed, reducing our cycle-29 data set to 143 388 data points. The resulting data coverage is shown in Fig. 3, along with a histogram of the distribution of WW3 H values in our study.

We binned these data into 0.2-m-wide bins according to their WW3 H value and then computed means and variances of the binned data from the 1-Hz data averages and standard deviations, for each of the three data types, namely, SSH, SWH, and backscatter coefficient. The RMS expected variation of a 20-Hz retrieval around its corresponding 1-Hz mean, expectation taken over each H bin, was computed as $\bar{\sigma}$, where the overbar indicates averaging in an H bin, and the mean square value is computed via

$$\bar{\sigma}^2 = \frac{1}{M} \sum_i \sigma_i^2 \tag{2}$$

with M being the number of data points falling in the bin, σ_i being the individual 1-Hz standard deviation values falling in the bin, and the summation taken over all values in the bin. This was done for each H bin in 0.2-m steps, so long as the bin had at least 100 samples in it. This yielded data for bins with $H \leq 7$ m. Results are shown in Fig. 4 for both the 128-element (black) and 256-element (gray) values. The one-sigma error bars shown are $\bar{\sigma}/\sqrt{M}$. The backscatter results plot on top of one another, but the other results show clear differences at low wave heights, with the 256-element $\bar{\sigma}$ values being lower than the 128-element $\bar{\sigma}$ values. The reductions in variance, expressed in percent as $100(\bar{\sigma}_{128}^2 - \bar{\sigma}_{256}^2)/\bar{\sigma}_{128}^2$, are shown in the fourth panel in Fig. 4.

To test whether there was a systematic difference in the retrieved values, as opposed to their standard deviations, we estimated a mean difference as follows:

$$b_i = x_{i,128} - x_{i,256} \tag{3}$$

$$\sum_{i}^{n} b_{i} w_{i}$$
(1)

$$b = \frac{1}{\sum_{i} w_{i}}$$
(5)
$$\sigma_{b} = \left[\sum_{i} w_{i}\right]^{-1/2}.$$
(6)

Equation (3) expresses a difference estimate obtained from each 1-Hz average sample, where x stands for any of the geophysical retrievals: SSH, SWH, and backscatter. The variance in the difference of two uncorrelated quantities is the sum of the variances in each; thus, (4) gives a weight to each difference estimate that is inversely proportional to the variance of the difference estimate, assuming the two x values have uncorrelated errors. Equation (5) expresses the mean difference in a 0.2-m SWH bin, \bar{b} , as the weighted average of all the estimates in the bin. Equation (6) expresses the expected standard deviation in the mean difference estimate, σ_b .

The results are shown in Fig. 5, with labels ΔX replacing \bar{b} , and X one of SSH, SWH, or σ^0 , corresponding to SSH



Fig. 4. (First to third panels) Average uncertainty in 20-Hz geophysical retrievals of SSH, SWH, and backscatter, respectively, as a function of WW3 SWH. In these top three panels, black stars and error bars are obtained from conventional waveforms, and gray circles and error bars are obtained from zero-padded waveforms. (Fourth panel) Reduction in variance, in percent, achieved by employing zero-padding.

(range), SWH, and backscatter, respectively. The mean SSH difference is negative everywhere, indicating that, on average, the 256-element sea height is above the 128-element sea height and the 256-element range is shorter than the 128-element range. However, the difference magnitudes, 3–7 mm, are very small compared with the range resolution $\Delta r = c\Delta t/2$ of about 47 cm. The mean *H* difference is negative for H > 0.5 m, indicating that 256-element waveforms give slightly (a few cm) larger SWH estimates than 128-element waveforms. The backscatter difference, although very small, -0.051 dB, appears essentially constant at all SWH values. Its sign indicates that the 256-element waveforms give slightly higher backscatter estimates than the 128-element waveforms.

V. BANDWIDTH NECESSARY TO RESOLVE THE RANGE TO, AND ROUGHNESS OF, A GAUSSIAN ROUGH SURFACE

A. Assumptions

Brown [23] expressed the statistical expectation for the power received from a rough scattering surface in terms of a convolution of three functions

$$PTR(t) * [SSIR(t) * PDF(t)]$$
(7)



Fig. 5. (First to third panels) Average difference in geophysical retrievals of SSH, SWH, and backscatter, respectively, as a function of WW3 SWH. These differences are defined as the value from a conventional waveform minus the value from a zero-padded waveform.

which we will call here the point target response (PTR), the smooth surface impulse response (SSIR), and the probability density function (PDF) for the distribution of the scatterers. Equation (7) is routinely and successfully used to model averaged altimeter waveforms over ocean surfaces [23]–[28]. Here, we will examine the bandwidth and the spectral shape of each of the terms in (7) separately, to offer some insight into the results obtained above and the aliasing claim made in [1].

In (7), the PTR accounts for the instrument's resolution of the two-way travel time of an ideal Dirac delta function pulse, and thus, the bracketed function represents a theoretical mean power echo arising from a hypothetical Dirac delta function pulse. Echoes in the real world arise from the reflections of actively transmitted sources, which must necessarily have finite bandwidth, and thus, the idealized echo in brackets in (7) cannot exist in reality independently of an active radar instrument. However, here, we will suppose that it is valid to compare the spectra of the PTR and the PDF.

Expressing the ideal echo as the convolution in brackets in (7) requires assumptions [23], [25], [29] that an incoherent surface scattering process in the SSIR can be assumed separable from the PDF of the scatterers, that the scattering PDF is independent of location on the surface, and that the autocorrelation of two scatterers is a Dirac delta function of their separation distance in the surface. These assumptions cannot be made for altimeters operating over snow- and ice-covered surfaces [29], but are usually justified for ocean altimetry by arguing that the horizontal correlation scale of sea level undulations of geophysical interest is large, whereas the horizontal correlation scale of the scatterers is small, large and small being in comparison with the dimensions of the pulse-limited measurement



Schematic one-sided power spectral density showing normalized Fig. 6. power versus frequency, relative to the instrument's chirp bandwidth B. A conventional instrument digitizes the complex echo amplitude at a rate that will reconstruct frequencies in the light-gray rectangle below the Nyquist, B/2. An echo having such a rectangular amplitude spectrum would have a power spectrum shown as the dark gray and hachured triangle extending to two times the Nyquist frequency. This is the basis for the claim [1] that echoes should be resampled before amplitude is squared in order to avoid aliasing. We propose that whether aliasing occurs or not depends on whether the leading edge of an idealized theoretical echo waveform has significant power in the hachured portion of the triangle. Curves indicate the contributions to this theoretical echo spectrum from a Gaussian rough surface. Labels outside (respectively, inside) parentheses indicate the surface roughness in terms of the SWH H in meters corresponding to a bandwidth B equal to 320 MHz (respectively, 500 MHz), typical of a Ku-band (Ka-band) altimeter.

footprint. However, this justification rests on the assumption of a finite footprint area, and the area of the pulse-limited footprint depends on the combined widths of the PTR and the PDF [30]. Therefore, the usual argument does not justify the convolution in brackets in the limit of an infinitesimal pulse and a very narrow PDF. We will be careful here not to draw conclusions in the case that the width of the PDF is much narrower than the range resolution of the PTR.

B. Bandwidth and Shape of the PTR

The time-bandwidth product of the instrument's chirp is very large, $BT_{\rm RX} > 10^5$, and thus, the PTR is very nearly an ideal sinc² function

$$PTR(t) = \frac{\sin^2(\pi Bt)}{(\pi Bt)^2}$$
(8)

and its Fourier transform (we will use the Fourier transform definitions and normalizations of [31]) is the triangle function shown in Fig. 6

$$PTR(f) = \begin{cases} 1 - |f|/B, & |f| \le B\\ 0, & |f| > B \end{cases}.$$
 (9)

Equations (8) and (9) are the expressions for the instrument's resolution of the power p and not the complex echo amplitude z. Because $p = |z|^2$, the instrument's time resolution for z is simply $\operatorname{sinc}(Bt)$, and its spectral resolution is simply the rectangular spectrum shown in Fig. 6. The bandwidth of z is simply B, the bandwidth of the chirp.

C. Problem That Zero-Padding Can Solve

Jensen [1] noted that the hachured portion of the triangle in Fig. 6 would exceed the Nyquist frequency of the sampling of p

unless the zero-padding step was included. Since zero-padding is not conventionally performed, he claimed that conventional waveforms are aliased. This need not be the case, however, if the spectrum of the bracketed terms in (7) has very little energy in the hachured portion in Fig. 6. This section aims to investigate what conditions, if any, would cause the spectrum to fall in the hachured portion in Fig. 6. It seems intuitive that only when the hachured portion is important to the echo will the zero-padding be important to avoiding aliasing.

D. Theoretical Echo

Let $p(\tau)$ represent the bracketed term in (7) referred to a range delay, $\tau = t - 2h/c$, the two-way travel time in excess of that required to make the round trip to the mean elevation of the scattering surface at nadir, a distance h below the antenna. [In ranging to grounded ice, one must consider that the point of closest approach ("POCA") between the antenna and the mean scattering surface may be significantly displaced from nadir [32]. On an ocean surface, the separation between POCA and nadir is small compared with the footprint diameter because the surface slopes are very small [33], [34]. In either case, the mathematics given here is the same.]

We will write the convolution in the brackets in (7) as

$$p(\tau) \propto s(\tau) * g(\tau) \tag{10}$$

in the time domain and

$$P(f) \propto S(f)G(f) \tag{11}$$

in the frequency domain, with proportional signs, rather than equality signs, because we will omit leading amplitude coefficients where convenient, as our interest here is not in the amplitudes of these functions or their physical units, but only the shape of their time or frequency dependence.

E. SSIR

The SSIR takes the form [28]

$$s(\tau) \propto u(\tau) \exp(-f_A \tau)$$
 (12)

in which $u(\tau)$ is a Heaviside unit step function, and f_A is a decay constant having units of frequency and depending on the altitude h, beam width β , and off-nadir pointing ξ of the antenna

$$f_A = \frac{c}{h\kappa\beta^2} \left(2 - \frac{\xi^2}{\beta^2}\right). \tag{13}$$

Here, $\kappa = 1 + h/R$ accounts for the curvature of the Earth, R [8], [26]. The beam width parameter is related to the antenna's half-power beam width, $\theta_{\rm HPBW}$, via

$$\beta = \frac{\theta_{\rm HPBW}}{2\sqrt{\ln(2)}} \tag{14}$$

and the antenna's gain pattern is assumed circularly symmetric. CryoSat's antenna pattern is actually slightly elliptical [35], but the above assumption does an excellent job of modeling the CryoSat echoes if the square of the half-power beam width above is set to the harmonic mean of the squares of CryoSat's major and minor axis half-power beam widths [16], [35]. This results in a θ_{HPBW} about 1.16°, so that β is about 0.016 rad. For CryoSat's altitude *h* around 730 km and typical mispointing angles less than 0.1°, the decay rate f_A is about 2.9 MHz.

The Fourier transform of (12) is

$$S(f) = (f_A + j2\pi f)^{-1}$$
(15)

and we can define its normalized magnitude as

$$S_M(f) = f_A |S(f)| = \frac{f_A}{\sqrt{f_A^2 + (2\pi f)^2}}$$
(16)

so that $S_M(f) = 1$ at f = 0 and decays as f^{-1} for $f \gg f_A$. The f^{-1} decay rate is so slow that definitions of bandwidth involving integrals of (16) [31] do not converge; however, the product S(f)G(f) in (11) can have finite bandwidth if G(f)is sufficiently compact. In particular, a Gaussian function will suffice.

F. PDF

The PDF for the SSH ζ is very well approximated by a Gaussian PDF with standard deviation equal to one fourth of the SWH *H* [36], [37]. Converting height ζ to range delay *t* via $\zeta = c\tau/2$ and expressing the standard deviation in terms of *H*, we may write

$$g(\tau) \propto \exp\left[-\frac{1}{2}\left(\frac{2c\tau}{H}\right)^2\right]$$
 (17)

so that the standard deviation of $g(\tau)$ is H/2c.

These Gaussian functions are a Fourier transform pair [31]

$$\exp\left[-\pi\left(\frac{t}{t_W}\right)^2\right] \Leftrightarrow \exp\left[-\pi\left(\frac{f}{f_W}\right)^2\right] \tag{18}$$

in which $t_W = 1/f_W$, and t_W and f_W are the "equivalent widths" of these Gaussian functions. The equivalent width of a Gaussian PDF is simply its standard deviation multiplied by $\sqrt{2\pi}$ [31]. Therefore, we have

$$G(f) \propto \exp\left[-\frac{1}{2}\left(\frac{\pi H f}{c}\right)^2\right] = \exp\left[-\pi \left(\frac{f}{f_W}\right)^2\right]$$
 (19)

and the equivalent width of this frequency PDF is inversely proportional to the wave height, or surface roughness

$$f_W = \sqrt{\frac{2}{\pi}} \left(\frac{c}{H}\right). \tag{20}$$

In physical units, f_W is about (240 MHz/H), for H in meters.

G. Combined Theoretical Echo

Combining (16) and (19), we can write the normalized spectral shape for the expected theoretical echo in the bracketed terms in (7)

$$P(f) \propto \frac{f_A}{\sqrt{f_A^2 + (2\pi f)^2}} \exp\left[-\frac{1}{2} \left(\frac{\pi H f}{c}\right)^2\right]$$
(21)



Fig. 7. Portion of the Gaussian echo bandwidth in Fig. 6 that is outside the Nyquist of a conventionally sampled echo, as a function of SWH. The horizontal axis labels on the lower (respectively, upper) axis indicate wave height in meters corresponding to a chirp bandwidth of 320 MHz (respectively, 500 MHz).

and the equivalent width of this theoretical echo

$$f_E = \int_{-\infty}^{\infty} S_M(f)G(f)df = \frac{f_A}{2\pi} \exp(\nu_H)K_0(\nu_H)$$
(22)

in which K_0 is the modified Bessel function of the second kind of order zero and

$$\nu_H = \left(\frac{f_A H}{4c}\right)^2 \tag{23}$$

is a nondimensional parameter depending on the wave height H.

For most sea states, ν_H is a quantity on the order of 10^{-6} , and thus, (21) and (22) suggest that the theoretical echo in brackets in (7) has a bandwidth of only a few megahertz. This is because the spectrum of the overall echo is dominated by the slow decay of its plateau at the rate f_A ; the convolution with the PDF has significant effect on the waveform only at its leading edge. These considerations suggest that the bandwidth of the instrument's chirp is sufficient to resolve the echo overall.

H. Leading Edge of the Waveform

The shape of the waveform's leading edge is of critical importance to the retrieval of range and surface roughness. Since f_A is small, the SSIR is effectively a Heaviside step in the vicinity of the leading edge, and thus, the median of the PDF locates the midpoint of the rise of the leading edge, whereas the standard deviation of the PDF determines the duration of the rise. Although there are theoretical difficulties with trying to define the bandwidth or the spectrum of a narrow portion of a function, it seems intuitively clear that good resolution of the leading edge of the waveform should require a bandwidth adequate to characterize the first and second moments of the PDF. Therefore, we have plotted the spectra of Gaussian PDFs in Fig. 6 using (19). How much of this Gaussian spectrum falls outside the rectangular bandwidth representing the Δt sampling rate is then calculated by a complementary error function, as shown in Fig. 7.

We find that, for a typical Ku-band altimeter with B = 320 MHz, if H > 2 m, then the sampling rate for z_t is sufficient

that simple squaring without resampling should not produce significant aliasing. Conversely, for $H \ll 1$ m, the standard deviation of the surface roughness, $\sigma = H/4$, is much less than 47 cm, the intrinsic range resolution of the instrument, $\Delta r = c\Delta t/2$, and thus, the instrument should have difficulty resolving the shape of the waveform's leading edge whether or not zero-padding is employed. For intermediate H values, we anticipate that the sampling rate for z_t will be adequate, but simple squaring of z_t without resampling should cause some aliasing of p_t .

I. Relationship Between SWH and Bandwidth

Equating f_W in (20) with B, one may define a "bandwidth equivalent wave height"

$$H_B = \sqrt{\frac{2}{\pi}} \left(\frac{c}{B}\right). \tag{24}$$

This H_B is the SWH at which the PDF has the same bandwidth as the instrument's chirp. One may suppose that leading edge resolution is poor, whether or not zero-padding is applied, when H is near or below H_B . For B = 320 MHz, $H_B = 0.75$ m.

J. Performance Differences Between Ku-Band and Ka-Band

Fig. 7 is labeled with two values of H. The first value assumes that the chirp bandwidth B = 320 MHz, as in CryoSat and typical of Ku-band altimeters. The second value applies to a chirp bandwidth B = 500 MHz, as in the Ka-band "AltiKa" altimeter [38], which was launched on the "SARAL" satellite in February 2013. Because a larger bandwidth is available in Ka-band, the instrument should have better range resolution, and problems due to a lack of zero padding should arise at smaller sea states, better and smaller being with respect to a Ku-band altimeter. In particular, for B = 500 MHz, $H_B = 0.48$ m.

VI. FINDINGS, DISCUSSION, AND CONCLUSION

When CryoSat is in its SAR and SARIn modes, the raw digitized output from the reception of each echo is transmitted to Earth for processing in the mission's ground segment and made available in the FBR data product. In these modes, the Level-1b SAR and SARIn data products contain a waveform, which is not the pulse-limited waveform treated here but instead is the result of coherent aperture synthesis followed by incoherent "multilooking" [2]. In the first ("Baseline A") version of the CryoSat mission's ground segment processor, no zero-padding was included in this process. Experience with the resulting multilooked waveforms showed that they did not have sufficient sampling of the leading edge to support good range retrievals to leads in sea ice, which generate very specular echoes (that is, very sharply rising leading edges). Therefore, the second ("Baseline B") version of the processor was adapted to include zero-padding as the first step, prior to aperture synthesis and multilooking, so that "B" version Level-1b multilooked waveforms are sampled at twice the rate of the "A" version waveforms.

The CryoSat mission team thus arrived at the result that zero-padding is helpful when multilooked SAR echoes have steep leading edges, as they do over leads in sea ice, a key objective of the CryoSat mission. In this paper, we have extended and generalized that result by showing what happens when zero-padding is employed in pulse-limited echoes over the open ocean. We found that zero-padding makes substantial reductions in the variance of geophysical signals important in ocean altimetry (see Fig. 4). Zero-padding also changes the mean retrievals of range and surface roughness (see Fig. 5). Although these are small, millimeters of range and centimeters of SWH, compared with the intrinsic range resolution of the chirp bandwidth, $\Delta r = 47$ cm, they are systematic.

Theoretical considerations in Section V and Figs. 6 and 7 suggest that one should not expect zero-padding to be important when $H \gg 2$ m, it should be increasingly important as H decreases below 2 m, but at some point, roughly around $H = H_B = 0.75$ m, limitations due to bandwidth will be felt that will not be overcome by zero-padding. Theory also suggests that all these considerations scale to different H values in Ka-band than in Ku-band, due to the wider chirp bandwidths permissible in Ka-band.

We have empirically found that zero-padding seems to be important over a broader range of H values than the theoretical considerations would have suggested. However, the mere fact that zero-padded waveforms yield different results does not prove Jensen's [1] claims. It may be simply that zero-padding improves the retrievals by increasing the number of data points to be fit in the geophysical modeling.

We also have shown that zero-padding has a greater effect, and over a broader range of H values, on the estimation of surface roughness (SWH) than on range (SSH). This should perhaps not be surprising, as measuring SSH requires only that the altimeter's waveform resolves the first moment of the surface roughness PDF, whereas measuring SWH requires that the waveform resolve the second moment of the PDF.

Range, roughness, and backscatter retrievals from an altimeter require several corrections and calibrations before their accuracy can be compared with other data [30]. Discussion of these is beyond the scope of this paper. Since nearly all of these are external to the process of geophysical retrieval by retracking, the same kinds of correction would apply to either the conventional or zero-padded estimates, in general. Therefore, we have only attempted to investigate the change in level in retracked retrievals between the conventional and zeropadded waveforms.

It is interesting to note that the altimeters on Seasat, Geosat, TOPEX, and Envisat had specialized hardware and firmware that generated a few additional "in-between gates"; they delivered conventionally averaged waveforms as described here with sampling rate Δt throughout their waveform, plus, usually, three additional samples interleaved (effective sampling rate $\Delta t/2$) right around the track point, that is, the point where the waveform leading edge should rise if the target tracking loop is functioning well. One could generalize the calculations done here to include a feature that would double the sampling rate by zero-padding and then throw away most of the resulting additional samples, saving only those that correspond to the few in-between gates. We do not know enough about the details of these other altimeters to know if this would properly simulate their performance, and such a simulation is beyond the scope

of this paper. We speculate that their in-between gates may have served as a partial guard against some of the problems suggested in [1].

Estimates of wind speed (derived from backscatter) and wave height from conventional altimeters are used in marine forecasts. These forecasts have significant impact on ship routing and decisions about safety only when the forecast H is large. It is fortunate, then, that large-H conditions are those where conventional altimetry should have no aliasing problem.

Although the effects found here are small, they are not trivially small. Altimeter data are now routinely used to monitor and interpret regional and global sea level rise signals as small as 1 mm/year [39]. Therefore, a 5-mm reduction in the standard deviation in 20-Hz range measurements is significant. In addition, errors in estimating H and σ^0 will couple to errors in estimating SSH through the sea state bias correction [30]. This correction is of order 3%–4% of H, so that reducing the error in H by 9 cm will contribute an additional 3–4 mm of error reduction in sea level.

The altimeters planned for the future missions Sentinel-3 and Jason-CS inherit features from CryoSat's SAR mode. In the SAR modes of all three of these missions, although the final geophysical retrievals are made in on-the-ground processing of digitized echoes, where zero-padding can be included, the instruments still employ a conventionally processed waveform on board to drive the AGC and range tracking loops. Our results suggest that future instruments' tracking loops could operate on better waveforms if zero-padding were included in the onboard processing.

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