## Wind instability of a foam layer sandwiched between the atmosphere and the ocean

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The Kelvin-Helmholtz (KH) instability of short gravity waves is examined in order to explain the recent findings of the decrease in momentum transfer from hurricane winds to sea waves. A foam layer between the atmosphere and the ocean is suggested to provide significant stabilization of the sea-water surface by the wavelength shift of the instability towards smaller scales. It is conjectured that such stabilization leads to the observed drag reduction. The problem of a three-fluid system with large differences in densities provides an extension to the fundamental KH problem in fluid mechanics.

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Introduction. — Results of direct measurements extrapolated from weak to strong winds predict a linear increase of momentum transfer from wind to sea waves. The present study is motivated by recent findings of saturation and even decrease in the drag coefficient (capping) in hurricane conditions that is accompanied by production of a foam layer on the ocean surface [1]. A possible explanation for the phenomenon is the development of a foam layer at the air-sea interface. The principal role of such an air-water foam layer in energy dissipation and momentum transfer from hurricane wind to sea waves has been first suggested in [2]. Winds generate waves on the ocean surface with a wide spectrum of wave lengths. The longest waves, hundreds meters of length, attempt to catch up with the wind, while the steeper short waves break out and play a dominant role in drag production [3]-[4]. When the wind speed exceeds storm force (24m/s), wave breaking creates streaks of bubbles near the ocean surface. As the wind exceeds the hurricane force (32m/s), streaks of bubbles combined with patches of foam cover the ocean surface. When the wind speed exceeds 50m/s, a foam layer completely covers the ocean surface [1].

Nowadays, there is a little hope for a comprehensive numerical calculations of the drag coefficient reduction that includes a detailed description of the wave breaking and foam layer production. Indeed, up to now there is no complete understanding of the phenomenon. In the present study, the intermediately short wave Kelvin-Helmholtz instability (KHI) [5]-[6] of a foam layer between the atmosphere and the ocean is investigated in order to qualitatively explain the drag reduction phenomenon. Such three-layer system exhibits a high contrast in densities of constituting fluids  $\rho_a \ll \rho_f \ll \rho_w$ . The present study is not concerned with the formation mechanism of the foam layer by the hurricane but rather focuses on how a foam layer isolates the lower atmosphere from the sea surface The existence of the foam layer on the ocean surface is postulated and supported by observations (see [1], [7] and references therein). The present modeling demonstrates a new effective mechanism to stabilize the sea surface by a thin foam layer between the atmosphere and the ocean. However, beyond that particular application, the current work addresses a fundamental problem in fluid mechanics which provides a generalization of the classic KHI. Thus, the peculiarities of three-layer systems with large differences in the densities may be of interest to a wide range of applications in the laboratory as well as in geophysics and astrophysics.

The physical model.— A piecewise constant approximation for the equilibrium densities and for the longitudinal velocities of the water, foam and air  $\rho_j$  and  $U_j$ , (j = a, f, w) is employed:

$$\rho = \rho_w, U = U_w \equiv 0 \quad \text{for } y < 0,$$
  

$$\rho = \rho_f, U = U_f \quad \text{for } 0 < y < L_f,$$
  

$$\rho = \rho_a, U = U_a \quad \text{for } y > 0.$$
(1)

Here  $U_a$  is the known constant velocity of the wind, while the constant foam layer thickness  $L_f$  and velocity  $U_f$ are the widely unknown parameters of the foam layer in hurricane conditions. In addition, it is assumed that the equilibrium state is in hydrostatic equilibrium, namely,  $\partial P_i/\partial y = -g\rho_i$  (g is the gravity acceleration).

The equations of motion that govern the dynamics of the system in each of the three layers, and the appropriate boundary conditions are applied at the foam layer interfaces. The equilibrium state is perturbed as follows:

$$\Phi(x, y, z) = F(y) + F'(x, y, t),$$
(2)

where  $\Phi$  stands for any of the physical variables, and F

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and F' denote the equilibrium and perturbed values, respectively. The latter are assumed to be of the form  $F' = f'(y)exp(-i\omega t + ikx)$  with real k and complex  $\omega = \omega_r + i\omega_i$ . Thus, the amplitudes f' that satisfy the boundary conditions at  $y = \pm \infty$  are given by:

$$f'_{a} = \tilde{f}_{a} \exp(-ky), \ f'_{w} = \tilde{f}_{w} \exp(ky), 
 f'_{f} = \tilde{f}_{-f} \exp(-ky) + \tilde{f}_{+f} \exp(ky), 
 (3)$$

where tilde denotes constant magnitudes.

Finally, capillary and viscosity effects are neglected for both the equilibrium and perturbed states (see the section *Results and discussion*). Substitution of Eqs. (1)-(3) into the linearized Euler equations and applying the continuity conditions of normal velocity and pressure at the foam interfaces, yields the quartic dispersion relation for phase velocity C [8]:

$$2(H_a + H_w) + (E - 1)(H_a + 1)(H_w + 1) = 0, \qquad (4)$$

where

$$C = \frac{\omega}{k}, \quad H_w = \frac{\rho_w (U_w - C)^2 - (\rho_w - \rho_f)g/k}{\rho_f (U_f - C)^2},$$
$$E = exp(2kL_f), H_a = \frac{\rho_a (U_a - C)^2 - (\rho_f - \rho_a)g/k}{\rho_f (U_f - C)^2}.$$
 (5)

Before turning to the study of the foam layer effect, it is noticed that in the limit  $L_f = 0$ , or, equivalently, either  $\rho_f = \rho_w$  or  $\rho_f = \rho_a$ , Eq. (4) is reduced to the classic dispersion relation  $H_a + H_w = 0$  for KHI [5]:

$$\rho_w (k_0 U_w - \omega_0)^2 + \rho_a (k_0 U_a - \omega_0)^2 = k_0 g (\rho_w - \rho_a), \quad (6)$$

where the subscript 0 denotes the foam-free parameters. Asymptotic analysis.— First, the limit of low air-water density ratio,  $\rho_a/\rho_w = \epsilon^2 \ll 1$ ,  $(\epsilon^2 \approx 10^{-3})$  is applied to the classic two-layer case described by Eq. (6) with  $U_w = 0$ , in order to obtain an estimate for the various physical parameters:

$$\omega_0 = \sqrt{gk_0 - \epsilon^2 k_0^2 U_a^2} + O(\epsilon^2 k_0 U_a, \epsilon g k_0 / \omega_0).$$
(7)

Doing so, it can be concluded that the classic two-fluid KHI is excited in the short wavelength regime:

$$k_0 L_* \sim k_0^* L_* = 1/\epsilon^2, \ \omega_0 L_*/U_* \sim 1/\epsilon, \ C_0/U_* \sim \epsilon, \ (8)$$

where  $U_* = U_a$ ,  $L_* = U_a^2/g$ , while the superscript asterisk denotes the marginal values of the parameters.

Back to the general case of three-fluid systems, it is assumed that the water content in the foam,  $\alpha_w$ , is small (low water content is a characteristic feature of air-water foams). As a result,  $\alpha_w \sim 0.05$  is scaled with  $\epsilon$  and yields

$$\frac{\rho_f}{\rho_*} \approx \alpha_w \sim \epsilon, \quad \frac{\rho_a}{\rho_f} \approx \frac{1}{\alpha_w} \frac{\rho_a}{\rho_*} \equiv \frac{\epsilon^2}{\alpha_w} \sim \epsilon.$$
 (9)

Here  $\rho_* = \rho_w$ ,  $\rho_f = \alpha_a \rho_a + \alpha_w \rho_w$ ,  $\alpha_a = 1 - \alpha_w$ . Assuming now that the three-fluid system operates in the same regime that gives rise to the KHI in the classic air-water system, the following scales are adopted:

$$kL_* \sim \frac{1}{\epsilon^2}, \ \frac{\omega L_*}{U_*} \sim \frac{1}{\epsilon}, \ \frac{C}{U_*} \sim \epsilon.$$
 (10)

Further assuming that the foam layer thickness is much less than the characteristic length,  $L_f/L_* \ll 1$ ,  $(L_* \sim 250m$  for  $U_a \sim 50m/s)$ , while the foam velocity is much less then the wind velocity and much larger the phase velocity  $\epsilon \sim C/U_* \ll U_f/U_* \ll 1$ :

$$U_f/U_* \sim \epsilon^a, \ L_f/L_* \sim \epsilon^b, \ 0 < a < 1, \ 0 < b,$$
 (11)

which yields the following estimates for Eq. (4):

$$H_a \sim H_w \sim \epsilon^{1-2a}, \ E \sim exp(\epsilon^{b-2}).$$
 (12)

Inserting the scaling (12) into Eq. (4), and applying the principle of the least degeneracy [9] of the three-fluid problem, results in a = 1/2, b = 2, which means:

$$\frac{U_f}{U_*} \sim \epsilon^{1/2}, \ \frac{L_f}{L_*} \sim \frac{\lambda_0^*}{L_*} \sim \frac{1}{k_0^* L_*} = \frac{\rho_a}{\rho_*} \sim \epsilon^2,$$
 (13)

where  $\lambda_0^* = 2\pi/k_0^*$ . Following relations (13), the wave number and frequency are rescaled as follows:

$$\hat{k} = k/k_0^* \sim \epsilon^0, \quad \hat{\omega} = \omega/\sqrt{gk_0^*} \sim \epsilon^0.$$
 (14)

This yields the dispersion relation to leading order in  $\epsilon$ :

$$\hat{\omega} = \sqrt{\frac{2(\hat{k} - \hat{k}^2) - (E - 1)(\hat{k}^2 K_f - \hat{k})(K_f^{-1} + 1)}{2 + (E - 1)(K_f^{-1} + 1)}}, \quad (15)$$

where  $E = exp(2\hat{k}\hat{L}_f)$ , while the rescaled foam thickness  $\hat{L}_f$ , and the equilibrium ratio of the foam-to-air dynamic pressure  $K_f$  ( $0 < K_f < 1$ ) are given by:

$$\hat{L}_f = k_0^* L_f \sim \epsilon^0, \quad K_f = \frac{\rho_f U_f^2}{\rho_a U_a^2} \sim \epsilon^0.$$
(16)

Thus, the system stability is parameterized by the dimensionless foam velocity and thickness or, equivalently,  $K_f$ and  $k_0^* L_f$ , which has a meaning of a bulk foam Richardson number  $Ri_f$  scaled by  $\rho_a/\rho_f = \epsilon^2/\alpha_w \sim \epsilon$ :

$$\hat{R}i_f = k_0^* L_f, \ Ri_f = -g \frac{\Delta \rho}{\rho_f} \frac{L_f}{\Delta U^2} \approx \frac{\rho_a}{\rho_f} \hat{R}i_f$$

where  $\Delta U = U_a - U_w \equiv U_*$  and  $\Delta \rho = \rho_a - \rho_w \approx -\rho_*$ .

Two particular limits of Eq. (15) are readily obtained, namely, the foam-free limit  $(H_w + H_a = 0 \text{ for } L_f = 0)$ :

$$\frac{\omega_0}{\sqrt{gk_0^*}} = i\sqrt{\frac{k^2}{k_0^{*2}} - \frac{k}{k_0^*}}, \ \hat{R}i_f = k_0^* L_f = 0.$$
(17)

and the foam-saturated limit  $(H_w + 1 = 0 \text{ for } L_f = \infty)$ :

$$\frac{\omega_{\infty}}{\sqrt{gk_{\infty}^*}} = i\sqrt{\frac{k^2}{k_{\infty}^{*2}} - \frac{k}{k_{\infty}^*}}, \ \hat{R}i_f = k_0^* L_f = \infty,$$
(18)

which differs from Eq. (17) by replacing  $k_0^*$ ,  $\omega_0$  with  $k_{\infty}^* = k_0^*/K_f, \ \omega_{\infty} \ (0 < K_f < 1).$  Comparison of these two limits demonstrates the stabilizing effect of the foam due to the decrease of the marginal wavelength from the foam free  $\lambda_0^* = 2\pi/k_0^*$  to the foam-saturated  $\lambda_{\infty}^* = 2\pi/k_{\infty}^*$  value. The growth rate  $\omega_i$  decreases from the foam-free  $\omega_{i0}$  to the foam-saturated  $\omega_{i\infty}$  value. The definition for  $k_{\infty}^* = k_0^*/K_f$  is used in order to express  $K_f$  through  $\lambda_{\infty}^*$ :  $K_f = k_0^*/k_{\infty}^* \equiv \lambda_{\infty}^*/(2\pi\epsilon^2 L_*)$ . The intermediate wavelength value  $\lambda \approx \lambda_{\infty}^* \approx 1m$  is chosen for further estimations from the wavelength range of the drag responsible waves  $\sim 0.1 - 10m$ . In turn, a typical height  $h \approx 0.1m$  is expressed from Stokes heuristic rule for the critical steepness of breaking waves [10]. Consequently, the value  $K_f \approx 0.5$  is adopted that results in  $U_f = \epsilon U_a \sqrt{K_f / \alpha_w} \approx 5m/s$  at  $\alpha_w \approx 0.05, U_a \approx 50m/s.$ Figure 1 depicts the growth rate as a function of the



FIG. 1: Growth rate  $\hat{\omega}_i = \omega_i / \sqrt{gk_0^*}$  vs wave number,  $\hat{k} = k/k_0^*$ , for the typical foam-layer thicknesses,  $\hat{R}i_f \equiv \hat{L}_f = k_0^* L_f$  and the ratio of the foam/air dynamic pressure  $K_f = 0.5$ .

wavenumber. As can be seen, the growth rate decreases as the foam layer thickness is increased and approaches its saturated limit already at  $k_0^* L_f \approx 1$ . The depen-



FIG. 2: Growth rate  $\hat{\omega}_i = \omega_i / \sqrt{gk_0^*}$  vs foam-layer thickness,  $\hat{R}i_f \equiv \hat{L}_f = k_0^* L_f$ , for the typical wave number  $\hat{k} = k/k_0^*$  and the ratio of the foam/air dynamic pressure  $K_f = 0.5$ .

dence of the growth rate  $\omega_i/\sqrt{gk_0^*}$  on the foam-layer

thicknesses is depicted in Fig. 2. For sufficiently short waves  $(k/k_0^* > 1/K_f)$  the growth rate strongly drops from the foam free value at  $k_0^*L_f=0$  to its saturation level at foam-layer thickness  $k_0^*L_f \approx 1$ . The growth rates of perturbations with longer waves  $(k/k_0^* < 1/K_f)$  sharply decrease with the increase  $k_0^*L_f$ , till total stabilization at a finite value of  $k_0^*L_f$  is achieved. These two cases are separated by the threshold curve  $(k/k_0^* = 1/K_f)$  for which the growth rate vanishes at  $k_0^*L_f >> 1$ .

The marginal wave number  $k^*$  satisfies the eigenvalue equation for the three-layer system:

$$exp(2k^*L_f) = 1 - \frac{2}{1 + K_f^{-1}} \frac{1 - k^*/k_0^*}{1 - k^*/k_\infty^*}.$$
 (19)

As in the classic two-fluid system, to leading order in  $\epsilon$ , the waves propagate with phase velocity  $C = \omega/k$  without amplification for  $k/k^* < 1$ , and amplify with zero phase velocity for  $k/k^* > 1$ . The value  $k^*$  monotonically increases with  $k_0^* L_f$  from the foam-free value  $k^* = k_0^*$  to the foam-saturated value  $k^* = k_{\infty}^* \equiv k_0^*/K_f$ .

*Results and discussion.*— The atmosphere-ocean interaction in hurricane conditions creates a foam layer between the atmosphere and the ocean. This provides for an effective mechanism of the sea surface stabilization.

The analysis of the KHI is treated asymptotically in two small parameters: air-water density ratio  $\sim \epsilon^2$  and water content in the foam  $\sim \epsilon$ . The system stability is parameterized by the dimensionless foam velocity  $U_f$ and thickness  $L_f$  (or, equivalently, the dynamic pressure ratio  $K_f$  and Richardson number  $\hat{R}i_f$ ). Due to lack of observations or modelling data in hurricane environment, they are first estimated as  $L_f/L_* = \epsilon^2$  and  $U_f/U_* \sim \sqrt{\epsilon}$ by applying the asymptotic principle of least degeneracy of the problem. Then  $L_f^{(ef)} \approx 0.25m$  at  $U_* \approx 50m/s$  is evaluated by the condition that the growth rate approaches its minimal saturated value at  $L_f^{(ef)} = \epsilon^2 L_*$ , and further increase  $L_f$  is ineffective, as if the foam layer is of infinite thickness. The value  $L_f^{(ef)}$  is of the order of the experimentally observed values ([7] and references therein). The single fitting parameter of the model  $K_f = \lambda_{\infty}^* / \lambda_0^* \approx 0.5 \ (U_f \approx 5m/s)$  had been estimated through an intermediate value of length of drag responsible waves ( $\lambda \approx 1m$ ). The value of the wavelength ratio exhibits the instability shift towards smaller wavelength scales. Thus, the foam layer reduces the foam-free wavelength  $\lambda_0^*$  approximately by a factor 2 to the foam saturated limit  $\lambda_{\infty}^*$  already at  $L_f \approx L_f^{(ef)}$ . This scale-down in the characteristic unstable length scales provides a qualitative link between the linear stability modeling and the role of the foam layer in the air-sea momentum exchange. To see that, the local correlation, based on the dimensional grounds,  $z/\lambda = F(h/\lambda)$  between the ocean surface roughness  $z/\lambda$  and the wave steepness  $h/\lambda$ , is examined in a vicinity of intermediate values of height and length of drag responsible breaking waves. It is similar to the correlation [11] for pre-hurricane conditions, but with the local values  $h, \lambda$  instead of significant wave height and peak wavelength. Noting that the breaking process does not completely destroy the waves, but rather tears off their tops, when their steepness exceeds a critical value ( determined by nonlinear effects), the complete covering of the ocean surface by the foam layer occurs when the critical steepness 1/10 [10] is achieved for drag responsible breaking waves. As a result, the roughness is reduced along with the wave length by a factor  $\lambda_0^*/\lambda_\infty^* = K_f^{-1} \approx 2$ (for  $\lambda_{\infty}^* \approx \lambda \approx 1m, h \approx 0.1m, K_f \approx 0.5$ ) due to the foam effect. Remarkably, such simple scalings are supported by the observations of the roughness and drag reduction presented in [1]. The results are physically transparent, since in the foam saturated system the foam layer totally separates the air flow from the sea surface, and the threefluid system becomes close to a two-fluid foam-water system. Formally this corresponds to substituting the foam density and velocity instead of those parameters for the air in the classic two-fluid model.

Finally, the main assumptions adopted in the present study are discussed. First, it is noted that the assumption of foam thickness uniformity, breaks down due to the foam accumulation in troughs of high and long ocean waves (seen in a photograph of the sea surface before its complete coverage by a foam [1]). The drag of the hurricane induced surface waves comes mainly from intermediately short waves (of  $\sim 0.1 - 10m$  [12]). Thus, although the breaking waves under consideration (of  $\sim 1m$ ) are strongly modulated by long waves (of  $\sim 10^2 m$ ), such a slow variation of the foam thickness may be taken into account in the next approximation. It is assumed that the air-sea exchange of the highly solvable carbon dioxide or oxygen leaves the equilibrium water content to be small in compliance with the determining feature of gas-liquid foams. Zero compressibility and viscosity approximation is commonly accepted in KHI of air-water systems [5]-[6]. The foam compressibility may be ignored within the same accuracy as the air one. Indeed, using the smallness of the air Mach number  $M_a = U_a/C_a$  and noting that the foam-to-air sound velocity ratio  $C_f/C_a \sim \sqrt{\rho_a/\rho_f} \sim \sqrt{\epsilon}$ [13] is of the same order as  $U_f/U_a \sim \sqrt{\epsilon}$  (see Eqs. (13)), it is obtained that  $M_f = U_f/C_f \sim M_a \ll 1$ . Although the foam viscosity data in hurricane environment is unavailable, artificial foam viscosities are known to be significantly larger than the viscosity of its liquid and gas constituents. On the other hand, natural sea foams are expected to have lower viscosity than their artificial counterpart due to lack of man-made surfactants and a larger effective size of the foam bubbles (of  $\sim 0.2 - 2mm$  [4]). In any case, the stability behavior regarding the growth rate of the shear viscosity (see e.g. [14]) ignored in the present study can only enhance the foam stabilizing effects in the range of the intermediately short waves. Ignoring the capillary effects is valid at the water-foam interface since the foam is composed of the same sea-water. At the airfoam interface the value of the surface-tension coefficient may be naturally assumed equal to the surface tension between air and sea-water. Since the latter is much less than the surface tension between air and fresh-water due to the effect of the surfactants, its influence on KHI is rather small for the short waves (of  $\sim 1m$ ). The bubbly liquid, spray and foam coexist in hurricane environment. The high contrast in three-fluid densities is the principal feature of the foam-layer system  $\rho_a \ll \rho_f \ll \rho_w$ , distinguishing it from the layers of bubbly liquid or spray, with  $\rho_b \approx \rho_w$  or  $\rho_s \sim \rho_a$ , when the system will be close to the two-fluid air-water configuration. Indeed, the bubbly liquid superposed the water hard to be distinguished due to a small contrast in densities ( $\rho_b \approx \rho_w$ , since  $\alpha_w \approx 1$ ), and the bubbly liquid layer may be dropped from the KHI study. The spray between the air and the foam is well distinguished from them by a contrast in densities. Indeed, typically  $\rho_f \approx \alpha_w \rho_w$ , and  $\rho_s = \alpha_w \rho_w + \alpha_a \rho_a \approx 2\rho_a$ at  $\alpha_w \approx 0.001$  for spray and  $\alpha_w \approx 0.05$  for foam, and  $\rho_s \approx 2\rho_a \ll \rho_f$ . Thus, the spray may be described by a more detailed velocity and density air profiles within the present air-foam-water configuration. This however will increase the system uncertainty, since the spray layer thickness and velocity are rather unknown.

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