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Discussion

Discussion of “Wave setup and setdown generated by obliquely incident waves” by T.-W. Hsu et al., Coastal Engineering, 53, 865–877, 2006

Fengyan Shi *, James T. Kirby

Center for Applied Coastal Research, University of Delaware, Newark, DE 19716, USA

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ABSTRACT

In the recent paper by Tai-Wen Hsu, John R.-C. Hsu, Wen-Kai Weng, Swun-Kwang Wang, and Shan-Hwei Ou (Coastal Engineering, 53, 865–877, 2006), the authors derived theoretical formulations for calculating the wave setup and setdown induced by obliquely incident waves on a beach. The derivation of an expression for setdown contains errors which would lead to an imbalance in longshore momentum flux outside the surfzone. We correct their derivation and give results in terms of the radiation stress concept in a general case including an oblique wave incidence. We also point out that the correct form of wave setdown is important to describe the zero-net force in the momentum balance outside the surfzone.

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1. Setdown in shoaling zone

The wave setdown in the shoaling zone, prior to the onset of wave breaking dissipation, can be derived using either the concept of radiation stress (Longuet-Higgins and Stewart, 1962, 1964) or the wave-averaged Bernoulli equation applied to the free surface (Mei, 1983; McDougal and Hudspeth, 1983). The appropriate form of the Bernoulli equation (following the notation of Hsu et al., 2006) is given by

$$-\frac{1}{g}\frac{\partial\phi}{\partial t}+\frac{1}{2g}\left(u^2+v^2+w^2\right)+\frac{p}{\rho g}+\eta=C_b,\quad z=\eta \tag{1}$$

where u and v are velocity components in the cross-shore x and longshore y directions and w is vertical velocity. After expansion about the still water level, use of the linear kinematic boundary condition, and averaging over a wave period, a general expression for setdown $\overline{\eta}$ is given by

$$\overline{\eta}=-\frac{1}{2g}\left(\overline{u^2}+\overline{v^2}-\overline{w^2}\right) \tag{2}$$

Hsu et al. (2006) give the velocity potential of an oblique propagating wave as

$$\phi=\frac{ag}{\omega}\frac{\cosh k(h+z)}{\cosh kh}\sin(kx\cos\theta+ky\sin\theta-\omega t) \tag{3}$$

where a represents local amplitude and θ represents local wave angle, with neither being connected to conditions at infinity by shoaling or refraction. Substituting Eq. (3) in Eq. (2) then gives the usual expression for wave setdown,

$$\overline{\eta}=-\frac{a^2k}{2\sinh 2kh} \tag{4}$$

This result makes no reference to the orientation of the wave relative to the coastline, which is expected since the balance being explored is local and the approximate expression for the potential is appropriate for a plane progressive wave in a very slowly varying environment. Mei (1983), for example, presents the generic case without assuming normal wave incidence. A similar approach, applying the Bernoulli equation at sea floor, is shown by Jonsson (1998) and gives the same result.

In contrast, Hsu et al. (2006) neglect the longshore velocity component v in their expression for the Bernoulli equation, which is totally unjustified since the equation is scalar and makes no reference to any preferred direction. As a result, they obtain the directionally dependent expression

$$\overline{\eta}=-\frac{a^2k}{2\sinh 2kh}\cos^2\theta \tag{5}$$

which is incorrect. The remaining results in the paper are in error by amounts proportional to the value of cosθ, which will be close to unity in the nearshore region.

Refraction and shoaling may be included in Eq. (4) by referencing a and θ to incident values a_0 and θ_0 in deepwater, using the energy flux conservation, giving

$$\overline{\eta}=-\frac{1}{2}a_0^2k_0\frac{\coth^2 kh}{2kh+\sinh 2kh}\left(\frac{\cos\theta_0}{\cos\theta}\right) \tag{6}$$

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* Corresponding author.
E-mail address: fyshi@coastal.udel.edu (F. Shi).

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Noting that $\cos\theta_0/\cos\theta < 1$ and decreases with wave incident angle θ_0, wave setdown should decrease with an increase in wave angle θ_0 for a given deep water height a_0.

The results obtained here may be verified for obliquely incident waves using the momentum balance between the gradients of radiation stresses and pressure. Longuet-Higgins and Stewart (1964) and Phillips (1977) described the wave setdown in terms of the radiation stress concept for normally incident waves. For the case of oblique wave incidence, the S_{xx} component of the radiation stress tensor can be written as (Longuet-Higgins and Stewart,1962, 1964)

$$S_{xx} = E\frac{C_g}{C}\cos^2\theta + E\left(\frac{C_g}{C} - \frac{1}{2}\right) \tag{7}$$

where E represents wave energy; C_g and C are wave group velocity and phase velocity, respectively. The radiation stress gradient in x direction is given by

$$\frac{\partial S_{xx}}{\partial x} = \frac{EC_g\cos\theta}{\sigma}\frac{\partial k\cos\theta}{\partial x} + \frac{\partial}{\partial x}\left[E\left(\frac{C_g}{C} - \frac{1}{2}\right)\right] \tag{8}$$

where energy flux is conserved according to

$$\frac{\partial}{\partial x}\left(EC_g\cos\theta\right) = 0 \tag{9}$$

The conservation of wave crests (Phillips, 1977, neglecting the mean current) is given by

$$C_{gx}\frac{\partial k_x}{\partial x} = -\frac{\partial\sigma}{\partial H}\frac{\partial H}{\partial x} \tag{10}$$

where H represents the total water depth, i.e., $H=h+\bar{\eta}$, and C_{gx} and k_x represent x components of group velocity and wave number vector, respectively. Noting that

$$\frac{\partial\sigma}{\partial H} = \frac{\sigma}{H}\left(\frac{C_g}{C} - \frac{1}{2}\right) \tag{11}$$

Eq. (10) becomes

$$\frac{\partial k\cos\theta}{\partial x} = -\frac{\sigma}{C_g H\cos\theta}\left(\frac{C_g}{C} - \frac{1}{2}\right)\frac{\partial H}{\partial x} \tag{12}$$

substituting Eq. (12) in Eq. (8) yields

$$\frac{\partial S_{xx}}{\partial x} = -E\left(\frac{C_g}{C} - \frac{1}{2}\right)\frac{1}{H}\frac{\partial H}{\partial x} + \frac{\partial}{\partial x}\left[E\left(\frac{C_g}{C} - \frac{1}{2}\right)\right] = H\frac{\partial}{\partial x}\left[\frac{E}{H}\left(\frac{C_g}{C} - \frac{1}{2}\right)\right] \tag{13}$$

The x direction momentum balance reads

$$\frac{\partial\bar{\eta}}{\partial x} = -\frac{1}{\rho g H}\frac{\partial S_{xx}}{\partial x} \tag{14}$$

Substituting Eq. (13) into Eq. (14) and integrating with respect to x, we get

$$\bar{\eta} = -\frac{E}{\rho g H}\left(\frac{C_g}{C} - \frac{1}{2}\right) \tag{15}$$

where the integration constant is 0 corresponding to $\bar{\eta}=0$ in deep water. Noticing $E = \frac{1}{2}\rho g a^2$ and assuming $\bar{\eta} << h$ in the shoaling zone, the setdown can be written as

$$\bar{\eta} = -\frac{ka^2}{2\sinh 2kh} \tag{16}$$

which is as same as Eq. (4).

2. Consequences for a spatially inhomogeneous case

Although wave setdown is small compared to the total water depth in the shoaling zone, it plays a main role in momentum balance outside the surfzone. Previous studies on wave-induced nearshore circulations indicated that the pressure field associated with the wave-induced setdown should be balanced by the gradient of the radiation stress in the shoaling zone. There are no net forces outside the surfzone that might produce circulation patterns. Bowen (1969) presented a case in which waves approach normal to a plane beach and the wave height varies along the beach. In the wave shoaling zone, the longshore pressure gradient associated with the longshore varying wave setdown caused by the longshore non-uniformity of waves can be balanced by the pressure gradient force.

Corresponding to this discussion we extend Bowen's (1969) case to oblique wave incidence and also assume a longshore non-uniform wave amplitude $a(y)$. The depth-integrated and wave-averaged momentum equation in the y direction can be written as

$$\frac{\partial\bar{v}(h+\bar{\eta})}{\partial t} + \frac{\partial\bar{u}\bar{v}(h+\bar{\eta})}{\partial x} + \frac{\partial\bar{v}\bar{v}(h+\bar{\eta})}{\partial y} = -g(h+\bar{\eta})\frac{\partial\bar{\eta}}{\partial y} + F_y^w + \overline{\tau_y^b} \tag{17}$$

where $\bar{u}$ and $\bar{v}$ are wave-averaged current velocities in x and y directions, respectively; $\overline{\tau_y^b}$ is the bottom friction in the y direction; F_y^w is the y component of wave forcing that can be expressed by

$$F_y^w = -\frac{\partial S_{yy}}{\partial y} - \frac{\partial S_{xy}}{\partial x}. \tag{18}$$

Using

$$S_{xy} = E\frac{C_g}{C}\sin\theta\cos\theta \tag{19}$$

and

$$S_{yy} = E\frac{C_g}{C}\sin^2\theta + E\left(\frac{C_g}{C} - \frac{1}{2}\right) \tag{20}$$

the wave forcing F_y^w may be written as

$$F_y^w = -\left(\frac{C_g}{C} - \frac{1}{2}\right)\frac{\partial E}{\partial y} - \frac{\sin\theta}{C}\left(\frac{\partial EC_g\cos\theta}{\partial x} - \frac{\partial EC_g\sin\theta}{\partial y}\right) - EC_g\cos\theta\frac{\partial}{\partial x}\left(\frac{\sin\theta}{C}\right) \tag{21}$$

Assuming that there is no energy input and dissipation, i.e.,

$$\frac{\partial EC_g\cos\theta}{\partial x} + \frac{\partial EC_g\sin\theta}{\partial y} = 0 \tag{22}$$

and using Snell's law (or wave crest conservation)

$$\frac{\partial}{\partial x}\left(\frac{\sin\theta}{C}\right) = 0 \tag{23}$$

Eq. (21) can be written as

$$F_y^w = -\left(\frac{C_g}{C} - \frac{1}{2}\right)\frac{\partial E}{\partial y} = -\rho g\frac{kh}{\sinh 2kh}a(y)\frac{\partial a(y)}{\partial y} \tag{24}$$

Using Eq. (4) the pressure gradient term in Eq. (17) is

$$-\rho g(h+\bar{\eta})\frac{\partial\bar{\eta}}{\partial y} = \rho g\frac{kh}{\sin 2kh}a(y)\frac{\partial a(y)}{\partial y} \tag{25}$$

with an assumption $\bar{\eta} << h$ outside the surfzone. Eqs. (25) and (24) show that the longshore gradient of pressure is balanced by the alongshore wave forcing. It can also be proved that, in the more general case with a longshore non-uniform distribution of wave angle, the longshore pressure gradient can be balanced by the longshore gradient of radiation stresses.

It is obvious that, from Hsu et al.'s (2006) expression of wave setdown, the pressure gradient $\partial\bar{\eta}/\partial y$ with $\cos^2\theta$ in the expression may not be balanced by the gradient of radiation stresses, which would lead to the presence of wave-driven mean flows outside the surfzone.

Another consequence from the incorrect setdown expression is that the wave setup would be over-estimated for a case of oblique wave incidence because the integration constant in the wave setup calculation (C_1 in (12), Hsu et al., 2006) is determined by the wave setdown at the breaker line. The final expression (24) in Hsu et al. (2006) for wave setup and setdown may be corrected as

$$\frac{\eta}{h_b} = \begin{cases} \frac{\kappa \sin^2\theta_b}{2(8+3\kappa^2-2\kappa^2\sin^2\theta_b)}\left[\left(\frac{h}{h_b}\right)^2-1\right]-\frac{3\kappa^2-2\kappa^2\sin^2\theta_b}{8+3\kappa^2-2\kappa^2\sin^2\theta_b}\left(\frac{h}{h_b}-1\right)-\frac{\kappa^2}{16} & x \le x_b \\ -\frac{\kappa^2}{16}\left(\frac{h_b}{h}\right)^{3/2} & x > x_b \end{cases} \tag{26}$$

3. Conclusion

The theoretical expression of shoaling wave-induced setdown derived by Hsu et al. (2006) is corrected. Although the wave setdown could be omitted in engineering applications its derivation may reflect the physical mechanism of the wave setdown generation. The correct expression of wave setdown is important to describe the momentum balance outside surfzone. We extend Bowen's (1969) analysis of momentum balance outside surfzone to a case of oblique wave incidence. Our analysis with the corrected form of wave setdown confirms that there are no net forces outside the surfzone to drive nearshore circulations.

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