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INSTRUMENTS AND METHODS

HF radio measurements of surface currents

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Abstract—HF radio waves backscattered from the ocean surface can be used to measure ocean surface currents. Measurements of the range-Doppler spectrum of these signals yield the wavenumber k and the frequency ω of an ocean surface wave and its phase velocity $v = \omega/k$ relative to the radar. Subtraction of the phase velocity of the wave in still water, $c = \sqrt{(g/k)}$, yields a measure of the average current from the surface to a depth of the order of $(2k)^{-1}$. A measure of the current shear is obtained by observing at more than one radio frequency. To test these ideas, surface currents were measured using both a conventional and the HF technique, and reasonable agreement was found.

INTRODUCTION

OVER THE past 5–10 years there has been a rapid growth in both the theoretical and experimental study of radar scatter from the sea, demonstrating the possibility of remotely sensing the state of the sea surface. We consider here the application of back-scattered HF radio waves to the measurement of ocean surface currents.

RADIO THEORY

Radio scatter has been observed for both vertically and horizontally polarized radio waves, from grazing to vertical incidence, over a frequency band ranging from 2 MHz to 70 GHz. Over most of this range the height of the sea-surface roughness elements or waves is large compared to the radio wavelength, and the theory for the scatter is complex, but at the lower frequencies, in particular 2–30 MHz, commonly called HF or dekameter radio waves, the scatter is relatively simple. Here the radio wavelength is much greater than the height of the surface waves and a perturbation expansion of the scattered field in terms of the Fourier components of the surface wave field accurately predicts the observed scatter (BARRICK, 1972). Accordingly, vertically polarized HF radio waves propagating at grazing incidence (ground-wave propagation) are scattered by a resonant interaction, with a particular Fourier component of the surface, whereby an incident radio wave of wavenumber $\vec{k}_i$ and frequency ω_i is scattered into two waves of wavenumber $\vec{k}_s$ and frequency ω_s such that the resonant or Bragg conditions are satisfied (HASSELMANN, 1966, equation 1.20), namely

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$$\vec{k}_s = \vec{k}_i \pm \vec{k}, \quad \omega_D = \omega_s - \omega_i = \pm\,\omega, \tag{1}$$

where ω_D is the Doppler shift of the scattered radio wave and $\vec{k}$ and ω are the wave-number and frequency of the surface wave. For the particular case of backscatter, $\vec{k}_s = -\vec{k}_i$ so that $2\vec{k}_s = \pm\,\vec{k}$. In the following discussion we will denote that Fourier component of the sea surface that scatters the radio wave as the ocean wave.

Consider now a simple experiment: we transmit a narrow beam of pulsed, coherent radio waves in a particular direction φ. This beam is backscattered from the sea surface and is received at the transmitter, i.e. we form a pulsed-Doppler radar. The narrow beam ensures that we know the direction of $\vec{k}_i$ and of $\vec{k}$ precisely, and the radio wave-length gives the magnitude of $\vec{k}$. The travel time of the pulses gives the distance (range) to the scattering area, and the coherence allows us to calculate the Doppler spectrum of the received signal at each range. Theoretically, this spectrum should have two lines at $\pm\,\omega$, and herein lies the essence of the method for measuring currents. If there are no currents the lines are observed at their theoretical positions; but if there is a current, they are displaced to a new position ω_p, and the amount of this displacement is a measure of the current.

We calculate the surface current by subtracting the observed ocean wave velocity c_p from the velocity c_1 it should have if it were travelling through still water. In the next section we show that this is a weighted average (over depth) of the surface current. Now by definition $c_p = \omega_p/k$, $c_1 = \omega/k$. Let their difference be

$$v = k^{-1}(\omega_p - \omega) = k^{-1}\,\Delta\omega.$$

We calculate ω from the dispersion relation for water waves (LAMB, 1932, p. 365), $\omega = (gk)^{1/2}$, where k is known from the Bragg conditions. This is subtracted from the wave frequency observed in the Doppler spectrum of the scattered radio waves to calculate v.

The difference frequency $\Delta\omega$ is small and we must consider the limits of the theory. They are of two types, electromagnetic and hydrodynamic. The former is the weaker. The perturbation expansion of the scattered field requires the surface roughness height be small compared to the radio wavelength. This is generally true below 30 MHz (wavelength of 10 m). Calculations to second-order in surface roughness show the Bragg line in the Doppler spectrum has sidebands (BARRICK, 1971). There is no shift in the position of the line.

The hydrodynamic effects are more important. The radio wave scatters from that Fourier component of the sea that is coherent over one-half of the radio pulse length. Typically the pulse is 15 km long and contains hundreds of wavelengths, so the Fourier component of the sea surface behaves as a linear wave. To first order in wave amplitude it is independent of all other components of the sea spectrum. At second order, pairs of components interact with each other to produce a forced wave having sum and difference frequencies. This wave does not obey the dispersion relation, but can satisfy (1) to produce scatter. As before, this results in sidebands to the Bragg line (BARRICK, 1971; HASSELMANN, 1971; STEWART, 1971). The interaction also produces a current (commonly called a Stokes current), and the ocean wave interacts with the current produced by all pairs of waves in the spectrum just as though it were a wind-driven current. That is, the ocean wave cannot distinguish between currents produced by wave–wave interactions and wind-driven currents. To clarify this, we will show in the next section that the change in velocity of a finite-amplitude (Stokes) wave is the same as the effect of its Stokes current on itself.

OCEAN WAVE THEORY

To relate the Doppler measurements to a surface current, we calculate the phase velocity of a wave on a mean current $\hat{U}$. We assume the current is small compared to the wave speed (typical values for $\hat{U}$ are 30 cm s^{-1}, while c_p is always greater than 300 cm s^{-1} for our measurements). In formulating the problem, Squire's theorem (see LIN, 1966, pp. 28, 77; PHILLIPS, 1966, p. 92) allows us to consider only that component of $\hat{U}$ in the direction of wave propagation. The perturbation velocity field is thus reduced to two dimensions, and the problem is reduced to finding the solution of the inviscid Orr–Sommerfeld equation (Rayleigh equation) of stability theory.

Consider the velocity field (u, w) in a Cartesian coordinate system (x, z) where z is vertically upward. Assume that the wave travels in the x direction, and that the component of the mean velocity in this direction is $\hat{U}(z)$. The motion of a wave on a shear flow is a problem in stability theory, and a formulation of the theory pertinent to our problem can be found in YIH (1972), using z and w in place of his y and v. We use his equations without derivation. The stream function of the wave is $\psi = f(z) \exp[ik(x - ct)]$, and the linear, inviscid equation of motion is

$$f'' + [\hat{U}''/(c - \hat{U}) - k^2]f = 0. \tag{2}$$

At great depths, $z \to -\infty$ and $f \to 0$. The combination of the free surface dynamic and kinematic boundary conditions yields the boundary condition on f,

$$f'_0 = [g/(\hat{U} - c)^2 + \hat{U}'/(\hat{U} - c)]f'_0, \tag{3}$$

where the subscript 0 means evaluation at the mean surface $z = 0$.

We seek an approximate solution of (2) assuming the surface current is small compared to the wave phase velocity, i.e. $\hat{U}(z) = \varepsilon c_1 U(z)$ where $\varepsilon \ll 1$, c_1 is the dimensional wave velocity in the absence of $\hat{U}$, and $U(z)$ is the 0(1) nondimensional mean current velocity in the direction of the wave propagation. We expand the solution of (2) in powers of ε, $f = f_1 + f_2 + \ldots$, $c = c_1 + c_2 + \ldots$, then

$$f''_1 - k^2 f_1 = 0, \tag{3a}$$

$$f''_2 - k^2 f_2 = \varepsilon U'' f_1, \tag{3b}$$

and the boundary conditions at $z = 0$ are

$$f'_1 - g/c_1^2 f_1 = 0, \tag{4a}$$

$$f'_2 - g/c_1^2 f_2 = [2g\varepsilon/c_1^2(U - c_2/c_1) - U']f_1. \tag{4b}$$

The first-order solution is $f_1 = a \exp(kz)$ with $c_1^2 = g/k$. Substituting this into (3b), using the method of variation of parameters to find the particular solution, and integrating the resulting integral once by parts, we obtain

$$f_2 = a \exp(kz) - \varepsilon a \exp(-kz) \int_{-\infty}^{z} U'(z) \exp(2kz)\, \mathrm{d}z. \tag{5}$$

Substituting this into (4b) yields the phase velocity of the wave:

$$c = c_1 (1 + 2k\varepsilon) \int_{-\infty}^{0} U(z) \exp(2kz)\, \mathrm{d}z. \tag{6}$$

To second order in $\hat{U}$, this is the phase velocity measured by the radar, so we equate $c = c_p$.

It is useful to consider the special case

$$\hat{U}(z) = \varepsilon c_1 e^{mz}. \tag{7}$$

Substituting this into (6) gives

$$c_p = c_1 [1 + 2k\varepsilon/(m + 2k)]. \tag{8}$$

To obtain the expression used to compare $c_p - c_1$ with other measurements of the current, we note that $2k\varepsilon c_1/(m + 2k) \approx \hat{U}(\lambda/4\pi)$, where $\lambda = 2\pi/k$ is the wavelength of the scattering ocean wave. The approximation is correct within 25% as long as $m < 2k$, and it is independent of m. In fact, it should be insensitive to the form of U so long as the velocity decreases monotonically with depth and is not confined to a layer less than $\lambda/4\pi$ thick. We note that this approximation is exact when $\hat{U}$ varies linearly with z. We rewrite (8)

$$c_p - c_1 \approx \hat{U}(\lambda/4\pi). \tag{9}$$

In the next section, we will compare currents measured at a depth d with $c_p - c_1$ measured at various radio frequencies. Equating $d = \lambda/4\pi$ gives the wavelength, and thus the radio frequency, at which this comparison should be made.

Finally, we use (8) to calculate the change in the phase velocity of a finite amplitude wave (Stokes wave) resulting from its mean Lagrangian current. From LAMB [1932, Section 250(16)], this current is $\hat{U}(z) = k^2a^2c_1e^{2kz}$. Using $\varepsilon = k^2a^2$, $m = 2k$ in (8) gives $c_p = c_1(1 + k^2a^2/2)$. This agrees with Lamb's Section 250(16) when k^2a^2 is small. That is, use of (8) gives the correct phase velocity of a finite amplitude wave when its second-order current is known.

EXPERIMENTAL WORK

To test this technique, we conducted two experiments to compare radar measurements of ocean surface currents with conventional measurements of the same current. The experiments were conducted in January and May of 1973 off San Clemente Island in the Pacific Ocean near California.

Radio data were provided by an HF radar operated by the Institute for Telecommunication Sciences, Office of Telecommunications, U.S. Department of Commerce, under contract from the U.S. Naval Research Laboratory. This radar has two antennae of 15° beamwidth pointed towards 270°T and 240°T (Fig. 1), and observed a scattering area 7·5 km deep approximately 20 km offshore. We assume the ocean current is the same at both areas, and use the two antenna directions to measure two different components of the current. Later we will test whether the data are consistent with this hypothesis. Water depth in the scattering region was much greater than the wavelength of the scattering waves, so they were deep-water waves.

The radar produced 50-μs pulses at a number of radio frequencies (Table 1). Data were recorded coherently for 1000 s to provide a frequency resolution of $2\pi \times 10^{-3}$ rad s^{-1} in the Doppler spectrum of the scattered signal. Several spectra were averaged to reduce the variance of the spectral values. The deviation $\Delta\omega$ of the first-order Bragg line from its expected position was used to calculate the radial component of the current as a function of radio frequency (Figs. 3–5). A positive velocity indicates a current toward the transmitter (from 270° or 240°).

The weather on both days was calm. The wind speed was 3 m s^{-1} in January and 1 m s^{-1} in May. The significant wave height was around 1 m. On both occasions the days prior to collecting data had stronger (up to 15 m s^{-1}) winds.

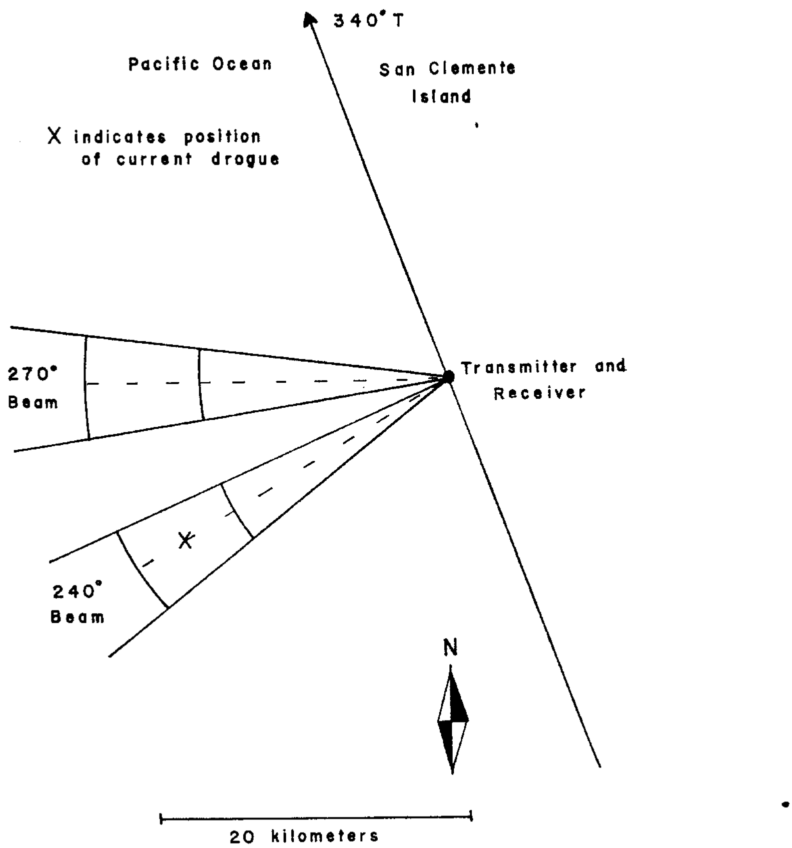


Fig. 1. Plan view of experiment site showing radar scatter geometry and current drogue positions. San Clemente Island is to the right of the diagonal line which represents schematically the shore line.

Table 1. *Experimental parameters: ocean waves (period, frequency, and wavelength which backscatter radio waves (wavelength, frequency) used in experiment.*

Radio frequency (MHz)	*Radio wavelength* (m)	*Ocean wavelength* (m)	*Ocean wave frequency* (Hz)	*Ocean wave period* (s)
2·43	123·5	61·7	0·159	6·3
3·25	92·3	46·2	0·184	5·4
4·60	65·2	32·6	0·219	4·6
6·80	44·1	22·1	0·276	3·8
9·36	32·1	16·0	0·312	3·2
13·44	22·3	11·2	0·374	2·7
17·42	17·2	8·6	0·426	2·3
20·45	14·7	7·3	0·461	2·2

To measure the surface current we used a method comparable to the radio technique; nevertheless, the two methods are not identical. The current was measured by tracking the positions of a parachute drogue placed near the surface of the ocean and close to the centerline of the 240° radar beam 20 km offshore. In January, a small drogue was used and it measured the water velocity at a depth of 4 m. In May we used a large drogue that measured the mean current from the surface to a depth of

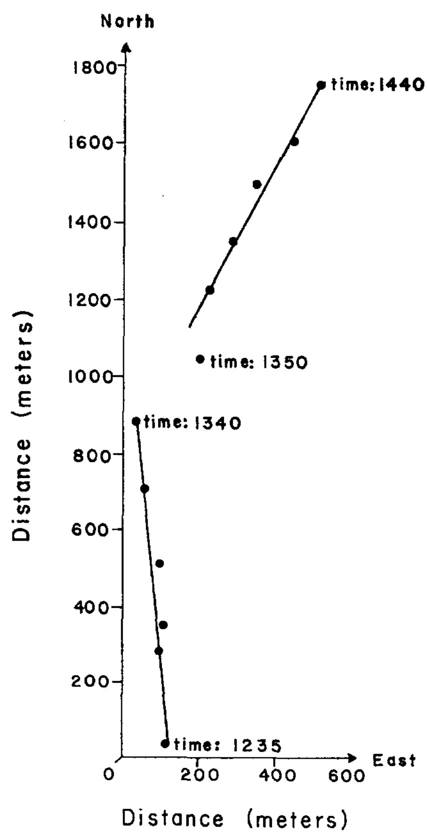


Fig. 2. Current drogue positions versus time in May.

2 m. The drogues had a surface float that was tracked with an accurate microwave radar. Their positions were measured approximately every 10–20 min over a 2-h period, with a relative accuracy of ± 2 m.

The drogue positions for the May experiment (Fig. 2) indicate the current was constant for about 1 h, then abruptly changed direction and remained constant for another hour. The velocity in January (not shown) was constant for the entire observation period. The mean direction of the current was obtained from a straight line drawn through the positions, the mean speed from the distance the drogue moved in 1 h. The measured velocities were 15 cm s^{-1} toward 144°T in January and 22 cm s^{-1} toward 356°T and 25 cm s^{-1} toward 30°T in May. The accuracy of these velocities is limited by the accuracy with which the drogue followed the current and not by tracking errors. Only the small drag of the surface float disturbed the drogue velocity, which we estimate to be correct within 10%.

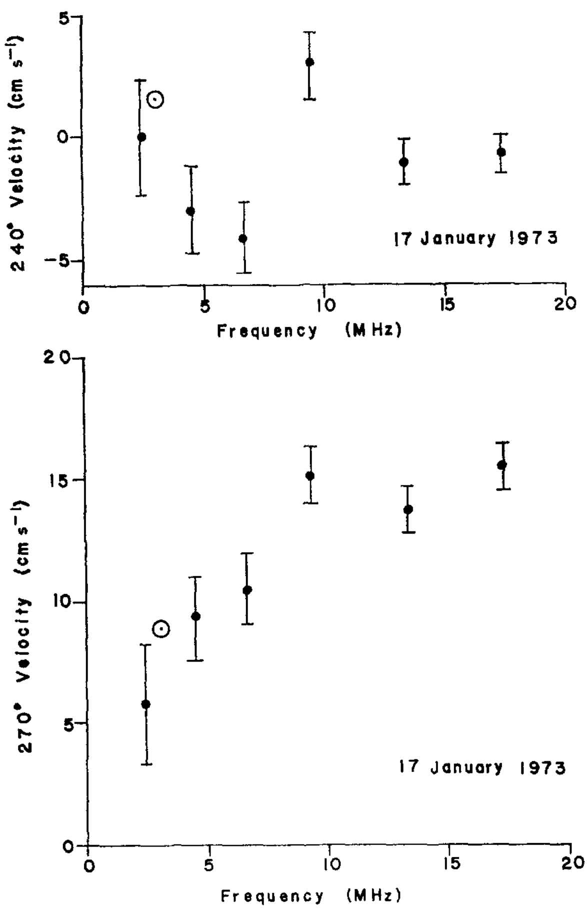


Fig. 3. Surface current velocity measured by the two radar beams as a function of radio frequency on 17 January 1973. Positive velocity indicates current toward the transmitter (from 240 or 270°). The vertical bars represent a resolution of $\pm\pi \times 10^{-3}$ rad s^{-1} in the position of ω_p. Open circles indicate values of current component observed by the current drogue. Their positions with frequency is determined by the depth of the drogue using equation (9).

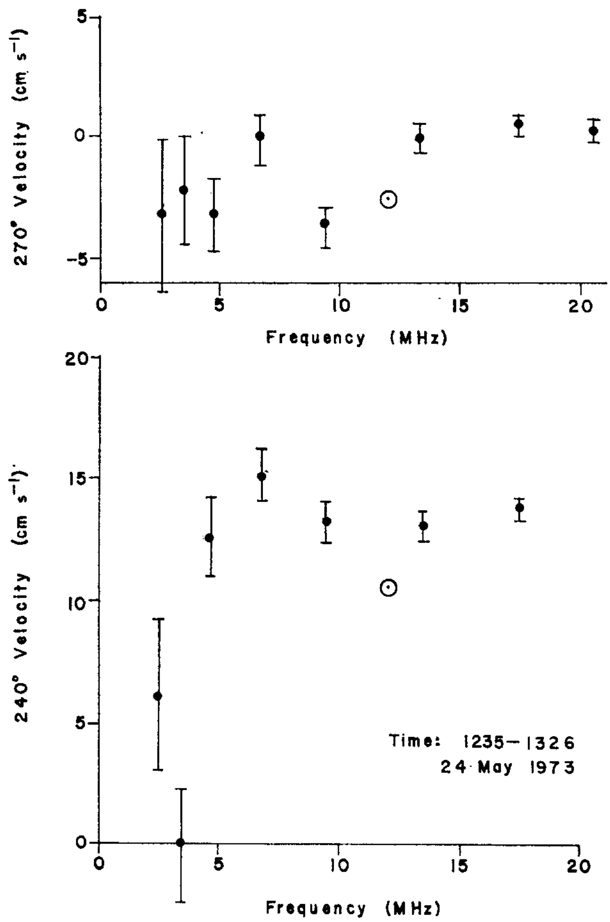
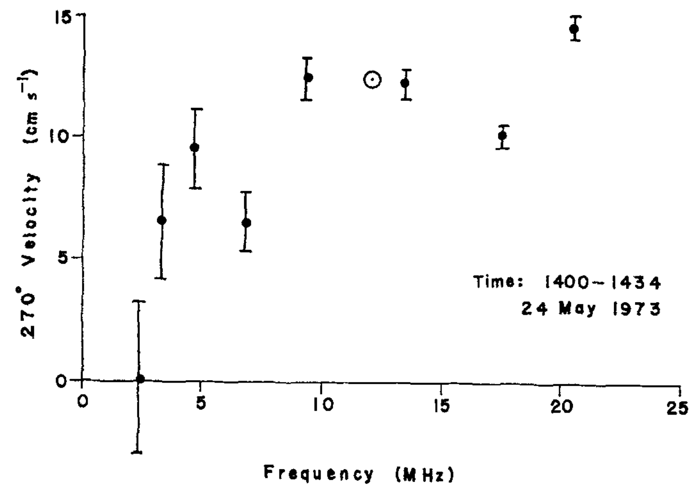


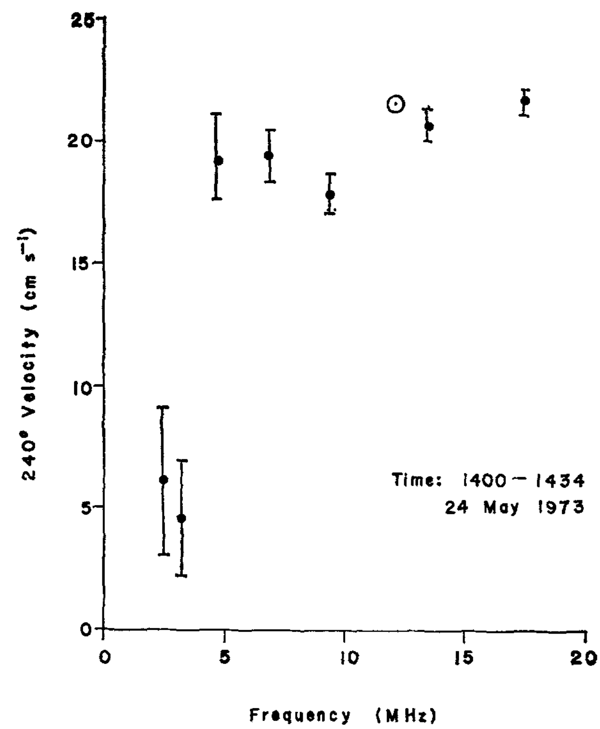
Fig. 4. Same as Fig. 3. Data are from the first hour on 24 May 1973.

COMPARISON OF DATA

The comparison of the drogue currents with the radio scatter requires care. The radio measures the weighted average of the current with depth, the drogue measures the same current at a particular depth. A unique comparison of the two sets of data involves the solution of an inverse problem. The radio data must be used to determine the velocity at the drogue depth. Since we are not prepared to obtain a rigorous solution, nor are we certain a unique solution exists, we must rely on some model for the current. We then show that the data are consistent with this assumption. As a model we take any distribution of current with depth such that (9) is valid. We use the depth of the drogue d to determine the wavelength $\lambda = 4\pi d$, and thus the radio frequency to be used for the comparison. Velocities measured at all other frequencies are used to determine whether the profile is consistent with (9). The problem is not as complicated as it might seem. First, the integral in (6) is heavily weighted toward the surface, and we can neglect currents below some depth D. Secondly, because windy conditions existed for some days before we collected data we expect the current



(a)



(b)

Fig. 5. Same as Fig. 3. Data are from the second hour on 24 May 1973.

structure above D to be simple.* We certainly do not expect strong stratification with the possibility of current sheets going in different directions.

To estimate the thickness of the layer that contributes significantly to $c_p - c$, consider the relative error E involved in limiting the integral in (6) to a layer of thickness D. Using $U(z) = 1$,

$$E = \int_{-\infty}^{-D} e^{2kz} dz \Big/ \int_{-D}^{0} e^{2kz} dz.$$

Carrying out the integrations and solving for D,

$$D = \lambda/(4\pi) \quad \ln[(1 + E)/E].$$

If $E = 0{\cdot}25$, then $D = \lambda/8$. That is, the neglect of currents below 1/8 of a wavelength introduces an error of 25% in the calculated phase velocity of the wave on a uniform current. Of course, other current profiles would give somewhat different values, but the essential point is that only a thin surface layer influences the velocity of the wave.

Some numerical values used in the comparison are: in January, $d = 4$ m, $\lambda = 16\pi$ m, so $D \approx 2\pi$ m (this λ scatters a 3·0-MHz radio wave). In May, $d = 1$ m, $\lambda = 4\pi$ m, and $D = \pi/2$ m. The scattered radio frequency is 12 MHz.

The data from May are the easier to interpret. Radio frequencies above 5 MHz all give essentially the same velocity but with some scatter. We infer from this that the current is nearly constant to a depth of 1/8 of the wavelength that scatters 5 MHz signals, or about 4 m (Table 1). This layer of nearly constant velocity includes the drogue, and we expect equation (9) to be valid. This idea is strengthened by the agreement between drogue and radio measurements. Components of drogue motion toward 240 and 270°T agree with velocities measured by radio frequencies near 12 MHz. If we account for the uncertainty in the radio measurements, as shown by the bars in the figures, the two methods agree within 2 cm s^{-1} (out of a total velocity of 10–20 cm s^{-1}).

The comparison for the January data is a little more complicated. The model requires that the drogue velocity be compared with radio data at 3 MHz, and measurements at 2·4 and 3·3 MHz do bracket the velocity components measured by the drogue. The only question concerns the validity of the model. The current observed by the radio tends to increase with increasing radio frequency. Since each frequency samples the current over a different depth, this implies $U(z)$ was not constant, but varied with depth, being larger near the surface than deeper. This is to be expected if the current were wind driven. The model requires that the current not be confined to a layer thinner than approximately $\lambda/4\pi \sim 4$ m. Since the drogue was at this depth and measured a current slightly more than 1/2 the current near the surface (measured at the highest radio frequency), we conclude that the current was not confined to a thin layer, and the model should apply.

The apparent agreement between the two measures of the surface current has some important implications. Firstly, the agreement is not likely to be the result of chance. We have measured two components of the velocity at three different times using two different drogue depths for a total of six independent measurements. We found fair agreement on all occasions. Secondly, the velocities measured with antennae pointing

*For reasons entirely unrelated to this work we attempted to obtain data during storms. On both days we were delayed, and the wind and waves had just died down by the time data were collected.

in two different directions agree with the components of the drogue motion in these directions. This implies that the current field was the same in both scattering areas. Thirdly, if we agree that the current measured by the radar adequately predicts the current at a single depth, then it may be possible to measure the velocity shear near the surface, and thus the momentum flux to the sea by the wind. Certainly, if the wind were blowing strongly for some time we might expect the current distribution to fit a simple model, i.e. the profile to be logarithmic with depth. This model should allow a unique inversion of the radio data to find $\hat{U}'$ (o).

In summary: we have used radio scatter measurements and a simple model to predict the current near the ocean surface. This velocity agrees, within a few centimeters per second, with the velocity of a drogue placed at this depth. Repeated applications of the technique under different conditions yield the same result. This strongly suggests that the radio technique is a valid way to measure surface currents.

Acknowledgements—As is often the case in applying HF scatter to the sea, the first suggestion that this scatter might be used to measure currents came from conversations with D. D. Crombie. We also wish to thank R. Bogle of the Naval Research Laboratory for supplying the radar data. This research was supported by the Advanced Research Projects Agency of the Department of Defense and was monitored by ONR under Contract No. N00014-69-A-0200-6012.

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