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*THE PRINCIPLES OF TIME SERIES ANALYSES APPLIED TO OCEAN WAVE DATA**

BY H. R. SEIWELL†

WOODS HOLE OCEANOGRAPHIC INSTITUTION, WOODS HOLE, MASSACHUSETTS

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Introduction.—In the original report on the auto correlation analysis of ocean wave data the apparent advantages of that method over periodogram and Fourier series applications to finite amounts of data were brought out.[1] Because of practical restrictions in that only finite amounts of observational data are available for study, the fitting of wave heights as functions of time by means of trigonometric components is only in special circumstances suitable for describing the physical characteristics of the data itself. In the case of ocean wave data, *a priori* knowledge of frequency is not available, and results from this type of analysis have resulted in claims that sea surface roughness patterns are composed of large numbers of frequencies which have traveled with independent velocities from their regions of generation. In other words, mathematical representations of the data have been assumed to be physically significant, a situation which may lead to apparent misconceptions of the behavior of the phenomenon and of the dynamics causing it. In fact, the fitting of data by any of the various methods of cycle analyses, in the absence of *a priori* knowledge of the physical properties, is no more effective than the fitting of that same data, say, by orthogonal polynomials.[2]

It appears that the problem of understanding the behavior of a physical phenomenon which varies with time is directly associated with that of predicting its future course from its past behavior, and the distance into the future which the phenomenon may be accurately forecasted may be taken as an indication of the amount of dynamics associated with the phenomenon itself. Thus, in the case of sea surface wave data, while a mathematical theory may be set up from the results of a periodogram or Fourier series analysis to explain a record, it has little physical meaning unless it can be extrapolated into the future. It was from this point of view that Wiener[3] investigated the problem of prediction and defined the dynamics of a time series in terms of the possibility of prediction into the future. The auto correlation of a time series provides a means of doing this. It can be shown that the auto correlation function may be obtained from the spectrum of the series, and the spectrum may also be obtained from the auto correlation function. In this process the spectrum and the auto correlation function of a discrete sequence of observations preserve information by retaining the frequency and the amplitudes of all frequencies from zero to infinity, whereas, on the other hand, it discards information relative to phase relationships.

TABLE 1

RECORD NO.	LOCATION	DEPTH, FT.	DATE	STARTING TIME	RECORD LENGTH, SECS.
B-713a	Bermuda	120	2/21/47	1840	350
B-12a	Bermuda	120	10/25/46	1405	350
1-5B	Cuttyhunk	75	6/14/46	1149	60
39-L	Cuttyhunk	75	7/ 4/46	0627	350
53-W	Cuttyhunk	75	9/15/46	0050	350
53-X	Cuttyhunk	75	9/15/46	0650	350
53-Y	Cuttyhunk	75	9/15/46	1250	350
56-E	Cuttyhunk	75	10/ 1/46	1244	350

The method of auto correlation analysis is employed in the analysis of many types of scientific problems. In addition to its use as a prediction mechanism, the correlograms and the Fourier transforms of the correlograms into power spectra serve to describe the basic data by measuring its physical properties of period, amplitude and damping coefficients which, in turn, may be employed in an attempt to discover something of the physical causes. The results throw new light on the physical properties of the sea surface and on the behavior of ocean wave patterns. The method provides a means for distinguishing between mathematical and realistic physical representations of sea surface roughness patterns.

Computation of the Auto Correlation Coefficient.—Theoretical formulae defining the auto correlation coefficient as developed by Wiener,[3] Khinchine[4] and others are not strictly applicable to practical computations.[5] This is because wave heights are not observed as infinite functions of time (very frequently only short series of observations are obtainable), and also because the means of such finite series frequently depart from zero. In our computations it was found necessary to use a running mean rather than extracting a grand mean at the very beginning of the computation. To compute the auto correlation coefficient we compute the following statistics from N equally spaced observations for k equally spaced time lags.

$$T_0 = \sum_{i=1}^{N} x_i, \quad T_k = \sum_{i=1}^{N-k} x_i, \quad T_k' = \sum_{i=1}^{N-k} x_{i+k}$$

$$S_0 = \sum_{i=1}^{N} x_i^2, \quad S_k = \sum_{i=1}^{N-k} x_i x_{i+k}$$

$$\mathcal{S}_0 = \frac{NS_0 - T_0^2}{N^2}, \quad \mathcal{S}_k = \frac{(N-k)S_k - T_k T_k'}{(N-k)^2}$$

$$r_k = \mathcal{S}_k/\mathcal{S}_0$$

The correlogram is obtained by plotting the auto correlation coefficient (r_k) as ordinate against k as abscissa. In this particular investigation the correlogram provides the basis for analysis of the data, and may be transformed into a power spectrum by a Fourier transform of the auto correlation coefficients. This spectrum is identical with that which would be obtained by periodogram analysis of an infinite series.

In the applications considered in this paper the auto correlation function frequently occurs as a composite of a pure sinusoidal wave and one producing a continuous spectrum. The period of the former is given by the correlogram and its amplitude computed from the original data by least squares. The residue after subtraction from the original data consists of a continuous spectrum. This provides a practical example of theorem 3 of Khinchine's theory of correlation[4] in which the auto correlation function is split into a line and a continuous spectrum.

The Data.—This paper presents results of auto correlation analysis of eight ocean wave records. The records are from underwater pressure recorders located at Cuttyhunk, Massachusetts (depth 75 feet), and at Bermuda (depth 120 feet) and represent pressure pulses at the sea bottom transmitted from the sea surface.[6] For the analyses, the records were scaled for pressure values at intervals of one second and were not reduced to sea surface wave heights.[7]

Bermuda Record B-713a is discussed separately. Table 1 summarizes the records analyzed.

The Analysis of Wave Record B-713a.—This record is discussed in more detail than the others for comparison of the results from cycle analyses with those of auto correlation analyses on finite amounts of data. Cycle analysis refers both to mechanical periodogram analysis and to the fitting of data by sums of sines and cosines. Results obtained by the former method, to which the data had already been subjected,[8] are first discussed in the light of their usual interpretation. This is then supplemented by computation of certain statistical properties of the data and then followed by fitting a series of cosine waves to the data over the critical part of the spectrum. Results obtained by the above methods are then compared with those obtained by the method of auto correlation analysis.

a. Mechanical Periodogram Analysis: The mechanical periodogram analysis of this 20-minute wave record (Fig. 1) shows a continuous spectral distribution with about 32 defined peaks between 8.8 and 40 seconds. The greatest development occurs in the 8- to 10-second band, and maximum energy is indicated for 8.8, 9.0 and 9.5 seconds. The customary interpretation given this type of analysis is that the basic wave pattern is made up of

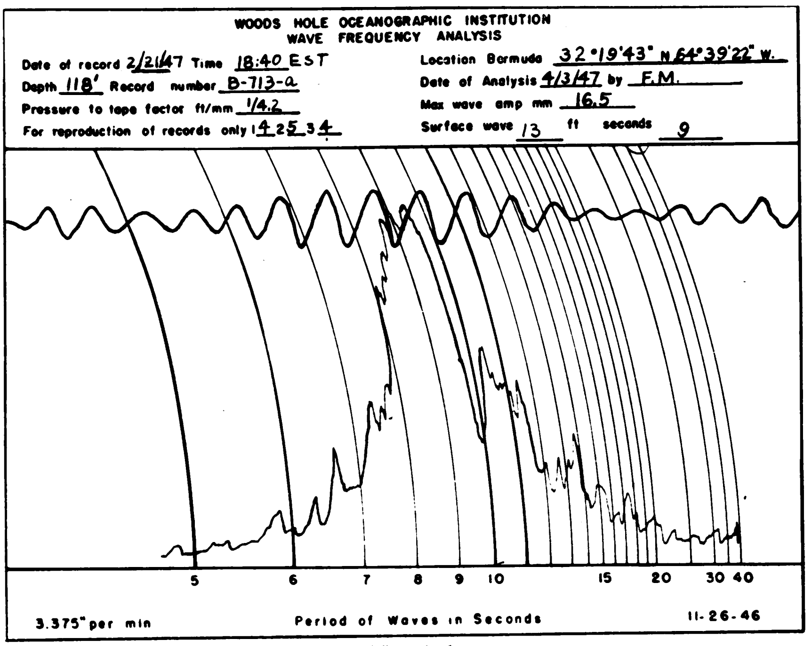

FIGURE 1

Results of mechanical periodogram analysis of Wave Record B-713a. See text.

the frequencies indicated. The principal wave periods lie between 6 and 15 or 20 seconds, with the maximum period band of the "wave train" extending from 8 to 10 seconds. The characteristics of different wave records are usually compared by the widths and locations of the maximum band development, together with locations of the individual maximum points. Special consideration is frequently given to indications of longer periods, say above 20 seconds, which are sometimes presumed to be waves generated by oceanic storms at great distances but which have "out-run" the storm itself.

The above physical interpretation is doubtful in that periodogram analyses of finite amounts of data are not necessarily physical, but may be only mathematical representations of the data. Its acceptance as a physical representation without *a priori* reason may lead to a misconception of both the physical properties and the causative mechanism of the sea surface roughness. This question is discussed later after additional results are developed.

b. Statistical Characteristics: The frequency distribution of periods, compiled from measurements of the times between successive wave crests on the record, has the following properties:

Standard deviation = 1.135 seconds
Mean = 8.59 ± 0.077 second
Mode = 8.75 seconds

Thus, the mean and modal values of distances between wave crests in the original record lie in the range of maximum energy brought out by the mechanical periodogram analysis (Fig. 1).

TABLE 2

PERIOD	RELATIVE AMPLITUDE	PHASE ANGLE, °
7.50	1.00	156.6
8.00	2.31	111.9
8.25	2.97	23.6
8.50	4.01	30.1
8.75	4.60	18.9
9.00	5.29	69.6
9.25	10.93	276.9
9.50	3.49	231.2
9.75	2.20	210.2
10.00	0.77	16.9

Amplitude and phase values obtained by fitting cosine waves to Wave Record B-713a.

c. Cosine Wave Analysis: Results obtained by independent least square fits of ten cosine waves (periods from 7.5 to 10.0 seconds) to the basic data are tabulated in Table 2. The spectrum is similar to that obtained by mechanical periodogram analysis, increasing from a minimum at 7.5 seconds to a maximum value at 9.25 seconds and then diminishing rapidly to a minimum at 10.0 seconds. This representation, obtained from a finite amount of data, is like that of the mechanical periodogram analysis—of mathematical significance only and without physical reality. The rôle of this type of analysis is a secondary one, to be applied after the period, or periods, have been established by other means.

d. Auto Correlation Analysis: The correlogram of the primary data of Wave Record B-713a (Fig. 2) maintains a period of 8.75 seconds and damps

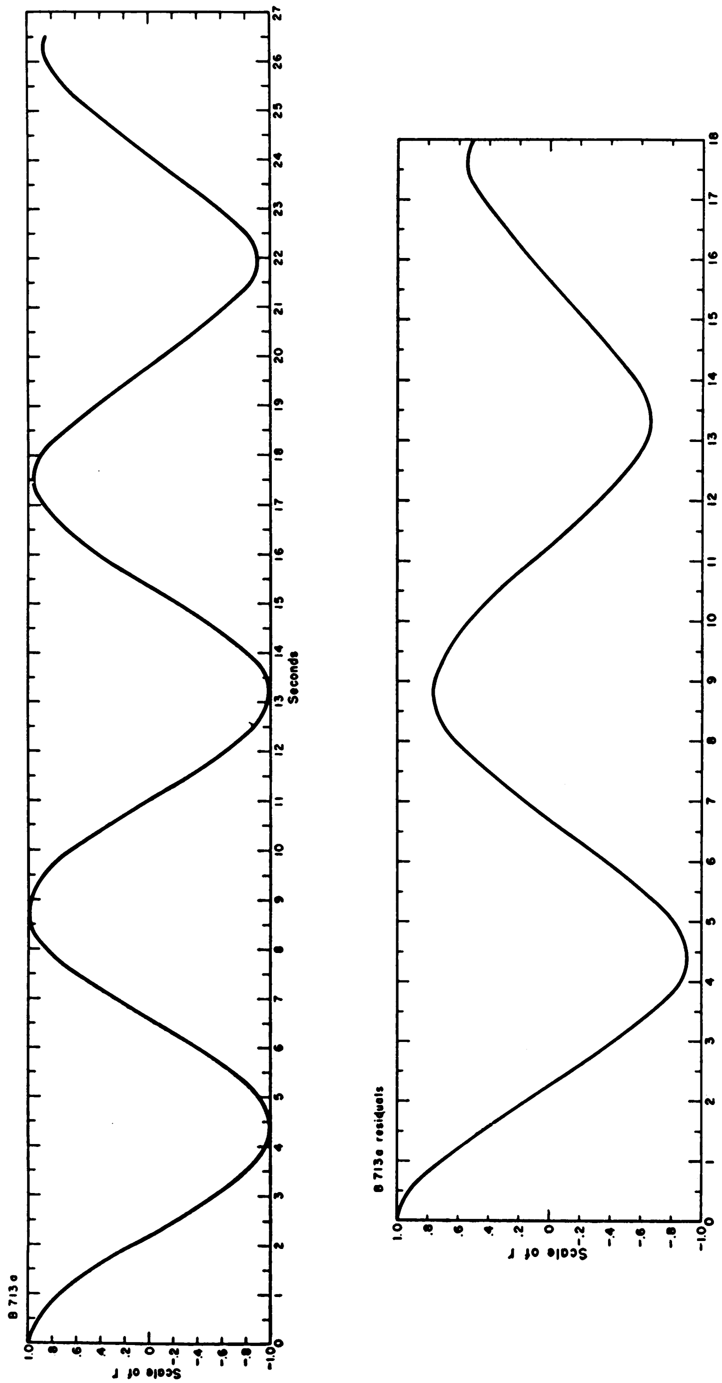

FIGURE 2

Correlograms of Wave Record B-713a. Curves have been drawn through values of r computed for each second. The curves have not been smoothed.

slowly; at the end of the third cycle the amplitude is reduced to a value of $r = 0.87$, after which it continues to damp slowly until reaching a terminal value (r_T). The terminal amplitude is maintained by a cyclical component which proceeds undamped through the basic data. It is the fundamental wave to which energy is supplied by an external force at a rate equal to that at which it is dissipated in the system. Its amplitude, obtained from a least square fit of an 8.75-second sine wave to the primary data, is 0.720 foot.

Subtracting the cyclical component ($T = 8.75$ seconds, $A = 0.720$ foot) from the original data, the variance reduces from a value of 0.492 square foot to 0.216 square foot for the residuals. Recomputing the auto correlations of the residuals, the correlogram then damps in a fashion characteristic of an autoregressive series; at the end of the second cycle, the correlogram amplitude is $r = 0.54$ as compared to $r = 0.95$ for that of the primary data (Fig. 2).

The value of the terminal amplitude of the original correlogram is

$$r_T = \frac{\text{Variance cosine}}{\text{Total variance}} = \frac{A^2/2}{\sigma_y^2} = 0.56$$

for Wave Record B-713a. This value also represents the per cent reduction due to the cyclical component itself, and may be expressed as the ratio of the residual (σ_r^2) and original (σ_y^2) variances. Thus:

$$r_T = 1 - \frac{\sigma_r^2}{\sigma_y^2}$$

The numerical solutions of the above and other relations, for cases where the original series consists of a sine term and a random residual, provides a means of checking the computations.

The significant difference between the results of periodogram analyses and correlogram analyses of finite amounts of data is that the former are not the power spectra of the latter. The large number of peaks in the periodogram of Record B-713a (Fig. 1) are not real periods, and they provide only a mathematical representation of the data. The spectrum is similar to that obtained by the arbitrary fitting of cosine waves to the data, and is not to be interpreted as having physical reality.

On the other hand, the use of auto correlation analysis reveals that for Record B-713a the auto correlation function is broken into two parts—one having a line, and the other a continuous spectrum. The line spectrum is that of the cyclical component present in the basic data, whereas the continuous spectrum represents a wide band of frequencies, with an average frequency equal to that of the cyclical components, but which damp out rather rapidly.

The physical interpretation to be given results from the two types of analyses is equally significantly different. The auto correlation analysis suggests that the physical characteristics of the sea surface wave pattern comprise one fundamental wave with a period of 8.75 seconds and an amplitude of 0.72 foot, on which is superimposed a series of local impulses. The former, generated presumably by a meteorological situation acting at some distance from the target, is the predictable component in the data. It accounts for an estimated 56% of the total variability of the sea surface pattern (obtained from the value of the terminal amplitude of the correlogram) and lends itself to being forecast from knowledge of wind action on the sea surface. The remainder of the data represents a superimposed

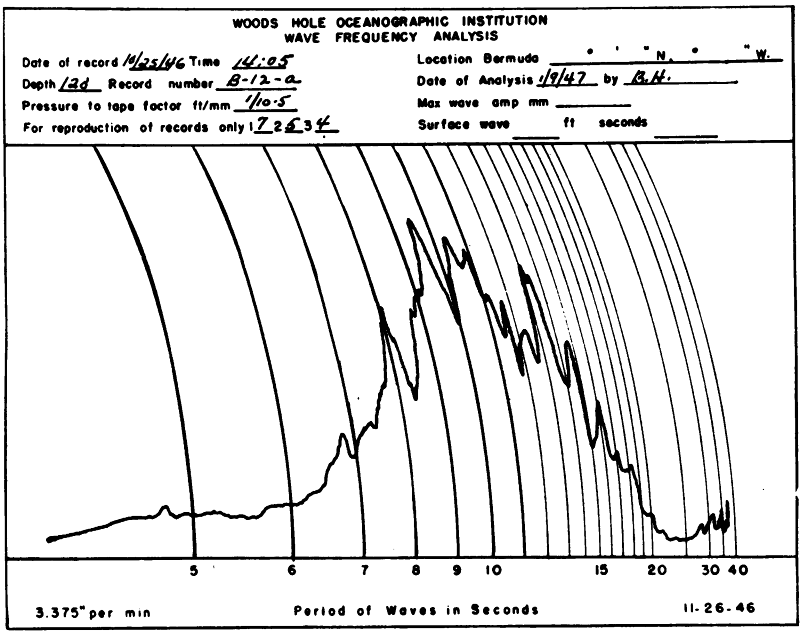

FIGURE 3

Results of mechanical periodogram analysis of Wave Record B-12a. See text.

series of local impulses which die out rather rapidly. They would appear to be generated by local winds in the area of the target, and their relative significance to the pattern is also evaluated from the terminal amplitude of the original correlogram.

Auto Correlation Analyses of Additional Ocean Wave Records.—The results of auto correlation analyses of seven additional ocean wave records from Cuttyhunk and Bermuda are tabulated in Table 3. The mechanical periodogram analyses of these records gave results similar to those for Record B-713a, each showing approximately 30 peaks. Figures 1 and 3 are representative of this analysis.

The original correlograms do not damp completely (Figs. 2 and 4),[9] terminal amplitudes range from 0.08 to 0.66 and all show well-defined periods ranging from 6.50 to 15.5 seconds. Amplitudes of the cyclical components associated with the data range from 0.10 to 0.72 foot, and their relative quantitative significance to the total sea surface roughness patterns are revealed by the terminal amplitudes (r_T). Five of the records show r_T values

TABLE 3

RECORD NUMBER	ORIGINAL DATA σ_y^2	AD/σ	T	A	r_T	RESIDUAL DATA σ_r^2	AD/σ	A'
B-713a	0.492	0.78	8.75	0.72	0.56	0.216	0.78	0.74
B-12a	0.018	0.80	9.07	0.10	0.22	0.014	0.83	0.09
53-W	0.109	0.80	15.50	0.35	0.57	0.047	0.79	0.35
53-X	0.185	0.84	12.25	0.51	0.66	0.064	0.77	0.49
53-Y	0.070	0.82	11.00	0.10	0.08	0.065	0.80	0.10
56-E	0.128	0.80	6.50	0.38	0.56	0.057	0.80	0.38
39-L	0.049	0.82	8.29	0.15	0.25	0.035	0.80	0.15
I-5B	0.030	0.84	8.00	0.19	0.63	0.011	0.83	0.19

Results of auto correlation analyses of ocean wave records. σ_y^2 = variance original data, AD/σ = ratio of average deviation to standard deviation, T = period of correlogram, A = amplitude of cyclical component by harmonic analysis, σ_r^2 = variance residual data, r_T = terminal amplitude of original correlogram, $A' = \sqrt{2(\sigma_y^2 - \sigma_r^2)}$.

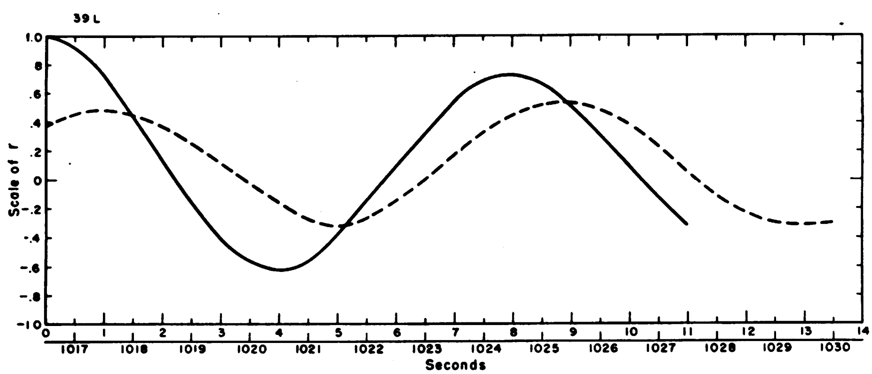

FIGURE 4

Correlogram of Wave Record 39-L, 0 to 11 seconds (solid line) and 1017 to 1030 seconds (dashed line). Curves have been drawn through values of r computed for each second. The curves have not been smoothed.

exceeding 0.5, indicating more than 50% of the roughness pattern resulting from the cyclical component. For the remainder, local influences dominate the pattern, and in one of these, 53-Y, the terminal amplitude of the correlogram is so low (0.08) as to indicate the data to be almost autoregressive. In the complete absence of sea swell the original correlogram damps rapidly to zero.[10]

The results are typical of those obtained from the analyses of more than 30 wave records from the North Atlantic, Mediterranean and the eastern American coast. After extraction of the cyclical components, the residual variances are diminished, and the correlograms of the residuals damp more rapidly in the fashion of an autoregressive series (Fig. 2).

A numerical result of apparent significance is that the ratios of the mean deviations to the standard deviations (for both original and residual data) is approximately equal to $\sqrt{2/\pi}$, a relationship which usually characterizes unimodal curves approaching symmetry. This relationship enables an approximate estimate of the mean sea surface roughness (expressed as twice the average deviation of the original data) to be made directly from the variance of the original data.

The properties of the sea surface roughness pattern, as brought out by auto correlation analysis of a finite amount of data, describe its physical characteristics. They indicate that the instantaneous sea surface pattern is not an interference pattern composed of many wave frequencies which have traveled with independent velocities from their generation areas, such as may be concluded from results of a cycle analysis of finite amounts of data, but rather that it seems to consist of a single cyclical component (sea swell) on which is superimposed random frequencies (local sea) of the same average frequency. The sea swell appears to have been generated by a dominating meteorological situation at some distance from the target and is maintained by energy supplied at a rate equal to that dissipated by the system. It is to be expected that this period changes with time and at a rate dependent on the stability of the off-shore meteorological situation causing it. On the other hand, the local sea is induced by local winds and other factors tending to disturb the sea surface and dies out fairly rapidly.

It may thus be reasoned that the ocean itself acts as a filter which, under specific conditions, passes only certain fundamental frequencies, which we call sea swell. This fundamental frequency at any time is perhaps a function of the characteristics of the ocean basin over which it has traveled, plus wind velocity, direction, fetch and other properties of an established wind-generating system. Its constancy is related to the stability of the meteorological situation. At any particular time local conditions may also produce a continuous band of frequencies with an average frequency the same as that of the sea swell. These latter die out rapidly in a manner similar to the effect of random impulses on a sluggish oscillator as the ocean basin continues to act as a filter.

However, regardless of the exact nature of the physical explanation of the mechanism maintaining the state of the sea, a quantitative separation into sea swell and local sea is fundamental to an understanding of its physical properties. This has an added practical significance for operational purposes as well as for aerial depth determination and for controlled experiments involving sea surface properties. To this end, the method of auto correlation analysis is an improvement over other procedures for the analysis of finite amounts of data in which not even the length of the periods or the recurrence intervals are known or suspected at the outset. The results in themselves provide a more realistic geophysical explanation of the phenomena and avoid the pitfalls from claims of large number of periods and cycles which may arise from procedures less adapted to the available data.

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† The methods and conclusions in this paper have partly resulted from a collaboration with Dr. George P. Wadsworth, of the Massachusetts Institute of Technology, who is preparing a theoretical account of the adaptations of generalized harmonic analysis to the specific problems of ocean waves.

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[7] The ratio of amplitudes of instantaneous pressure fluctuations at the sea bottom ΔP_h to those at the sea surface ΔP_s is

$$\frac{\Delta P_h}{\Delta P_s} = 1.35 \frac{1}{\text{Cosh } kh}$$

where h is depth to bottom and $k = 2\pi/\text{wave-length}$. The factor 1.35 has been experimentally determined for the Woods Hole–Bermuda area.

[8] These analyses were performed on the ocean wave analyzer at this Institution. The term mechanical periodogram analysis is used to distinguish it from other types of periodogram analyses. See reference 6.

[9] Correlograms for Record 53-X have been published in reference 1.

[10] Examples of this situation have been encountered, although they are not discussed in this paper.