A Study of Fetch-Limited Wave Spectra with an Airborne Laser

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An airborne laser wave-profiler was used to estimate fetch-limited, one-dimensional wave number spectra by flying seaward of Cape Henlopen, Delaware, to a distance of 95 nautical miles, following passage of a cold front associated with a mean horizontal wind speed of 14 meters sec^{-1}. The equilibrium range constant was estimated at 4.2×10^{-3}, and it is shown that the constant may be overestimated or underestimated depending on assumed angular spreading of the waves. 'Overshoot' observed in these one-dimensional wave number spectra is associated with angular spreading. Linear and exponential growth parameters estimated from fetch-limited wave spectra are in reasonable agreement with other field investigations. Observations of exponential wave growth generally support the Miles-Phillips predictions in the range $12 \leq C/U^* \leq 21$ where C is the phase speed of the wave and U^* is the friction velocity.

Well-documented field experiments associated with reasonably homogeneous wind conditions continue to be extremely valuable as a means of testing and amplifying the applicability of ocean-wave generation theories, particularly in determining the extent to which linear theory is useful for the development of ocean-wave forecasting schemes. The purpose of this paper is to present results of airborne wave-profile estimates of the sea surface in terms of a wave number spectrum. These estimates illustrate some characteristics of the equilibrium range of wave numbers and the applicability of the linear aspects of wave generation theory in explaining part of observed wave growth. Nonlinear mechanisms, of course, must be included in ocean wind-wave generation theory; however, it is difficult to assess the quantitative importance of various nonlinear interaction terms, and successful wave forecasting approaches (e.g., the methods of W. J. Pierson at New York University and T. P. Barnett at Westinghouse Ocean Research Laboratory) continue to evolve by parameterizing nonlinear terms.

Since *Miles'* [1957] shear-flow theory and *Phillips'* [1957] resonance theory were proposed, two field experiments have been executed to compare predictions of theory to observations in the presence of a known wind-field. In the experiment of *Snyder and Cox* [1966], four wave-recorders were towed in tandem at constant speed so that one component (17-meter wavelength) could be studied in great detail. The experiment of *Barnett and Wilkerson* [1967], identical to the one described here, was conducted with a Naval Oceanographic Office aircraft equipped with a radar altimeter to profile the ocean surface seaward of a coastline. Both Snyder and Cox and Barnett and Wilkerson found observation of linear wave growth reasonably consistent with Phillips' theory and found that Miles' shear-flow theory underestimated observed wave growth. *Phillips* [1966], stressing that an interplay of various wave-generating mechanisms is involved, modified the shear-flow theory to include the effect of 'undulations in the turbulent flow' and, with the usual assumption of a logarithmic wind profile, exponential wave growth was investigated in terms of the ratio of the friction velocity $U^* = (\tau/\rho)^{1/2}$ (τ is wind stress and ρ is air density) to the phase speed c of a particular spectral component. This modified form of the Miles-Phillips mechanism was used by *Inoue* [1967] to develop a system of partially developed wave spectra based on experimental data with the limiting spectral configuration for a given steady-state condition represented by the spectral form of *Pierson and Moskowitz* [1964]. A compilation of early field results by *Volkov* [1968] indicated generally good agreement between observations and the Miles-Phillips theory.

In his recent studies, *Cardone* [1969] synthesized later results and developed a planetary boundary-layer model for wave forecasting in which effects of atmospheric stability on the shape of the wind profile were included. Cardone parameterized the Miles-Phillips theory in terms of a log-linear wind profile so that the dimensionless growth parameter of Phillips could be investigated from presently available meteorological data. Thus the present status of ocean-wave generation theory, as addressed to the wave prediction problem, appears to be in reasonable agreement on the general accuracy of the modified Miles-Phillips mechanism, with uncertainty remaining concerning the structure of the interplay of wave-generating mechanisms starting from an undisturbed sea surface and evolving to a 'fully developed' state or possibly to a limiting form of wave spectrum whose structure changes very slowly at the smallest wave numbers.

Barnett and Sutherland [1968], however, present arguments based on field and laboratory observations of the spatial growth of the individual spectral components of waves for a tendency to 'overshoot and undershoot' the eventual equilibrium value of that spectral component. *Mitsuyasu* [1969] found not only a tendency to overshoot and undershoot (analogous to underdamped vibration of a mechanical system) but also attributed such behavior to proximity of the overshoot component to the spectral peak. In addition, Mitsuyasu found that the equilibrium range constant was dependent on a dimensionless fetch. In the present study, observations of fetch-limited wind waves obtained by a low-flying aircraft will be used to illustrate properties of the equilibrium range, including effects of assumed angular spreading, and to compare predictions of the Miles-Phillips theory to these field observations.

FETCH-LIMITED WAVE MEASUREMENTS

The fetch-limited wave observations were obtained by profiling the sea surface with a laser Geodolite flying 91.4 meters above the sea surface in a Cessna 310 aircraft out to a distance of 95 nautical miles (176 km), seaward of Cape Henlopen, Delaware, following passage of a cold front about twenty-four hours earlier. In this respect the experiment is identical to the downwind part of the fetch-limited experiment of *Barnett and Wilkerson* [1967]. There are some differences, however. In their experimental setup, a radar altimeter was used to profile the sea surface from a Super Constellation, with higher-frequency plane motion removed by monitoring vertical motions with an accelerometer and very-low-frequency components removed by a high pass filter. Their experimental evidence suggested that aircraft roll and pitch angles did not influence observations of the profile of the sea-surface. In the present experiment, however, a smaller aircraft was used and a lower flight altitude was chosen. The return laser signal was derived from an 'illuminated strip' of width .91 cm and length 4.6 meters associated with an aircraft speed of 91.4 m/sec to provide a detailed profile of the sea surface. Aircraft motions did influence observations of lower wave-number components, and these were removed by a high pass filter [*Ross et al.*, 1968]. It is fairly certain, however, that higher wave-number components were relatively unaffected by platform motion. Consequently, results discussed here apply to one-dimensional radian wave-number components, $k_1 = 2\pi/L_1$, which range from 0.015 ft^{-1} to 0.11 ft^{-1} (0.048 m^{-1} to 0.35 m^{-1}).

WEATHER

The weather situation shown in Figure 1 illustrates a well-developed fetch area of several hundred miles associated with northwest winds. As is evident from the 1800 UT weather map (the actual flight took place at 1610 UT), a fairly steady wind-field existed. This requirement of a homogeneous wind-field and uncertainty on the nature of the surface drag remains one of the most ambiguous attributes associated with field studies of the wind-swept sea surface, and no completely satisfactory way to ameliorate such difficulties was found in our study. Wind-field statistics were approximated from wind observations made from the aircraft, from merchant ships, and from Coast Guard light towers. In addition, geostrophic wind-speed estimates were made independently by two of the authors. A composite of all the above indicated an average horizontal wind speed of 14 m sec^{-1} referred to a level of 10 meters. Turbulent wind conditions were reported by the pilot at the flight altitude of 91.4 meters; this fact, combined with an average air–sea temperature difference of $-8.2°C$ from ship reports in the fetch area, suggested that atmospheric stratification was moderately unstable, and results of this study should be interpreted with this in mind. Based on recent drag coefficient results from Argus Island tower (Bermuda) for unstable atmospheric conditions, a value for the friction velocity U^* of 70 cm sec^{-1}, was used to compare observed exponential wave growth to results of other investigators.

SPECTRUM ANALYSIS

Good data obtained over the first 95 nautical miles (176 km) of the downwind flight track were recorded on FM tape and were partitioned into 10 nonoverlapping sections each of length 9.5 nautical miles (17.6 km). The 10 sections were then digitized at an interval of 0.04 sec, after having been filtered with a low pass Butterworth filter with a response time of 0.1 sec corresponding roughly to a sample every fifteen feet (4.6 meters). Forty-two hundred points from each of the 10 records were used to calculate a one-dimensional wave number spectrum for each section by the methods of *Blackman and Tukey* [1959]. These spectra of encounter, obtained from the aircraft flying at 190 knots, were mapped into spectral estimates relative to fixed coordinates in order to obtain estimates of one-dimensional wave-number spectra with 90% confidence limits of 0.78 and 1.27. Spectral estimates for the ten runs were also calculated by fast Fourier transform [*Cooley and Tukey*, 1965] with practically identical results.

Nine of the ten downwind spectra are shown in Figure 2. The high wave-number end is truncated at a wave number of 0.11 feet^{-1} (0.35 m^{-1}), assumed to represent the higher wave-number end of the equilibrium range, but which is considerably removed from the region of capillary waves or even very short gravity waves. Spectral peak densities, except run 5, lie close to the reference line whose slope is -3. Run 9 had no well-defined peak.

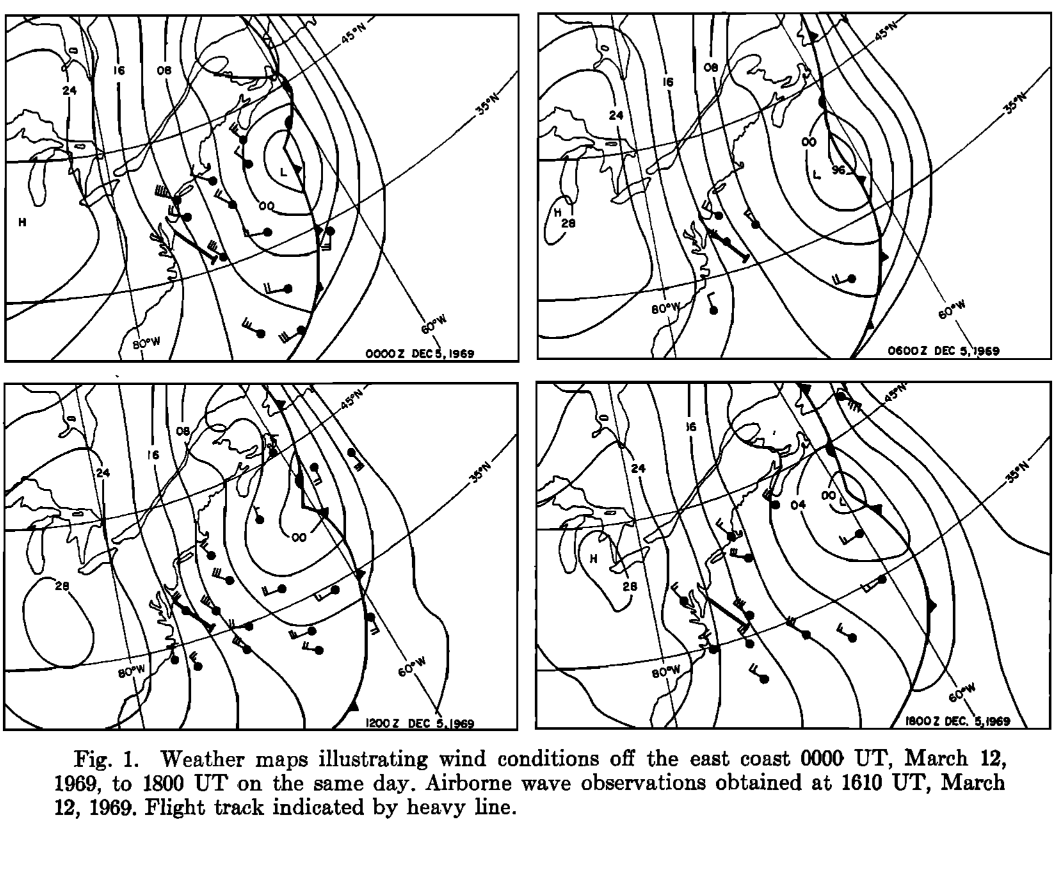

Fig. 1. Weather maps illustrating wind conditions off the east coast 0000 UT, March 12, 1969, to 1800 UT on the same day. Airborne wave observations obtained at 1610 UT, March 12, 1969. Flight track indicated by heavy line.

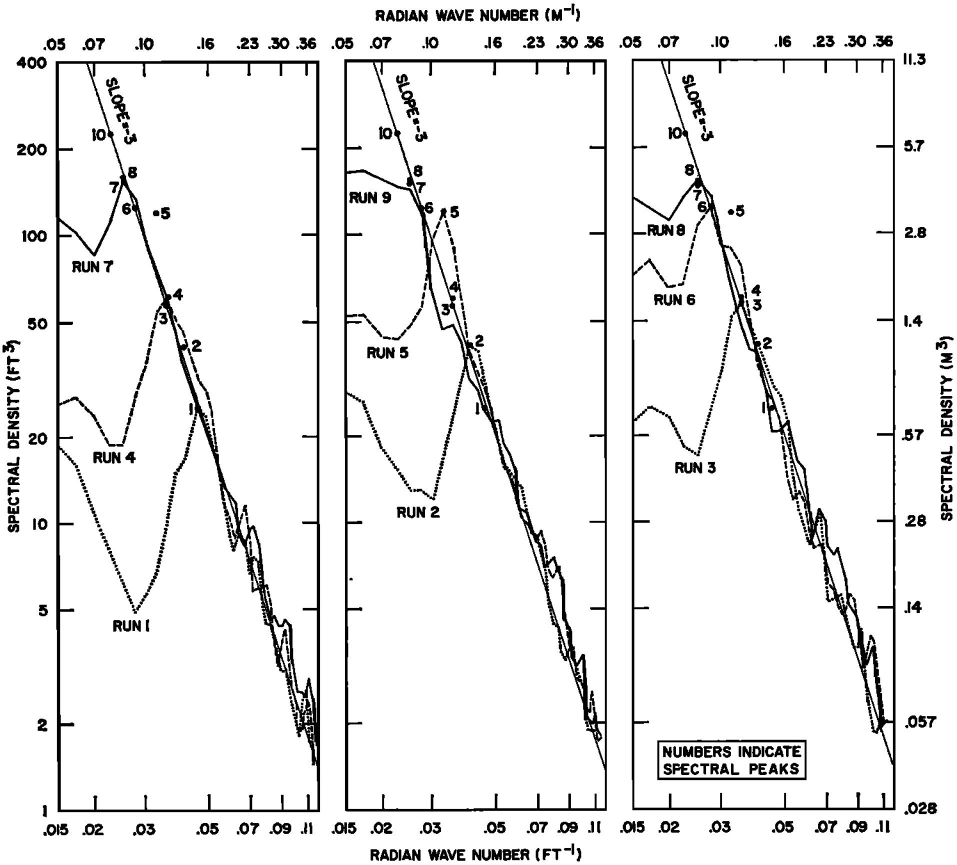

Fig. 2. One-dimensional wave-number spectra plotted as spectral density versus radian wave-number. Numbers indicate spectral peaks for each fetch (run).

THE EQUILIBRIUM RANGE

Perhaps the part of the gravity wave spectrum that has been experimentally investigated most frequently both in the field and laboratory is the saturated region at frequencies higher than the spectral peak governed by an equilibrium range. *Phillips* [1958] showed that the only dimensionally correct functional form for this range is given by equation 1

$$S(\omega) = Dg^2\omega^{-5} \qquad (1)$$

$$\omega_0 \ll \omega \ll \omega_c$$

where D is constant, g is acceleration of gravity, and ω is radian frequency; ω_0 is frequency of the spectral peak and ω_c is the frequency at which capillary waves become important. Equation 1 is an upper limit to the value of the spectral density imposed by the condition that downward acceleration of the water particles should not exceed g. The existence of an equilibrium range has been demonstrated to hold over several decades of wave spectral density [*Phillips*, 1966; *Volkov*, 1968]. It may not hold over very short gravity waves and capillary waves, particularly when strong winds occur [*Pierson*, 1970].

Barnett and Sutherland [1968] compare growth curves from field (Barnett) and laboratory (Sutherland) experiments which they refer to as 'overshoot' of a particular spectral component past its equilibrium value at a particular fetch, followed by a return of that spectral component to its eventual equilibrium value at a longer fetch. It is suggested by them (also *Hasselmann* [1968]) that 'white-capping', being highly directional, may be associated with the tendency to 'overshoot', and the equilibrium range may be more complicated than suggested by *Phillips* [1958].

In the present case, however, implicit in the nature of the one-dimensional wave-number spectrum is a tendency for its components to overshoot, which does not imply overshoot of the true wave-number spectral components. The behavior of a one-dimensional spectral component is related to the angular spreading factor associated with directional spreading of the waves. As the spectral density of a one-dimensional component of a wave number is tracked downwind in a limited fetch, as in the present case, it is observed to grow to its equilibrium value at a particular fetch distance. If the component is assumed to grow as if the directional spread were effectively unidirectional, then it will be shown below that under this assumption $S(k_1) \rightarrow Bk_1^{-3}$ where $S(k_1)$ is the one-dimensional wave-number spectral density and B is a constant. If the fetch is somewhat longer and the directional distribution of this wave-number spectral component is assumed to widen toward isotropy, then $S(k_1) \rightarrow B/2k_1^{-3}$.

EFFECTS OF ANGULAR SPREADING

As an illustration of how directional spreading may result in differing estimates of the equilibrium range constant and may explain evolution of the spatial history of a wave-number component (at least in the one-dimensional, fetch-limited case), several beamwidth functions are presented covering cases from isotropic to unidirectional spreading.

In terms of the vector wave-number $\mathbf{k} = (k_1, k_2) = (k, \theta)$ an equilibrium range can be represented by a vector wave-number spectrum

$$S(\mathbf{k}) = S(k, \theta) = BF(\theta)k^{-4} \qquad (2)$$

$$k_p \ll k \ll k_c$$

where B is a constant, θ is an angle specifying the direction of the wave-number vector $\mathbf{k}$ and is equal to zero along the mean wind direction. $F(\theta)$ is functionally determined by the directional distribution of wave numbers much higher than k_p, the wave number at the spectral peak, and much lower than k_c, the wave number at which capillary waves become important. Equation 2 implies the existence of sharp wave crests at the sea surface as the wave field approaches a saturated condition. *Phillips* [1958], however, excluded from consideration the type of instability in which sharp crests may be blown off by very strong winds. Under these conditions, functional forms for higher wave-number spectra may require considerable modification. *Pierson* [1970] discusses some of these possible forms; however, very little data are available on higher wave-number structure of wave spectra when very strong winds occur.

From a single downwind flight of the type described here, very little can be inferred about $F(\theta)$. An estimate of the constant B for these particular data will illustrate the numerical range of the one-dimensional wave-number spectrum as the range of $F(\theta)$ varies from wide to narrow beamwidth. Let equation 2 be written

$$S(\mathbf{k}) = B\left[\frac{1}{\pi}\int_0^\infty A(\sigma)(\cos\theta)^\sigma\, d\sigma\right]k^{-4} \qquad (3)$$

where $F(\theta)$ is given by the bracketed term and

$$(\cos\theta)^\sigma = \left(\frac{k_1}{k}\right)^\sigma$$

The function $A(\sigma)$ will be assigned particular forms having properties of a delta function associated with a value of σ. The integrand of equation 3 is somewhat similar to the cosine power law which *Longuet-Higgins et al.* [1963] and *Ewing* [1969] found to be a best fit to the directional spectrum observed with a pitch-roll buoy.

Now spectra from the downwind aircraft data allow estimates of the one-dimensional wave-number spectrum $S(k_1)$

$$S(k_1) = \int_{-\infty}^{\infty} S(\mathbf{k})\, dk_2 = \frac{B}{\pi}\int_0^\infty A(\sigma)k_1^\sigma\, d\sigma \cdot \int_{-\infty}^{\infty} \frac{dk_2}{(k_1^2 + k_2^2)^{\frac{\sigma+4}{2}}} \qquad (4)$$

In the discussion below, wave propagation is limited in the general direction $|\theta| \leq \pi/2$, i.e., the half-plane in which $\theta = 0°$ corresponds to the mean wind direction. The normalizing condition requires

$$\int_{-\pi/2}^{\pi/2} F(\theta)\, d\theta = \frac{1}{\pi}\int_0^\infty A(\sigma) \cdot \int_{-\pi/2}^{\pi/2} (\cos\theta)^\sigma\, d\theta\, d\sigma = 1 \qquad (5)$$

The integral of the cosine function with respect to θ can be easily evaluated in terms of the gamma function

$$\int_{-\pi/2}^{\pi/2} (\cos\theta)^{\sigma}\, d\theta = (\pi)^{1/2} \frac{\Gamma\left(\frac{\sigma+1}{2}\right)}{\Gamma\left(\frac{\sigma+2}{2}\right)}$$

so that equations 4 and 5 can be expressed

$$S(k_1) = \frac{Bk_1^{-3}}{(\pi)^{1/2}} \int_0^{\infty} \frac{\Gamma\left(\frac{\sigma+3}{2}\right)}{\Gamma\left(\frac{\sigma+4}{2}\right)} A(\sigma)\, d\sigma \qquad (6)$$

$$\int_{-\pi/2}^{\pi/2} F(\theta)\, d\theta$$

$$= \frac{1}{(\pi)^{1/2}} \int_0^{\infty} A(\sigma) \frac{\Gamma\left(\frac{\sigma+1}{2}\right)}{\Gamma\left(\frac{\sigma+2}{2}\right)}\, d\sigma = 1 \qquad (7)$$

Hence, to express the one-dimensional wave-number spectrum in terms of a particular angular spreading, we need only choose $A(\sigma)$ in equation 6 subject to the normalizing condition of equation 7.

CASE 1: ISOTROPIC SPREADING

$A(\sigma) = A_0 \cdot \delta(\sigma)$ where $\delta(\sigma)$ is a delta function with $A_0 = 1$ from the normalizing condition 7. Therefore, equation 6 reduces to $S(k_1) = B/2k_1^{-3}$ which agrees with *Phillips* [1966, p. 130] for the form of a one-dimensional wave-number spectrum in the equilibrium range. At least one condition necessary for the existence of an equilibrium range is independence of $S(k_1)$ on fetch as illustrated in the observations shown in Figure 3. Isotropic spreading was assumed over the range 0.069 feet^{-1} $\leq k_1 \leq$ 0.11 feet^{-1} (0.23 meters^{-1} $\leq k_1 \leq$ 0.35 meters^{-1}), considerably above the spectral peaks, but at wave numbers low enough to ensure freedom from the noise level of the laser system. A regression fit

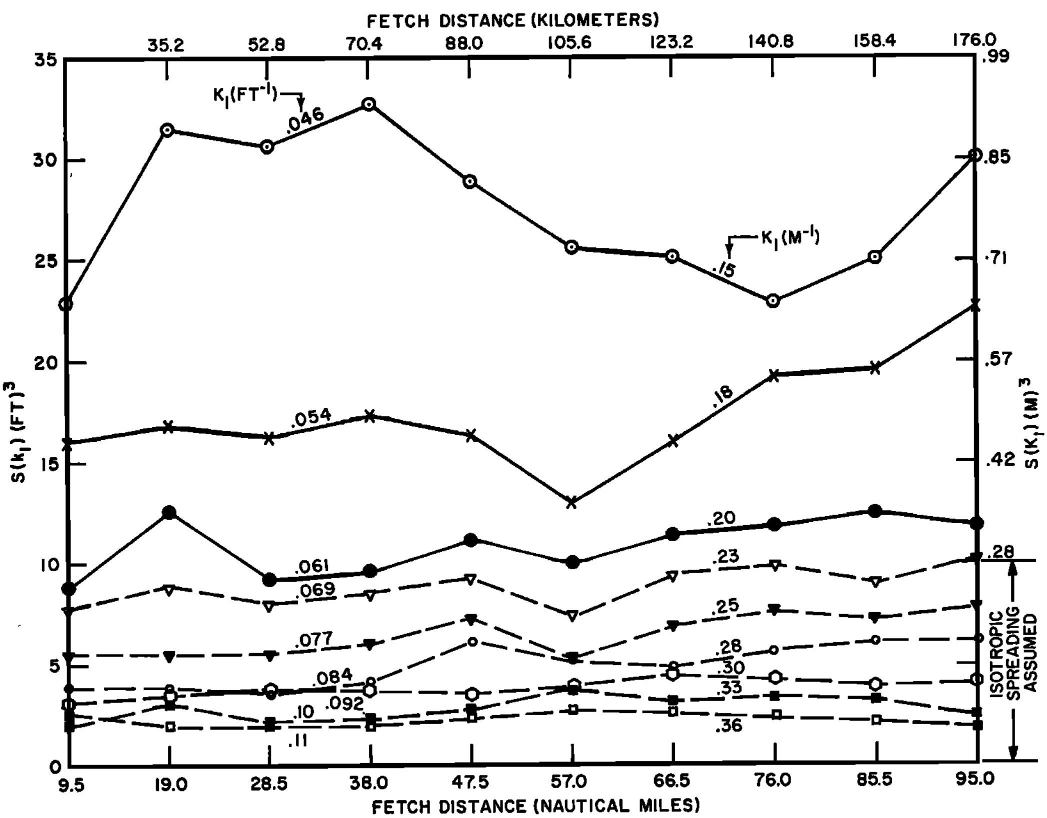

Fig. 3. One-dimensional wave-number spectra versus fetch for typical higher wave numbers.

to the average of all spectra (except run 6) over this range resulted in a value of $B = 4.2 \times 10^{-3}$ and -3.2 for the exponent. Spectral densities over this range of wave numbers in run 6 (57-nautical-mile-long fetch) did not satisfy a k_1^{-3} behavior, but did so at somewhat lower wave-numbers. Results of other field experiments, summarized by *Phillips* [1966], indicate a range of 2 to 8 $\times 10^{-3}$ for B.

CASE 2: SWOP

Pierson [1962] found that the directional spread of waves in the stereo wave observation project (Swop) followed a cosine square law of the form $A_0 + A_2 \cos^2 \theta$; hence, $A(\sigma) = A_0\delta(\sigma) + A_2\ (\delta - 2)$ with the normalizing condition requiring $A_0 + A_2 = 1$. Pierson found $A_0 = 1/2$ as a best fit; hence, $A_2 = 1$.

Table 1 summarizes the range of values attained by $S(k_1)$ subject to the choice of beam-width function $A(\sigma)$. A cosine power law, somewhat similar to case 5, was used by *Longuet-Higgins et al.* [1963] and *Ewing* [1969] to describe the directional distribution observed with a pitch-roll buoy.

In the limiting case as S, in Table 1, becomes very large, unidirectionality is approached and

$$S(k_1) \rightarrow Bk_1^{-3} = Bk^{-3}$$

i.e., when the sea surface approaches a unidirectional wave system. Thus, if a spectral wave-number component is assumed to be unidirectional (for this instance, denote the equilibrium constant by B_u) and it actually is unidirectional, then $S(k_1) = B_u k_1^{-3} = Bk_1^{-3}$. Thus, $B_u = B$ where B is the true value of the constant. However, if the unidirectional assumption was incorrect, then $S(k_1) = B_u k_1^{-3} = \gamma B k_1^{-3}$ where $1/2 \leq \gamma < 1$. Thus $B_u = \gamma B < B$, and we obtain an underestimate of the constant B. Similarly, if the angular distribution of a spectral wave-number component is assumed to be isotropic (denote the constant by B_i) and it actually is isotropic, then $S(k_1) = 1/2 B_i k_1^{-3} = 1/2 B k_1^{-3}$. Thus, $B_i = B$. However, if the isotropic assumption was incorrect, then $S(k_1) = 1/2 B_i k_1^{-3} = \gamma B k_1^{-3}$ where $1/2 < \gamma \leq 1$. Thus, $B_i = 2\gamma B > B$, and we obtain an overestimate of the constant.

Figure 4 illustrates the growth of a normalized spectral component in terms of a dimensionless fetch similar to Figure 2 of *Barnett and Suther-*

TABLE 1

Case	Type of Angular Spread	$A(\sigma)$	$S(k_1)$
1	Isotropic	$\delta(\sigma)$	$B/2\ k_1^{-3}$
2	Swop $A_0 + A_2 \cos^2 \theta$	$\frac{1}{2}\delta(\sigma) + \delta(\sigma - 2)$	$\frac{5B}{8} k_1^{-3}$
3	$(\cos \theta)^2$	$2\delta(\sigma - 2)$	$\frac{3B}{4} k_1^{-3}$
4	$(\cos \theta)^4$	$\frac{8}{3}\delta(\sigma - 4)$	$\frac{5B}{6} k_1^{-3}$
5	$(\cos \theta)^S$	$(\pi)^{1/2} \dfrac{\Gamma\left(\frac{S+2}{2}\right)}{\Gamma\left(\frac{S+1}{2}\right)} \delta(\sigma - S)$	$\left(\frac{S+1}{S+2}\right) B k_1^{-3}$
6	Delta Function $\delta(\theta)$ Undirectional	$\lim_{S\to\infty} (\pi)^{1/2} \dfrac{\Gamma\left(\frac{S+2}{2}\right)}{\Gamma\left(\frac{S+1}{2}\right)} \delta(\sigma - S)$	$\lim_{S\to\infty} \frac{(S+1)}{(S+2)} B k_1^{-3} = B k_1^{-3}$

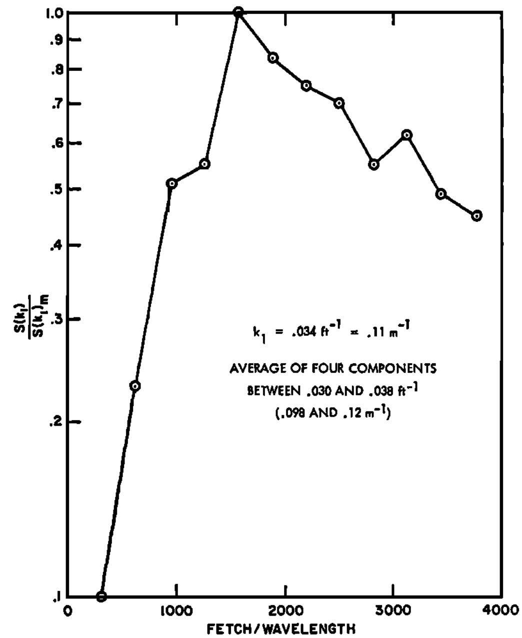

Fig. 4. Normalized spectral growth as a function of fetch divided by wavelength.

land [1968]. The ordinate is the ratio of the spectral density of the wave-number component $S(k_1)$, to the maximum value $S(k_1)_m$ attained by that component, and the abscissa is formed from the ratio of fetch to the wavelength of the component. Thus, as a component evolves as a function of fetch, and directional spreading ranges from narrow beamwidth associated with $S(k_1)_m$ to a widening beamwidth associated with $S(k_1)$, the ratio approaches one-half. *Barnett and Sutherland* [1968] observed a similar ratio in their data and concluded that a wave-wave interaction mechanism was associated with this phenomena.

Spectral Wave Growth Characteristics as a Function of Fetch Distance

There is an accumulating body of evidence that the theoretical framework initiated by theories of *Phillips* [1957] and *Miles* [1957] and subsequent papers by both these authors have provided essential guidelines required to develop a rational theory for the generation of ocean wind waves. In Phillips' theory, a flat water-surface experiences wave generation through resonant interaction of uncorrelated turbulent pressure fluctuations. Some of the generated waves travel at just the speed of the advected atmospheric pressure components, and these waves grow linearly with time. In Miles' theory the pressure field induced over existing waves results in a transfer of energy to waves that grow exponentially with time. *Miles* [1960] extended and combined his theory with the resonance model and showed that each generating mechanism dominated at a different stage of wave development. *Phillips* [1966] modified Miles' theory to include undulations in turbulent flow, stressing that the final result will undoubtedly require an interplay of various mechanisms, some of which may still be undetermined. A very general approach has been developed by *Hasselmann* [1968] in which an energy balance is governed by the radiative transfer equation. In Hasselmann's scheme, for example, processes described by Miles and Phillips are particular applications of a general theory of weak interactions that yield energy transfer for expansible interactions between the wave field and the atmosphere, and the Phillips and Miles mechanism may be viewed as two lower-order source terms associated with linear aspects of wave growth. In this paper, we shall discuss some observations of growth of spectral wave components in terms of such linear properties and assume that wave growth has not reached the stage at which nonlinear terms are dominant.

To compare our field results to the results of other investigators, wave spectra associated with one-dimensional wave-number components in the range

$$0.015 \text{ ft}^{-1} \leq k_1 \leq 0.028 \text{ ft}^{-1}$$
$$0.048 \text{ m}^{-1} \leq k_1 \leq 0.091 \text{ m}^{-1}$$

are assumed to be associated with a spreading beamwidth sufficiently narrow so that the dispersion relation $k = \omega^2/g$ can be used to map wave number spectra onto the frequency domain.

The assumption is made that a linear functional form of a continuity equation can be used to describe the evolution of the variance spectral density of a unidirectional sea-surface wave displacement $S(f, x, U, t)$ and is given by

$$\frac{\partial S(f, x, U, t)}{\partial t} + V(f)\frac{\partial S(f, x, U, t)}{\partial x} = \alpha(f, U) + \beta(f, U)S(f, x, U, t) \quad (8)$$

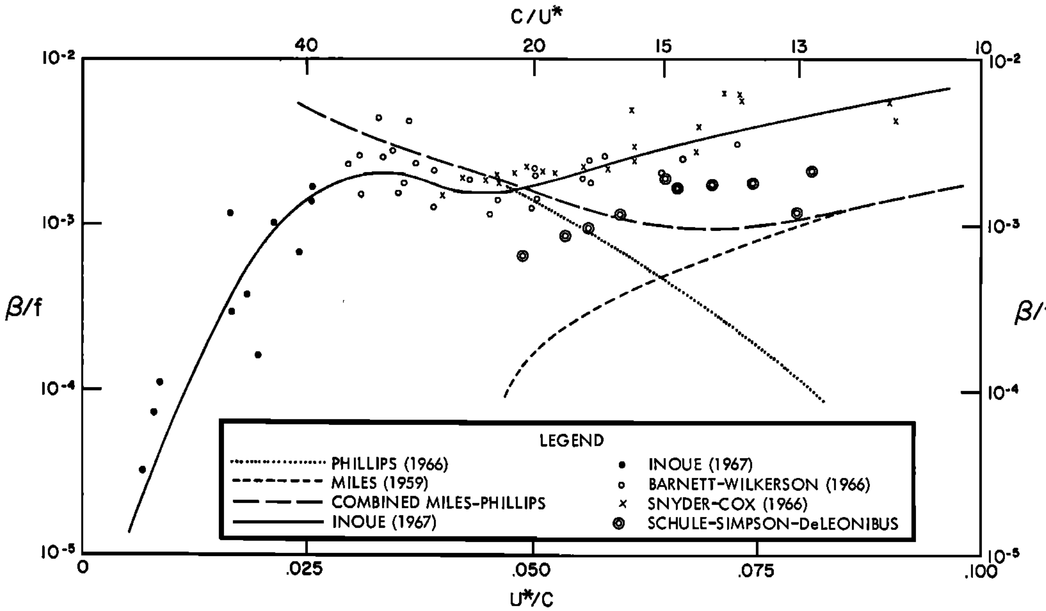

Fig. 5. Summary of studies of dimensionless growth function β/f versus dimensionless friction velocity C/U^* including results of this study (after *Inoue* [1967] and *Cardone* [1969]).

where f is frequency in cycles $\sec^{-1}$; x is downwind fetch distance seaward from the coast oriented parallel to the steady-state mean direction of the wind; $V(f)$ is group velocity of the component whose frequency is f; U is wind speed over the given fetch, and $\alpha(f, U)$, $\beta(f, U)$ are the linear and exponential growth functions, respectively. In the steady-state, fetch-limited case, $\partial S/\partial t$ is 0 and U is constant over the fetch, and equation 8 reduces to

$$V(f)\,\frac{\partial S(f, x)}{\partial x} = \alpha(f) + \beta(f)S(f, x) \tag{9}$$

It is further assumed that each component is well below its saturated value. With the boundary condition $S(f, 0) = 0$, the solution to equation 9 is expressed in closed form

$$S(f, x) = \frac{\alpha(f)}{\beta(f)}\,\{\exp\,[\beta x/V(f)] - 1\} \tag{10}$$

where $V(f) = g/4\pi f$. At sufficiently large X, the logarithmic derivative of equation 10 reduces determination of β to

$$\beta = g/2\omega\,\frac{\partial\,\ln\,S(f, x)}{\partial x} \tag{11}$$

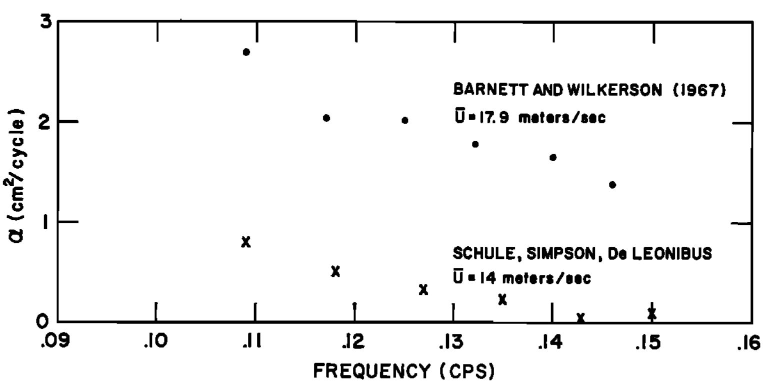

Fig. 6. Observations of linear growth function α as a function of frequency at two different wind speeds.

and β can be determined from the linear part of semi-log plots of $S(f, X)$ obtained from the aircraft data. Values of α are subsequently determined from equation 10 by a least squares regression.

RESULTS

In their recent studies, *Inoue* [1967] and *Cardone* [1969] synthesized theory and field observations by investigating a dimensionless β/f as function of C/U^* as suggested by *Phillips* [1966]. Results of this study, superimposed on their composite graph, are shown in Figure 5. These observations of β/f lie somewhat above the Miles-Phillips prediction and below the empirical relationship determined by *Inoue* [1967] as the best fit through combined field results of *Snyder and Cox* [1966], *Barnett and Wilkerson* [1967], British weather ship data, and Argus Island tower data.

Comparatively few frequencies were available to study the linear growth function α since much of the data lay in the range $12 \leq C/U^* \leq 21$, where Miles' mechanism is particularly effective and spectral growth of these components is rather efficiently 'tripped' over into the exponential growth region. The observed values in Figure 6 are comparable with, but lower than, values determined by Barnett and Wilkerson, since their mean wind speeds were nearly 18 m sec^{-1} compared to 14 m sec^{-1} in the present study.

No attempt was made to evaluate these observed values of α versus frequency in terms of atmospheric pressure measurements conducted over a land site by *Priestley* [1965]. The re-

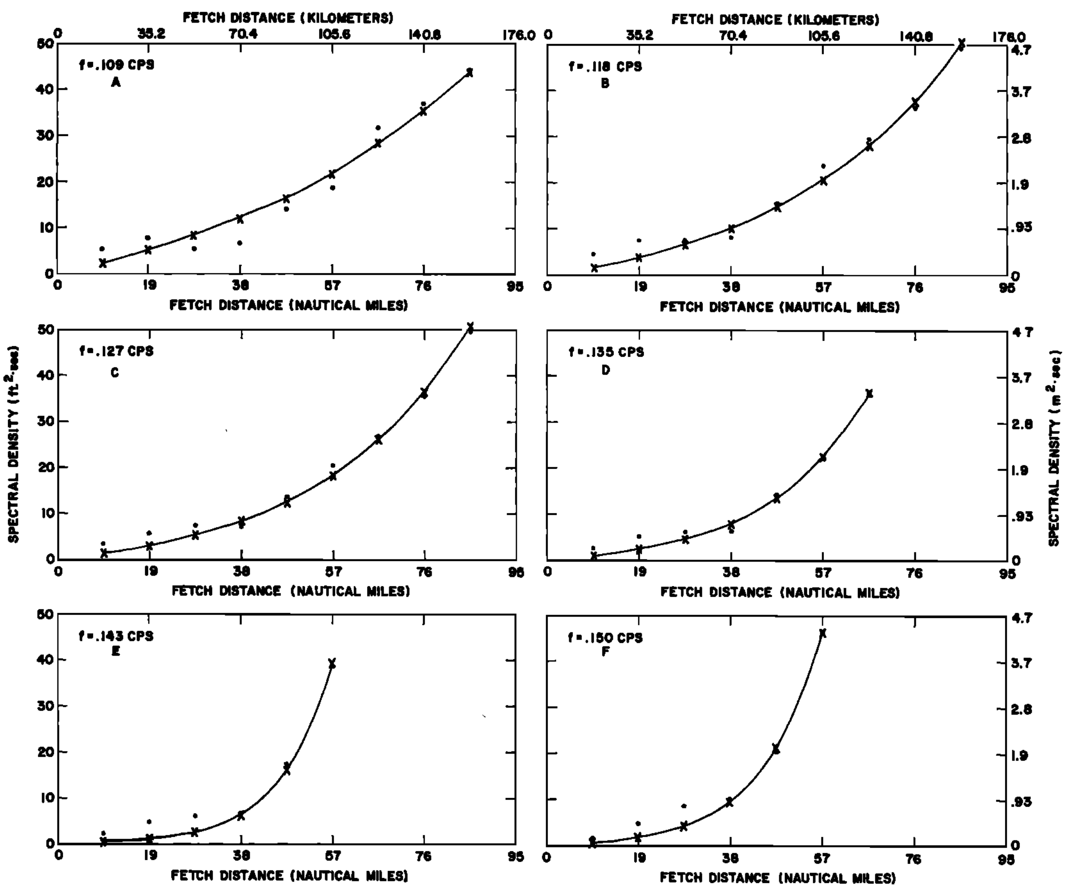

Fig. 7. Comparison of observed spectral growth (closed circles), to growth curves computed by equation 10 (solid line).

ports by Snyder and Cox and Barnett and Wilkerson have confirmed the functional form of α-dependence on frequency through Priestley's data, and it would seem that further confirmation would require measurements of atmospheric pressure spectra simultaneously with wave spectra over an ocean surface.

Observed values of α and β were used to generate spectral growth curves by equation 10 and compared to observed growth curves as a function of fetch for six different frequencies. Six examples, in Figure 7, illustrate the range in which linear theory is in agreement with observations. At the very shortest fetch distances, predicted growth curves underestimate observed wave growth. For example, in Figures 7*C* through 7*F*, wave growth out to 38 nautical miles appears to be influenced by some other factor, in addition to external sources indicated by equation 8. However, these are rather low spectral-density values, and it is possible they include some residual wave energy associated with past disturbances.

CONCLUSIONS

Interpretation of one-dimensional wave-number spectra is critically dependent on assumed angular spreading factors. Over the wave number range $0.069\ \text{ft}^{-1} \leq k_1 \leq 0.11\ \text{ft}^{-1}$ ($0.23\ \text{m}^{-1} \leq k_1 \leq 0.35\ \text{m}^{-1}$), where $S(k_1)$ was independent of fetch, an assumption of isotropy could be made with reasonable confidence. For this wave-number range, the equilibrium range constant was estimated at $B = 4.2 \times 10^{-3}$.

A possible explanation, based on angular spreading characteristics, is offered for the overshoot observed in one-dimensional wave-number spectra. The existence of overshoot observed in one-dimensional wave-number spectra does not imply its existence in the true wave-number spectra.

Reasonably good agreement between observations of the exponential growth of a wave and predictions of the modified Miles-Phillips theory was found in the range $12 \leq C/U^* \leq 21$.

Observed values of the linear growth function α in the frequency range $0.11 \leq f \leq 0.15\ \text{sec}^{-1}$ appear to be consistent with and comparable to observations of Barnett and Wilkerson. This function, however, requires further study over an oceanic site.

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