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CURVATURE DISTRIBUTIONS OF WIND-CREATED WATER WAVES

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Abstract--Measurements made using a small water-wind tunnel, having a maximum fetch of 26 inches, indicate that water waves are not formed until the wind velocity reaches approximately seven knots. After this threshold there is a sudden build-up of the waves. Between seven and 20 knots, the mean curvature of the surface c in cm^{-1} is related to the wind velocity v in knots by the expression $c = 0.046v$. The standard deviation σ in cm^{-1} of the curvature is related to the wind velocity by the expression $\sigma = 0.036v - 0.11$. The momental skewness of the curvature distribution was positive and averaged about 0.8 standard deviation unit.

Introduction--The reflection and scattering of light, radio, and sound waves impinging upon a wind-disturbed water surface depend upon the orientation and sizes of the scattering elements. It is therefore desirable to measure the distributions of the sizes and slopes of facets of the sea surface.

COX and MUNK [1954] and SCHOOLEY [1954] have studied the statistical distribution of water surface slopes for various wind velocities by two different methods and are in substantial agreement on this aspect of the problem. The distributions of facet sizes still need to be determined.

In this paper a simple optical method is described for taking data related to the distribution of water facet sizes. The term 'facet size' is really ambiguous because a wind-disturbed water surface has perfectly flat facets only over infinitesimal areas. Thus, there is a need for a more meaningful measure of facet size that includes the concept of how much the various facets deviate from being perfectly flat. It appears that this can be done by measuring the radii-of-curvature of the facets.

Method--Figure 1 is a schematic presentation of the photographic method that has been used for determining the distributions of curvature and radius of curvature of water waves. C represents a camera with film F and lens L directed towards water surface W. The water surface is illuminated by a light source of diameter ℓ_o. The light source is shown co-axial with L indicating that actual parallax between the light source and the lens has been neglected. An image (diameter ℓ_i) of the light source is formed by reflection from the water surface. The camera records an image of diameter ℓ_c on the film F. It can be shown that

$$\ell_c \cong (r\,\ell_o d_c)/[2\,d_o\,(r \pm d_o)] \quad \ldots\ldots (1)$$

where

ℓ_c = camera image size
r = radius of curvature of the water facet
ℓ_o = size of illuminating source
d_c = camera lens to film distance
d_o = lens-to-water distance

The second term in the denominator is positive for a convex facet and negative for a concave facet.

The following assumptions were made in deriving (1): (a) the facets may be represented as spherical mirrors having small apertures; (b) the lens-to-water distance is a constant; (c) the lens aperture is small; (d) there is negligible parallax between the light source and the lens; and (e) the light and the lens are at the same distance from the water surface.

The water facet shown in Figure 1 is on the axis of the camera and hence ℓ_c is in the center of the film F. Projected images of facets at angles off the axis will be approximately proportionally displaced from the center of the film if the angles are less than about 20° [SCHOOLEY, 1954].

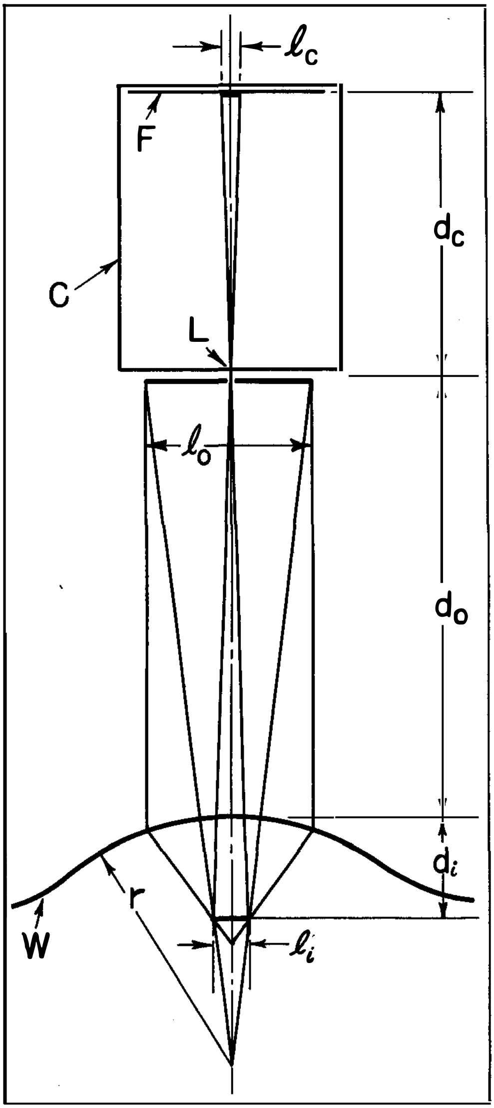

Fig. 1--Geometry for measuring radius of curvature

Figure 2 shows (1) plotted with camera image size ℓ_c as abscissa for the conditions of convex and concave facets. The constants indicated on the figure were used because preliminary experiments had indicated that the region of principal interest was between 0.03 and 0.3 ft radius of curvature. With the constants chosen, the curves for convex and concave facets are substantially the same in the region of interest, and the minimum camera image size is resolvable with readily available photographic equipment.

Apparatus--In order to meet the conditions set forth in the preceding paragraph, the simple water-wind tunnel of Figure 3 was constructed. The clear plastic-walled channel between the two end containers is three inches wide. The water depth is about seven inches. Above the water is an air space of about three inches. Air is drawn from left to right by the two vacuum-cleaner blowers at the upper right. The wind velocity was measured by a Type P Portable Air Meter of the Hasting Instrument Company pictured in the right foreground. The probe for this meter is shown inserted in the airstream and held by the ringstand clamp just behind the water-wind channel. The velocity of the airstream was adjusted by means of a Variac used to control the speed of the blower motors. The waves traveling into the right container were largely absorbed by a screen at its entrance. The photographic equipment (not shown) consisted of a 4 × 5-inch speed Graphic with a six-inch Ektar lens set at f8. A focal plane shutter speed of 1/1000 sec was used. The camera was mounted directly above the center of the channel. The light source was a GE no. 31 flash bulb masked down to 2.25-inch diameter and held adjacent to the camera lens. The slight tilt of the transparent top of the channel shown in Figure 3 was used to throw the reflections from its surfaces outside the field of view of the camera.

Figure 4a shows a picture of the quiet water surface with no wind blowing. The diameter of the center light image corresponds to the right hand flat portion of Figure 2. The other images are due to the curvature of the water at the walls of the channel and may be disregarded. As the wind speed is gradually increased, the central image will remain intact until it suddenly breaks up in the region of seven knots. Figure 4b shows the onset of this breakup. For higher wind velocities, the pictures resemble Figure 4c which was taken at a wind speed of about 11 knots.

Data analysis--Examination of Figures 4b and 4c shows that the images are irregular and not circular. Thus it is evident that the surface is not made up of small portions of many perfect spheres, and the spherical concept is only a first approximation. The images are usually longer across the channel than along the channel. This indicates that the average radius of curvature of the waves is greater when measured across the wind than when measured with the wind. All measurements presented in this paper were made to the outer edges of the facets in the direction of the wind.

About 24 pictures were analyzed by measuring the size of the image of the individual facets for various wind velocities. The maximum widths of the facets were measured in the direction of the wind for a central one degree wide strip extending 10° upwind and 15° downwind. In the water-

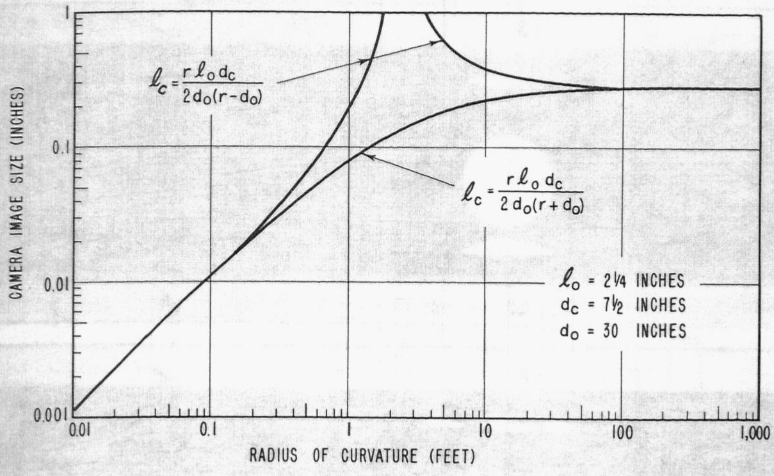

Fig. 2--Relation between radius of curvature and camera image size

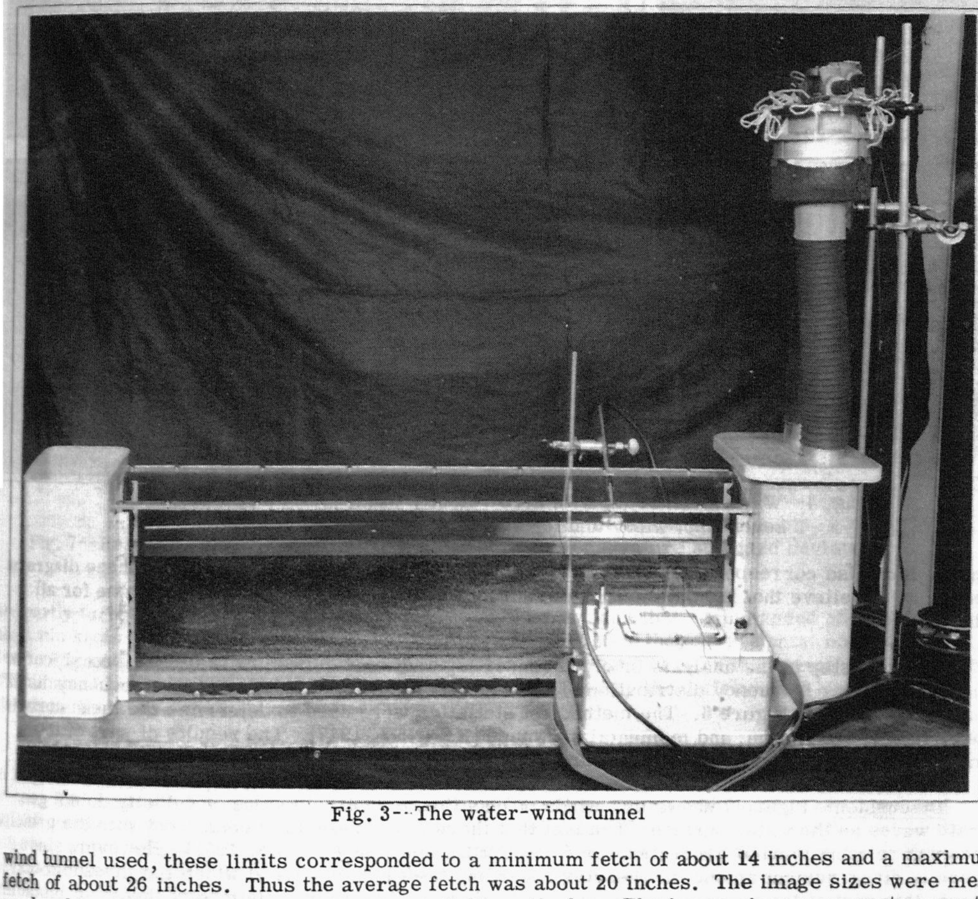

Fig. 3--The water-wind tunnel

wind tunnel used, these limits corresponded to a minimum fetch of about 14 inches and a maximum fetch of about 26 inches. Thus the average fetch was about 20 inches. The image sizes were measured under a low power microscope using suitable reticules. The image sizes were next converted to radii of curvature by means of Figure 2, and made into spot diagrams, one of which is shown in Figure 5. Each spot on this diagram represents the radius of curvature of a facet at a particular

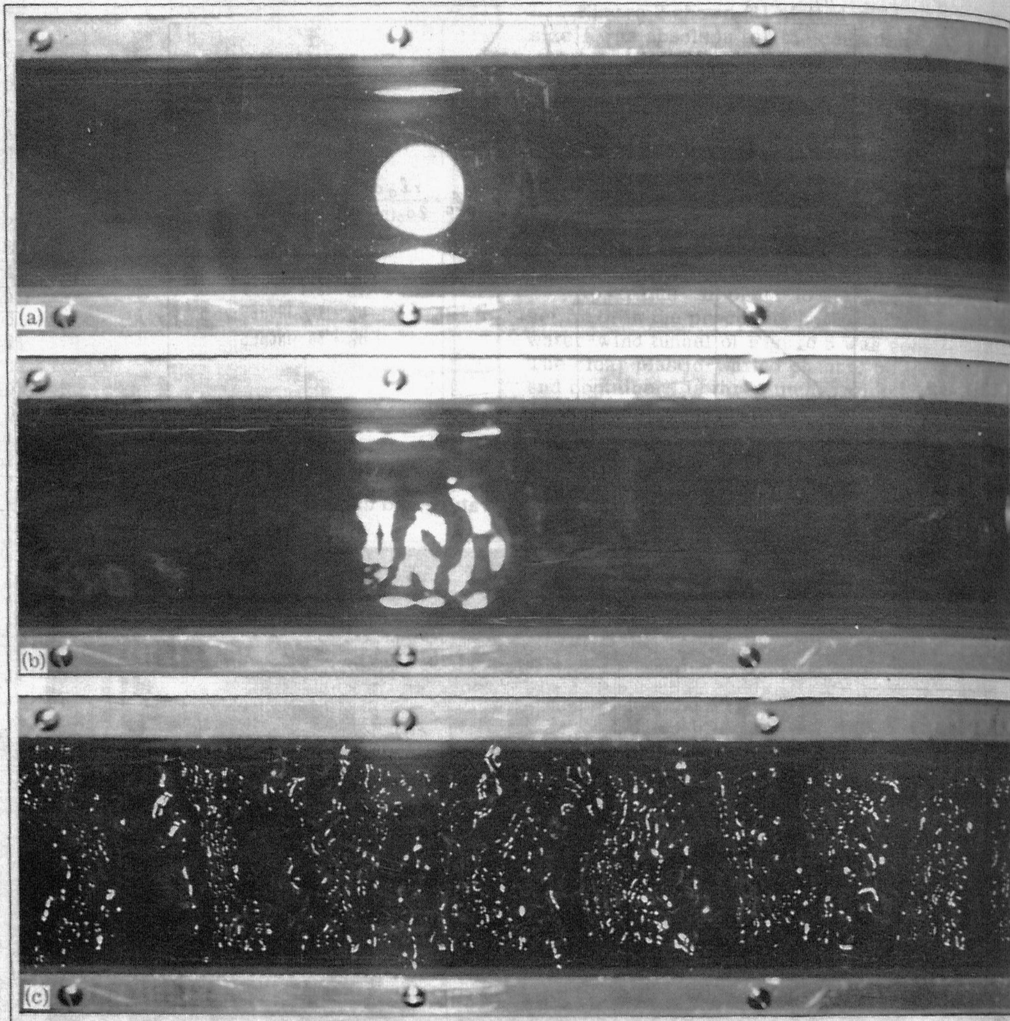

Fig. 4--Water surface (a) with zero velocity wind; (b) with approximately seven-knot wind; and (c) with approximately 11-knot wind

angle which also corresponds to the slope of the facet. An examination of many of these diagrams leads one to believe that the distribution of radii of curvature is substantially the same for all slopes.

The next step in the analysis involved converting radius of curvature to its reciprocal, curvature, and making frequency distributions from the spot diagram data. One of the frequency distributions is shown in Figure 6. The methods of statistics were used to determine the mean curvature, standard deviation, and momental skewness [KENNEY, 1947]. The results of part of this analysis are summarized in Figures 7 and 8.

Discussion--Figure 7 shows that winds of less than a threshold value of velocity do not generate waves on the water surface. The fact that there is a threshold is consistent with the principal theories relating to wind-generated waves [RUSSELL and MACMILLAN, 1953]. However, since there is disagreement among the theories regarding the wind velocity at which the threshold takes place, this part of the experimental curve is of interest. The Kelvin-Helmholtz theory of instability which assumes laminar flow, predicts that waves should start to appear at a wind velocity of about 13 knots. The sheltering theory of Jeffreys, based on the assumption that the flow is partly laminar

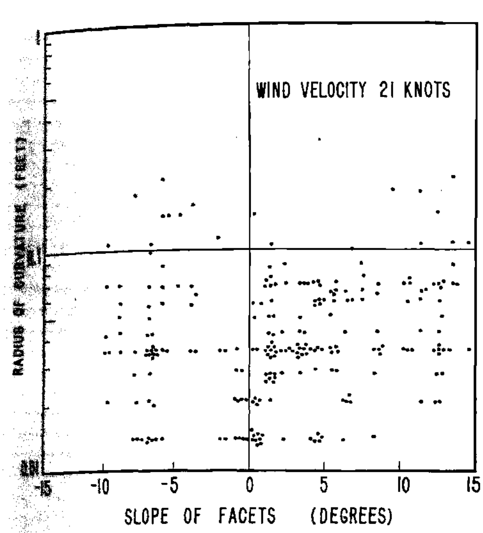

Fig. 5--Distribution of radii of curvature for various facet slopes

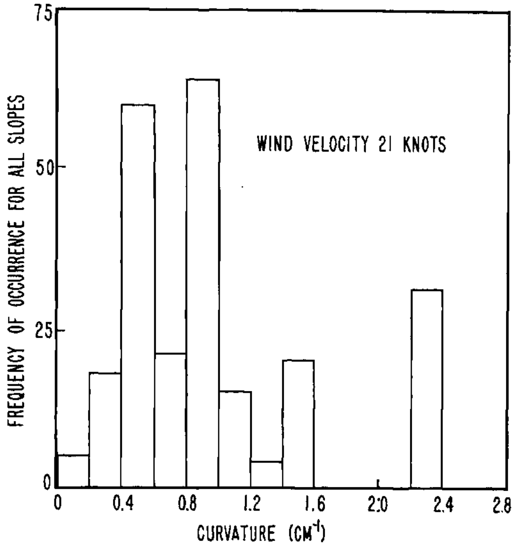

Fig. 6--Frequency distribution of curvature

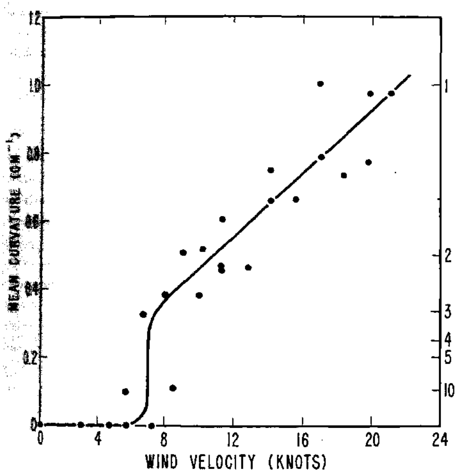

Fig. 7--Relation between wind velocity and mean curvature

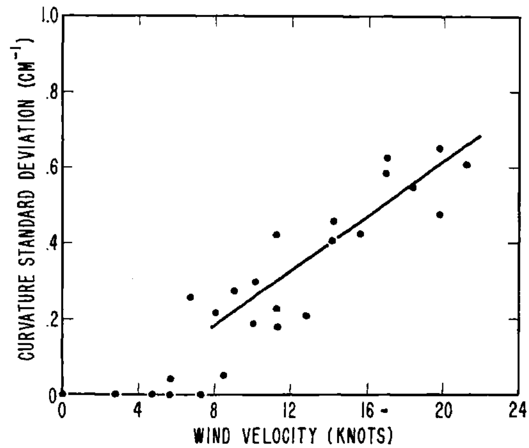

Fig. 8--Relation between wind velocity and curvature standard deviation

and partly turbulent, has been used to explain some observations that a wind speed of as low as about two knots will cause waves to appear on still water. Under the experimental conditions that lead to Figure 7, the critical wave-forming wind velocity appears to be in the region of seven knots. This is about half way between the velocities that are justified by the Jeffreys and by the Kelvin-Helmholtz theories.

The experimental scatter of the points in Figure 7, combined with the lack of adequate theory to help determine what the trend of the data should be, makes it difficult to draw the curve that is shown. The straight-line portion of the curve could have been drawn with a range of slopes and still be reasonably consistent with the small sampling of data. For simplicity, a straight line was selected, which, if projected, would intersect the origin. The equation of this line is $c = 0.046v$, where c is the curvature in cm^{-1} and v is the wind velocity in knots.

In Figure 8 the curvature standard deviation σ is plotted as ordinate with wind velocity as abscissa. The threshold effect is still evident but not in as striking a manner as in Figure 7. The approximate average curve for the data above seven knots still appears to be a straight line. However, its intercept with the σ-axis appears to be less than zero. The equation for the line that is shown is $\sigma = 0.036v - 0.11$.

According to private correspondence with Munk, Neumann's energy spectrum proposed on the basis of amplitude observations may be used to predict a standard deviation or rms value of 0.1 cm^{-1} curvature that is independent of wind speed. It is interesting to note that Figure 8 shows that the curvature standard deviation was not found to be constant for wind velocities above the threshold. The rate of increase was 0.036 cm^{-1}/knot. Figure 8 also indicates that above the threshold the curvature standard deviation is always larger than predicted by Neumann's theory.

The momental skewness of the curvature distributions for various wind velocities was calculated. Above the threshold velocity of about seven knots, the skewness points scattered in a more or less random fashion between about 0.3 and 1.3 standard deviation units. The skewness was consistent in that all points were positive.

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