A Simple Optical Method for Measuring the Statistical Distribution of Water Surface Slopes

ALLEN H. SCHOOLEY
U. S. Naval Research Laboratory, Washington, D. C.
(Received May 18, 1953)

A flash-photography technique for determining the statistical distribution of slopes of wind-created waves is described. The technique consists of taking photographs with a flash camera directed vertically downward toward the water surface. The resulting photographs are flash reflection patterns that can be readily interpreted in terms of wave slopes. Wave-slope dispersion curves, as determined from daytime flash photographs taken from a bridge about 45 feet above the water, are given for wind velocities up to 20 knots. For example, the standard deviation of the across-wind wave slopes was found to be approximately 2.5, 5, and 7.8 degrees for wind velocities of 5, 10, and 20 knots, respectively. Under the same conditions the with-wind wave-slope standard deviation was found to be 4.2, 7.5, and 10 degrees.

THE reflection and scattering of light, radio, and sound waves as they impinge upon a water surface depend upon the orientation of the scattering elements and the sizes and shapes of these elements. Therefore, if a satisfactory understanding of the reflection and scattering phenomena is to be attained, the distributions of the sizes, shapes, and slopes of the facets of the sea surface must be measured.

In this paper a simple optical method is described for taking data on the distribution of water surface slopes. It is a special case of the method for measuring the roughness of the sea surface from photographs of the sun's glitter which has been used by Hulburt,[1] and by Cox and Munk.[2] The special case is accomplished by placing the "sun" at the camera position through the use of a flash gun attached to the camera. The technique is illustrated by Fig. 1. C represents a camera with film F and lens L directed vertically downward towards the water surface W. It is assumed that the flash bulb is approximately a point source located close enough to the lens, as compared with the distance from the camera to the water, so that parallax can be neglected. Thus, the light and the lens are both considered to be at point L. The light from L is projected downward and is reflected from the water surface. If the water surface is perfectly calm, the light reflected from the portion of the surface directly below the lens of the camera will enter the camera and be focused as a spot in the center of the film. All other rays from the flash reflected from other portions of the surface will not be intercepted by the lens and hence will not reach the film.

As the surface of the water becomes rough, it is evident that reflections from waves having a given slope are intercepted by the lens only if the wave facet is perpendicular to a line drawn between the facet and the lens. Consequently, the distance d of the sparkle image from the center of the film is related to the slope α of the water wave facet by the equation

$$\alpha = \tan^{-1} d/l, \tag{1}$$

where l is the focal length of the camera lens.

It is interesting to note that the height of the camera above the water is not a factor in Eq. (1). Also, for $\alpha < 20°$, Eq. (1) can be approximated as

$$\alpha \simeq d/l, \tag{2}$$

with a resulting error of less than about 5 percent.

Pictures of the surface of the Anacostia River were taken from the South Capitol Street Bridge in Washington, D. C. to try out the flash-camera technique for measuring the distribution of wave slopes. Figure 2 shows the reflection from the water when the surface is relatively calm. The sparkle image in the center of the 10°-diameter circle subtends about $\frac{1}{2}$ degree. Figure 3 shows a sample of the wave-slope distribution with a 7-knot wind blowing in the direction of the arrowhead. It will be noted that the distribution has greater dispersion with the wind then across the wind.

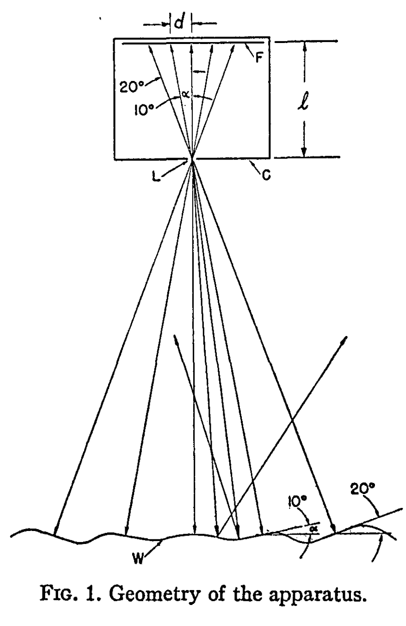

FIG. 1. Geometry of the apparatus.

[1] E. O. Hulburt, J. Opt. Soc. Am. 24, 35–42 (1943).
[2] C. Cox and W. Munk, "The measurement of the roughness of the sea surface from photographs of the sun's glitter; Part 1—The Method," Scripps Institution of Oceanography Report, November 24, 1952, on U. S. Air Force Contract No. AF19(122)–413.

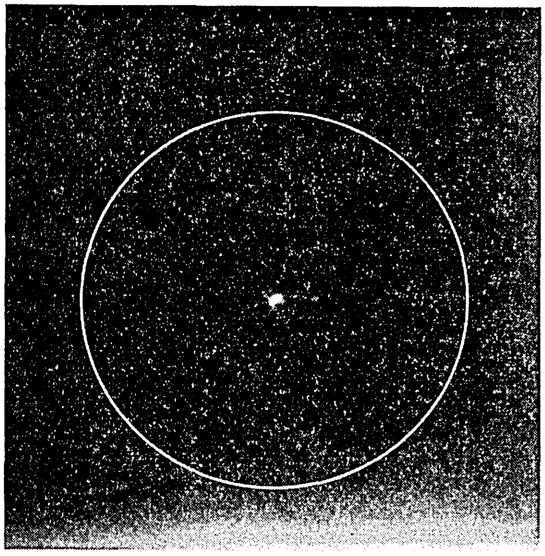

FIG. 2. Night flash picture of calm water surface of Anacostia River.

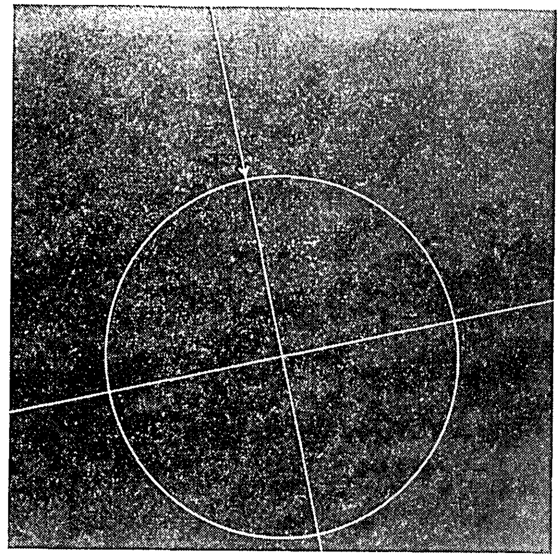

FIG. 4. Daytime flash picture of Anacostia River, wind 6 knots.

The pictures in Figs. 1 and 2 were exposed at night with a $2\frac{1}{4}$-in. $\times 3\frac{1}{4}$-in. view camera using open flash with Wabash No. 0 flash bulbs. The 10.5-cm lens was focused at ∞ and set at f/6.3. The film was Ansco Supreme having an ASA esposure index of about 80. Normal developing was used. The camera-water distance was about 45 feet.

If the flash method for determining wave slopes is to be useful in sea-state studies, it is highly desirable that pictures be taken during the day as well as at night. Figure 4 is such a picture; it was taken at 10 A.M. on October 13, 1952. The sky was clear and the wind was about 6 knots, as determined by the nearby Anacostia Naval Air Station. Film with an exposure index of 80 was used in a Speed Graphic camera having a lens of 15.2-cm focal length. Synchronized flash was used with a shutter speed of 1/100 second at f/16 and a Sylvania No. 2 flash bulb. The negative was developed normally, but contrast was enhanced in making the print. Both the sparkle pattern and the usual visual wave pattern are visible in the print. It is interesting to observe that the reflections at the greater angles tend to be from the tops of the larger waves. It is obvious that the sun-glitter pattern was outside the field of view of the camera in Fig. 4. Flash-sparkle pictures will be confused and of limited value at times when sun glitter falls within the field of view of the camera.

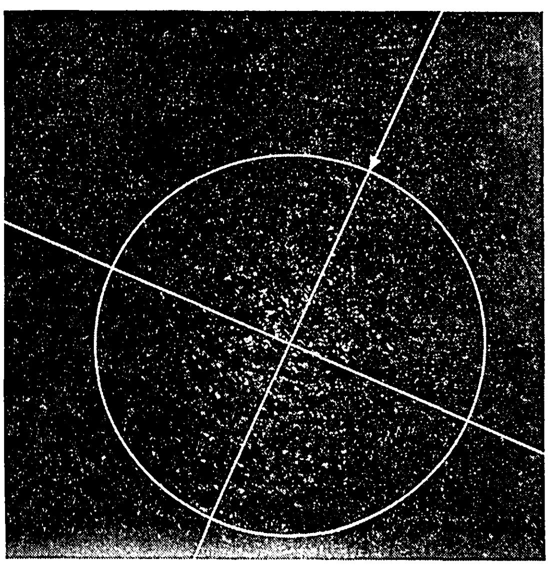

FIG. 3. Same as Fig. 2 except wind 7 knots.

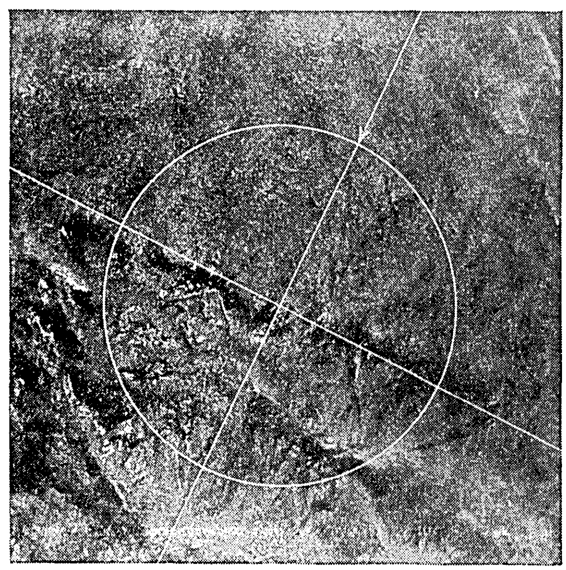

FIG. 5. Same as Fig. 4 except wind 30–35 knots.

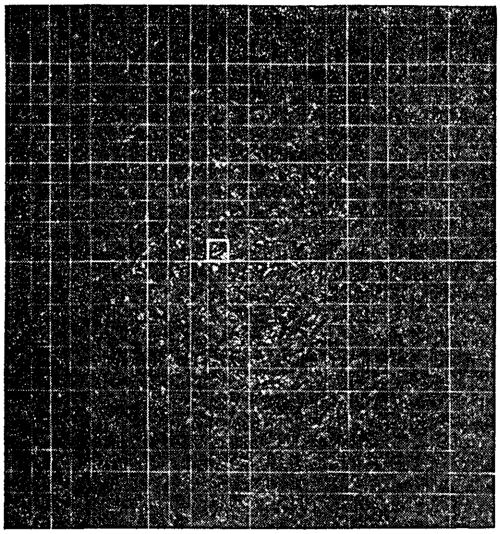

FIG. 6. Flash-sparkle picture of Anacostia River with 8-knot wind and an overlay ruled to 1-degree intervals.

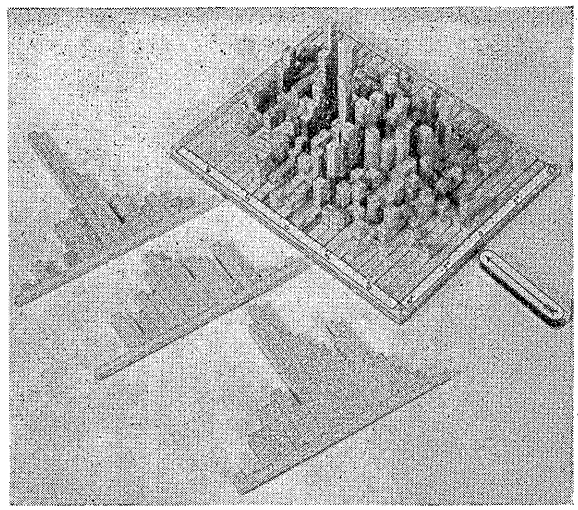

FIG. 8. Model representing wave-slope-probability density of Fig. 6.

Figure 5 shows how the texture of the sparkle pattern is changed by the reduction of facet size when a gust of 30 to 35 knots churns up the water surface. This change in texture was not noticeable for wind velocities of less than about 25 knots.

Casual observation of the sparkle-pattern pictures yields some qualitative information. Figures 6 and 7 show a method whereby quantitative information can be obtained. Figure 6 is a picture which was taken on an overcast day with the wind blowing at about 8 knots. Overlaying the picture is a Lucite grid having lines ruled at approximately one-degree intervals in both directions. Figure 7 shows an enlarged view of the accentuated square of Fig. 6 with a small movable reticle placed so that it divides an area of approximately one square degree into 100 parts. By viewing this reticle with a low-power microscope it is possible to estimate the relative area of sparkles for any given area on the sparkle picture. After this is done over the area of the complete picture, a three-dimensional model such as that shown in Fig. 8 may be constructed. The height dimension is in arbitrary units of area of sparkle per unit area at intervals of one degree. If a sample were used of a calm surface with no wind, the model would consist of a single one-degree square shaft in the center. The arrow in Fig. 8 indicates the approximate direction of the 8-knot wind. Three one-degree slices

FIG. 7. Enlargement of accentuated square in Fig. 6 with reticle dividing one square degree into 100 parts.

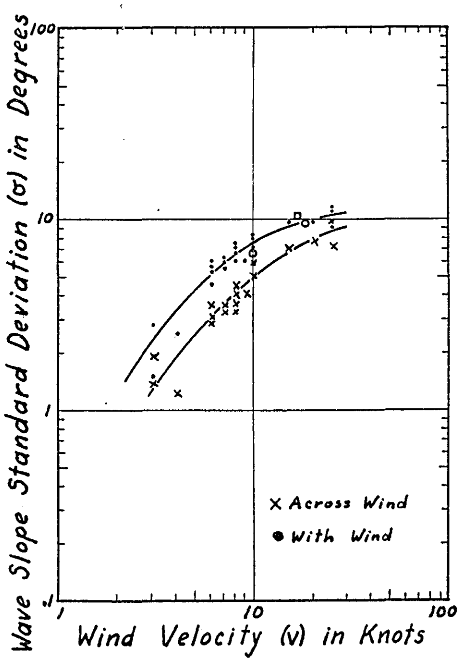

FIG. 9. Experimental with-wind and across-wind wave-slope standard deviation curves for various wind velocities.

have been removed from the model and laid on their side so that representative distributions across the wind can be seen. It is indicated that the average probability density of slopes can probably be approximated by a normal distribution in the cross-wind direction. It is noticeable in the actual model of Fig. 8 that the probability density of slopes in the direction of the wind is not normal but is slightly skewed away from the direction the wind is blowing. This confirms the observations of Duntley.[3]

Suppose the simplifying assumption is made that the probability density of slopes can be approximated by a two-dimensional normal distribution having greater dispersion in the direction of the wind than across the wind. Then by the methods of statistics[4] it is possible to determine the with-wind and across-wind standard deviations (σ) for any particular flash-sparkle picture. This requires analyzing only the central one-degree slices in the with-wind and across-wind directions. If this is done for a series of pictures taken with various wind velocities, the wave-slope dispersion as measured by the standard deviation may be plotted as a function of wind velocity. Figure 9 is such a plot made from selected pictures taken from the South Capitol Street Bridge. It must be realized that the data presented were taken under rather special conditions and therefore are of limited general value. For example, the fetch was always less than about 1500 feet, and there were boundary effects and obstacles, such as bridge piers, to interfere with wind flow and wave motion. The two points indicated by the circles on Fig. 9 are derived from Hulburt's[1] sun-path data and are seen to agree quite well with the recent flash-picture data. The one point indicated by a square is a with-wind point derived from Duntley.[3]

It is interesting to note in Fig. 9 that the standard deviation of the across-wind wave slopes is 5 degrees, and the with-wind standard deviation is 7.5 degrees for a 10-knot wind. Thus, under the conditions given, the with-wind standard deviation is 50 percent greater than the across-wind standard deviation. There is some evidence that the two curves of Fig. 9 tend to level off at the higher wind velocities. This might be expected because there is certainly an upper limit to the wave-slope standard deviation which can be attained without elimination of the definite water-air boundary.

[3] S. Q. Duntley, "The visibility of submerged objects; Part 1—Optical effects of water waves," Massachusetts Institute of Technology Report, December 15, 1950, on U. S. Office of Naval Research Report No. N5 ori-07831.

[4] J. F. Kenny, *Mathematics of Statistics* (D. Van Nostrand Company, Inc., New York, 1947), Part 1.

JOURNAL OF THE OPTICAL SOCIETY OF AMERICA VOLUME 44, NUMBER 1 JANUARY, 1954

On the Absorption Law for Samples of Nonuniform Concentration with Special Reference to the Spectroscopy of Irradiated Glasses*

IAN G. ROSS
Department of Chemistry, Florida State University, Tallahassee, Florida

(Received October 7, 1953)

The dependence of optical density upon extinction coefficient is examined for light absorption measurements in which the number of absorbing molecules encountered by each ray of the measuring light is not constant. Some general results applicable to all such situations are derived. Particular attention is given to measurements of the absorption spectra of molecules produced by transverse irradiation of rigid solutions (triplet-triplet spectra). The conditions are determined under which such absorption is most intense, and the deviation from proportionality between optical density and extinction coefficient is least.

INTRODUCTION

CONVENTIONAL methods of measuring the extinction coefficients of dissolved substances employ techniques in which each ray of the measuring light beam passes through an equal number of absorbing molecules. If this condition is not complied with, uncritical use of the ordinary absorption law leads to errors, whose nature is the subject of this communication. The conclusions reached are applied in the analysis of one type of absorption-spectrum measurement (triplet-triplet spectra) in which this question is inevitably encountered.

It is proposed to consider the absorption of light by solutions which conform to the familiar Beer-Lambert (or Bouguer) law. The term "solution" is used in a general sense, and includes vapors and glasses, as well as liquid solutions, while the conclusions reached have an obvious application to crystals as well. In the most general form, the absorption law states that, for light of any particular wavelength,

$$d\mathcal{I}/d\mathcal{I}_0=\exp\left[-\epsilon\int_0^l c(x)dx\right]=e^{-\epsilon C}. \qquad (1)$$

* The work described here was carried out under Contract NR-015-318 between the Office of Naval Research, Department of the Navy and the Florida State University.