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MARINE PHYSICS

Spatio-temporal Variability of the Amplitude-Phase Structure of Storm Waves in the Coastal Zone of the Sea

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Abstract—The spatio-temporal variability of individual storm waves in the coastal zone is studied. Wave chronograms were presented as sums of the first and second harmonics with amplitudes slowly varying in time, which are envelopes for the waves of the corresponding frequency bands. The amplitude-frequency structure of individual waves was estimated from the variations in the amplitude of the envelopes. Bi-spectral analysis was applied to estimate the phase composition of the waves. It is shown that, in the initial stage of transformation, the fluctuations of the waves of the first and second harmonics are synchronous. The amplitudes of the second harmonics are proportional to the squared amplitudes of the first harmonics. Thus, a simple model is possible for the description of individual waves based, for example, on the generalization of the Stokes theory. As the waves approach the coast, the ratio of the amplitudes of the first and second harmonics in individual waves varies in time chaotically, thus it is not possible to describe individual waves on the basis of a simple model. The main cause of the chaotic variation of the amplitudes is the backward transfer of energy from the highest harmonics to the first harmonics due to the sub-near resonant triad interaction close to the resonance. The phase composition of individual waves depends on the distance propagated by the waves and on the stage (main or backward) of the energy exchange between the nonlinear harmonics.

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INTRODUCTION

Many specific problems of coastal zone dynamics, such as the suspension of sediments and transport of deposits; the protection of coasts and artificial and natural beaches; and the construction and designing of sea channels, ports, pipelines, and other constructions require exact descriptions of wave field. Recent investigations have demonstrated that the processes of suspension and transport of deposits, as well as extreme wave loads on coastal constructions, are determined by individual waves and can never be estimated on the basis of average wave parameters. For example, the statistical moments of the highest orders and their spatio-temporal variability determine the breaking of individual waves or groups of waves, the transport of sand, and the generation of low-frequency oscillations (infragravity waves) and currents [16].

Practically all the wind and storm waves in nature are irregular. This means that the parameters of individual waves, for example, the height and the period of the waves (or the amplitude-frequency composition), vary from one wave to another. This is clearly seen in Fig. 1, which demonstrates waves (Fig. 1a) and their wavelet transformation (Fig. 1b), which is a spectral scan in time reflecting the amplitude-frequency composition of individual waves (based on the data of the Shkorpilovtsy 2007 experiment, series 37, 150 m from the coast). Usually, time averaged parameters of waves are used in practical engineering calculations, which are obtained from the spectral characteristics of the waves in a deep zone by recalculating them for the conditions of the coastal zone using semi-empirical relations (for example, [3]). However, one can easily note that the spectral composition of individual waves differs significantly from their time averaged spectra (Fig. 1c).

The greatest part of the coastal zone is considered to be an intermediate depth for the waves (with boundaries, for example, $0.005 < h/L < 0.5$, where h is the depth, and L is the wavelength). The nonlinear processes in the waves at these depths are determined by near resonant triad wave interactions when the resonance conditions are approximately satisfied for the frequencies (for example, [1]) or for the wave vectors [11]. For example,

$$\pm k_1 \pm k_2 \mp k_3 = \delta, \pm\omega_1 \pm \omega_2 \mp \omega_3 = 0,$$

where $k = k(\omega)$ is determined by the dispersion relation (k is the wavenumber, and ω is the angular frequency). In the literature, triad interactions and the resulting nonlinear harmonics are frequently called bound waves because it is considered that resonance-free waves exist simultaneously with bound waves, and δ is the differ-

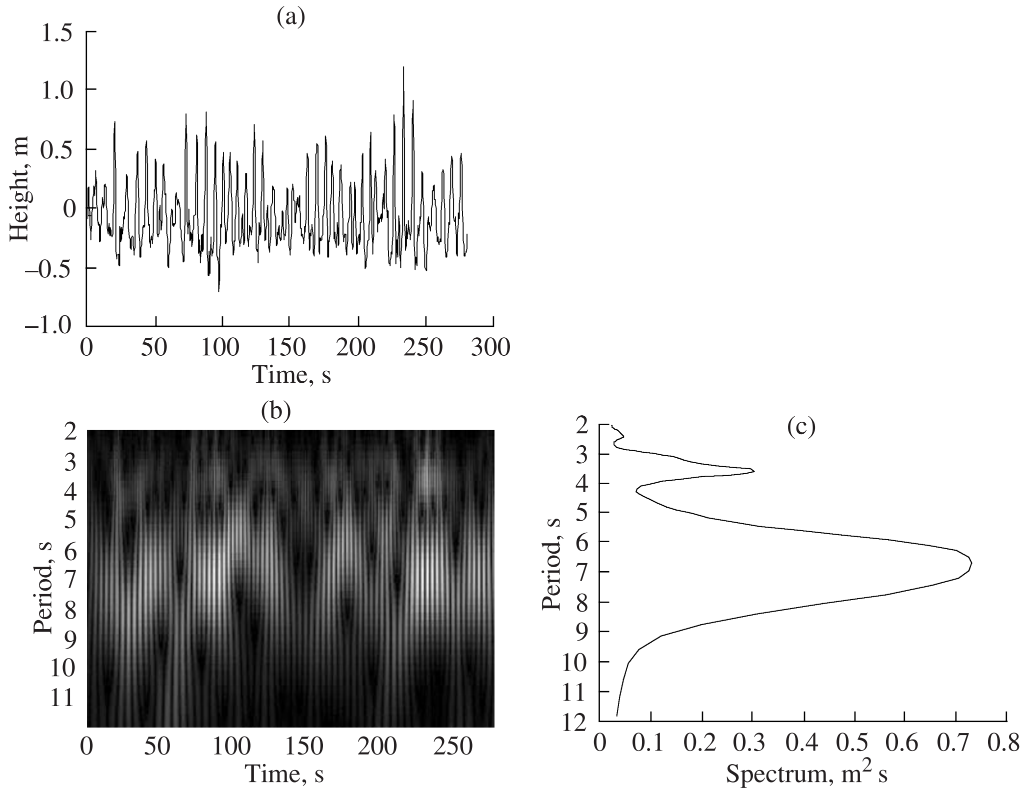

Fig. 1. Free surface elevation waves (a), their wavelet transforms (b), and the wave spectrum (c).

ence between their wavenumbers. There are currently no exact experimental proofs of the fact that the existing mismatch δ is related only to the wavenumbers or only to the frequencies of the interacting harmonics.

The main property of near-resonant nonlinear interactions is energy transfer not only from the main harmonics to higher and lower harmonics formed as a result of the total and differential interactions and backward energy transfer. This occurs periodically (the period is determined by δ) and leads to fluctuations in the amplitude of the harmonics and, correspondingly, to fluctuations of all the statistical wave moments, which are well observed experimentally.

Numerous investigations demonstrate that the waves (the amplitude-frequency structure of the waves) can change significantly during their propagation in the coastal zone owing to the depth decrease and also due to nonlinear interactions (for example, [7, 8]). When waves propagate in the coastal zone, energy exchange between the first and second harmonics, which is periodic in space, can lead to the formation of underwater bars [5]. Thus, the time and space variability of the parameters of individual waves is very important for the description and calculation of the dynamic processes in the coastal zone.

Despite evident progress in the numerical and analytical modeling of the propagation and transformation of waves, their spatio-temporal regularities of variations in the amplitude-frequency composition of individual waves are not known [4, 17]. It was shown for weakly deformed waves in the coastal zone that waves with large amplitudes of the first harmonics also have large amplitudes of higher harmonics [6]. However, it was also found that the ratio between the amplitudes of the first and high harmonics changes in time so that the waves with small amplitudes of the first harmonics can have large amplitudes of higher harmonics, but the regularities of these variations remain unknown [2, 15].

The objective of this work is to find how the amplitude-frequency composition of individual irregular waves changes in space and time, what are the regularities and physical mechanisms of these variations, and whether it is possible to construct a simple model for the description of the variability of parameters of individual waves during their transformation in the coastal zone.

METHODS OF RESEARCH

Since the waves in the coastal zone are irregular and have a group structure, the description of the amplitude-frequency composition of individual waves is a difficult problem. Therefore, we shall limit our research by considering only the first and second wave harmonics while neglecting nonlinear harmonics of higher orders. In order to describe the variations in the amplitude-frequency composition of individual waves, we shall apply the following general concept.

Using the similarity with the Stokes wave theory, the free surface elevations (wave) at an arbitrary distance from the coast can be presented as a sum of multiple (first and second) nonlinear harmonics with slowly varying in time instantaneous amplitudes a_1 and a_2 and the frequency ω:

$$w(t) = a_1(t)\cos(\omega t) + a_2(t)\cos(2\omega t + \varphi). \qquad (1)$$

In reality, the instantaneous amplitudes a_1 and a_2 are envelopes of the corresponding frequency ranges of the first and second harmonics. The frequency ranges of the first and second harmonics were determined visually on the basis of localization of the minima and maxima of the wave spectra. Such a presentation allows us to describe the time and space variability of individual waves when analyzing the variations in the phase shift φ between the first and second harmonics and the variations in the amplitudes of their envelopes.

Envelopes of the frequency range of the first and second harmonics were calculated from relation

$$e_{1,2}(t) = \sqrt{\{\mathrm{L}[w(t)]\}^2 + \{\mathrm{H}[\mathrm{L}[w(t)]]\}^2}, \qquad (2)$$

where H[.] is Hilbert's transform, L[.] is the operator of the linear filtration in the necessary frequency ranges, and $w(t)$ are free surface elevations measured in the experiment.

The variations in the amplitude-frequency composition of individual irregular waves during their propagation to the coast were estimated by analyzing the evolution of the wave spectra, the spectra of the envelopes, and the coherence functions between the envelopes of the first and second harmonics.

The correlation function was calculated as

$$C_{12}(f) = \frac{|S_{12}(f)|^2}{S_1(f)S_2(f)}, \qquad (3)$$

where $S_{12}(f)$ is the cross spectrum of the envelopes of the first and second harmonics, and $S_1(f)$ and $S_2(f)$ are spectra of the envelopes of the corresponding harmonics.

We applied bispectral analysis to estimate the nonlinear correlations between the harmonics appearing due to triad nonlinear interactions.

The bicoherence (b^2) and biphase (β) were calculated from relations [10]:

$$b^2(f_1, f_2) = \frac{|B(f_1, f_2)|^2}{E[|W(f_1)W(f_2)|^2]E[|W(f_1 + f_2)|^2]}, \qquad (4)$$

$$\beta(f_1, f_2) = \arctan\left[\frac{\mathrm{Im}\{B(f_1, f_2)\}}{\mathrm{Re}\{B(f_1, f_2)\}}\right], \qquad (5)$$

where $B(f_1, f_2)$ is the bispectrum, E[.] is the averaging operator, and $W(f)$ is the Fourier transform of the measured free surface elevations $w(t)$.

BICHROMATIC WAVES: LABORATORY EXPERIMENT

Let us analyze the behavior of the first and second harmonics ($a_1(t)$ and $a_2(t)$) using a simple example of the transformation of waves with a group structure of the bichromatic waves over a flat bottom with constant depth ($kh \approx 1$).

We shall use the data of the laboratory experiment carried out in wave flume of the Institute of Hydroengineering of the Polish Academy of Sciences (Gdansk) in November 2005. The length of the wave flume is 64 m; its width is 0.8 m. The waves (free surface elevations) were measured synchronously at 15 points of the wave flume starting at a distance of 4 m from the wave generator with an interval of 3 m up to a distance of 47 m. Wire gauges were used for the wave recording: eight capacity gauges and seven resistance gauges. The duration of the measured series of the free surface elevation was 4 min; the sampling rate was 200 Hz.

Let us consider a typical example of propagation of bichromatic waves. At the initial moment, they consisted of two sinusoidal waves with frequencies of 0.5 and 0.52 Hz and amplitudes of 4 cm over a horizontal bottom at a depth of 0.3 m. The waves propagated without breaking.

The evolution of the spectra of the initially bichromatic waves during their propagation in the wave flume is shown in Fig. 2. It is seen clearly that, as the waves transform, periodic energy exchange occurs between the first and second harmonics, which is manifested in the periodic variations of their amplitudes. For example, at a distance of 4 m from the wave generator, the wave spectrum has clearly pronounced first harmonics (frequency range 0.5–0.52 Hz) and second harmonics (frequency range 1–1.04 Hz). At a distance of 7 m, the amplitudes of the first harmonics increased, while the amplitudes of the second harmonics decreased owing to the backward energy transfer caused by the differential near resonant nonlinear triad interactions. After this, at a distance of 10 m, the amplitudes of the second harmonics increased again, and the amplitudes of the first

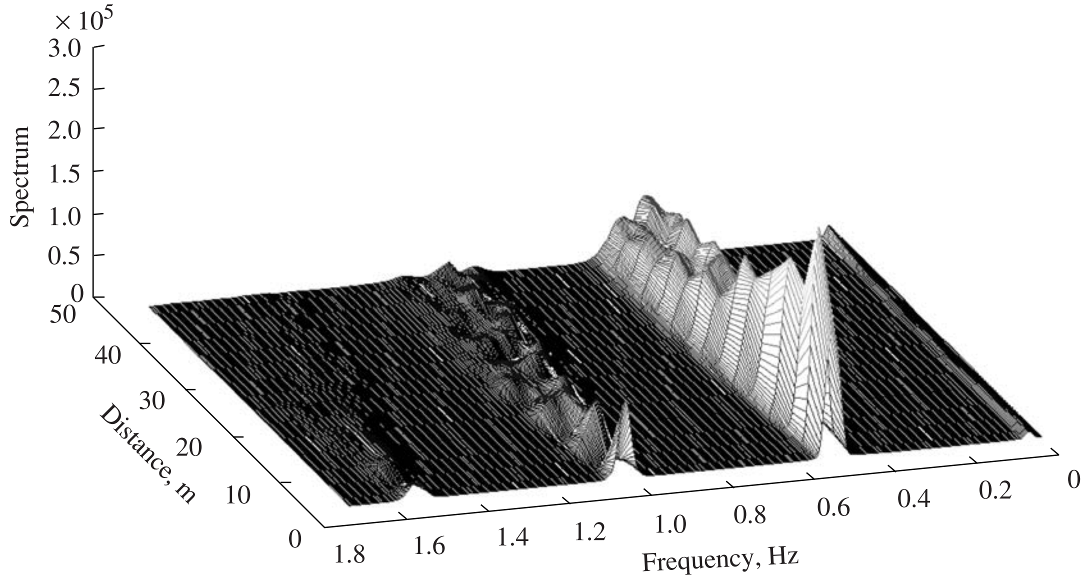

Fig. 2. Spatial evolution of the spectrum of initially bichromatic waves over a horizontal bottom at intermediate depth. The spectra are normalized by the dispersion of the free surface elevation.

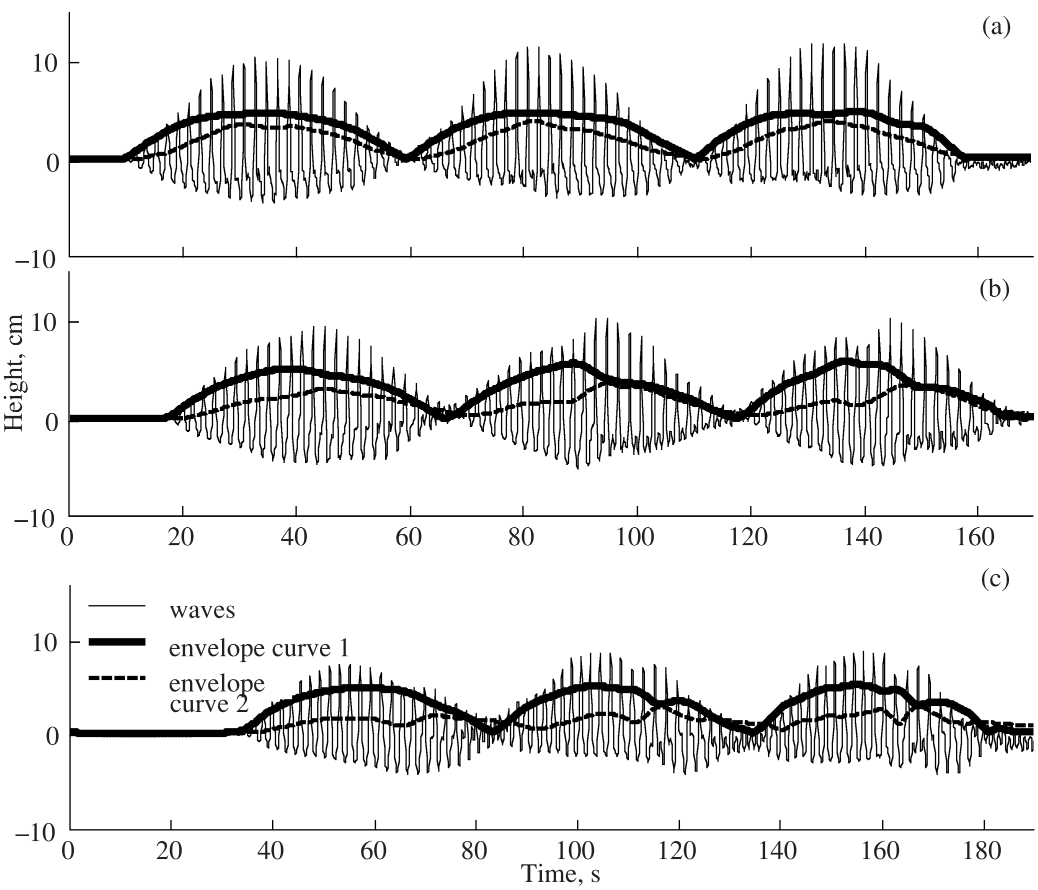

Fig. 3. Transformation of initially bichromatic waves (thin line) and envelopes of the frequency ranges of the first (thick line) and second harmonics (dashed line) at different distances from the wave generator: (a) 7 m; (b) 19 m; (c) 44 m.

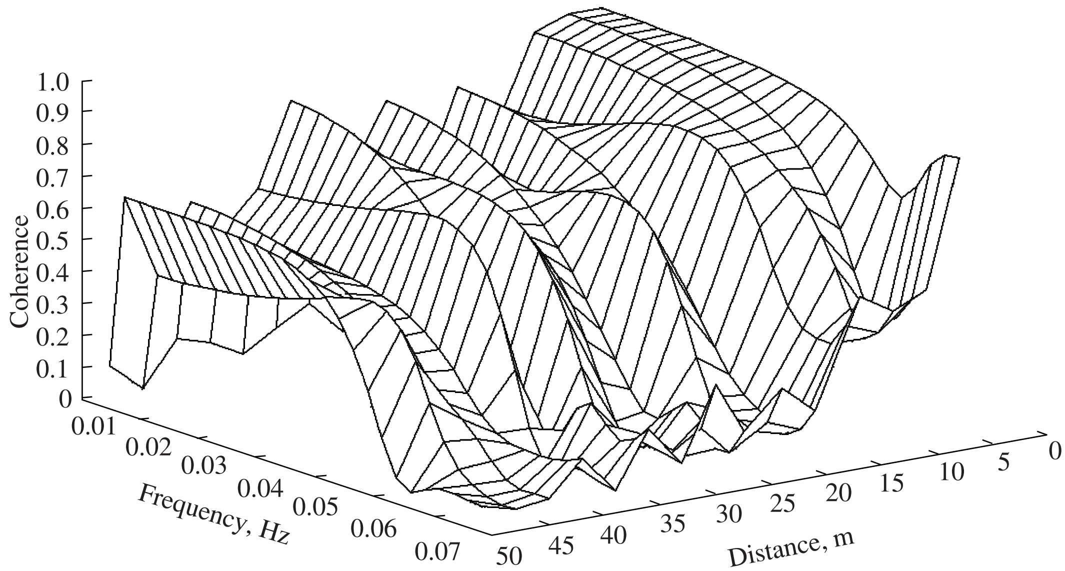

Fig. 4. Spatial evolution of the coherence function between the envelopes of the frequency range of the first and second harmonics of initially bichromatic waves over a horizontal bottom at intermediate depth.

harmonics decreased due to the total nonlinear triad near resonant interactions. As seen from Fig. 2, as the waves transform in the wave flume, the process of energy exchange between the harmonics repeats periodically. It is worth noting that gradual widening of the frequency range of the first and second harmonics occurs as a result of these periodic nonlinear interactions. In the initial stage of the wave transformation (a distance of 4 m), the frequency ranges of the first and second harmonics were 0.5 – 0.52 and 1–1.04 Hz. At a distance of 47 m, they were already 0.4–0.65 and 0.85–1.15 Hz, respectively.

Let us consider in more detail how nonlinear interactions induce such widening of the spectrum. Initially, the waves consisted of two first harmonics:

$$f_{11} = 0.5 \text{ Hz and } f_{12} = 0.52 \text{ Hz},$$

As a result of the total triad nonlinear interactions, they generate three (!) bound second harmonics at a distance of 10 m from the wave generator:

$$f_{21} = 2f_{11} = 1 \text{ Hz}; f_{22} = f_{12} + f_{11} = 1.02 \text{ Hz and}$$

$$f_{23} = 2f_{12} = 1.04 \text{ Hz}.$$

The following frequencies (or overtones of the first harmonics) appear due to sub-triad nonlinear interactions (backward energy transfer from the second harmonics to the first ones) at a distance of 20 m from the wave generator, which widen the frequency range of the first harmonics:

$$f_{10} = f_{21} - f_{12} = 2f_{11} - f_{12} = 0.48 \text{ Hz};$$

$$f_{13} = f_{23} - f_{11} = 2f_{12} - f_{11} = 0.54 \text{ Hz},$$

and so on. After each full cycle of near-resonant nonlinear interactions, new frequencies appear in the wave spectrum. In the same manner, the spectrum of the frequency range of the first harmonics additionally widens due to the backward energy transfer from the harmonics higher than the second. The frequency range of the second harmonics widens due to the total nonlinear interactions with the newly appearing harmonics of the frequency range of the first harmonics and also due to the differential nonlinear interactions with the harmonics whose frequencies are higher than the second ones.

In turn, such widening of the spectrum leads to spatial variations in the form of the envelopes of the first and second harmonics and, correspondingly, to the variation in their spectrum. Thus, in general, as one can see from Fig. 3, at different points of the space, the envelopes vary in time nonsynchronously. However, in the initial stage of the wave transformation, when the widening of the frequency ranges of the first and second harmonics (as well as the variation in the spectrum) is not strong, the envelopes vary almost synchronously (Fig. 3a). At these distances from the coast, individual waves have practically the same amplitude-frequency structure, when the amplitudes of the second harmonics are proportional to the amplitudes of the first harmonics, which is clearly seen in Fig. 3a. During the further wave transformation, the synchronism in the variation of the envelopes of the frequency ranges of the first and second harmonics is gradually broken due to the appearance of the time shift between the maxima of their envelopes and the periodic variations in the spectra of the envelopes, which occur due to the direct and backward energy transfer between the first and second harmonics [2, 15].

The asynchronous variations in the envelopes evidence that the proportionality between the amplitudes of the first and second harmonics in individual waves breaks. It is clearly seen from Fig. 3b that waves of the first harmonics with large amplitudes and waves of the second harmonics with small amplitudes alternate with the waves of the first and second harmonics with large amplitudes. Later, during further transformation of the waves, the amplitude-frequency structure of individual waves begins to change chaotically (Fig. 3c).

The synchronism in the fluctuations of the envelopes for the frequency range of the first and second harmonics can be estimated from the coherence functions between them. Variations in the coherence function in the process of the transformation of bichromatic waves are shown in Fig. 4. A frequency of 0.02 Hz corresponds to the maximum of the spectra of envelopes of the frequency range both of the first and second harmonics. It is determined by the difference in the frequency of the harmonics of the initial bichromatic waves: $0.52 - 0.5 = 0.02$ Hz. It is seen that the value of the coherence function at 0.02 Hz changes periodically in space and generally decreases (from 0.9 to 0.04) as the waves propagate and their spectrum widens.

The dependencies of the amplitude (envelope curve) of the second harmonics frequency range on the amplitude of the first harmonics frequency range are shown in Fig. 5.

As was mentioned earlier, in the initial stage of the wave transformation, the amplitudes of the second harmonics are proportional to the amplitudes of the first harmonics, and this dependence is quadratic as in the Stokes wave theory (Fig. 5a). During the further transformation of the waves and widening of their spectrum, the values of the coherence function decrease, and the dependence between the amplitudes of the second and first harmonics ceases to be quadratic and cannot be approximated by any other polynomial function (Figs. 5b and 5c). This means that the ratio between the amplitudes of the second and first harmonics changes chaotically in time and the amplitude-frequency characteristic of one individual wave differs strongly from that of another wave.

Thus, a simple qualitative description of the variability of the amplitude-frequency structure of individual waves is possible only in the initial stage of the wave transformation at distances at which the values of the coherence function between the envelopes of the frequency range of the first and second harmonics are high and the spectra are similar. This can be done, for example, on the basis of the generalization of the Stokes wave theory, when the amplitudes of the second

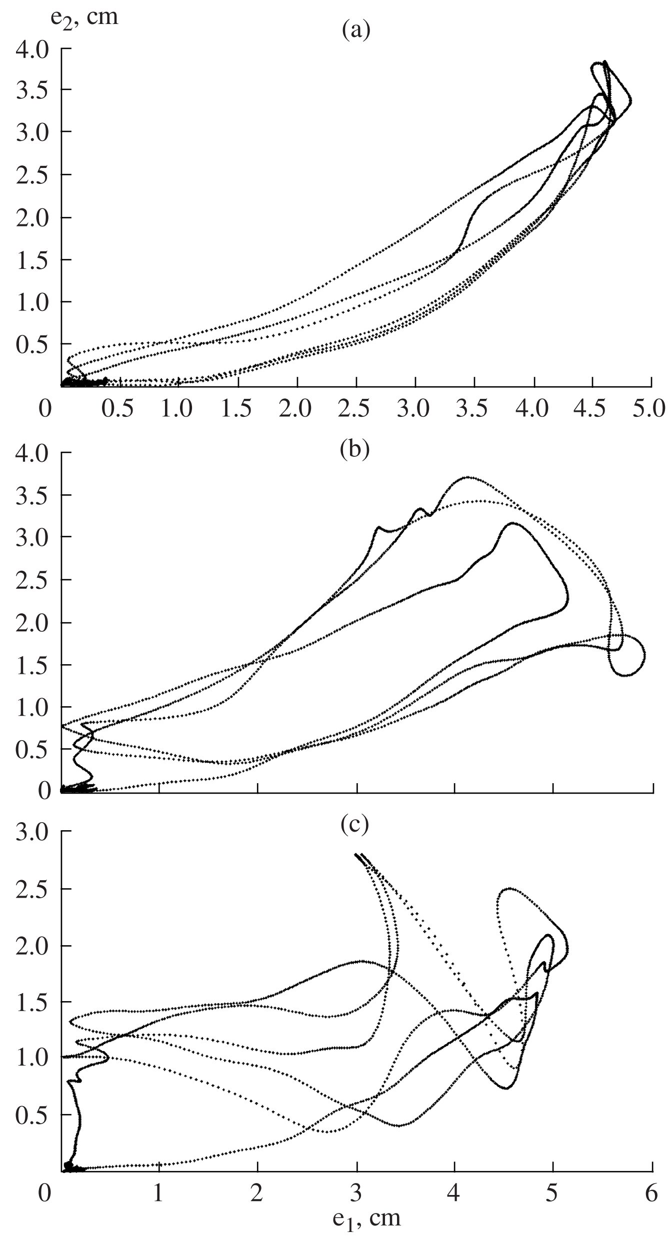

Fig. 5. Dependences of the amplitude of the envelope curve of the frequency range of the second harmonics on the amplitudes of the envelopes of the frequency range of the first harmonics for initially bichromatic waves at different distances: (a) 10 m; (b) 19 m; (c) 44 m.

harmonics are proportional to the square of the amplitudes of the first harmonics.

IRREGULAR WAVES: FIELD EXPERIMENT

In nature, waves are irregular and their spectrum is continuous. Is it possible that the regularities found in the experiment described above can be observed with real waves in a coastal zone?

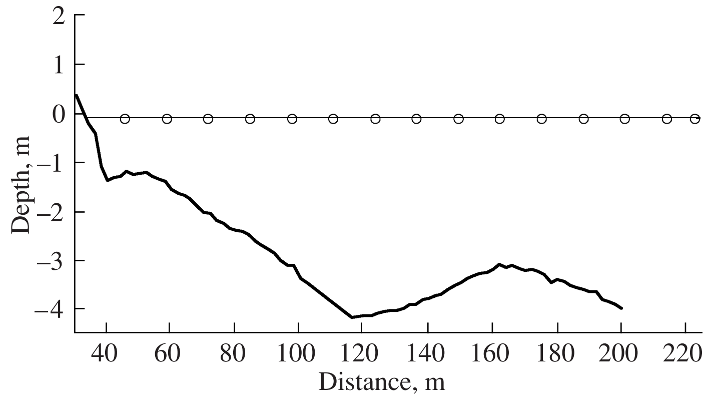

Fig. 6. Bottom topography and scheme of the location of the wave gauges in the Shkorpilovtsy 2007 field experiment.

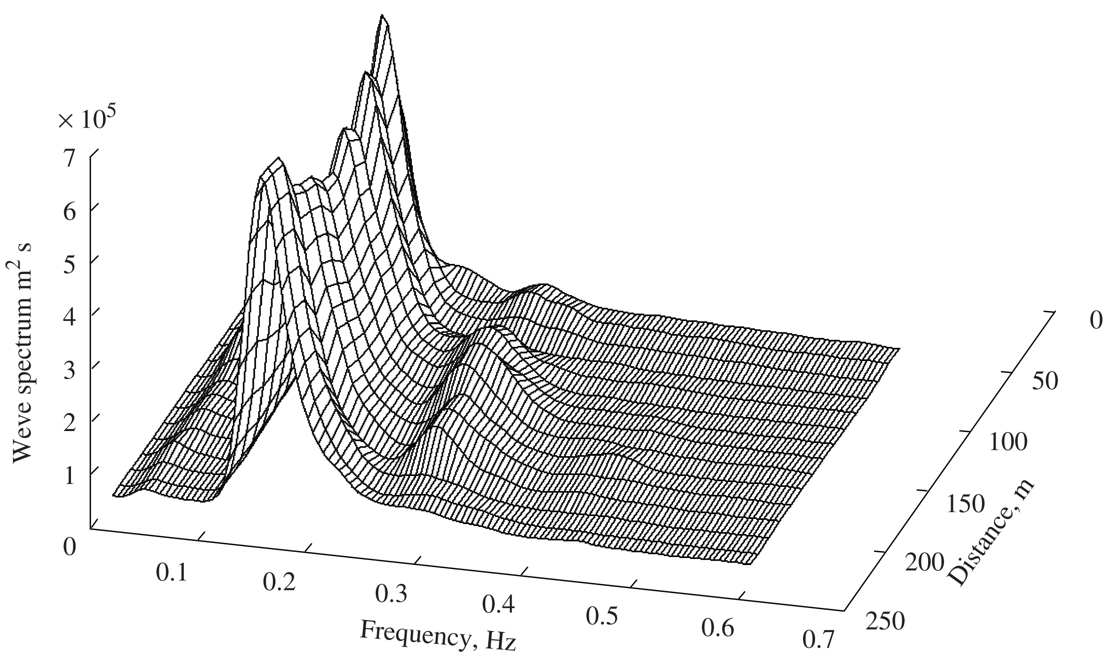

Fig. 7. Spatial evolution of the spectra of irregular waves (record no. 39; field experiment Shkorpilovtsy 2007). The wave spectra are normalized by the corresponding dispersion of the free surface elevation.

In our analysis, we used the data of the international field experiment in the Black Sea in Bulgaria (town of Shkorpilovtsy) in 2007. Fifteen wire wave gauges were used for recording waves: seven capacity gauges and eight resistance gauges. The locations of the wave gauges and the bottom profile are shown in Fig. 6. The measurements at 15 points were carried out synchronously with a sampling rate of 5 Hz. The duration of the series of measurements was 1 h.

The main features of the variations in the amplitude-frequency structure of the irregular waves will be discussed based on the example of typical wave record no. 39 with parallel wave fronts to the coastline. Wave breaking (plunging) occurred at a distance of 84 m from the coastline.

The measured free surface elevations were studied using the methods described above, and the analysis confirmed the results obtained for regular waves in the laboratory experiment.

Figure 7 presents the evolution of the wave spectra as the waves approach the coast. The frequency of the wave spectral maximum f_p = 0.135 Hz, and the fre-

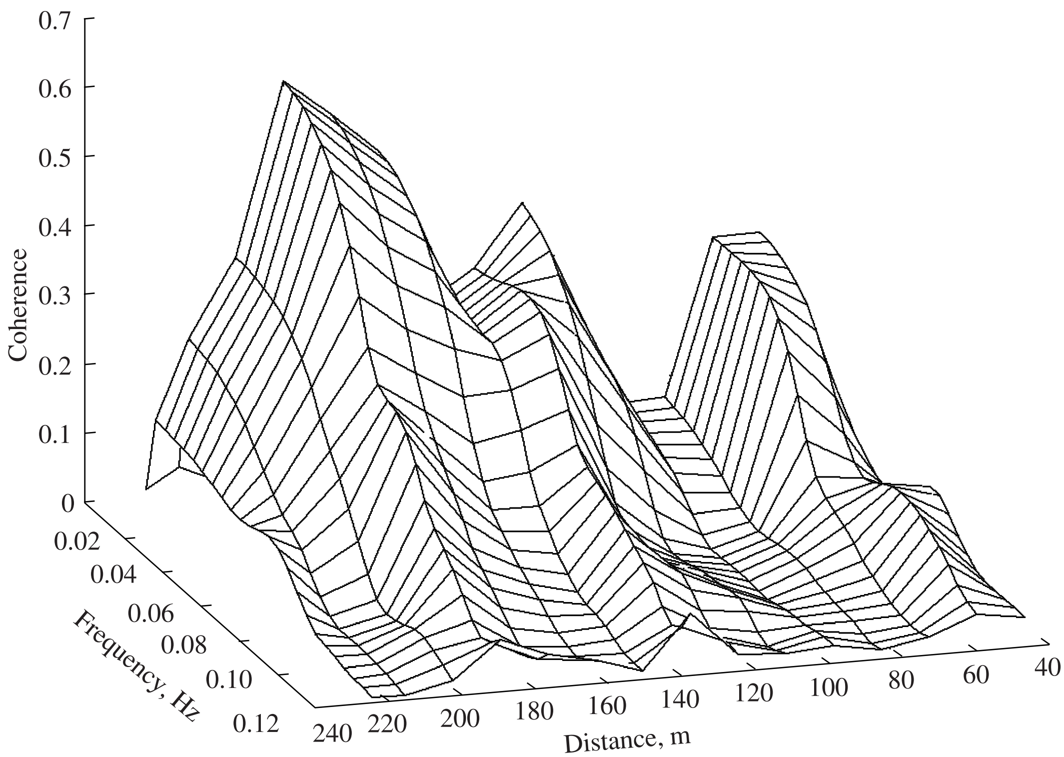

Fig. 8. Spatial evolution of the coherence between the envelopes of the frequency range of the first and second harmonics (record no. 39; Shkorpilovtsy 2007 field experiment).

quency ranges of the first and second harmonics are equal to 0.1–0.23 Hz and 0.23–0.35 Hz, respectively (Fig. 7). It is clearly seen from the spectra that, similarly to the case of bichromatic waves, secondary harmonics are formed as a result of nonlinear interactions as the waves approach the coast. Periodic energy exchange between them exists, which is reflected in the periodic variation of their amplitudes.

The coherence function between the envelopes of the frequency range of the first and second harmonics is shown in Fig. 8. In the initial stage of the wave transformation (distances of 201–149 m), its values are quite high (approximately 0.7). This means that the envelopes of the frequency range of the first and second harmonics change synchronously, while the amplitudes of the second and first harmonics in individual waves are proportional.

Similarly to the case of bichromatic waves, the coherence function changes periodically and generally decreases to the coast. The greatest decrease in the coherence starts at a distance of 136 m from the coast, and, at a distance of 97 m, its value is 0.2. If we analyze the evolution of the spectrum (Fig. 7), we see that, at distances from 223 to 136 m, the maximum of the amplitudes of first harmonics decreases, while the maximum of the amplitudes of the second harmonics increases. Later, backward energy transfer starts. At distances from 136 to 97 m, the amplitude maximum of the second harmonics decreases, while the maximum of the first harmonics increases. Thus, similarly to as for the regular waves, the backward energy transfer decreases the coherence function between the envelopes of the frequency range of the first and second harmonics and breaks the synchronism of their time variation.

Does the absence of correlation between the envelopes mean that the wave motion becomes completely chaotic and not only the linear but also the nonlinear relations between the harmonics of the wave motion disappear? In order to answer this question, we performed a bispectral analysis, which makes it possible to determine the relations of the second order.

Figure 9 shows the squared bicoherence function, whose values estimate the share of quadratic relations between the harmonics in the wave spectrum and their distribution over frequencies at different distances from the coast.

It was found that relations of the second order between the first and second harmonics exist at all stages of the wave transformation. This is proved by the maximum value of the bicoherence at the frequency of the spectral maximum f_p = 0.135 Hz (the nonlinearly interacting frequency pair (f_p, f_p), which generates the second harmonic). The values of the squared bicoher-

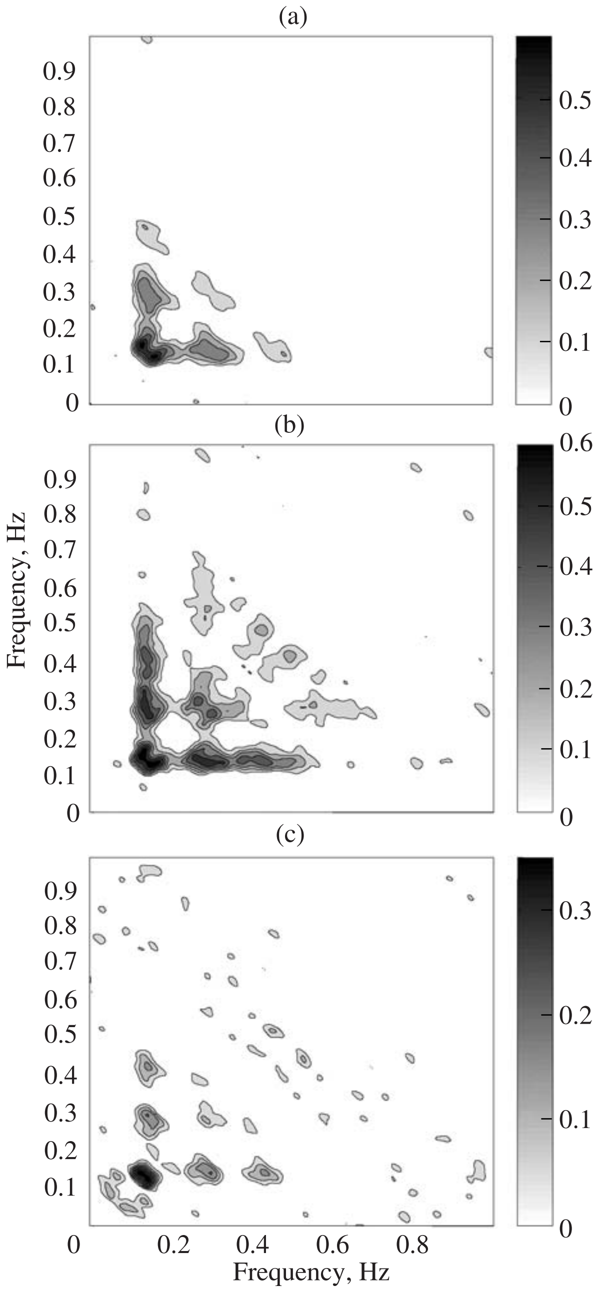

Fig. 9. Squared bicoherence function for record 39 (field experiment Shkorpilovtsy 2007) at different distances from the coast: (a) 201 m; (b) 136 m; (c) 97 m.

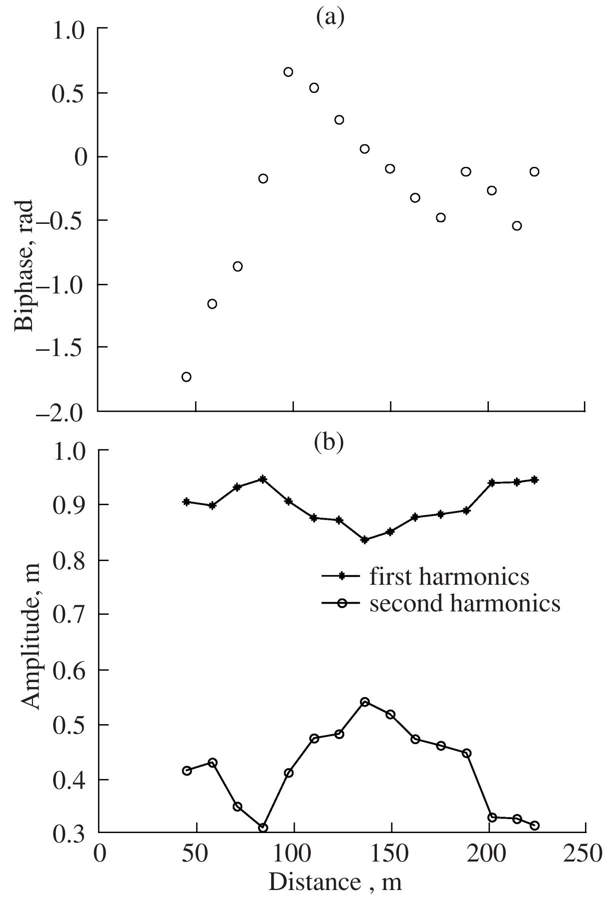

Fig. 10. Spatial evolution of the biphase (a) and time averaged amplitudes of the first and second harmonics (b) (record 39; field experiment Shkorpilovtsy 2007).

ence at this frequency are sufficiently high (about 0.6), including the stage of backward energy transfer at distances from 123 to 97 m (0.35) (Fig. 9a). The high values of the bicoherence evidence that the first and second harmonics of the waves are correlated waves appearing as a result of total near resonant triad interactions. The decrease of the bicoherence during the backward energy transfer can be explained by the appearance of a multitude of new harmonics with different phases. The difference between their frequencies is close to the frequency resolution of the bispectral analysis.

Relatively high bicoherence values allow us to estimate the relative phase shift (biphase) between the first and second harmonics of waves and thus determine φ in relation (1).

The biphase of the interacting frequency pair generating the second harmonics in nonlinear Stokes waves is zero [8, 12, 13]. The biphase evolution for the frequencies of the bispectra maximum near the first harmonics as a function of the distance from the coast is shown in Fig. 10a. Figure 10b shows the variations of the normalized amplitudes of the first and second harmonics determined as the mean value of the envelopes for the corresponding frequency ranges at the same spatial points.

One can see that the biphase is negative at distances from 197 to 123 m. It only slightly differs from zero, which can be interpreted as waves close to Stokes waves. This also means that, in the initial stage of the wave transformation, at the moment of the energy transfer from the first harmonics to the second ones due to the total nonlinear near-resonant triad wave interactions (Fig. 10b), the second harmonics are shifted slightly ahead relative to the first ones. This occurs due to the nonlinear processes generating the second harmonics shifted ahead relative to the first harmonics by $-\pi/2$ preventing dispersion processes, which shift the second harmonics backwards relative to the direction of propagation of the first harmonics. The shift between the harmonics fluctuates slightly in space and remains small because, owing to the relatively great depth dispersion, it approximately balances the nonlinearity effect. At distances from 110 to 58 m, when backward energy transfer begins from the second harmonics to the first ones (Fig. 10b), the values of the biphase tend to $\pi/4$, because, in this case, the nonlinearity and dispersion act in one direction shifting the second harmonics backwards [14]. At the next stage of the wave transformation, when the backward energy transfer from the first harmonics to the second begins again (at a distance of 84 m from the coast), the biphase tends to the classical value $-\pi/2$ due to the domination of nonlinear processes over relatively shallow depths [9]. This corresponds to strongly asymmetric saw-shaped waves, which are in the stage before breaking, breaking, or their transformation is determined only by nonlinear processes.

Thus, the waves remain nonlinearly correlated over the entire length of the coastal zone and, as the waves approach the coast, their biphases change quasi-periodically with their period being equal to the characteristic time of the nonlinear interactions, which is the main cause that leads to the differences in the amplitude-frequency composition of individual waves.

CONCLUSIONS

The spatio-temporal variations in the amplitude-frequency composition of individual waves were investigated on the basis of experimental data analysis. It was found that, as a result of nonlinear near-resonant triad interactions, the amplitude-frequency structure of individual waves changes quasi-periodically both in space and time. The phase shift between the first and second harmonics strictly depends on the direction of the energy transfer between the first and second harmonics and the ratio between the intensities of the nonlinear and dispersion processes. Therefore, the construction of a simple qualitative physical model (parameterization) describing the variability of individual waves in the coastal zone is possible only in the initial stage of their transformation, for example, on the basis of the generalization of the Stokes wave theory.

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