TRANSFORMATION OF IRREGULAR WAVES AND WAVE GROUPS DUE TO BREAKING AND NONLINEAR PROCESSES

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On a basis of field and numerical experiments the evolution of irregular waves with group structure from deep to shallow water are studied. It has been found that on deep water the wave group structure is determined by non-linear properties of waves and looks as automodel process. The non-linear processes are the basic reason of degradation of wave group structure towards to the shore. That is determined by non-linear reorganisation of a wave spectrum in the frequency band of the first harmonics and filling intervals between wave groups of the first harmonics by wave groups of highest harmonics. Wave breaking does not influence directly on wave group structure change. It is revealed three types of dependencies of energy dissipation on frequency during wave breaking: uniform (in outer part of surf zone), square and selective (in inner part of surf zone), which one is determined by asymmetry of waves and bed slope. It is established, that the energy dissipation of breaking waves compensates processes of linear and non-linear transformations of waves.

1. Introduction

Ocean waves always are irregular and have a group structure, when high waves alternate by low waves. The group structure of waves and its variability in time and space is extremely important for the description of many dynamic processes in a coastal zone. In spite of evident advantage of wave modelling, the clear qualitative understanding of non-linear wave evolution in coastal zone is absent due to on the one hand variety of sceneries of wave transformation over a bottom relief depending on input wave parameters and on the other hand that most of the models are checked only against time-averaged parameters of waves. It sometimes results in wrong interpretation of experimental data and using of the erroneous premises for construction of the simplified analytical models.

The physical origin of wave groups is still unknown and it is not clear, whether wave groups can be a sequence of wave irregularities due to random phases (Elgar et al., 1984), or a result of non-linear instability (Tulin, Waseda, 1999). The lack of understanding a wave group's origin does not permit to establish the correct initial conditions for a phase resolving models. The differences in spectral composition of groups with small and big waves (as well as a difference in spectral composition of consecutive groups of big waves) are unknown also. In the published points of view there is an obvious contradiction on the reasons of change of wave group structure in shallow water. The most popular idea is its degeneration due to wave breaking, which diminishes mostly the highest wave smoothing the group structure (Madsen et al, 1997). But in nature wave groups exist up to the shore and sometimes become more pronounced in inner part of surf zone (Mase, 1989; List, 1991).

Process of energy dissipation during wave breaking is studied insufficiently. Now it is not present not only conventional and well tested model of energy dissipation of irregular waves in surf zone, but also clear physical description of changes of wave properties during breaking is absent. For an estimation of wave breaking influence on amplitudes and phases of wave harmonics two approaches are used: 1) wave breaking does not change the form of a wave spectrum, and only reduces spectral density in proportion equally for all frequencies (Battjes, Beji, 1993; Eldeberky, Battjes, 1996); 2) the spectral density during wave breaking decreases depending on a square of frequency (Mase, Kirby, 1992; Kirby, Kaihatu, 1996).

The aim of this research is the analysis of characteristic features and reasons of irregular waves and wave group structure transformation in a coastal zone on the basis of field data and numerical modelling.

2. Parameters of wave group structure and methods of data processing

For the allocation of wave groups structure the envelopes of various frequency bands were calculated: $H_e(t) = \sqrt{\{L[H(t)]\}^2 + [X\{L[H(t)]\}]^2}$, where X - Hilbert transform, L- the operator of a linear filtration of required frequencies band, $H(t)$- free-surface elevations.

Height and period of wave groups were estimated from envelope fluctuations using dimensionless integral parameters: the groupiness factor GF and average number of waves in groups NW (List, 1991; Solovyev, 1989). $GF = 1.41\sigma_{He} / \overline{H}_e$, where $\overline{H}_e$ – time average value of envelope, equal to half of mean height of waves, σ_{He} - standard deviation of envelope. The large values of GF correspond to more expressed wave group structure, GF = 0 indicate its absence.

$NW = m_1/m_0$, where $m_i = \int_0^\infty f^i S(f)df$, S – spectrum, f- frequency.

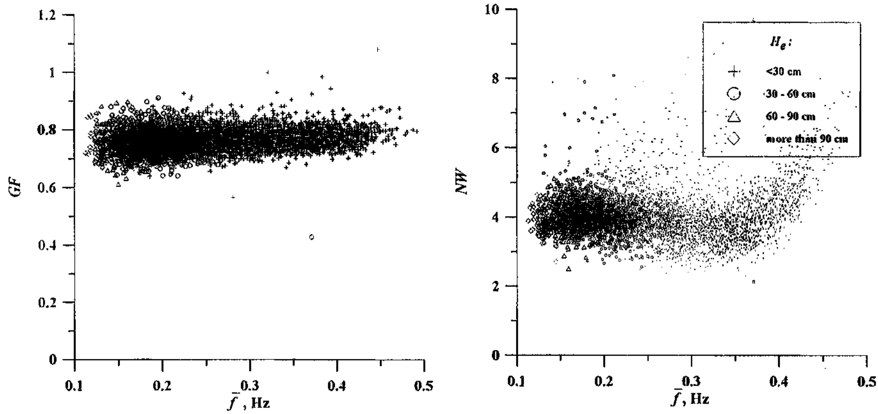
GF and NW were calculated for the all frequency band of wave spectrum and separately for frequency bands of the first and higher harmonics.

The spectra of waves and envelopes were calculated on Welch method. To investigate spectral structure of individual waves and mutual arrangement of wave groups of different frequency bands wavelet-analysis with wavelet function Morlet was applied (Torrence, Compo, 1998).

3. Wave group structure on a deep water

To define parameters of irregular waves and of wave groups on deep water a continuous two-year data of waverider directional buoy was used (Kos'yan et al., 1998). Buoy was placed in 7 km from a coast of Black sea over depth of 85 m and worked in an independent mode, transferring the information by the radio channel. Free surface elevations were registered each three hours at small heights of waves and each hour during storm. The length of a series of measurements was 20 minutes with sampling frequency 1.28 Hz. 6100 records of free surface elevations were used for the analysis. To exclude influence of infragravity waves and high-frequency noise, the envelopes were calculated for frequencies $0.6f_m < f < 2f_m$, where f_m - frequency of maximum of wave spectrum.

The obtained dependencies testify about probably automodelity of wave group structure on deep water, as it is shown on Fig.1. Really, *GF* does not depend on parameters of waves and changes in rather narrow range of values from 0 65 till 0.85. Average value *GF* is equal to 0.76 and average value *NW* is equal 4. It was found absence of dependencies *GF* and *NW* on many wave parameters also. Thus, apparently, values *GF* and *NW* on deep water are defined only by non-linear properties of wave process and do not depend obviously on parameters of waves and conditions of wave formation.



a) b)

Figure 1. Independence of groupiness factor (*GF*) (a) and mean number of waves in groups (*NW*) (b) from the mean frequency and wave height.

4. Wave group transformation in the coastal zone

4.1. *Field experiment results*

To demonstrate main features of irregular waves and wave groups transformation in a nature the data of the field experiment “Shkorpilovtsy-88” (Black Sea, Bulgaria) were used. During the experiment 15 wire resistant type gauges were placed on depths from 10 to 0.5 m at the distances up to 350 m from the shore. Measured series was about 15 minutes with sampling frequency 3.33 Hz. Two experimental series (d16, k3), where waves propagate in normal direction to a coast, were chosen. Wave records d16 (a frequency of spectrum maximum f_m= 0.11 Hz and height of significant waves H_s = 1.8 m on depth 10

m) and k3 (f_m= 0.12 Hz and H_s= 1.3 m) differ with a location of wave breaking point. In k3 the most gauges were seaward from the surf zone, but in d16 most gauges were inside of the surf zone. Spectra of waves, bed profiles, locations of wave gauges and breaking points, groupiness factors and number of waves in groups are shown in Fig.2.

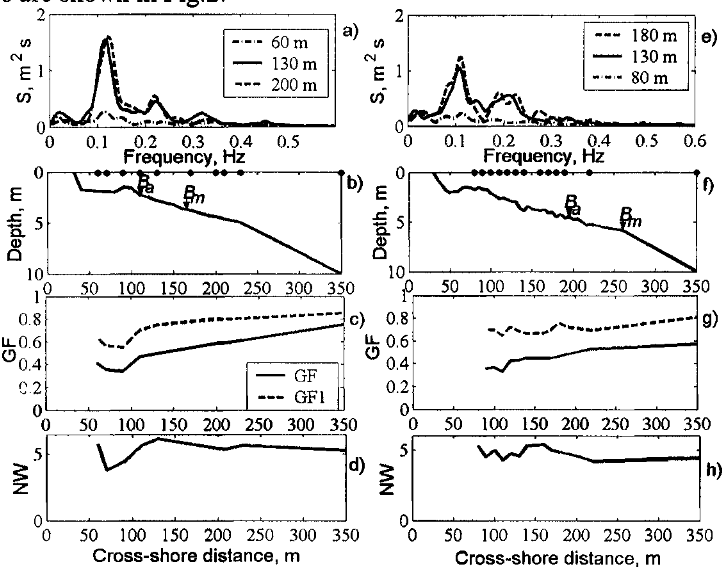


Figure 2. Wave spectra (a, e), bottom profile (b, f) (B_a – breaking point of highest waves, B_m – breaking point of mean waves; •- gauge locations), *GF* (c, g) and *NW* (d, h) for series k3 (left) and d16 (right).

NW changes in a range from 4 up to 6 for both series similarly (Fig.2d, h). Simultaneous increasing of the mean frequency of wave spectrum and the mean frequency of envelope towards to the shore provide minor changes of *NW*. *GF* mainly decreases towards to the shore (Fig.2c, g) similarly for both series, but in k3 decreasing begins long before wave breaking point. Comparing series k3 and d16 is possible to conclude, that wave breaking does not influence directly on decreasing *GF* and on changing *NW*. Notice that *GF1*, calculated from frequency band of the first harmonics, changes similarly to *GF*, calculated from all frequencies of waves, but exceeds it. It is obvious that decreasing of groupiness factor to a coast is caused by change of a spectrum in frequency band of the first harmonics due to non-linear processes. The excess *GF1* above *GF* occurs due to effect of "filling" of intervals between the groups of large wave of the first harmonics frequency band by groups of high-frequency waves take place.

The effect of "filling" is well seen in a Fig.3, where the fragment of wave record d16 and its wavelet transform are shown. The intensity of colour from white (maximum) to black (minimum) shows relative values of the modules of wavelet coefficients. It is visible that waves with large amplitude of first harmonics and small amplitude of highest harmonics (times 90 - 160 s and 230 - 300 s) alternate with waves with small first harmonics and large highest (times 170 - 210 s). The statistical reliability of "filling" effect proves to be true by correlation of envelopes of first and highest harmonics (Fig.3c). It is 0.45 for both examined series. In a Fig.3c the dependence of correlation coefficients shown by intensity of colour, versus of time shift and distance from coast for a series d16, is given. The time shift between maximums of envelopes of firsts and highest harmonics is changed to a shore from 80 to 50 s while the period of waves groups is about 110 s.

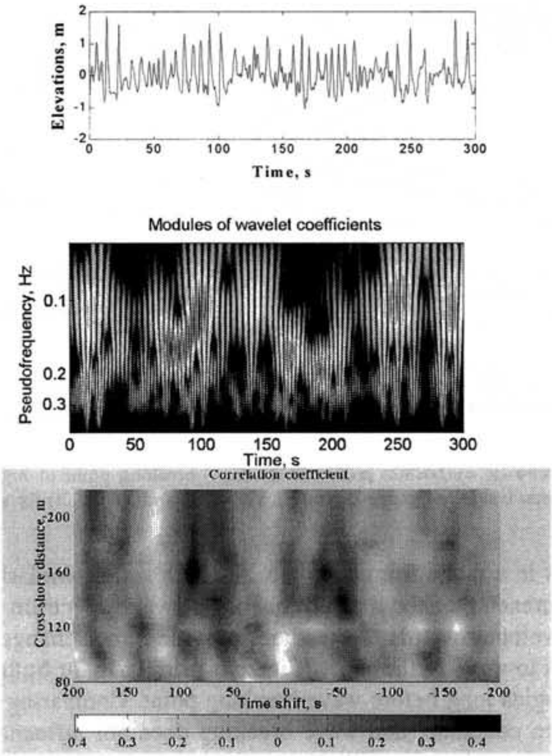


Figure 3. Alternation of first and higher harmonics groups in series d16: a) – wave chronogram at depth 4.1 m, b) – its wavelet transform, c) - cross-shore evolution of correlation coefficient of envelopes of first and higher harmonics.

The Fig.3b shows time variability of the relation between amplitudes of the first and highest harmonics. That contradicts classical ideas based, for example, on the theory of Stokes waves, where the amplitudes of highest harmonics are proportional to amplitude of the first harmonic.

To investigate in detail revealed features numerical simulation of simple example of waves with group structure – bichromatic waves above horizontal and slope bottoms was carried out.

4.2. *Model of wave transformation*

To modelling wave transformation, the Boussinesq-type equations with improved dispersion characteristics in frequency domain were applied. They are successfully used for simulation of waves transformation in a coastal zone in depths H/L_0<0.5, where L_0 is wavelength on deep water (Madsen, Sorensen, 1993; Eldeberky, Battjes, 1996):

$$\frac{dA_p}{dx} = -b_2 \frac{h_x}{h} A_p - i2g(F_p^+ + F_p^-) - \alpha_n A_p \cdot \tag{1}$$

The first term on the right represent linear shoaling, proportional to the bottom slope (h_x) and local amplitude. The second represents the triad sum and difference non-linear interactions. Third term describes spectral energy dissipation due wave breaking (Kirbi, Kaihatu, 1996):

$$\alpha_n = \alpha_0 + (f_n / f_m)^2 \alpha_1, \quad \alpha_0 = F\beta, \quad \alpha_1 = (1-F)\beta \frac{f_m^{\,2} \sum_n |A_n|^2}{\sum_n f_n^2 |A_n|^2}, \tag{2}$$

F - the parameter varied from 0 up to 1, (usual F=*0.5*, corresponding to partial squared dependence of energy dissipation on frequency), β – breaking coefficient; A_n - complex Fourier amplitude; f_n – frequency, n – rank of harmonics. $\beta = D_{tot} / F_{tot}$, where F_{tot} – total local rate of energy flux per unit width, D_{tot} – total local rate of energy dissipation. D_{tot} was determined from simple probabilistic energy dissipation model (Battjes, Janssen, 1978; Eldeberky, Battjes, 1996). The evolution equations (1) were solved numerically using 4-5 orders Runge-Kutta method.

4.3 *Model results*

4.3.1. *Transformation of bichromatic waves over a constant depth*

In a Fig.4 the modelling results of transformation of bichromatic waves at constant depth 4 m during 1000 m are given. Initial waves were from two harmonics with frequencies 0.10 Hz and 0.11 Hz and amplitudes 0.5 m (period of wave groups 0.01 Hz). The solution was obtained for cut-off frequency 0.45 Hz with discretization 0.01 Hz. Wave breaking was absent.

Amplitudes of initial harmonics and their second harmonics change quasi-periodical in process of wave propagating (Fig.4a). Groupiness factors *GF* (frequency band 0.08 - 0.45 Hz) and *GF1* (frequency band of first harmonics 0.08-0.14 Hz) have spatial fluctuations with the periods about 200 m, which visually are like to the periods of fluctuations of amplitudes (Fig. 4b). These fluctuations of *GF* are induced only by changes in spectral structure of waves determined non-linear interactions of harmonics because wave breaking was absent and the depth was constant. *GF* has fluctuation with the period about 900 m also, caused by effect of "filling" of intervals between groups of high waves with frequencies of the first harmonics by high-frequency waves. These features as well prove to be true by results of wavelet-analysis. Time delay between maximums of envelopes of first and highest harmonics increase during wave propagation. It becomes equal to the period of wave groups (100 s) on distance 1000 m, as it is visible from the correlation coefficients given on a Fig.4c.

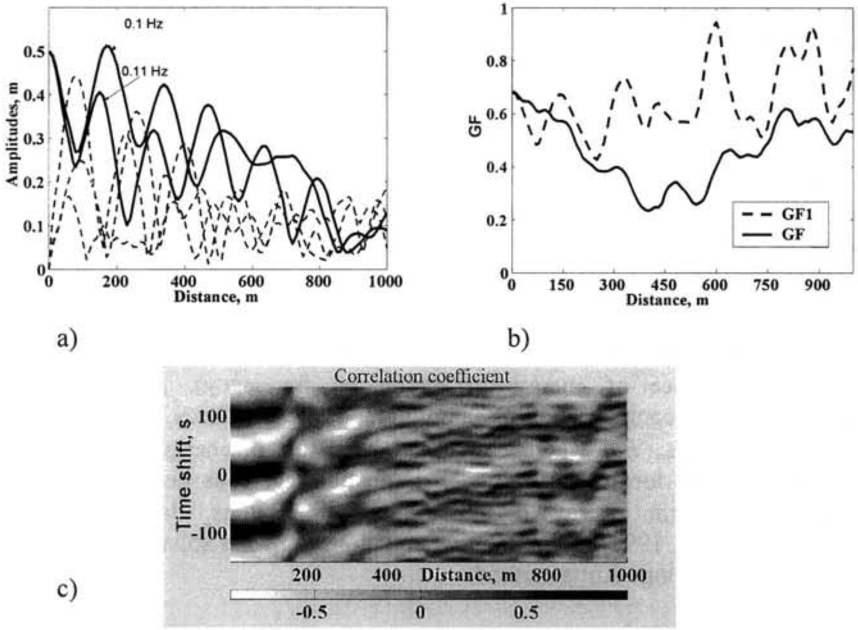


Figure 4. Transformation of bichromatic wave of initial harmonics 0.1 Hz and 0.11 Hz and equal amplitudes of 0.05 m at constant depth 4 m: a) amplitudes of harmonics evolution: solid line – first harmonics, dotted line – second harmonics; b) evolution of groupiness factor; c) evolution of correlation coefficient of envelopes of first and higher harmonics.

4.3.2. *Transformation of bichromatic waves over a sloping bottom*

The transformation of bi-chromatic waves above sloping bottom is similar to a case of horizontal bottom differing by the greater intensity of non-linear processes caused by depth decreasing. *NW* and *GF* change quasi-periodic also, but periods above sloping bottom are a little bit less, than above horizontal.

In a Fig.5 the influence of bottom slope on changes *GF* and *NW* towards to the shore is shown. *GF* and *NW* fluctuate quasi-periodically following changes of spectral structure of waves. The periodicity of *GF* and *NW* fluctuations is determined by two factors: by run of waves between the same difference of depths (dependence from slope), and by local depth. For small slope (0.01-0.02) the influence of run dominates, but for large slope (0.03-0.05) the influence of depth dominates. It is obvious, that above moderate sloping bottom we can observe one - two quasi-periods of changes *GF* and *NW* or only part of period depending on slope. This part often considered by investigators as a sustainable trend. Wave breaking begun in model for different slopes at depth from 3.5 up to 3.8 m, obvious does not change behaviour *GF* and *NW*.

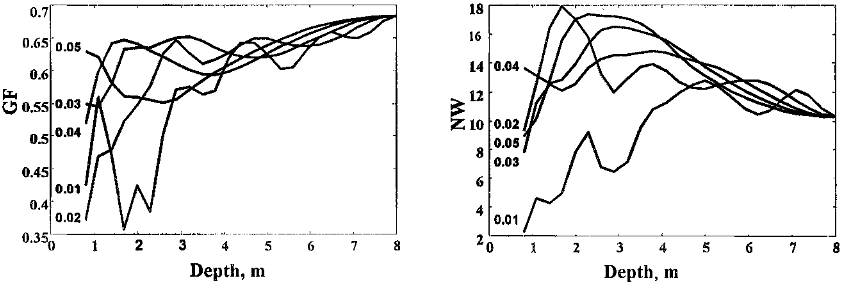


Figure 5. *GF* and *NW* for initially bichromatic waves (0.1 and 0.11 Hz) above different slopes of bottom. Numbers show values of slopes.

5. Influence of wave breaking on irregular waves

In previous sections was observed that wave breaking is not the main reason of wave group structure changing. Consider how the wave breaking influences on irregular waves, particular on its amplitude and frequency content, by estimating the frequency dependence of damping coefficient α_n in equation (1).

5.1 *Method of estimation of damping coefficient*

A method of estimation of damping coefficient is based on comparison model (without account of dissipation process, i.e. $\alpha_n=0$) and experimental (measured, naturally taking into account both non-linear processes, and dissipation) spectra of irregular waves (Mase, Kirby, 1992):

$$\alpha_n = \frac{(S(x_{start} + \Delta x)_{calc} - S(x_{start} +, \Delta x)_{meas})}{S(x_{start})_{meas} \times (2\Delta x)} \qquad (3)$$

where S - spectrum of waves at different distances from the coast, the indexes *calc* and *meas* denote the model (without dissipation) and measured experimental spectra accordingly; x - spacing interval, x_{start} - initial start point.

In a used method, the energy dissipation of breaking waves occurs not uniformly and permanently, but is a single-pass event at the end of a test site, that probably does not correspond to a real physical process of wave breaking. But our estimation has shown that this simplification is quite suitable for the qualitative description of dependence of a damping coefficient on frequency with ratio errors about 20%.

5.2. *Types of frequency dependencies of damping coefficient*

The data of two field experiments: "Shkorpilovtsy-88", described in section 4.1, and "Norderney-94" (North sea, Germany) with different types of wave breaking (spilling and plunging) were used. In experiment "Norderney-94" four pressure gauges were placed at the horizons of 0.60 - 0.80 m above the bottom. The tidal fluctuation of water levels have allowed to make measurements in range of depth from 1 up to 2.5 m. Length of records were 60 minutes, with sampling frequency 4.55 Hz. The free surface elevations were calculated using Fourier transformation and wave linear theory.

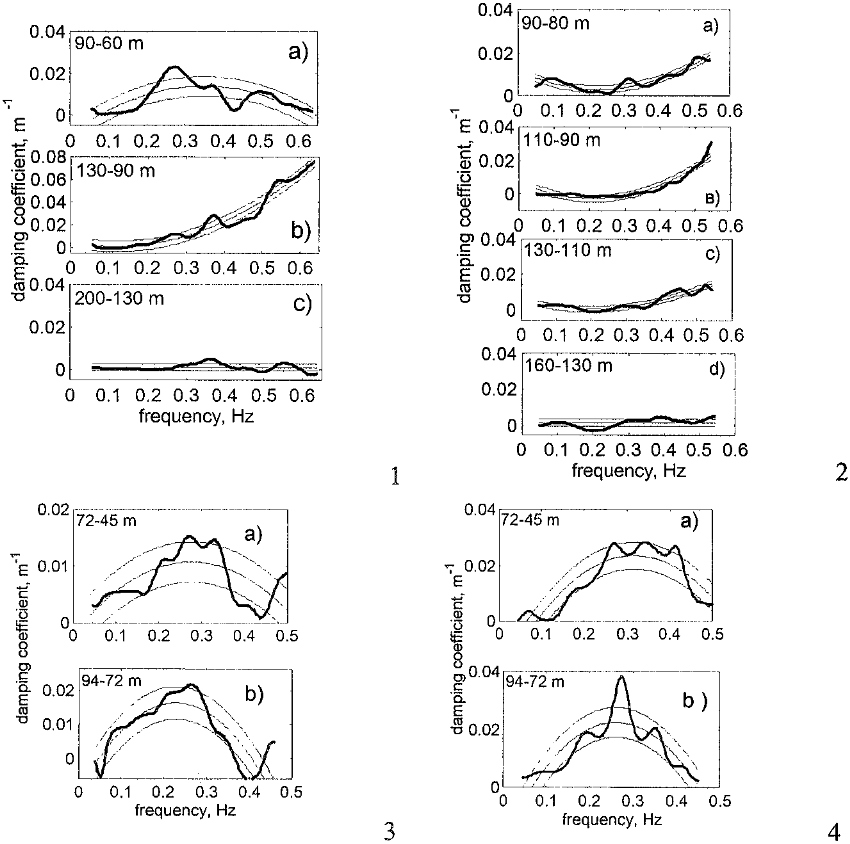


Figure 6. A damping coefficient estimated on the formula (2) on some offshore distances for a series: 1) k3, 2) d16, 3) №10, 4) №28. The smooth line is a curve of approximating, dotted lines-50 % confidence interval of approximating.

For the analysis the regimes of waves approaching on a normal to a coast measured during a maximum tidal level were selected: series №28 (f_m = 0.11 Hz, H_s = 2 м) and №10 (f_m = 0.098 Hz, H_s = 1.5 м). The selected series have covered an inner part of a surf zone in experiment "Norderney-94" and practically all coastal zone in experiment "«Shkorpilovtsy-88". The main difference in conditions of these two experiments is mean bed slope: about 0.027 in " Shkorpilovtsy-88" and about 0.014 in "Norderney-94". The changes of damping coefficient in different parts of surf zone estimated on the expression (3) are shown in Fig.6.

A damping coefficient has different kinds of frequency dependence, but can be qualitatively divided on to three types: uniform (or frequency independent) (Fig. 6.1c, 6.2d), depending on the frequency squared (Fig. 6.1b, 6.2a, b, c) and selective dependence on frequency (Fig. 6.1a, 6.3, 6.4). It is possible to say, that kind of damping coefficient does not depend on a type of breaking (spilling or plunging), but it depends on a part of surf zone.

In an outer part of surf zone, where an initial wave breaking take place and only highest waves break, energy dissipation is distributed over all frequencies practically uniformly (Fig. 6.1c, 6.2d). In the inner part of surf zone the values of damping coefficient increase and become to be proportional to square of frequency, i.e. the energy of highest harmonics of waves decreases greater, than first. A minimum of energy dissipation also is well seen in the frequency band of a first harmonic. However in an inner part of surf zone selective dependence on frequency is observed also. Some hypotheses about conditions of formation of different types of energy dissipations in breaking wave will be discussed in next sections.

5.3. *Influence of wave shape*

In various parts of surf zone waves are differed in shape. The asymmetry of waves is traditionally characterized by the third moments of wave motion:

$$A = \langle X(H)^3 \rangle / \langle H^2 \rangle^{\frac{3}{2}} \quad , \quad S = \langle H^3 \rangle / \langle H^2 \rangle^{\frac{3}{2}}$$

where the brackets denote a time averaging. Skewness (S) characterizes wave symmetry about a horizontal plane, asymmetry (A) - wave symmetry about a vertical plane. The positive values S correspond to sharp waves crests and the flat trough, negative values A correspond to waves with abrupt forward and flat back fronts. In a Fig. 7 both moments for all experimental series are shown. At the calculation of the moments, high frequency noise (f>0.7 Hz) and infragravity waves (f< 0.06 Hz) were excluded from the wave data. From Fig.6 and 7 it is visible, that the frequency selective of energy dissipation of breaking wave corresponds to strongly asymmetrical about a vertical plane waves (A<-0.5), nearest to "sawtooth" shape.

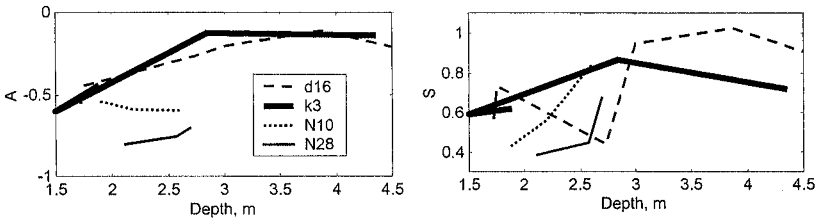


Figure 7. Asymmetry (A) and skewness (S) of experimental waves.

The uniform dissipation corresponds to gentle vertical asymmetry of waves ($A \approx -0.2$) at the moment of initial rearrangement of a wave from symmetrical to asymmetrical shape. The values of vertical asymmetry $-0.2<A<-0.5$ correspond to intensive rearrangement of a waves to "sawtooth" shape and then square frequency dependence of damping coefficient is observed. Skewness in all cases varies within wide range: from 1 up to 0.37, and does not influences visible on type of a frequency dependence of wave energy dissipation.

5.4. *Influence of bed slope*

In series d16 at depth from 2 up to 1.7 m (110-80 m from shore) waves have large vertical asymmetry, but at the same time frequency selectivity of damping coefficient is not observed (Fig.6.2a, b). This fact testifies that selectivity of frequencies during wave breaking is connected with distinctions in transformation of waves above various bed slopes. For series №10 and №28 mean bed slopes were 0.012 and 0.014 accordingly. For series k3 and d16 the bed slope at distance from 60 up to 90 м from shore was negative because the bottom relief has a submerged bar, and seaward of this distances it was 0. 027. Obviously frequency selective type of energy dissipation will be observed above small bed slopes.

Consider distinctions of waves transformation above different bed slope in detail, modeling wave propagations with "cutoff" terms of equations (1) described linear and non-linear transformations of waves. In reality linear and non-linear transformation of waves occur jointly, and it is impossible strongly to make such artificial separation, but qualitatively such simulation can demonstrate the role of different terms depending on bed slope. In Fig.8 model results of linear and non-linear transformations of waves above bed slopes 0.06 and 0.014 are shown. As initial spectrum of series № 10 in 72 m from shore on depth of 2.75 m was taken. The relative changes of a spectrum were estimated through 20 m of wave propagation. It is obvious, that non-linear processes during wave transformation at bed slope 0.06 transfer energy of first harmonic basically to higher harmonics (0.5-0.6 Hz) (Fig.8a), and at slope 0.014 - basically to frequencies of second and third harmonics (0.25-0.35 Hz). The increasing of bed slope leads to significant increasing of wave energy near the peak frequency during only linear wave transformation (Fig.8b).

The influence of non-linear energy transfer inside a spectrum on a type of frequency dependence of damping coefficient is marked in many scientific publications. It was supposed, that square type of frequency dependence of damping coefficient is result of fast nonlinear transfer, and low transfer leads to frequency independence of damping coefficient (Chen et al., 1997; Kaihatu, Kirby, 1996). Following to this ideas and our results it is possible to conclude, that the energy dissipation during wave breaking occurs so what to compensate consequences of linear and non-linear transformation of waves above an inclined bottom. A kind of damping coefficient is determined by a ratio of linear and non-linear changes of waves, arising at its transformation. In outer part of surf zone, the linear and non-linear changes can give the approximately equal contribution that is determined by relatively deep water and results in frequency independent dissipation of energy. In inner part of surf zone at large bed slopes the linear processes dominate in first harmonic frequency band, but in the band of higher harmonics - the non-linear processes dominate, it results in square frequency dependence of damping coefficient with a minimum dissipation approximately near mean frequency of a spectrum. On small slopes, where the non-linear changes exceed linear, transferring energy basically to second and third frequency harmonics (Fig. 8a), there is a heightened dissipation on frequencies of these harmonics.

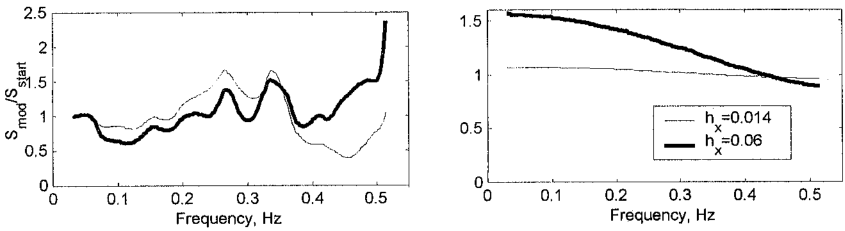


Figure 8. Relative non-linear (a) and linear (b) changes of model wave spectra above different bed slope.

6. Conclusions

Group structure of irregular wave on deep water is automodel process. In coastal zone parameters of wave group structure changes in space quasi-periodical due to non-linear processes and wave breaking does not influence directly on its changing. The decreasing of groupiness factor towards to the shore is determined by non-linear reorganisation of a wave spectrum in the frequency band of the first harmonics and by filling intervals between wave groups of the first harmonics by groups of highest harmonics.

The wave breaking changes the shape of wave spectrum to compensate processes of linear and non-linear wave transformation. It was found three types of frequency dependence of energy dissipation of breaking waves: uniform, squared and selective. It is determined by asymmetry of waves and by bed slope.

Acknowledgments

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