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LONG WAVES IN DIRECTIONAL SEAS

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ABSTRACT

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In recent years the group-induced long waves have received an enhanced degree of attention. Especially in nearshore regions, the long waves can be of considerable height, and consequently the influence on harbour resonance, on the operation of ship terminals, on moorings of large vessels, etc. is obviously very important. It is the grouping of natural wave fields that generates the long waves, and they are proportional to the square of the short-wave height. Therefore, the expressions for the long-wave elevations can be found to include the short-wave components of the wave field and a second-order transfer function. This function is presented in a diagram with dimensionless parameters. For practical purposes a formula for rough estimate of the long-wave height is proposed.

The second-order equations show that the long waves are determined by the difference of the wave-number vectors of the short waves. This is shown to imply that the spread of the long waves is larger than that of the short waves, and that the wave lengths of the long waves are dependent on the short-wave spread. Hereby it is possible to change the long-wave lengths, which seems to be a quality of great practical importance.

The long waves are also expressed in spectral terms. That is, a formula for the directional long-wave spectrum is shown to comprise the transfer function squared and the short-wave amplitudes and phases.

INTRODUCTION

Long-period oscillations are often responsible for harbour resonance, for disturbing operations in ship terminals, and for breaking the moorings of large vessels. One of the sources that generate long waves is the wave grouping. The long waves in a regular wave group were shown by Longuet-Higgins and Stewart (1964) to be produced by the radiation stress, which acts in the direction of wave propagation. The radiation stress is an internal compressive force proportional to the square of the wave height. This is the reason why the large waves induce a trough of a very long wave, and the small waves between the groups produce the corresponding crest. The resulting long-period oscillations are of second order, and they are tied to the groups. Hence, these group-induced long waves are of periods equal to that of the group, and they travel with the group velocity.

During a storm in the North Sea in 1980, the Danish Hydraulic Institute took a 4-hour record of the wave elevations, $\eta(t)$. Since a reversed echo sounder was used for the measurements, also the long-wave elevations, $\xi(t)$, appear in the record. The comparison of about 2.5 min of the short and the long-wave elevations in Fig. 1 confirms the qualitative description above.

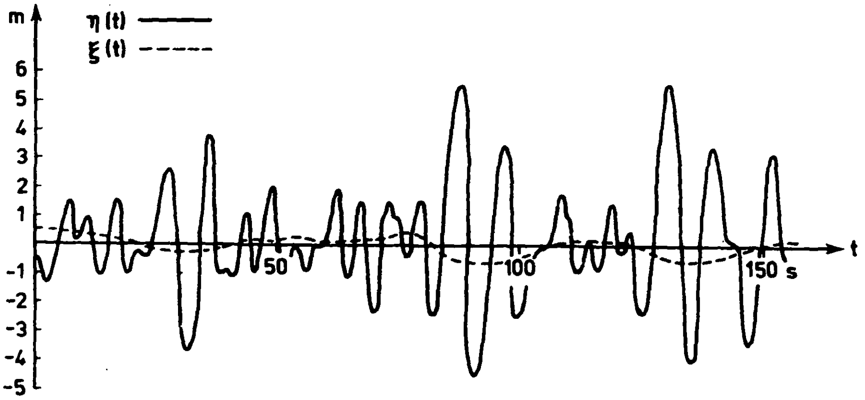

Fig. 1. Short- and long-wave measurements in the North Sea. The significant wave height was 6.5 m and the water depth 40 m. The long waves have been doubled for clearness.

The range of periods of interest for harbours and ships is usually from 20 sec to 2—3 min. Typically, the groups comprise from 4 to 10 waves with the most probable number being 5 according to Sedivy (1978). The amplitudes of the second-order, group-induced long waves are of the order of centimeters, rarely reaching 1 m. However, as, for instance, the damping of the long-period movements of a moored vessel is rather low, a critical level of resonance response can be obtained even with small long-wave amplitudes.

The long waves also play an important role in laboratory tests with harbours, terminals and the like. Sand (1981a) showed that several undesired long-wave effects are introduced if the wave generators are controlled by a traditional first-order signal. Also Bowers (1980) has discussed these long-period model phenomena.

The group-induced long waves can, under some circumstances, be released and propagate as free waves. For instance, Tucker (1950) measured, off a beach in Cornwall, long-period waves, which were out of phase (a lag of some minutes) with the groups. They seemed to be free waves produced by wave trains that travelled into the breaker zone. There the short-wave energy was emitted, and the group-induced long waves were reflected from the beach. This is known as surf beat. Furthermore, Bowers (1977) has shown that harbour resonance can be excited by free long waves also. Due to the different wave heights inside and outside the harbour, an imbalance in group-induced long waves arises at the entrance. Thus, free long waves necessarily appear, and it was shown that these are the waves that are amplified in resonance situations.

Instead of second-order long waves (of frequency $f_1 - f_2$, i.e. the group frequency), Biésel (1963) studied the diffraction around a breakwater of second harmonic waves (of frequency $f_1 + f_2$, i.e. corresponding to twice the frequency of a regular wave). He experimentally verified that the diffraction caused the release of the second-harmonic waves that are otherwise tied to the primary waves. Hence, free waves of twice the basic frequency penetrated into the harbour, and in some cases these second-order phenomena were the dominating wave disturbances.

It is believed that only drastic changes in the wave pattern, the bottom profile, etc. can cause the emission of second-order free waves. As regards group-induced long waves, the propagation as free waves has been discussed in connection with surf beat and harbour resonance, but from the above analogies it also seems likely that diffraction around obstacles, e.g. breakwaters, piers etc., and diffraction caused by entrance channels will release the tied long waves. The following sections will, however, consider only the general problem of describing the directional long waves tied to wave groups propagating over a horizontal bottom.

As briefly indicated above, the long waves are of great practical importance, and they should be accounted for in wave theories and forces used for design, in numerical models and in physical model tests. Usually natural wave fields are, as a first approximation, represented by uni-directional wave trains, and the long waves are calculated on that basis. Below, the three-dimensional expressions and transfer functions for the group-induced long waves will be given, and comparisons are made with the results obtained from the assumption of unidirectional waves. The directional spectrum of the long waves will also be derived.

DIRECTIONAL LONG-WAVE SOLUTION

The one-dimensional solution of the Laplace equation (or what is equivalent: the momentum equations) with respect to long waves was given by Bowers (1977) and Ottesen Hansen (1978). If two Fourier components constituting a regular wave group are considered, the second-order long wave, $\xi(x,t)$, generated by this group can be found to be:

$$\xi_{nm}(x,t)/h = G_{nm} h \{[(a_n a_m + b_n b_m)/h^2] \cos(\Delta\omega_{nm} t - \Delta k_{nm} x) + [(a_m b_n - a_n b_m)/h^2] \sin(\Delta\omega_{nm} t - \Delta k_{nm} x)\} \quad (1)$$

where G_{nm} is a transfer function applied to the frequencies indicated by subscripts n and m. These two frequencies are represented by cosine and sine terms with the amplitudes a and b, respectively. The water depth is h, and the long wave is characterized by the cyclic frequency $\Delta\omega_{nm} = \omega_n - \omega_m$ and the wave number $\Delta k_{nm} = k_n - k_m$. To find the long-wave elevations in an irregular wave train all the contributions similar to (1) are added. Then:

$$\xi(x,t) = \sum_{n-m=1}^{\infty} \sum_{m=m^*}^{\infty} \xi_{nm}(x,t) \qquad \text{with} \qquad m^* = f^*/f_0 \tag{2}$$

That is, all combinations of pairs are included. The lowest frequency in the short-wave spectrum is denoted f^*, while $f_0 = 1/T$ is the basic Fourier frequency, T being the length of the time series. The quantity f^* is often equal to about 0.05 Hz since this is the lower limit for measuring equipment like wave-riders. It is seen from (1) that the long waves are characterized by the frequency difference $f_n - f_m$, and the wave-number difference $k_n - k_m$.

In a natural wave field the wave numbers are vectors, while the frequencies, wave heights, etc. are scalars. When directional seas are considered the long waves must, therefore, be expected to propagate in the direction determined by $\vec{k}_n - \vec{k}_m$, i.e. the wave-number vector difference of the short waves, cf. Fig. 3. The long-wave frequency must, however, still be the simple difference $f_n - f_m$ as in the one-dimensional case.

The characteristics of the second-order, group-induced long waves are derived mathematically below. A definition sketch is given in Fig. 2. When

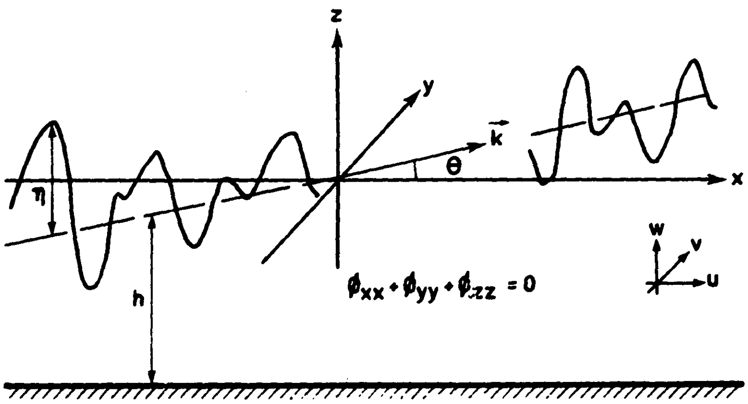

Fig. 2. Definition sketch showing a wave train with wave number $\vec{k}$ in direction θ.

the velocity potential $\phi(x,y,z,t)$ is defined as:

$$\phi_x = u, \qquad \phi_y = v \qquad \text{and} \qquad \phi_z = w \tag{3}$$

where u, v and w are the velocity components indicated in Fig. 2, the Laplace equation can be written:

$$\phi_{xx} + \phi_{yy} + \phi_{zz} = 0 \qquad \text{for} \begin{cases} -h < z < \eta \\ -\infty \leqslant x,y \leqslant \infty \end{cases} \tag{4}$$

This equation should be solved with the surface conditions:

$$\left. \begin{aligned} &\eta_t + \phi_x \eta_x + \phi_y \eta_y = \phi_z \\ &g\eta + \tfrac{1}{2}(\phi_x^2 + \phi_y^2 + \phi_z^2) + \phi_t = 0 \end{aligned} \right\} \quad \text{for } z = \eta \tag{5}$$

These can be combined, and η can be eliminated so that (5) appears in terms of only the velocity potential ϕ. At the bottom the vertical velocity, w, should be zero, i.e.:

$$\phi_z = 0 \qquad \text{for } z = -h \tag{6}$$

Usually a perturbation technique is applied for solution of the Laplace equation with the small parameter in the expansions being the water surface slope. Initially, this results in the well-known first-order equations with a very simple solution. That is, a regular wavelet of amplitude a in the direction θ appears, viz.:

$$\eta(x,y,t) = a\cos(\omega t - kx\cos\theta - ky\sin\theta + \varphi) \tag{7}$$

where the k-vector is projected on the horizontal axes, and φ is the phase.

If the regular wave in (7) is the input to the second order equations, only the usual second-harmonic wave would appear. Consequently, in order to find the group-induced long waves (difference terms) the first-order input must obviously consist of at least two frequencies, i.e.:

$$\begin{aligned}\eta_{nm}(x,y,t) = {} & a_n\cos[\omega_n t - \vec{k}_1\cdot(x,y)] + b_n\sin[\omega_n t - \vec{k}_2\cdot(x,y)] \\ & + a_m\cos[\omega_m t - \vec{k}_3\cdot(x,y)] + b_m\sin[\omega_m t - \vec{k}_4\cdot(x,y)]\end{aligned} \tag{8}$$

where the k-vectors are applied directly instead of their projections in (7), and $\vec{k}_1$ indicates the wave number and direction of the a_n component, $\vec{k}_2$ the wave number and direction of the b_n component, etc. The second-order solution now consists of two terms, viz. one corresponding to the second harmonic waves of frequency $f_n + f_m$, and one giving the long waves of the group frequency, $f_n - f_m$. The total solution can be seen in Sharma and Dean (1979). If their results are rewritten for the first-order input corresponding to (8), and only the long-wave terms are retained, the long-wave elevations $\xi_{nm}(x,y,t)$ become:

$$\begin{aligned}\xi_{nm}(x,y,t)/h = {} & a_n a_m G_{\eta\xi}(\Delta\theta_{13})h\cos[\Delta\omega_{nm}t - \Delta\vec{k}_{13}\cdot(x,y)]/h^2 \\ & + b_n b_m G_{\eta\xi}(\Delta\theta_{24})h\cos[\Delta\omega_{nm}t - \Delta\vec{k}_{24}\cdot(x,y)]/h^2 \\ & + a_m b_n G_{\eta\xi}(\Delta\theta_{32})h\sin[\Delta\omega_{nm}t - \Delta\vec{k}_{32}\cdot(x,y)]/h^2 \\ & - a_n b_m G_{\eta\xi}(\Delta\theta_{14})h\sin[\Delta\omega_{nm}t - \Delta\vec{k}_{14}\cdot(x,y)]/h^2\end{aligned} \tag{9}$$

where $G_{\eta\xi}$ is a transfer function from first-order short-wave to second-order long-wave elevations, and $\Delta\vec{k}_{ij}$ denotes the wave number vector of the long waves, i.e. $\vec{k}_i - \vec{k}_j$, generated by the short waves with $\vec{k}_i$ and $\vec{k}_j$, cf. Fig. 3. As expected, the solution shows that the frequencies of the group-induced long waves are determined simply by the difference $f_n - f_m$, while the wave number and the direction of propagation follow from $\vec{k}_i - \vec{k}_j$. The latter has four different values in (9) because of the individual directions of the short-wave components in (8). Of course, the sine components could have been given the same directions as their respective cosine components, i.e. $\vec{k}_2 = \vec{k}_1$ and $\vec{k}_4 = \vec{k}_3$.

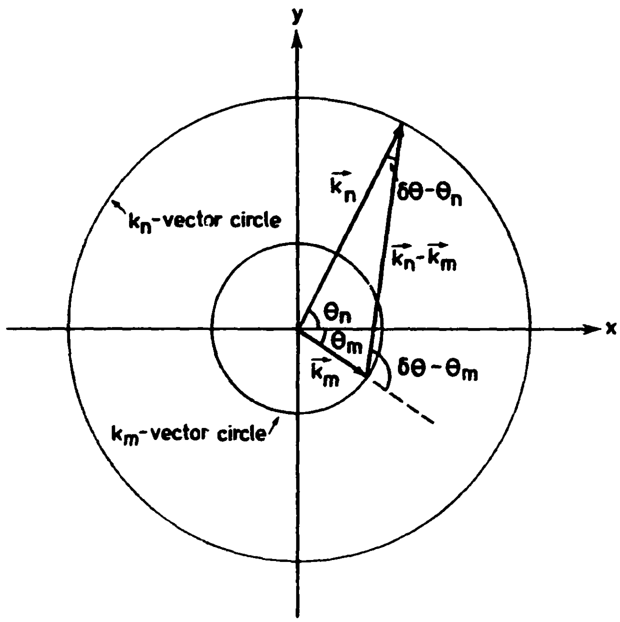

Fig. 3. Wave number vectors $\vec{k}_n$, $\vec{k}_m$ of two short-wave components, and the resulting long-wave vector $\vec{k}_n - \vec{k}_m$. The associated angles of travel are used for directional spectrum calculations.

This would simplify the expression in (9) so a form like that of (1) was reached. The reason why individual directions are maintained is to fit the theory for Fourier decomposition of directional site wave records given by Sand (1979). In this, two directional components (a_n and b_n) are attached to each frequency in very narrow intervals. Thus, the form of (9) is only a matter of principle. A detailed discussion of the function $G_{\eta\xi}$, which is dependent on the difference of the short-wave directions $\Delta\theta_{ij} = \theta_i - \theta_j$, is given in the following section.

In a natural, directional sea the total long-wave elevation, $\xi(x,y,t)$, becomes the sum of contributions from all pairs of frequencies. Due to the structure of the decomposition by Sand (1979), mentioned above, the result is in this case simply obtained by combining (9) and (2). However, if alternatively each frequency comprises infinitely many directional components, additional sums must be introduced. Thus, if the directional components are numbered 1,2,3, . . . p at each frequency, the total long-wave elevations become:

$$\xi(x,y,t) = \sum_{n-m=1}^{\infty} \sum_{m=m^*}^{\infty} \sum_{p_n=1}^{\infty} \sum_{p_m=1}^{\infty} \xi_{nm}(x,y,t) \tag{10}$$

It is seen from (9) that the directional long-wave amplitude, ξ_a, is of the order:

$$\xi_a = G_{\eta\xi}(\Delta\theta_{nm})\, hA_nA_m/h \tag{11}$$

where A_n, A_m represent the amplitude expressions. This will later be applied for rough estimates of the long wave amplitude in directional seas.

THE DIRECTIONAL TRANSFER FUNCTION $G_{\eta\xi}$

Solving the Laplace equation to second order gives a rather complicated directional transfer function, $G_{\eta\xi}$. As indicated in (9) it is a function of frequency and direction, i.e. $G_{\eta\xi}(f_m, \Delta f_{nm}, \Delta\theta_{nm})$. It is stressed that the difference of the short-wave directions, $\Delta\theta_{nm} = \theta_n - \theta_m$, is not equal to the long-wave direction, $\delta\theta$, which is determined by $\vec{k}_n - \vec{k}_m$. The dimensionless expression for $G_{\eta\xi}$ applied in (9) is:

$$G_{\eta\xi}h = \frac{1}{2}\left[\frac{G'h^2 - k_nh\, k_mh \cos\Delta\theta_{nm} - 16\,\pi^4 D_m^2 D_n^2}{4\,\pi^2 D_n D_m} + 4\pi^2(D_n^2 + D_m^2)\right] \tag{12}$$

where another dimensionless function has been inserted, viz.:

$$\begin{aligned} G'h^2 = {} & \{(D_n - D_m)\,[D_m(k_n^2h^2 - 16\,\pi^4 D_n^4) - D_n(k_m^2h^2 - 16\,\pi^4 D_m^4)] \\ & + 2(D_n - D_m)^2\,[k_nh\,k_mh \cos\Delta\theta_{nm} + 16\,\pi^4 D_n^2 D_m^2]\}/ \\ & \{(D_n - D_m)^2 - K^{-}h/4\,\pi^2\} \end{aligned} \tag{13}$$

with:

$$K^{-}h = |\vec{k}_n - \vec{k}_m|\, h \tanh(|\vec{k}_n - \vec{k}_m|\, h) \tag{14}$$

The dimensionless frequency, D_m, applied above is:

$$D_m = \sqrt{\frac{h}{g}}\; f_m \tag{15}$$

The expression in (12) can be shown to equal the one-dimensional G_{nm} function in (1) for $\Delta\theta_{nm} = 0$. With the $G_{\eta\xi}$ function the elevations of the group-induced, second-order long waves can be determined, and the angle of propagation, $\delta\theta$, is easily found by means of an expression for the vector difference, i.e.:

$$\tan\delta\theta = (k_nh \sin\theta_n - k_mh \sin\theta_m)/(k_nh \cos\theta_n - k_m h \cos\theta_m) \tag{16}$$

The directional transfer function $G_{\eta\xi}h$ is shown in Fig. 4 together with the angles of propagation $\delta\theta$ of the resulting long waves. $G_{\eta\xi}$ is a function of the difference $\Delta\theta_{nm}$, whereas $\delta\theta$ is a function of each of the short-wave directions θ_n and θ_m. The lower part of the diagram has, therefore, been made for wave components travelling symmetrically about the x-axis, i.e. $\theta_n = -\theta_m$. Also shown is the G_{nm} function (applied in one-dimensional cases) for two values of $\Delta f_{nm}/f_m$. It can be concluded that the long-wave amplitudes in natural wave fields are clearly smaller than in plane wave trains, even for small angles between the short-wave components. In shallow water there seems, typically, to be a factor 5—10 between long-wave amplitudes generated by directional and uni-directional short waves. However, the factor decreases with increasing water depth.

As regards the directions of propagation, $\delta\theta$, of the long waves, it can be

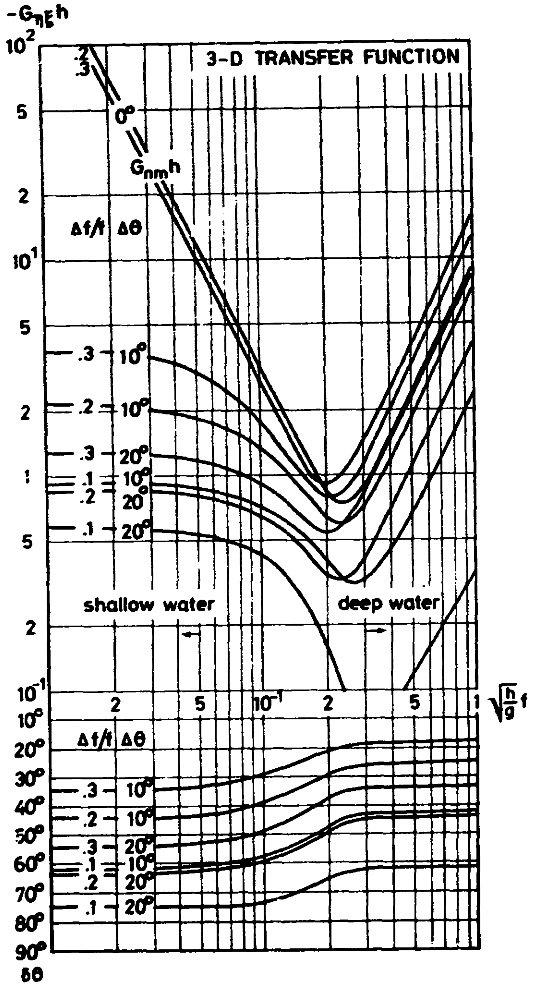

Fig. 4. The directional transfer function $G_{\eta\xi}$ for a wave group consisting of two frequencies $f_m = f$ and $f_n = f + \Delta f$ with angle $\Delta\theta$ between one another. Lower part indicates long wave direction of travel, $\delta\theta$, for short waves with $\theta_n = -\theta_m$.

seen that these are generally larger than the corresponding short-wave angles of travel. This is, indeed, true for shallow-water wave fields. The large spread of the long waves could also be deduced by considering Fig. 3. An example of the long-wave energy distribution is given in Fig. 5. The commonly used $\cos^2\theta$ distribution is applied to a North Sea wave spectrum, $S_{\eta\eta}(f)$, recorded during a storm. The $\cos^2\theta$ curve seen in the figure is thus valid for all short-wave frequencies. However, as indicated by the calculated spectrum of the long waves, $S_{\xi\xi}(f)$, the distribution of the directions of propagation of the long waves is clearly broader. The spectrum calculations are discussed in a later section.

QUALITIES OF DIRECTIONAL LONG WAVES

One of the characteristics of the long waves in a uni-directional wave train is the large amplification occurring in shallow water, cf. the G_{nm} function in

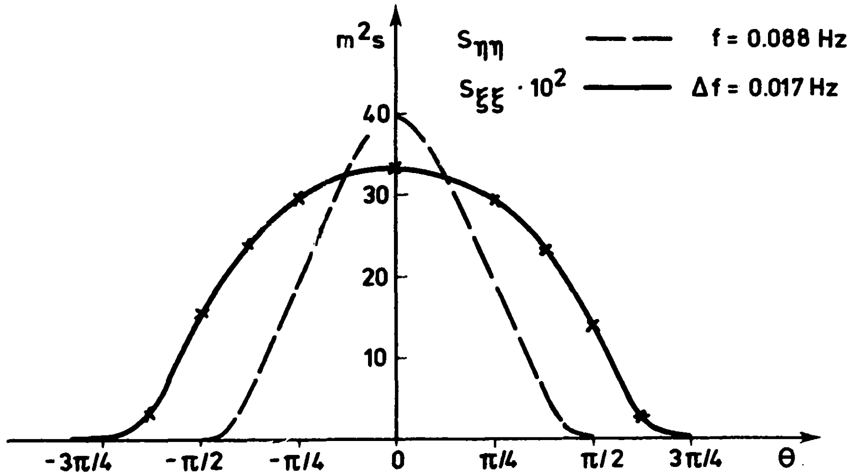

Fig. 5. A $\cos^2\theta$ distribution applied to a North Sea wave spectrum with peak frequency f_p = 0.088 Hz. The resulting long-wave spread is shown for the components with Δf = 0.017 Hz.

Fig. 4. This is, according to Ottesen Hansen et al. (1980), due to the small difference between the group velocity, c_g, and the phase velocity, c, making it possible for the individual waves in the group to follow the induced long waves. In case of directional seas, however, cause and effect cannot follow each other because of the directional spread. Therefore, no matter how close c_g comes to c, the long waves will not be amplified so much as in plane wave trains. This conclusion can also be drawn from Fig. 4.

The figure is also applicable for a rough estimate of the amplitudes of the long waves in a directional sea. Such estimates can often be valuable in the planning phase of harbours, ship terminals, etc. For instance, an evaluation of the annual number of inoperative days and of the critical mooring forces could be obtained. According to (11) the $G_{\eta\xi}$ function, the water depth and the short-wave amplitudes are needed. The peak frequency, f_p, is inserted in $\sqrt{h/g}\,f_p$ to give a reasonable estimate of one of the variables in Fig. 4. However, to determine a value of the $G_{\eta\xi}$ function also $\Delta f/f$ and $\Delta\theta$ of the short waves should be known. The simplest way to approximate directional differences and amplitudes of a wave field is indicated in Fig. 6a. Generally, a good estimate of the highest wave in a wave group is H_s, and the amplitude is therefore $H_s/2$. This may be distributed over three directions representing the peak energy distribution. Thus, a reasonable α should be determined from the actual short wave records, or from an assumed directional distribution. Since, however, the $G_{\eta\xi}$ function for $\Delta\theta = 2\alpha$ is much smaller than for $\Delta\theta = \alpha$, the contribution from the combination of the two oblique waves becomes rather small. It is, therefore, sufficient to consider the contributions from the main direction ($H_s/4$) with each of the waves in the α-directions ($H_s/8$). As regards long-wave amplitudes, this becomes equivalent with the situation shown in Fig. 6b. Hence, the wave field is approximated by two waves each of amplitude $H_s/4$ (to make a total of $H_s/2$) and with an angle α between one an-

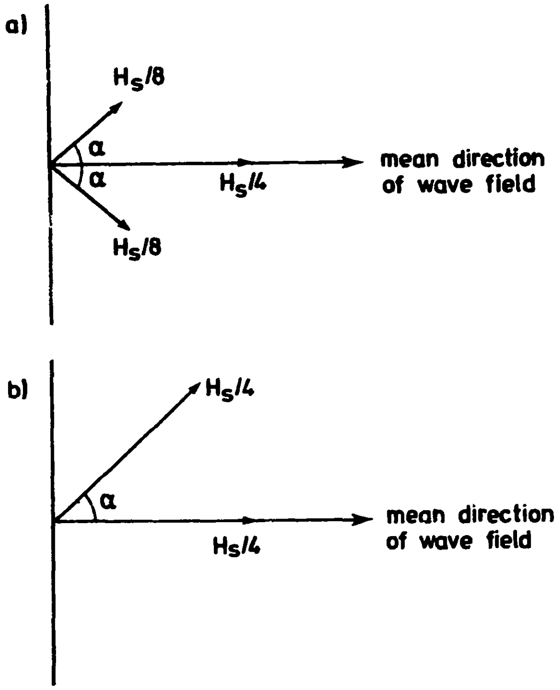

Fig. 6. a. Directional distribution approximated by three wave components. b. Equivalent distribution as regards long-wave amplitudes.

other. Since $G_{\eta\xi}$ is a function of f, $\Delta f/f$ and $\Delta\theta$, only a value of $\Delta f/f$ is left. Sedivy (1978) has analysed numerous records and found the most probable number of waves in a group to be 5, i.e. $\Delta f/f$ becomes typically 0.2. Thus, with $\sqrt{h/g}\,f_p$ a reasonable value of $G_{\eta\xi}(f_p, 0.2, \alpha)$ can be found from Fig. 4. Then for practical purposes a rough estimate of the long wave amplitude becomes according to (11):

$$\xi_a = G_{\eta\xi}h\, H_s^2/16\, h \qquad (17)$$

Consider, for example, a wave field with $H_s = 4$ m and peak frequency $f_p = 0.1$ Hz that propagates into a water depth of $h = 10$ m. The variables in Fig. 4 become $\sqrt{h/g}\,f_p = 0.1$, $\Delta f/f = 0.2$ and $\Delta\theta = 20°$, the latter being an assumed, typical spreading of the peak energy. The $G_{\eta\xi}h$ function is then found to be about 0.7 in Fig. 4. By means of (17) the long-wave amplitude in the wave field is roughly $\xi_a = 0.7 \cdot 16/16 \cdot 10 = 0.07$ m. For reasons of comparison, it can be seen that a uni-directional wave train with the same parameters would have generated long waves of amplitude $\xi_a = 0.35$ m. Since (17) represents a typical amplitude, it should be noted that an estimate of, for instance, the significant height of the directional long waves becomes approximately $H_{s,\xi} = 3$ to $4\,\xi_a$.

Another interesting property of directional long waves is connected with the fact that direction and wave number are determined by $\vec{k}_n - \vec{k}_m$. Thus, the wave lengths of the long waves are dependent on the directions of the short waves. This can also be deduced from Fig. 3. Since the smallest vector difference appears in uni-directional waves, the wave lengths of the long waves must obviously decrease for increasing spread of the short waves. This is shown graphically in Fig. 7. The wave field considered above gives $\Delta L/\Delta L_u = 0.5$, i.e. the wave length of the long wave is half the one appearing in uni-directional waves. This possibility of varying the long waves by changing the directional spread of the short waves has obviously practical importance in connection with, for instance, harbour resonance, cf. Sand (1981b).

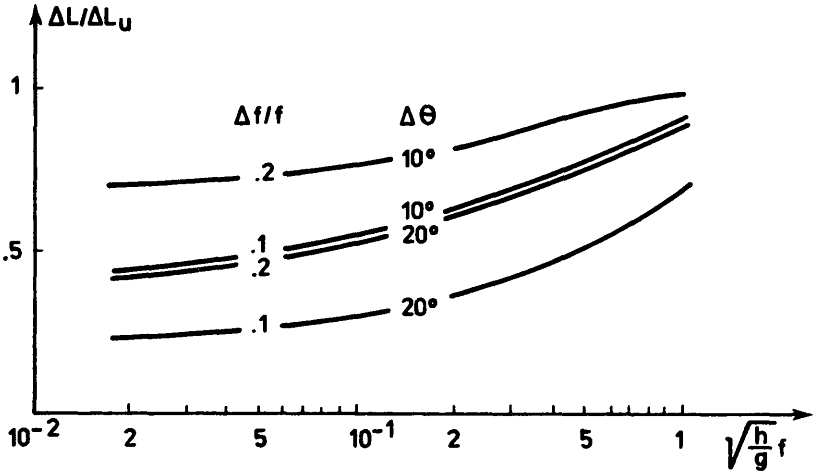

Fig. 7. Wave lengths, ΔL, of directional long waves to wave lengths, ΔL_u, of uni-directional long waves shown as function of a dimensionless frequency.

DIRECTIONAL LONG-WAVE SPECTRUM

Since the elevations of the group-induced long waves are already determined, it is possible to calculate the long-wave spectrum. The long waves depend on the square of the short-wave amplitudes, and the long-wave spectrum must, consequently, include the fourth power of these amplitudes. In uni-directional waves the long-wave spectrum, $S_{\xi\xi}(\Delta f)$, is quite easily found by means of (1). All the contributions to the frequency Δf should be added up and squared. That is, the simple, raw long-wave spectrum[1] becomes:

$$S_{\xi\xi}(\Delta f) = \left[\left(\sum_{m=m^*}^{\infty} G_{nm}(f,\Delta f)\,(a_n a_m + b_n b_m) \right)^2 + \left(\sum_{m=m^*}^{\infty} G_{nm}(f,\Delta f)\,(a_m b_n - a_n b_m) \right)^2 \right] \Big/ 2f_0 \qquad (18)$$

[1]This spectrum cannot be written as a function of the short wave spectrum as suggested by Ottesen Hansen (1978).

where the G_{nm} function from (1) is applied, and $n = m + \Delta f/f_0$.

The calculation of the directional long-wave spectrum is a little more complicated. The spectrum should be written as a function of the long-wave frequency, Δf, and the long-wave direction, $\delta\theta$, i.e. $S_{\xi\xi}(\Delta f, \delta\theta)$. Analogous with the principle in (18) contributions to a specific frequency and direction should be added before squaring. The problem is, however, to find the short wave directions that will generate a long wave with the angle $\delta\theta$. As mentioned before, two short waves travelling in the directions θ_n and θ_m, i.e. with the difference $\Delta\theta$, generate a long wave in the direction $\delta\theta$, where $\delta\theta \neq \Delta\theta$. The problem is solved by means of Fig. 3. If the angle $\delta\theta$ of the directional long wave and one short-wave component in direction θ_m are given, the direction θ_n of the remaining short-wave component has to be determined. The figure shows that:

$$\sin(\delta\theta - \theta_n)/k_m = \sin(\pi - \delta\theta + \theta_m)/k_n \tag{19}$$

and then θ_n is directly found as:

$$\theta_n = \delta\theta - \arcsin\left[\frac{k_m}{k_n}\sin(\pi - \delta\theta + \theta_m)\right] \tag{20}$$

Since the frequencies and directions of the short-wave components can now be found for given values of the long-wave parameters Δf and $\delta\theta$, it appears that the raw, directional long-wave spectrum can be written as:

$$S_{\xi\xi}(\Delta f, \delta\theta) = \left[\left(\sum_{m=m^*}^{\infty}\ \sum_{\theta_m=-\pi}^{\pi} G_{\eta\xi}(a_n a_m + b_n b_m)_p\right)^2 + \left(\sum_{m=m^*}^{\infty}\ \sum_{\theta_m=-\pi}^{\pi} G_{\eta\xi}(a_m b_n - a_n b_m)_p\right)^2\right] \Big/ 2 f_0 \Delta\theta \tag{21}$$

where the $G_{\eta\xi}(f, \Delta f, \Delta\theta)$ function from (12) is inserted, $n = m + \Delta f/f_0$, θ_n is found from (20), and $\Delta\theta$ is the step (discretization) in the summation over directions. Thus, the a and b coefficients in (21) should be read as functions of frequency (indicated by n and m), but also as functions of direction (indicated by the index p). That is, two frequencies n and m comprise a series of coefficients, representing wavelets with different directions, which implies that p runs as $p = 1, 2, 3, \ldots$ Hence, in principle all combinations of n, m and p should appear in (21). Dependent on the short waves and their decomposition a and b could, of course, be zero in some or many of the terms in (21), cf. for instance the structure of (9).

Both a short and a long-wave spectrum are shown schematically in Fig. 8. The total directional wave spectrum is denoted $S_{\zeta\zeta}(f, \theta)$, i.e. $S_{\zeta\zeta} = S_{\eta\eta} + S_{\xi\xi}$.

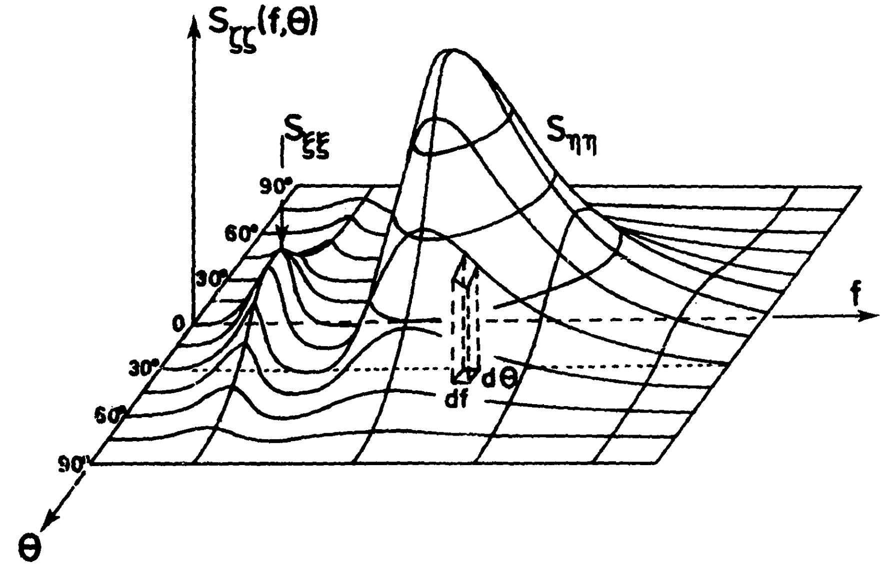

Fig. 8. Directional spectrum consisting of a short- and a long-wave part.

CONCLUSIONS

The directional long waves induced by the grouping of a natural wave field have been determined. The expressions are shown to include a directional transfer function from which important qualities of the long waves can be derived. Mathematically the directional long waves are determined by the differences of short-wave terms, i.e. the long-wave frequency is $f_n - f_m$, while the direction of propagation and the wave number are found from $\vec{k}_n - \vec{k}_m$.

Of practical interest are the facts that the amplitudes of the directional long waves seem to be significantly smaller than those of uni-directional waves, and that the wave lengths of the long waves can be altered simply by changing the directional spread of the short waves. Moreover, the spread of the long waves is clearly larger than that of the causative short waves.

In practice it is often desirable in, for instance, an early design stage to assess the long-wave amplitudes in a fast and simple way. Therefore, an attempt to derive a formula for rough estimates of the directional long-wave amplitudes was made.

Generally, the directional spectrum is convenient for the representation of a natural wave field. This, of course, also applies to the long waves. By considering the angles of travel of the short and the long waves an expression for the long wave directional spectrum appeared. It was found that this spectrum can be written as a function of the short-wave components and the directional transfer function.

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