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Modification of the damping function in the k–ε model to analyse oscillatory boundary layers

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Abstract

A simple relationship has been developed between the wall coordinate y^+ and Kolmogorov's length scale using direct numerical simulation (DNS) data for a steady boundary layer. This relationship is then utilized to modify two popular versions of low Reynolds number k–ε model. The modified models are used to analyse a transitional oscillatory boundary layer. A detailed comparison has been made by virtue of velocity profile, turbulent kinetic energy, Reynolds stress and wall shear stress with the available DNS data. It is observed that the low Reynolds number models used in the present study can predict the boundary layer properties in an excellent manner.

Keywords: Turbulence model; Oscillatory boundary layer; Shear stress; k–ε model

1. Introduction

A number of practical situations in fluid mechanics require an understanding of oscillatory boundary layers and turbulence associated with them. Many experimental and analytical studies have been carried out on this topic. Comprehensive reviews of these studies have been published by Sleath (1990) and Soulsby et al. (1993). The idea of using turbulence models to tackle this phenomenon is relatively new. With the availability of excellent computing facilities at affordable costs, this option is gaining more popularity among the researchers and the practicing engineers. The benefit of using a good turbulence model is that it produces detailed boundary layer properties at a reasonable cost within short time. For a turbulence model to be good, an essential requirement is computational economy with reasonable accuracy. The present study deals with the application of low Reynolds number k–ε models to an oscillatory boundary layer. The term *low Reynolds number* implies that this model is applicable over the whole cross-stream dimension including the low Reynolds number region (viscous sublayer).

The low Reynolds number k–ε model was originally developed by Jones and Launder (1972) and then various modifications were proposed to widen its scope of applications or improve the predictive ability of this model. Most of these modifications proved to be ad hoc in nature after applying these modified versions to the cases other than they were developed for. Some of these models were applied to oscillatory boundary layers as well. Patel et al. (1985) reviewed some of the versions of two-equation models in relation to steady flow phenomena. In this study, the two-equation models were reviewed for their correct near-wall behaviour as well. Moreover, a valuable experimental data gathered from various sources were provided that proved to be helpful in developing new models or modifying them. A number of modified versions of two-equation turbulence models were published based on the findings of Patel et al. (1985).

Although two-equation models were mainly developed for steady boundary layers, their application to oscillatory

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boundary layers has been successful too (Thais et al., 1999; Cotton and Stansby, 2000; Foti and Scandura, 2004; Shen et al., 2004; Sue et al., 2005). Tanaka and Sana (1994) reviewed some of the older versions with reference to oscillatory boundary layer properties by using the available experimental data. As a result of this study it was found that the original model by Jones and Launder (1972) performed better than the rest of the models tested, especially considering the transitional properties of oscillatory boundary layers. Sana and Tanaka (2000) concluded after reviewing the original and four newer versions with reference to the available direct numerical simulation (DNS) data for oscillatory boundary layers that the turbulence models proposed by Myong and Kasagi (1990) and Nagano and Tagawa (1990) showed better agreement with the DNS data by virtue of the shape of the turbulent kinetic energy profile. But it was noted that the expressions for the damping function used in many of the newer versions of low Reynolds number k–ε models involve the wall coordinates y^+ $(=yu_f/v$, $y=$ cross-stream distance, $u_f=$ shear velocity and $v=$ kinematic viscosity). In case of oscillatory flow, the bottom shear stress goes to zero twice in a wave cycle and at that time the damping function based on y^+ becomes zero in the whole cross-stream dimension, which is physically incorrect.

In the present study, a simple relationship has been developed and then utilized to transform wall coordinates to another suitable variable based on the DNS data for steady boundary layers. The damping functions of two of the popular versions proposed by Myong and Kasagi (1990) and Nagano and Tagawa (1990) are thus modified and tested against the DNS data for oscillatory boundary layers along with the original model by Jones and Launder (1972).

2. Methodology

2.1. Governing equations of k–ε model

Using the eddy viscosity concept, the equation of motion, for one-dimensional oscillatory boundary layer, may be written as

$$\frac{\partial u}{\partial t}=\frac{\partial U}{\partial t}+\frac{\partial}{\partial y}\left\{(v+v_t)\frac{\partial u}{\partial y}\right\}, \tag{1}$$

where u is the velocity in x-direction, U the free-stream velocity, t the time, y the cross-stream dimension and v_{t} the eddy viscosity. According to the general form of low Reynolds number k–ε model, the eddy viscosity is expressed as

$$v_t=C_\mu f_\mu\frac{k^2}{\tilde{\varepsilon}}, \tag{2}$$

where C_μ is a constant ($=0.09$ for the models considered here) and f_μ is the damping function. The turbulent kinetic energy, k, transport equation is

$$\frac{\partial k}{\partial t}=\frac{\partial}{\partial y}\left\{\left(v+\frac{v_t}{\sigma_k}\right)\frac{\partial k}{\partial y}\right\}+v_t\left(\frac{\partial u}{\partial y}\right)^2-\tilde{\varepsilon}-D. \tag{3}$$

And the transport equation of turbulent kinetic energy dissipation rate, $\tilde{\varepsilon}$, is

$$\frac{\partial\tilde{\varepsilon}}{\partial t}=\frac{\partial}{\partial y}\left\{\left(v+\frac{v_t}{\sigma_\varepsilon}\right)\frac{\partial\tilde{\varepsilon}}{\partial y}\right\}+C_1f_1v_t\frac{\tilde{\varepsilon}}{k}\left(\frac{\partial u}{\partial y}\right)^2-C_2f_2\frac{\tilde{\varepsilon}^2}{k}+E. \tag{4}$$

Here, C_1, C_2, σ_k and σ_ε are model constants and f_1, f_2, D and E are model functions.

Jones and Launder (1972) (JL model) proposed an expression based on turbulence Reynolds number only, i.e. $f_\mu=\exp\{-2.5/(1+R_t/50)\}$, $(R_t=k^2/(\tilde{\varepsilon}v))$. But, in some of the later versions, for example, Myong and Kasagi (1990) (MK model) and Nagano and Tagawa (1990) (NT model) used wall coordinate (y^+) in the expression for the damping function as follows:

MK model:

$$f_\mu=\left(1+3.45/\sqrt{R_t}\right)(1-\exp(-y^+/70)). \tag{5}$$

NT model:

$$f_\mu=(1+4.1/R_t^{0.75})(1-\exp(-y^+/26))^2. \tag{6}$$

In case of steady boundary layers, these models (MK and NT models) perform very well and sometimes better than the JL model, which proves that the wall distance should be somehow incorporated in the damping function. But in case of oscillatory boundary layers, the wall shear stress goes to zero twice in a wave cycle and results in the zero value of y^+ at those instants. From Eqs. (5) and (6) it may be readily noted that the damping function goes to zero at these instants leading to a zero eddy viscosity (from Eq. (2)). The experimental data for oscillatory boundary layers show that the turbulent kinetic energy is produced near the wall during acceleration phase and then spreads in the cross-stream direction during deceleration. Therefore, even when the wall shear stress is zero, the eddy viscosity is not zero over the whole cross-stream dimension. This physically incorrect behaviour of MK and NT models renders them to be unsuitable for not only oscillatory boundary layers but the boundary layers under adverse pressure gradient as well.

Abe et al. (1994) utilized Kolmogorov's length scale y^* $(=(v\varepsilon)^{1/4}y/v)$ based on the argument that in the close vicinity of the wall this length scale is very important due to its dependence on the dissipation rate of the turbulent kinetic energy ε. With the availability of DNS data for various types of boundary layers, it is now possible to develop the empirical relationships for different model parameters. In the present study, the DNS data by Kuroda et al. (1990) for steady boundary layer in a smooth channel is utilized to find the relationship between y^+ and y^*. By plotting these two variables it was observed that $y^+\cong 1.65y^*$ for $y^+\leqslant 3$. Therefore, Eqs. (5) and (6) may be re-written after replacing y^+ with y^* as follows:

$$f_\mu = (1 + 3.45/\sqrt{R_t})(1 - \exp(-y^*/42.42)), \quad (7)$$

$$f_\mu = (1 + 4.1/R_t^{0.75})(1 - \exp(-y^*/15.75))^2. \quad (8)$$

The models using Eqs. (7) and (8) are termed here as MK modified (MKM model) and NT modified (NTM model), respectively.

Fig. 1 shows the damping function f_μ against y^+ from DNS data by Kuroda et al. (1990) and the models under consideration. The experimental data given by Patel et al. (1985) are also shown. It may be observed that in the near-wall region, the modified damping functions shown as MKM and NTM models match very well with MK and NT models, respectively. Far from the wall, however, there is a difference in the variation of the damping function. It must be noted that none of the models except NT model shows an agreement with Patel et al. (1985) data. The JL model deviates considerably from the experimental data as well as the DNS data.

The function f_2 used in Eq. (4) was also modified to be expressed in terms of Kolmogorov's length scale. No other adjustment or modification in the original MK and NT model parameters or functions was done. The values of the model parameters, for the three versions considered here, are summarized in Table 1.

All the governing equations (Eqs. (1), (3) and (4)) were made dimensionless using the kinematic viscosity of the fluid v, amplitude of the free-stream velocity U_0, angular frequency χ $(=2\pi/T,\ T=$ period of oscillation), fluid density ρ, and the distance from the wall to the free stream y_h. In the dimensionless form, governing equations require wave Reynolds number $R_w\ (=U_0^2/(\chi v))$ and the reciprocal of Strouhal number $S\ (=U_0/(\chi y_h))$ as input values.

2.2. Boundary conditions

At the solid boundary, no slip boundary condition and at the free stream, gradients of velocity, turbulent kinetic energy and its dissipation rate were equated to zero. The

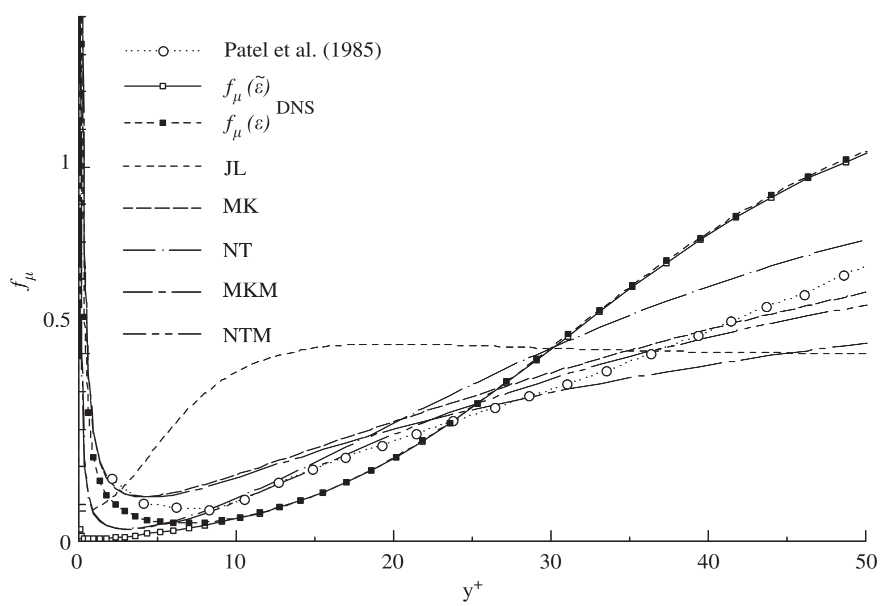

Fig. 1. The comparison of damping function from DNS data and the models.

Table 1
Model parameters for the low Reynolds number k-ε models used in the present study

Parameter	JL model	MKM model	NTM model
C_1	1.55	1.4	1.45
C_2	2.0	1.8	1.9
σ_k	1.0	1.4	1.4
σ_ε	1.3	1.3	1.3
f_μ	$\exp(-2.5/(1+R_t/50))$	$(1+3.45/\sqrt{R_t})\times(1-\exp(-y^*/42.42))$	$(1+4.1/R_t^{0.75})\times(1-\exp(-y^*/15.75))^2$
f_1	1.0	1.0	1.0
f_2	$1-0.3\exp(-R_t^2)$	$(1-(2/9)\exp(-R_t^2/36))\times(1-\exp(-y^*/3.03))^2$	$(1-0.3\exp(-(R_t/6.5)^2))\times(1-\exp(-y^*/3.64))^2$
D	$2v(\partial\sqrt{k}/\partial y)^2$	0.0	0.0
E	$2vv_t(\partial^2 u/\partial y^2)^2$	0.0	0.0
$\tilde{\varepsilon}_0$	0.0	$v(\partial^2 k/\partial y^2)$	$v(\partial^2 k/\partial y^2)$

wall boundary condition for turbulent kinetic energy dissipation rate $\tilde{\varepsilon}_0$ is given in Table 1. It may be observed that this boundary condition involves second derivative of the turbulent kinetic energy and therefore numerically rigid. A mathematically equivalent boundary condition may be derived as follows:

$$\tilde{\varepsilon}_0 = v\frac{\partial^2 k}{\partial y^2} = v\frac{\partial^2}{\partial y^2}\left(\sqrt{k}\right)^2 = 2v\left(\frac{\partial\sqrt{k}}{\partial y}\right)^2 + 2v\sqrt{k}\,\frac{\partial^2\sqrt{k}}{\partial y^2}. \tag{9}$$

But at the wall, $k = 0$, so the second term on the right-hand side vanishes and we get

$$\tilde{\varepsilon}_0 = 2v\left(\frac{\partial\sqrt{k}}{\partial y}\right)^2. \tag{10}$$

In the present study this mathematically equivalent wall boundary condition for $\tilde{\varepsilon}$ was used instead of the one with second derivative of k.

2.3. Numerical method

A Crank–Nicolson type implicit finite difference scheme was used here. The grid spacing near the wall was varied exponentially in order to achieve better accuracy. In space 100 and in time 6000 steps per wave cycle were used. The convergence was based primarily on velocity, k and ε and then on maximum wall shear stress. The convergence limit was set to 1×10^{-6} in the present study. Details of the finite difference method and the solution procedure are given by Sana (1997).

2.4. Reference data

Spalart and Baldwin (1989) carried out DNS simulation of a wave boundary layer under sinusoidal pressure gradient for various wave Reynolds numbers. Here, the results for $R_\delta = 1000$ are used for testing the models under consideration. In the data set the Reynolds number, $R_\delta = U_0\delta_1/v$, where, U_0 is the maximum free-stream velocity and $\delta_1(=\sqrt{2v/\chi})$ termed as Stoke's layer thickness. The corresponding values of input variables for the present turbulence model computations are $R_w = 500000$ and $S = 14.1884$. The free-stream velocity and pressure gradient for this case is shown in Fig. 2. As may be observed from this figure, the normalized free-stream velocity is $U/U_0 = \cos\chi t$ and therefore, the pressure gradient is expressed as $d(U/U_0)/d(\chi t) = -\sin\chi t$.

3. Results and discussion

3.1. Velocity profile

The velocity profile in oscillatory boundary layers shows a distinct overshooting which is efficiently predicted by the models used here as shown in Fig. 3. The cross-stream coordinate is normalized by the Stokes length δ_1. Although there is no distinct difference in the prediction of the velocity by the three models, NTM model performs better than the other models especially at $\chi t = 30°$, $150°$ and $180°$. At $\chi t = 150°$ all the three models show deviation from the DNS data to some extent, this shows the inability of these models to predict the velocity profiles under adverse pressure gradients.

3.2. Turbulent kinetic energy

The comparison of turbulent kinetic energy k (normalized by amplitude of the free-stream velocity U_0) profiles is shown in Fig. 4. All the models can reproduce the generation of turbulent kinetic energy near the wall during adverse pressure gradient phase ($\chi t = 90° \rightarrow 180°$) and its subsequent distribution in the cross-stream direction during favourable pressure gradient phase ($\chi t = 0° \rightarrow 90°$). In almost all the profiles, NTM model shows an excellent agreement with the DNS data far from the wall ($y/\delta_1 \geqslant 1$). In the near-wall region, all the models perform poorly except at $\chi t = 60°$ and $90°$. MKM and NTM models show considerable improvement over the JL model by virtue of the cross-stream variation of the turbulent kinetic

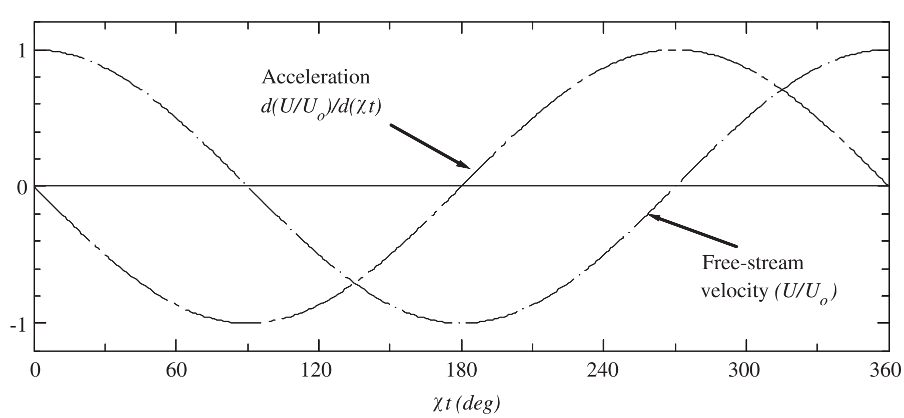

Fig. 2. Free-stream velocity and pressure gradient for DNS data.

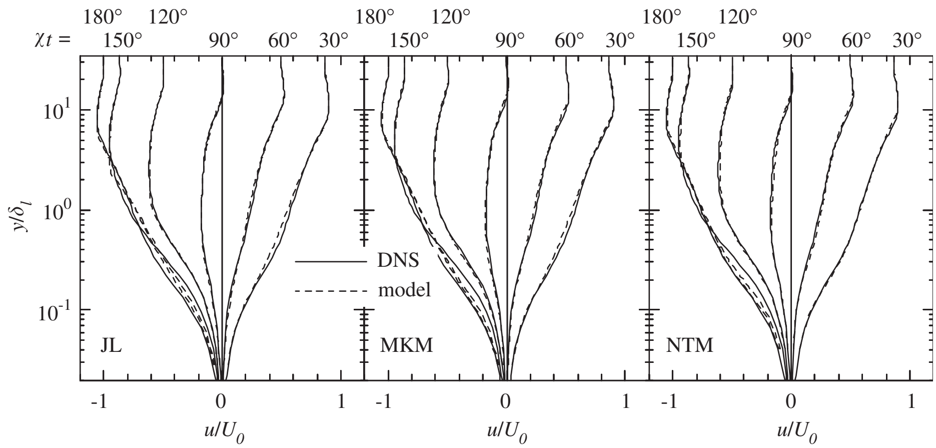

Fig. 3. Cross-stream velocity profiles.

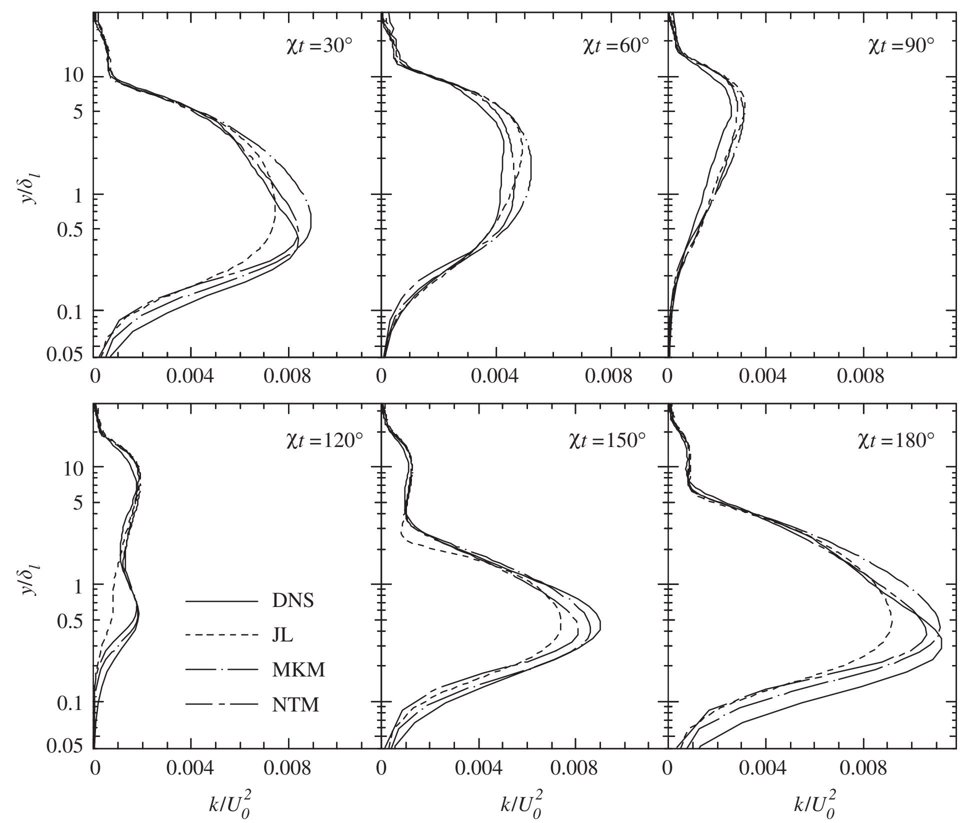

Fig. 4. Cross-stream profiles of turbulent kinetic energy.

energy especially at $\chi t = 120^\circ$ the JL model differs much from the DNS data. During the deceleration ($\chi t = 150^\circ$), although, none of the models could mimic the cross-stream variation, but the peak value of the turbulent kinetic energy is very well predicted by the MKM model.

3.3. Reynolds stress

In case of Reynolds stress $-\overline{u'v'}$ (normalized by amplitude of the free-stream velocity U_0), all the models can reproduce the cross-stream variation satisfactorily; however, JL model at $\chi t = 60^\circ$ and all the three models at $\chi t = 150^\circ$ show much deviation from the DNS data in the near-wall region. In general, the shape of the Reynolds stress profile in the cross-stream direction is predicted very well by three of the models used here (Fig. 5).

3.4. Wall shear stress and friction factor

An interesting feature of the transitional oscillatory boundary layers, i.e. a sudden increase in wall shear stress

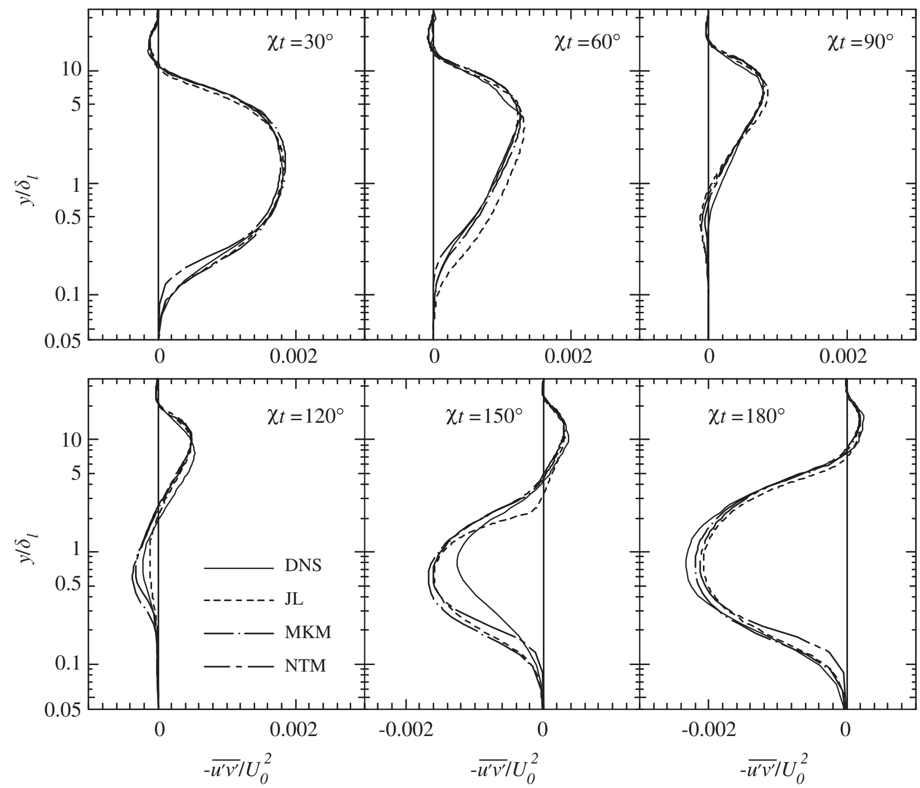

Fig. 5. Cross-stream profiles of Reynolds stress.

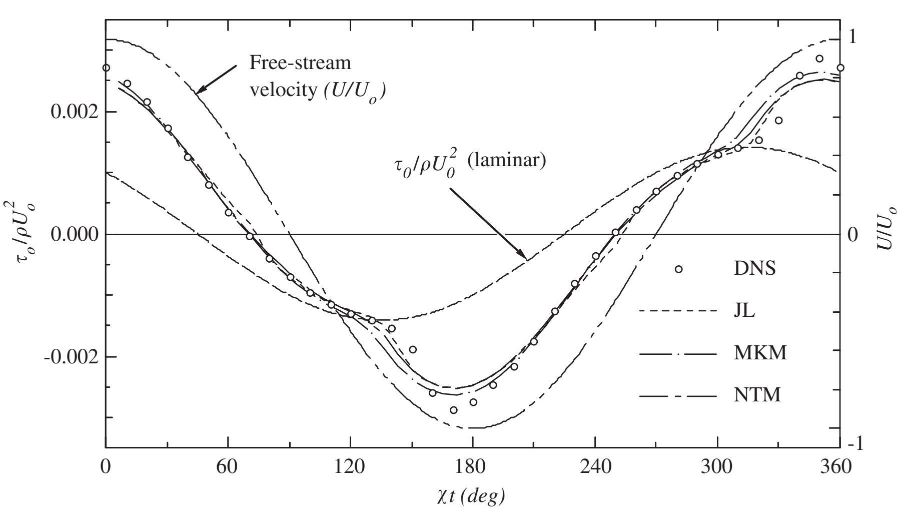

Fig. 6. Temporal variation of wall shear stress.

(τ_0) during adverse pressure gradient phase ($\chi t = 150^\circ$) has been very well predicted by the JL and NTM models and to some extent by the MKM model as may be observed in Fig. 6.

It can be observed that this sudden increase in shear stress occurs at the inflection point in the temporal variation of acceleration ($\mathrm{d}(U/U_0)/\mathrm{d}(\chi t)$). The wall shear stress is normalized by mass density ρ and U_0. The magnitude of the maximum wall shear stress predicted by the MKM model is closer to the DNS value than that by the JL and NTM models.

The laminar solution is also plotted using (Sleath, 1990)

$$\frac{\tau_0}{\rho U_0^2} = \frac{1}{\sqrt{R_\mathrm{w}}} \sin\left(\chi t + \frac{\pi}{4}\right). \tag{11}$$

Fig. 6 depicts that there is a phase difference of 45° between the free-stream velocity and the laminar wall shear stress, as suggested by Eq. (11). However, in the present

Table 2
Comparison of friction factor and phase difference for the present case

Parameter	Laminar solution	DNS	JL model	MKM model	NTM model
Friction factor f_w	0.00283	0.00574	0.00497	0.00529	0.00505
Phase difference ϕ	45°	10°	9.12°	9.48	8.4

DNS data the phase difference is reduced as a result of the turbulence. If the wave friction factor f_w is defined using the following relationship analogous to steady flow friction factor:

$$\tau_{0m} = \frac{\rho}{2} f_w U_0^2, \tag{12}$$

the value for wave friction factor for laminar flow can be obtained from Eq. (11) as $f_{wl} = 2/\sqrt{R_w}$. Here τ_{0m} is the maximum value of the wall shear stress. A comparison of the wave friction factor and the corresponding phase difference ϕ is shown in Table 2 for the models utilized in the present study.

From Table 2, it is evident that MKM model predicted the friction factor and phase difference very close to the corresponding DNS values. The JL and NTM models slightly underestimated the friction factor but for practical purposes any of the three models can be used.

4. Conclusions

The low Reynolds number k–ε models modified to analyse oscillatory boundary layers perform better than the JL model as far as near-wall velocities and turbulent kinetic energy profiles are concerned. During adverse pressure gradient phase, all the models used here could not predict the velocity as well as turbulent kinetic energy precisely. It was observed that near the wall further improvement of the model functions is required in order to get a better performance from these models. The DNS data for oscillatory boundary layer utilized in the present study pertain to mild pressure gradient variations. More challenging cases of steep pressure gradient are required to further investigate the predictive ability of these models.

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