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Full-range equation for wave boundary layer thickness

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Abstract

Full-range equation covering all the flow regimes in a wave boundary layer is proposed for the boundary layer thickness. The results are compared with the available experimental data and good agreement has been found. In case of wave boundary layers, there are three definitions of boundary layer thickness in use. Therefore, the full-range equation is derived for three of the definitions. The findings of this study may be useful in calculating suspended sediment transport in coastal environments and studying wave–current combined motion.

Keywords: Boundary layer thickness; Full-range equation; *k*-epsilon model

1. Introduction

In coastal environments, sediment transport plays an important role in changing the near-shore bottom topography. Due to the practical relevance and complexity associated with this phenomenon, a large number of studies have been carried out using experimental, analytical and numerical techniques. The influence of surface waves is transmitted to the sea bottom through the boundary layer. Hence, a significant part of the transportable suspended sediment is governed by the boundary layer thickness (Van Rijn, 1989). In addition, for wave–current combined motion, it is necessary to estimate the thickness of wave boundary layer, so that the wave and current dominated regions may be distinguished (Grant and Madsen, 1979). Thus, a precise determination of BL thickness is a matter of great importance.

A number of relationships have been proposed by previous researchers for wave boundary layer thickness in terms of known flow parameters, which are valid within specific regimes of flow. Therefore, in order to apply those relationships, it is necessary to first determine the regime of flow, i.e. laminar, smooth turbulent or rough turbulent. In real field situations however, the transitional flow is often encountered. If the conventional regime specific relationships are used, one must interpolate, for a particular BL property, between the two regimes. The need thus arises for a relationship that can cover all the flow regimes to predict various flow characteristics.

In the present study, a full-range equation is derived for the computation of wave boundary layer thickness δ covering all the flow regimes. There are three definitions of δ used in the contemporary literature, i.e. after Jonsson (1966), who defined δ to be the minimum distance from the bottom to a point where the ensemble-averaged velocity in x-direction; u equals free-stream wave velocity amplitude, U_0. According to Jensen et al. (1989) (JSF's definition), δ is the distance from the bottom to the point of maximum velocity amplitude. Sleath (1987) expressed δ to be the distance from the bottom to a point where defect velocity amplitude $\hat{u}_d$ is 5% of the free-stream velocity amplitude (Fig. 1). It may be readily noted that from a practical point of view the determination of wave BL thickness using Jonsson's or JSF's definition is quite easy because the velocity profile at the instant of maximum free-stream velocity may directly be used. Whereas Sleath's definition requires a detailed velocity data in order to plot the velocity defect amplitude with respect to cross-stream direction. Analogous to the steady boundary layers the wave boundary layer thickness can be

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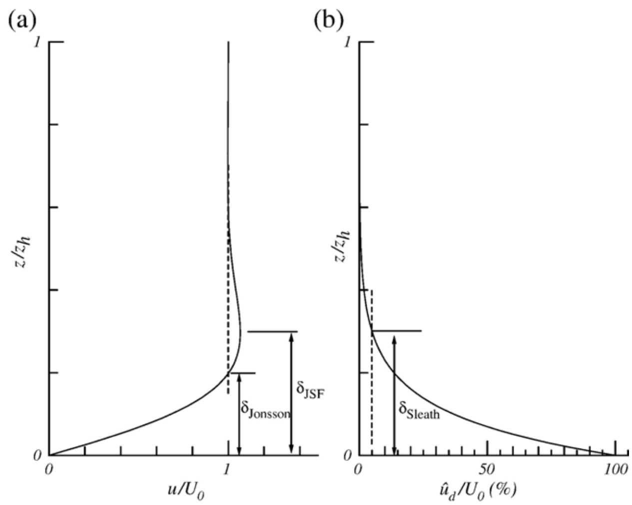

Fig. 1. Schematic of various definitions of wave BL thickness. (a) Jonsson's and JSF's definitions, and (b) Sleath's definition.

related to the friction velocity and thus to the wave friction factor as has been shown by Grant and Madsen (1986).

The full-range equations are derived for all of the above definitions in the present study.

The experimental data by Jonsson (1966), Sleath (1987) and Jensen et al. (1989) are used for comparison. Due to the scarcity of experimental data, low Reynolds number k–ε model by Jones and Launder (1972) is employed to supplement the existing data.

2. Methodology

2.1. Laminar flow

The velocity profile for a sinusoidal laminar wave boundary layer is given as (Sleath, 1990);

$$\frac{u}{U_0} = \cos\sigma t - \exp\left(-\frac{z}{\delta_l}\right)\cos\left(\sigma t - \frac{z}{\delta_l}\right) \tag{1}$$

Where, u is ensemble-averaged velocity in x-direction, U_0; the amplitude of free-stream velocity in x-direction, σ ($=2\pi/T$, $T=$ wave period); angular frequency, t; time, z; coordinate axis normal to the bottom and δ_l is Stokes' layer thickness ($=\sqrt{2\nu/\sigma}$, $\nu=$ kinematic viscosity).

Using the definition of BL thickness by Jensen et al. (1989), we get:

$$\delta_{(L)} = \frac{3\pi}{4}\delta_l \Leftrightarrow \frac{\delta_{(L)}}{a_m} = \frac{3\pi}{4}\left(\frac{2}{Re}\right)^{1/2} \tag{2}$$

Where, $\delta_{(L)}$ is the laminar wave boundary layer thickness, Re, wave Reynolds number $= U_0 a_m/\nu$, here, a_m is maximum particle excursion length ($=U_0/\sigma$ for sinusoidal wave boundary layers). In a similar manner, the respective definitions of δ can be used to derive the relationships for laminar wave boundary layer thickness. In general, $\delta_{(L)}$ can be expressed as:

$$\frac{\delta_{(L)}}{a_m} = C_1\left(\frac{2}{Re}\right)^{1/2} \tag{3}$$

2.2. Turbulent flow

In case of turbulent flow a number of empirical relationships are available for the above-mentioned definitions. For smooth turbulent flow Jonsson (1966) and Fredsoe and Deigaard (1992) have proposed the formulae for boundary layer thickness in terms of wave Reynolds number only. On the other hand, for rough turbulent flow the boundary layer thickness depends on the ratio of particle excursion length to Nikuradse's equivalent roughness height (a_m/k_s) as depicted in the experimental data by Jensen et al. (1989) and others. The general expression for smooth turbulent flow can be expressed in terms of wave Reynolds number as given below:

$$\frac{\delta_{(S)}}{a_m} = C_2(Re)^{C_3} \tag{4}$$

And that for rough turbulent flow can be given as:

$$\frac{\delta_{(R)}}{a_m} = C_4\left(\frac{a_m}{k_s}\right)^{C_5} \tag{5}$$

The subscripts S and R represent smooth and rough boundary layers, respectively.

2.3. Full-range equations

The idea of full-range equations for friction factor covering all the regimes of wave and wave–current boundary layer was first proposed by Tanaka and Thu (1994). Here the same approach has been followed by specifying the wave boundary layer thickness in the following general form:

$$\frac{\delta}{a_m} = f_2\left\{f_1\left(\frac{\delta}{a_m}\right)_{(L)} + (1-f_1)\left(\frac{\delta}{a_m}\right)_{(S)}\right\} + (1-f_2)\left(\frac{\delta}{a_m}\right)_{(R)} \tag{6}$$

The wave boundary layer thickness in the above equation is normalized by the particle excursion length. The weighting functions f_1 and f_2 govern the variation of BL thickness in the transitional zone.

2.4. k–ε model

One of the versions of two-equation turbulence models, i.e. k–ε model has proved to be very efficient in the prediction of steady boundary layers. Rodi (1984) has reviewed some of the flow phenomena for which this model has been applied successfully. Later, a number of researchers applied this model to wave boundary layers as well. Tanaka and Sana (1994) and Sana and Tanaka (2000) have carried out a comparative study of some of the popular versions of low Reynolds number k–ε model for predicting wave boundary layer properties. It was found that the original model by Jones and Launder (1972) could reproduce most of the boundary layer properties in an excellent manner. In the present study this model is, therefore, utilized for complementing the experimental data for wave boundary layer thickness.

3. Results

3.1. Laminar flow coefficients

As mentioned before the coefficient C_1 in Eq. (3) was found using the definitions of δ by Jonsson (1966), Sleath (1987) and Jensen et al. (1989) (JSF). The values of this coefficient for respective definitions are shown in Table 1.

3.2. Turbulent flow coefficients

For turbulent flow, the k–ε model by Jones and Launder (1972) was used to obtain complementary data for wave boundary layer on a smooth bottom. The values of δ were obtained for three definitions used in the present study from the predicted velocity profiles. The coefficients used in Eq. (4) were on the basis of this data. The values of these coefficients are given in Table 1. The coefficients of Eq. (5) are taken from Jonsson and Carlsen (1976) for Jonsson's definition without any modification. For the definition by JSF the experimental data from Jonsson and Carlsen (1976) and Jensen et al. (1989) were utilized and the coefficients were found by least square fitting technique. The same approach was utilized to determine the coefficients as per Sleath's definition using the experimental data by Sleath (1987). The coefficients obtained for Eqs. (4) and (5) are listed in Table 1.

3.3. Full-range equation for BL thickness

The weighting function f_1 which governs the transition from laminar to turbulent flow, was derived here for each definition of δ, but the function f_2 was kept the same as in Tanaka and Thu (1994). The general form of f_1 can be given as:

$$f_1 = \exp\left\{C_6\left(\frac{Re}{2.5\times10^5}\right)^{C_7}\right\} \tag{7}$$

Here, denominator of Re is in fact the critical Reynolds number for transition from laminar to smooth turbulent oscillatory flow. The value of critical Reynolds number in Eq. (7) is adopted from Tanaka and Thu (1994). The values of C_6

Table 1
Coefficients for the wave BL thickness formulae

Coefficient	Jonsson's definition	Sleath's definition	JSF's definition
C_1	$\pi/2$	3.0	$3\pi/4$
C_2	0.021	0.154	0.017
C_3	−0.068	−0.144	−0.011
C_4	0.072	0.896	0.111
C_5	−0.25	−0.469	−0.246
C_6	−0.759	−31.3	−0.941
C_7	5.0	9.97	2.01

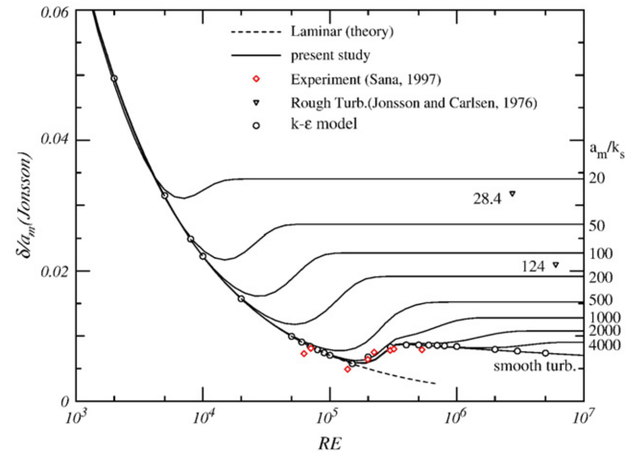

Fig. 2. Boundary layer thickness as per Jonsson's definition.

and C_7 are given in Table 1. The weighting function f_2 that governs the transition from smooth turbulent to rough turbulent flow is given as:

$$f_2 = \exp\left\{0.0101\left(\frac{Re}{R_1}\right)^{2.06}\right\} \tag{8}$$

Where, $R_1 = 25(a_m/k_s)^{1.15}$, k_s is the Nikuradse's equivalent sand roughness. The results obtained by the full-range equation for δ based on Jonsson's definition are plotted in Fig. 2. Experimental data for oscillatory boundary layers on a smooth bottom from Sana (1997) has also been plotted. For rough turbulent region only two experimental data points are available from Jonsson and Carlsen (1976) for this definition of δ. Therefore, it is hard to check the accuracy of the given equation over a wide range of Reynolds numbers and a_m/k_s values. However, the available data points confirm the validity of this equation in their domain of values.

The comparison between full-range equation and experimental data for δ as defined by Sleath is shown in Fig. 3. The overall agreement with the experimental data is good. A prominent difference of this definition from the other two is that

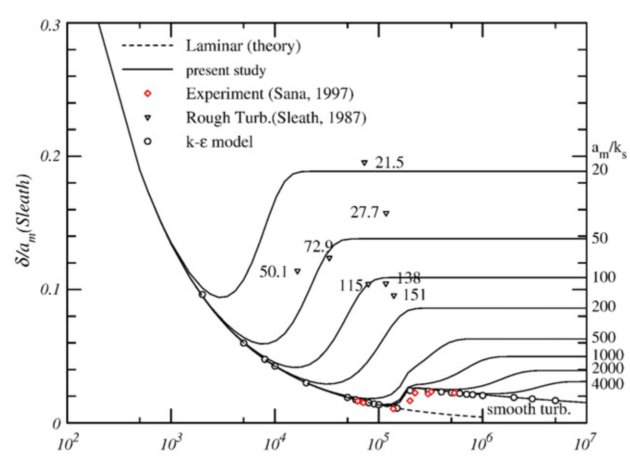

Fig. 3. Boundary layer thickness as per Sleath's definition.

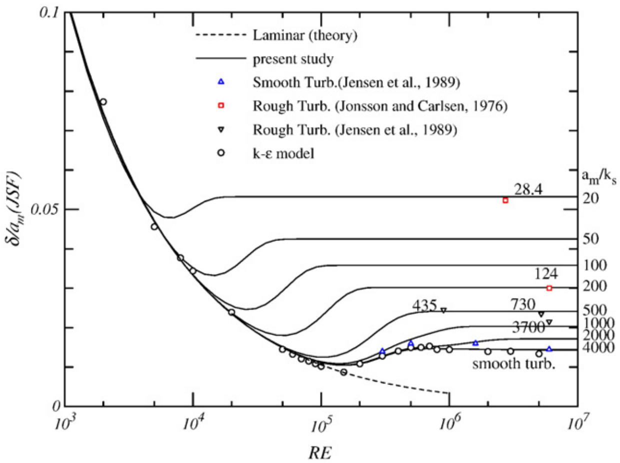

Fig. 4. Boundary layer thickness as per JSF's definition.

the transition from laminar to smooth turbulent flow is relatively abrupt. This behaviour is similar to that of friction factor as shown in other studies (Tanaka and Thu, 1994).

In Fig. 4, the comparison is made for JSF's definition. It is hard to draw the final conclusion because of the scarcity of experimental data in this case as well. However, the agreement between the available data is generally satisfactory in the medium range of a_m/k_s values.

4. Conclusions

An equation of practical convenience is proposed for wave boundary layer thickness covering all the regimes of flow, namely; laminar, smooth turbulent and rough turbulent, as per three different definitions in common use among coastal engineers. In order to complement the experimental data the original version of low Reynolds number k–ε model by Jones and Launder (1972) was used for smooth turbulent wave BL. This study will be useful for the practicing engineers in calculating suspended sediment transport in coastal environments and researchers interested in determining the wave boundary layer thickness to study wave–current combined motion.

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