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# Turbulent universality and the drift velocity at the interface between two homogeneous fluids

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# ABSTRACT

The drift velocity  $\mathbf{U}_0$  at the interface between two homogeneous turbulent fluids of arbitrary relative densities in differential mean motion is considered. It is shown that an analytical expression for  $\mathbf{U}_0$  follows from the classical scaling for these flows when the scaling is supplemented by standard turbulent universality and symmetry assumptions. This predicted  $\mathbf{U}_0$  is the weighted mean of the free-stream velocities in each fluid, where the weighting factors are the square roots of the densities of the two fluids, normalized by their sum. For fluids of nearly equal densities, this weighted mean reduces to the simple mean of the free-stream velocities. For fluids of two widely differing densities, such as air overlying water, the result gives  $\mathbf{U}_0 \approx \alpha \mathbf{V}_\infty$ , where  $\alpha \ll 1$  is the square root of the ratio of the fluid densities,  $\mathbf{V}_\infty$  is the free-stream velocity of the overlying fluid, and the denser fluid is assumed nearly stationary. Comparisons with two classical laboratory experiments for fluids in these two limits and with previous numerical simulations of flow near a gas–liquid interface provide specific illustrations of the result. Solutions of a classical analytical model formulated to reproduce the air–water laboratory flow reveal compensating departures from the universality prediction, of order 15% in  $\alpha$ , including a correction that is logarithmic in the ratio of dimensionless air and water roughness lengths. Solutions reproducing the numerical simulations illustrate that the logarithmic correction can arise from asymmetry in the dimensionless laminar viscous sublayers.

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#### I. INTRODUCTION

The flow adjacent to an interface between two homogeneous, turbulent fluids is a classical topic in fluid dynamics, which continues to stimulate current research because of its importance in a wide range of environmental and engineering contexts.<sup>1–7</sup> One aspect of this problem is the determination of the drift velocity: the mean interface-parallel motion at the interface. It is shown here (Sec. II) that a modest extension of classical scaling for these flows, supplemented by standard turbulent universality and symmetry assumptions, yields a quantitative, analytical prediction of this drift velocity for the general case of two homogeneous fluids of arbitrary relative densities (Fig. 1). Despite the classical nature of the derivation, this result does not seem to have been previously noted.

The main purpose of this article is to record this result and make it available in the literature. A second purpose is to explore briefly the nominal accuracy of the prediction and some possible sources and anticipated magnitudes of departure from it. While the analytical result for the drift velocity is exact under the universality and symmetry assumptions, it is important to recognize that those assumptions will be violated to some degree in any physical fluid system, and consequently, the result must be understood only to provide a starting point for further, more complete analysis. This is particularly true for flows supporting significant interfacial wave activity, the complex effects of which on flow near the interface are themselves a focus of much current research.<sup>2,8–15</sup> Nonetheless, the universality prediction suggestively reproduces a long-standing empirical rule for the ocean wind drift that was reported already more than half a century ago<sup>16,17</sup> and is still in operational use,<sup>18</sup> according to which the surface drift current speed is ~3% of the wind speed.

Consistent with the classical nature of the derivation, the result is compared with two classical laboratory experiments<sup>19,20</sup> that include explicit measurements of the velocity profiles on both sides of the interface and are only modestly affected by interfacial waves. For the air-water interface, the characteristic profile of



**FIG. 1.** Universality prediction (19) for the ratio of relative drift speed to relative free-stream speeds (solid line) as functions of (a) density ratio  $\rho_1/\rho_2$  and (b)  $\alpha = (\rho_1/\rho_2)^{1/2}$ . Five different specific fluid combinations are indicated (circles), with the corresponding analytical model results obtained using the model described in Sec. IV C for the Wu<sub>20</sub> flow configuration, with appropriately modified density ratios (crosses). The numerical results (pluses) from the simulations {R1, R10, R29} of Lombardi *et al.*<sup>26</sup> and the universality predictions (squares) for the corresponding density ratios and values of  $\alpha$  are also shown.

air velocity (perhaps above a wave boundary layer<sup>21,22</sup>) is generally regarded as well-enough known<sup>23</sup> that more recent laboratory studies generally measure the velocity profile only in the water and primarily focus instead on wave-state effects.<sup>24,25</sup> The result is also compared with a pioneering numerical study of coupled gas–liquid flow, which is conducted in a symmetric, dimensionless setting that anticipates the development by which the drift velocity prediction is obtained.<sup>26</sup>

# **II. UNIVERSALITY RESULT**

Consider two homogeneous fluid layers, the first with density  $\rho_1$  overlying the second with density  $\rho_2 > \rho_1$ , in mean differential motion, with turbulent boundary layers adjacent to the interface. Suppose that the flow in both layers is statistically homogeneous in the horizontal and stationary in time, and let the constant, horizontal free-stream velocities outside the upper and lower fluid boundary layers be denoted by  $\mathbf{V}_{\infty}$  and  $\mathbf{U}_{\infty}$ , respectively. In general, the interface will be distorted by interfacial waves induced by the turbulence and the shear layer. Let the averaging of the turbulent flow be conducted in a curvilinear coordinate system with a quasi-vertical coordinate measuring instantaneous distance from

the interface, with *z* being the corresponding mean vertical distance so that z = 0 defines the horizontally uniform position of the mean interface. Under the assumptions of homogeneity and stationarity, the Reynolds' or ensemble averaged velocities in the upper and lower layers can then depend only on the mean vertical position *z* and may be denoted by  $\mathbf{V}(z)$  and  $\mathbf{U}(z)$ , respectively. Note that the implicit use of an interface-relative coordinate implies restrictions on the degree of distortion and connectedness of the interface, which are discussed further in Sec. IV.

At the interface, standard conditions<sup>27</sup> of continuity of velocity and stress are required to hold for the instantaneous flow and so must also hold for the averaged flow,

$$\mathbf{V}(0) = \mathbf{U}(0) = \mathbf{U}_0, \quad \tau(z \to 0^+) = \tau(z \to 0^-) = \tau_0, \quad (1)$$

where the stress  $\tau(z)$  is the mean turbulent vertical flux of horizontal momentum and the one-sided limits, as the mean interface position is approached from above and below, are denoted by  $z \rightarrow 0^+$  and  $z \rightarrow 0^-$ , respectively. The interface drift velocity  $\mathbf{U}_0$  and the mean turbulent stress at the interface (i.e., the average force per unit area on the interface)  $\tau_0$  are defined by (1).

The magnitude of the mean turbulent stress at the interface may be expressed in terms of the respective friction velocities  $v_*$  and  $u_*$ for the two fluids,

$$\rho_1 v_*^2 = \rho_2 u_*^2 = |\tau_0|, \tag{2}$$

$$u_* = \alpha v_*, \tag{3}$$

where

which are related by

$$\alpha = \left(\frac{\rho_1}{\rho_2}\right)^{1/2}.$$
 (4)

Let  $\delta_1$  and  $\delta_2$  be intrinsic length scales, such as roughness lengths, which characterize the turbulent boundary layers adjacent to the interface. These friction velocities and characteristic length scales may be used to nondimensionalize the velocities  $\mathbf{V}(z)$  and  $\mathbf{U}(z)$  and the height *z*, giving dimensionless velocities  $\tilde{\mathbf{V}}(\zeta)$  and  $\tilde{\mathbf{U}}(\zeta)$  that are functions of the dimensionless height  $\zeta$ ,

$$\bar{\mathbf{V}}(\zeta) = \frac{\mathbf{V}(z)}{\upsilon_*}, \quad \bar{\mathbf{U}}(\zeta) = \frac{\mathbf{U}(z)}{u_*}, \quad \zeta = \begin{cases} z/\delta_1, & z \ge 0\\ z/\delta_2, & z \le 0. \end{cases}$$
(5)

The dimensionless continuity condition (1) at the interface is then

$$\bar{\mathbf{V}}(0) = \alpha \bar{\mathbf{U}}(0), \tag{6}$$

while the dimensionless free-stream velocities are

$$\bar{\mathbf{V}}_{\infty} = \frac{\mathbf{V}_{\infty}}{v_{*}}, \quad \bar{\mathbf{U}}_{\infty} = \frac{\mathbf{U}_{\infty}}{u_{*}}.$$
(7)

Consider now the dimensionless velocity deviations in the boundary layers,

$$\bar{\mathbf{V}}'(\zeta) = \bar{\mathbf{V}}(\zeta) - \bar{\mathbf{V}}_{\infty}, \quad \bar{\mathbf{U}}'(\zeta) = \bar{\mathbf{U}}(\zeta) - \bar{\mathbf{U}}_{\infty}, \tag{8}$$

which satisfy the boundary conditions

$$\tilde{\mathbf{V}}' \to 0 \quad \text{as} \quad \zeta \to \infty, \quad \tilde{\mathbf{U}}' \to 0 \quad \text{as} \quad \zeta \to -\infty,$$
 (9)

$$\bar{\mathbf{V}}'(0) - \alpha \bar{\mathbf{U}}'(0) = -(\bar{\mathbf{V}}_{\infty} - \alpha \bar{\mathbf{U}}_{\infty}).$$
(10)

The problem for the dimensionless velocity deviations depends on the dimensionless free-stream velocities only through the difference  $\tilde{\mathbf{V}}_{\infty} - \alpha \tilde{\mathbf{U}}_{\infty} = (\mathbf{V}_{\infty} - \mathbf{U}_{\infty})/v_*$  that appears in the interface condition (10). It will therefore be unchanged if the free-stream velocities  $\tilde{\mathbf{V}}_{\infty}$  and  $\tilde{\mathbf{U}}_{\infty}$  are replaced by a different pair of free-stream velocities that preserve that difference, that is, for the modified dimensionless free-stream velocities  $\tilde{\mathbf{V}}_{\infty}$  and  $\tilde{\mathbf{U}}_{\infty}$  such that

$$\tilde{\mathbf{V}}_{\infty} - \alpha \tilde{\mathbf{U}}_{\infty} = \tilde{\mathbf{V}}_{\infty} - \alpha \tilde{\mathbf{U}}_{\infty}.$$
 (11)

Condition (11) will be satisfied if

$$\tilde{\mathbf{V}}_{\infty} = \tilde{\mathbf{V}}_{\infty} + \Delta \tilde{\mathbf{V}}, \quad \tilde{\mathbf{U}}_{\infty} = \tilde{\mathbf{U}}_{\infty} + \frac{1}{\alpha} \Delta \tilde{\mathbf{V}}$$
 (12)

for any choice of  $\Delta \bar{\mathbf{V}}$ . Now, choose  $\Delta \bar{\mathbf{V}}$  such that

$$\tilde{\mathbf{V}}_{\infty} = -\tilde{\mathbf{U}}_{\infty},$$
 (13)

which requires that

$$\Delta \bar{\mathbf{V}} = -\frac{\alpha}{1+\alpha} (\bar{\mathbf{V}}_{\infty} + \bar{\mathbf{U}}_{\infty}). \tag{14}$$

The dimensionless modified problem with free-stream velocities defined by (12) and (14) has antisymmetric free-stream conditions and, by assumption and scaling, universal turbulent boundary layer structure on both sides of the interface. Given this symmetry and universality, it is consistent to assume that the dimensionless modified total velocity profile is itself antisymmetric across the interface. This, in turn, implies that the dimensionless modified total velocity vanishes at the interface, that is,

$$\tilde{\mathbf{U}}'(0) + \tilde{\mathbf{U}}_{\infty} = \tilde{\mathbf{U}}'(0) + \mathbf{U}_{\infty} - \frac{1}{1+\alpha} (\tilde{\mathbf{V}}_{\infty} + \tilde{\mathbf{U}}_{\infty}) = 0.$$
(15)

The sum  $\tilde{\mathbf{U}}'(0) + \mathbf{U}_{\infty}$  in (15) is just the dimensionless interface drift velocity  $\tilde{\mathbf{U}}_0 = \mathbf{U}_0/u_*$  for the original problem with dimensionless free-stream velocities  $\mathbf{V}_{\infty}$  and  $\mathbf{U}_{\infty}$ , and consequently that the drift velocity can be computed from (15),

$$\tilde{\mathbf{U}}_0 = \tilde{\mathbf{U}}'(0) + \tilde{\mathbf{U}}_\infty = \frac{1}{1+\alpha} (\tilde{\mathbf{V}}_\infty + \tilde{\mathbf{U}}_\infty).$$
(16)

The corresponding dimensional interface drift velocity is then

$$\mathbf{U}_{0} = u_{*} \, \bar{\mathbf{U}}_{0} = \frac{1}{1+\alpha} \left( \alpha \mathbf{V}_{\infty} + \mathbf{U}_{\infty} \right) = \frac{\rho_{1}^{1/2} \mathbf{V}_{\infty} + \rho_{2}^{1/2} \mathbf{U}_{\infty}}{\rho_{1}^{1/2} + \rho_{2}^{1/2}}.$$
 (17)

The fundamental universality result for the interface drift velocity is (17). It follows from (17) that the drift velocity relative to the lower fluid is proportional to the velocity of the upper fluid relative to the lower fluid,

$$\mathbf{U}_{0} - \mathbf{U}_{\infty} = \frac{\alpha}{1+\alpha} \left( \mathbf{V}_{\infty} - \mathbf{U}_{\infty} \right) = \frac{\rho_{1}^{1/2}}{\rho_{1}^{1/2} + \rho_{2}^{1/2}} \left( \mathbf{V}_{\infty} - \mathbf{U}_{\infty} \right).$$
(18)

The ratio of the relative drift speed to the relative free-stream speeds thus depends only on the relative densities of the two fluids (Fig. 1),

$$\frac{|\mathbf{U}_0 - \mathbf{U}_{\infty}|}{|\mathbf{V}_{\infty} - \mathbf{U}_{\infty}|} = \frac{\alpha}{1 + \alpha} = \frac{\rho_1^{1/2}}{\rho_1^{1/2} + \rho_2^{1/2}}.$$
 (19)

It follows from (17) to(19) that

$$\mathbf{U}_0 - \mathbf{U}_\infty \approx \alpha \, \mathbf{V}_\infty \quad \text{for} \quad \alpha \ll 1, \ |\mathbf{U}_\infty| \ll |\mathbf{V}_\infty|$$
 (20)

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$$\mathbf{U}_0 \longrightarrow \frac{1}{2} (\mathbf{V}_\infty + \mathbf{U}_\infty) \quad \text{as} \quad \rho_1 \longrightarrow \rho_2.$$
 (21)

The classical air–water<sup>20</sup> and freshwater–saltwater<sup>19</sup> cases considered below represent these two limits, respectively. The intermediate density ratios can be obtained for two-fluid systems consisting of water with other laboratory fluids (Fig. 1).

#### **III. LABORATORY AND NUMERICAL EXAMPLES**

#### A. Freshwater-saltwater

It is useful as a reference point to consider briefly the case of two fluids of similar densities, for which the dimensional mean velocity profiles should themselves have a nearly antisymmetric structure about the interface. Experiments on a two-layer, freshwatersaltwater system conducted by Lofquist<sup>19</sup> provide a specific illustration for which the maximum 7% density difference considered was similar to that for an olive–oil and water system (Fig. 1). The flow of the lower, saltwater layer was forced by a pump system, and the upper, freshwater layer was driven by momentum transport through the interface. Despite the small density differences between the fluids and the induced turbulence, relatively sharp density interfaces were maintained through the experiments, consistent with the theoretical configuration assumed in Sec. II.

The measured profiles of the mean horizontal velocity and its vertical derivative show the anticipated, approximately antisymmetric and symmetric structures, respectively (Fig. 2). Lofquist<sup>19</sup> noted explicitly that the upper-layer, freshwater flow profile approached antisymmetry with the flow profile in the turbulent lower layer more closely as the upper-layer flow strength and the associated degree of turbulence, as measured by a dimensionless parameter  $\beta$ , increased, with antisymmetry near the interface essentially achieved at  $\beta = 4$ . Comparisons (Fig. 2) of the measured flow profiles with predictions from a modified version of the analytical model described in Sec. IV C, for which the difference of the free-stream velocities was roughly 0.05 m s<sup>-1</sup>, confirm this antisymmetric structure near the interface. They also reveal a departure from antisymmetry of order 15% in the outer boundary layer, suggesting that the flow in the outer boundary layers was constrained by the limited size of the flume, but no measurements of the interface drift velocity were reported in a form that could be compared with the theoretical prediction, so this departure from the predicted antisymmetry could not be further quantified.

#### **B.** Air-water

For an air-water laboratory system with the limited wave effects, a systematic study of wind-induced drift currents using classical techniques was conducted by Wu.<sup>20,28</sup> Measurements were made of free-stream wind velocity, surface wind-drift current, near-surface wind and current profiles, and surface wave properties in a tank of length 22 m, width 1.5 m, and depth 1.55 m, filled to a depth of 1.24 m with water and open near the top at both ends to allow the throughflow of air driven by a variable-speed fan (see Table I and Appendix A for numerical data digitized from figures in Ref. 20). Air and water surface friction velocities were inferred from the measured log-layer wind and current profiles near the interface. Further details of the experimental methodology and the complete list and



**FIG. 2.** Dimensionless measured profiles (black lines and dots, respectively) of (left panel) horizontal velocity and (right) vertical shear of horizontal velocity (abscissa) for  $\beta = \{1, 2, 3, 4\}$  vs dimensionless distance from the interface (ordinate), with superposed results from a modified version of the analytical model described in Sec. IV C (blue and green lines). For the model, dimensionless freshwater (blue) and saltwater (green) profiles are shown, along with the reflection of the saltwater profile about the interface point and a freshwater profile multiplied by the square-root of the freshwater -saltwater density ratio, which by the universality argument, should be identical to the reflected saltwater profile. For this comparison, the dimensional model profiles were scaled by 0.8 times the model friction velocity and 0.75 times the maximum model shear. Adapted with permission from K. Lofquist, "Flow and stress near an interface between stratified liquids," Phys. Fluids **3**, 159–175 (1960). Copyright 1960 AIP Publishing LLC and with overlays added.

description of directly measured and derived quantities are available in Refs. 20 and 28.

Wu<sup>20</sup> reported total and "wind-induced" surface currents ranging from <10 cm s<sup>-1</sup> to over 50 cm s<sup>-1</sup>, both of which are in approximate agreement with the general theoretical prediction (20) for the surface wind drift (Fig. 3). The "wind-induced" surface current was computed by Wu<sup>20</sup> by removing an estimated surface Stokes' drift velocity from the measured, total surface current. Because of the small fetch and wave-damping baffles in the laboratory apparatus, wave amplitudes were less than 5 cm for the highest freestream velocities and decreased toward zero for lower free-stream velocities, so the Stokes' drift estimate was relatively small, generally less than 15% of the total surface current. The Wu<sup>20</sup> velocity data show a slightly better agreement with the general theoretical prediction (20) than with the alternate empirical relation  $|\mathbf{U}_0| \approx 0.53 v_*$  to which Wu<sup>20</sup> compared the laboratory surface drift measurements (Fig. 4), suggesting that the universality prediction (17) may be slightly more robust than the empirical proportionality to the friction velocity.

# C. Numerical simulations of Lombardi et al.26

Lombardi *et al.*<sup>26</sup> conducted pioneering numerical studies of coupled turbulent flow on both sides of a gas–liquid interface, with gas–liquid density ratios corresponding to the three values  $\alpha = \{1/29.9464, 1/10, 1\}$ , denoted as simulations {R29, R10, R1}, respectively. The simulations were conducted in a dimensionless setting, with Reynolds' number based on the friction velocity,

kinematic viscosity, and half-depth of each subdomain equal to 60.4 in each case, so the laminar viscous sublayers contained a significant fraction of the shear and were explicitly resolved by the numerical scheme. The kinematic viscosities  $v_L$  and  $v_G$  for the liquid and gas phases were further taken to be related by  $v_L = \alpha v_G$ .



**FIG. 3.** Wind drift current vs free-stream velocity for the theoretical relation (20) with the standard  $\alpha$  (thin solid line) and for the measured total (solid dot) and "wind-induced" (circles) drift from Ref. 20. Also shown are results discussed in Sec. IV: (20) with the modified coefficient  $\alpha' = 0.85\alpha$  (dashed), (32) with the corrected proportionality constant  $\tilde{\alpha}$  (thick solid), and the initial (pluses) and modified (crosses) sets of analytical model solutions.



**FIG. 4**. Ratios of wind drift current to (a) free-stream velocity and (b) air friction velocity. Symbols in (a) and (b) and lines in (a) are as in Fig. 3. In (b), the relation  $U_0 = 0.53v_*$  from Ref. 20 is shown (dashed line).

where  $\alpha = (\rho_G/\rho_L)^{1/2}$ , for which the resulting R29 case then corresponds approximately to air and water at temperature 320 K and standard atmospheric pressure.<sup>26</sup> The standard boundary conditions (1) were imposed on the instantaneous flow at the interface, which in these simulations was represented as a rigid plane surface and not allowed to deform. Flow was forced in the two layers by equal and opposite dimensional pressure gradients, with the result that no net momentum was imparted to the system by the forcing.

For the R1 simulation, with  $\rho_1 = \rho_2$  and  $\alpha = 1$ , Lombardi *et al.*<sup>26</sup> noted that the exact antisymmetry (neglecting a small deviation arising from the numerical implementation) of the resulting flow configuration requires that the velocity at the interface be exactly zero, just as in (15). It is then clear that for the R1 simulation, the drift velocity relative to the lower fluid is equal to one-half the difference of the free-stream (outer boundary) velocities (Fig. 1), just as in (19) and (21) and, essentially, as in the freshwater–saltwater example (Sec. III A).

For the R29 (air-water) and R10 simulations, corresponding to  $\alpha = \{1/29.9464, 1/10\}$ , Lombardi *et al.*<sup>26</sup> found instead a departure of the flow from antisymmetry, with resulting interface velocities approximately equal to 2.5 and 2.3 times the liquid friction velocity, respectively, in the direction of the liquid flow. Lombardi et al.<sup>26</sup> interpreted this effect as arising from differences in relative inertia of fluctuations that are coupled at the interface, which causes the interface roughly to resemble a no-slip boundary for the gas but a free-slip boundary for the liquid. However, expression (19) nonetheless provides an accurate first-order prediction of the relative interface drift velocities  $|\mathbf{U}_0 - \mathbf{U}_{\infty}|$  for both the R10 and R29 simulations (Fig. 1). The error associated with the departure from antisymmetry for these two cases is no more than  $2.5/18 \approx 15\%$ , as the free-stream (outer boundary) velocities  $(U_{\infty}, V_{\infty})$  for these flows are both ~18 times the respective friction velocities. For the R29 simulation, this result might be anticipated also from the empirical proportionality  $|\mathbf{U}_0|$  $\approx 0.5v_*$  cited by Wu<sup>20</sup> for the air-water case, by which the error might be estimated as  $2.5u_*/0.5v_* \approx 5\alpha \approx 15\%$ . Consistent with the Lombardi et al.<sup>26</sup> interpretation, it is shown in Sec. IV B that these small departures from universality for the R29 and R10 simulations

can be expressed as corrections to the interface velocity that are logarithmic in the ratio of effective roughness lengths for the outer, turbulent flow regimes.

#### IV. DEPARTURES FROM UNIVERSALITY

#### A. General considerations

The derivation of (17) is general but hides two important assumptions, beyond the standard turbulent universality conditions of homogeneity and stationarity. First, it assumes that the interface geometry is sufficiently simple that averaging can be conducted with respect to a quasi-vertical, curvilinear-coordinate distance from the interface. Second, it relies on a dimensional analysis argument that is incomplete if either or both boundary layer flows are characterized by more than a single intrinsic length scale, respectively. Some level of violation of these assumptions is inevitable in every observable fluid system, and the exact results (17)-(19) are thus best regarded as approximations or *a priori* estimates.

The assumption on the interface geometry should be satisfied as long as the wave activity at the interface is sufficiently weak that the fluctuating interface height has the form of a continuous, singlevalued function of horizontal position and time. It is reasonable to expect that it could more generally be satisfied as long as the interface remains a continuous surface, that is, prior to energetic wave-breaking events that cause either bubble and spray formation or irreversible mixing between the layers, depending on the fluid physics. Whether the result may provide useful guidance after the onset of energetic wave-breaking could be assessed empirically and presumably would depend on the fraction of the surface affected by the wave-breaking events. When the interfacial shear is sufficiently large relative to the density difference, frequent wave breaking may destroy the interface, in which case the definition of interface velocity itself becomes difficult or impossible. It is important to note, however, that a role of interfacial waves in supporting the stress  $\tau_0$ at the interface is not excluded, as the result (17) does not depend on any direct assumptions regarding the mechanisms of momentum transport.

An additional intrinsic length scale that would violate the dimensional analysis assumption will, in general, arise whenever the outer boundary layer scales are constrained independently of  $\delta_u$  and  $\delta_v$ . In laboratory flows, for example, the outer scales may be controlled by the dimensions of the apparatus. In rotating flows, with the rotation rate measured by  $\Omega$ , the outer scales may be constrained by the inertial depths  $(\delta_v, \delta_u) = (v_*, u_*)/\Omega$  for which  $\delta_u = \alpha \delta_v$ . Especially when  $\alpha \ll 1$  for which  $\delta_u \ll \delta_v$ , the ratio of the interfacial roughness or distortion scale to the respective outer boundary layer scales will be different in the two fluids, giving an additional dimensionless parameter on which the boundary layer structures may depend. Thus, a possible role of interfacial waves should also be recognized in this context.

# B. Corrections for the Lombardi et al.<sup>26</sup> gas-liquid flow

The dimensionless velocity profiles from the R29 Lombardi *et al.*<sup>26</sup> numerical simulation may be approximated by an analytical model (C1) with constant molecular viscosity in viscous sublayers near the interface and law-of-the-wall eddy viscosity in the outer, turbulent region (Fig. 5). In the

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**FIG. 5**. Dimensionless velocity profiles from the analytical model in Appendix C for gas (blue) and water (green), superimposed on profiles for (a) gas and (b) liquid for the R29 (air–water) numerical simulation of Lombardi *et al.*,<sup>26</sup> showing (left panels) the total velocity with both profiles from the full analytical model (C1) for ( $\zeta_{0a}^{G}$ ,  $\zeta_{1am}^{L}$ ) = (7.8, 5.2) and (right panels) the velocity relative to the interface velocity with both extended log-layer profiles from the model (C2) for ( $\zeta_{0}^{G}$ ,  $\zeta_{0}^{L}$ ) = (0.14, 0.42). The total velocity profiles for R10 and R1 are also shown (left panels), as well as those for a control simulation with a no-slip interface (CH; left and panels). Adapted with permission from P. Lombardi, V. De Angelis, and S. Banerjee, "Direct numerical simulation of near-interface turbulence in coupled gas-liquid flow," Phys. Fluids **8**, 1643–1665 (1996). Copyright 1996 AIP Publishing LLC and with overlays added.

Lombardi *et al.*<sup>26</sup> configuration, the universality theory would predict zero interface velocity, and consequently, the non-zero interface velocity  $\tilde{U}_0 \approx 2.5$  for case R29 indicates a departure from universality.

If logarithmic profiles are instead fit directly to the interfacerelative velocities in the outer, turbulent region, then the interface velocity  $\bar{U}_0$  can be expressed as a correction to the universality prediction that is logarithmic in the ratio of the corresponding effective roughness lengths. With the free-stream velocities defined as the velocities at the outer domain boundaries  $\zeta = \pm \zeta_1$ , the dimensionless logarithmic profiles (C2) are

$$\bar{V}_{\infty} = \alpha \bar{U}_0 + \frac{1}{\kappa} \ln \frac{\zeta_1}{\zeta_0^G}, \quad \bar{U}_{\infty} = \bar{U}_0 - \frac{1}{\kappa} \ln \frac{\zeta_1}{\zeta_0^L},$$
(22)

which fits the R29 numerical profiles when  $\zeta_0^G = 0.14$  and  $\zeta_0^L = 0.42$  (Fig. 5). It follows that

$$\bar{U}_0 = \frac{1}{1+\alpha} \left( \bar{V}_\infty + \bar{U}_\infty - \frac{1}{\kappa} \ln \frac{\zeta_0^L}{\zeta_0^G} \right),\tag{23}$$

and therefore, with  $\bar{U}_{\infty} = -\bar{V}_{\infty}$  for the Lombardi *et al.*<sup>26</sup> simulations,

$$\bar{U}_0 = \frac{1}{1+\alpha} \left( -\frac{1}{\kappa} \ln \frac{\zeta_0^L}{\zeta_0^G} \right).$$
(24)

For the fitted roughness lengths  $(\zeta_0^G, \zeta_0^L) = (0.14, 0.42)$  and  $\alpha = 1/29.9464$  for the R29 simulation, the correction (24) gives  $\tilde{U}_0 = -2.65$ . This is close to the R29 numerical result  $\tilde{U}_0 = -2.484$  from Table II of Lombardi *et al.*,<sup>26</sup> as it must be given the fitting procedure.

The departure of the ratio of effective roughness lengths from unity is a measure of the same physical effect that Lombardi *et al.*<sup>26</sup> characterized in terms of approximate no-slip vs free-slip behavior at the interface, resulting from the effects on coupled motions of the different relative inertias of the gas and the liquid. The larger effective roughness length on the liquid side may be understood physically from this point of view as resulting from the relatively greater turbulent activity close to the interface allowed on the liquid side by the relatively smaller inertia of the gas, which, in turn, allows greater turbulent momentum transport close to the interface on the liquid side for a given mean shear.

Velocity profiles for the R10 numerical simulations (Fig. 5) are nearly identical to those for the R29 simulations but according to Table II of Lombardi *et al.*<sup>26</sup> have a slightly smaller interface velocity  $\tilde{U}_0 = -2.2983$ , giving a value of 0.925 for the corresponding ratio of the R10 and R29 interface velocities. By (24), ~80% of this reduction in the interface velocity can be explained as arising from the ratio  $(1 + \alpha|_{R29})/(1 + \alpha|_{R10}) \approx 0.940$ , while only 20% should evidently be attributed to a change in the ratio of the effective roughness lengths, i.e., for the given R29 roughness length ratio  $(\zeta_0^L/\zeta_0^G)|_{R29} = 3$ , the predicted R10 ratio is  $(\zeta_0^L/\zeta_0^G)|_{R10} = 3^{(0.925/0.940)} \approx 2.95$ . This relatively small inferred change in effective roughness lengths is consistent with the near-equality of the R10 and R29 velocity profiles in the logarithmic outer layer (Fig. 5).

# C. Corrections for the Wu<sup>20</sup> air-water flow

It might be assumed from the agreement (Fig. 3) of the surface drift measured by  $Wu^{20}$  with the theoretical prediction (20) that the symmetry and universality assumptions are satisfied nearly exactly in this laboratory flow. The validity of this inference can be explored by examining the solutions of an explicit analytical model of the laboratory flow, which reveals that the apparent close agreement masks two compensating, small but non-negligible, departures from universality, each of order 15% in  $\alpha$ . These evidently arise from windwave disequilibrium and from differing air and water ratios of outer boundary layer scales to the effective interfacial roughness lengths. The former represents a departure from homogeneity that effectively modifies the stress condition at the interface, while the latter results in a correction that is logarithmic in the additional dimensionless parameter.

The model equations are classical but for completeness are provided in Appendix B. The equations and solutions for each layer are similar to those for the water-only model of Baines and Knapp,<sup>29</sup> who measured laboratory surface drift currents of ~2% of the air speed, somewhat smaller than but of the same order as those measured by  $Wu^{20}$  or predicted by (20). The model is formulated to support the classical wall-boundary-layer structures in both fluids, consistent with the logarithmic wind and current profiles reported by Wu,<sup>20,28</sup> with roughness lengths  $z_0^a$  for air and  $z_0^w$  for water.

The model air flow is forced by a uniform imposed pressure gradient. A condition of no depth-integrated water flow is enforced by diagnosing an opposite, uniform pressure gradient in the water layer, consistent with the confinement of the water in the laboratory tank apparatus and with a down-wind setup of the water surface; Wu<sup>20</sup> mentioned this setup but reported no measurements of it. No cross-tank flow is considered, so the velocities, V(z) for air and U(z) for water, are both scalar functions of height z with the interface located at z = 0. In each fluid, the pressure gradient force is balanced by the divergence of a turbulent stress that is parameterized by an eddy viscosity, where the latter is taken to increase linearly with distance from the interface and the top and bottom of the tank sufficiently near to those boundaries, reaching a constant maximum sufficiently far from those boundaries. For air or water roughness lengths sufficiently small that the corresponding eddy viscosity  $\kappa v_* z_0^a$  or  $\kappa u_* z_0^w$  is less than the air or water molecular viscosity  $v_a$  or  $v_w$ , a viscous sublayer region is included adjacent to the interface, as in the analytical model of the numerical simulations of Lombardi et al.<sup>26</sup> (Appendixes B and C).

The dimensional model solutions have an air velocity maximum near 0.20 m height, consistent with the air velocity profiles shown by Wu,<sup>28</sup> and a water velocity with narrow top and bottom boundary layers and a weakly sheared interior passing through zero near mid-depth (Fig. 6). The boundary layer widths in both air and water are of order 0.1 m. The velocity difference across the upper water boundary layer, adjacent to the interface, is several times greater than that across the lower water boundary layer.

The dimensionless model surface-relative velocity profiles,  $[V(z/z_0^a) - U_0]/v_*$  for air and  $[U(z/z_0^w) - U_0]/u_*$  for water, are effectively antisymmetric up to dimensional distances of order 10<sup>3</sup> roughness lengths from the interface, and approximately antisymmetric for distances approaching the height of the free-stream air velocity maximum (Fig. 7). This universal and antisymmetric structure is consistent with the assumptions from which the general theoretical results (17)–(20) are derived. As anticipated, the universal structure exhibited by these solutions is a classical logarithmic wall boundary layer (Fig. 7), consistent with the measured laboratory velocity profiles on both sides of the interface.<sup>20,28</sup>



FIG. 6. Model (a) air (V; z > 0), (a) and (b) water (U; z < 0) velocities, and (c) stress ( $\tau_a$ , z > 0;  $\tau_w$ , z < 0) vs height (z) for the solution with  $V_{\infty} \approx 7 \text{ m s}^{-1}$ . This solution was obtained with G = -0.918 N m<sup>-1</sup>,  $z_0^a$  = 1.125 × 10<sup>-4</sup> m, and  $z_0^w$  = 4.5 × 10<sup>-4</sup> m and had values  $V_{\infty}$  = 6.96 m s<sup>-1</sup> at height  $z_{max}$ = 0.187 m, v<sub>\*</sub> = 0.415 m s<sup>-1</sup>,  $U_0$ = 0.256 m s<sup>-1</sup>, F = 1.77 × 10<sup>-4</sup> N m<sup>-1</sup>, v<sub>\*H</sub> = 0.335 m s<sup>-1</sup>,  $u_{*D}$ = 0.0037 m s<sup>-1</sup>,  $z_2^a$  = 0.182 m, and  $z_2^w$  = -0.430 m (see text and Appendix B for parameter definitions).



**FIG. 7.** Model dimensionless (a) air  $[(V - U_0)/v_*]$  (solid line) and water  $[(U - U_0)/u_*]$  (dashed line) surface-relative velocities vs dimensionless height ( $\zeta = z/z_0^{a,w}$ ) for the solution with  $V_{\infty} \approx 7 \text{ m s}^{-1}$ . In (b), both profiles are again shown, but with the water profile replaced by its antisymmetric reflection,  $-[U(-\zeta) - U_0]/u_*$ . The reflected water profile (thin dashed line) is also shown in (a).

The logarithmic boundary layer structure and the theoretical result (20) can be combined to yield a prediction for the empirical proportionality  $|\mathbf{U}_0| \propto v_*$  reported by  $\mathrm{Wu}^{20}$  and the previous authors. The universal boundary layer solution takes the form  $|\mathbf{V}|/v_* \approx (1/\kappa) \ln(z/z_0^a)$ . If the free-stream velocity  $\mathbf{V}_{\infty}$  is approximated by the boundary layer solution at a height  $z_{\infty}^a$ , with  $\mathbf{U}_{\infty} \approx 0$  from the mean or mid-point water velocity, then (20) becomes

$$|\mathbf{U}_0| \approx \left(\frac{\alpha}{\kappa} \ln \frac{z_{\infty}^a}{z_0^a}\right) v_*,\tag{25}$$

with direction determined by the free-stream velocity. Madsen<sup>30</sup> [his Eq. (40), Sec. 4] used similar but converse reasoning to obtain a 3% proportionality of the wind drift to 10-m wind speed from his model prediction of wind-drift dependence on friction velocity for the ocean-atmosphere case. The constant of proportionality in (25),  $(\alpha/\kappa) \ln(z_{\alpha}^{*}/z_{0}^{0})$ , depends only on the ratio

of the free-stream velocity height to the roughness length and for the ocean-atmosphere case is directly related to the standard 10-m neutral drag coefficient.<sup>31</sup>

For the model solutions,  $z_{\infty}^a/z_0^a \approx 10^3$  (Fig. 7), so that (25) gives  $|\mathbf{U}_0| \approx 0.60v_*$ , similar to the relation  $|\mathbf{U}_0| \approx 0.53v_*$  reported by Wu.<sup>20</sup> The direct evaluation of the ratio  $|\mathbf{U}_0|/v_*$  from the model solutions also gives values that roughly match the individual observed values at each free-stream velocity, which range from 0.4 to 0.7 (Fig. 4). The larger variation with  $V_{\infty}$  [Fig. 4(a)] of the proportionality constant in the relation  $U_0 \propto v_*$ , compared to that for the equivalent  $\alpha$  in (20), evidently arises from the dependence seen in (25) on the roughness length  $z_0^a$ , which, in turn, depends on  $V_{\infty}$  (Fig. 8). If the roughness lengths are set to constant values, then the model can be seen to have a unique dimensionless solution that scales with the pressure-gradient forcing, and the ratios  $U_0/V_{\infty}$  and  $U_0/v_*$  are both constant, as can be verified by numerical solution.

One departure from universality is indicated by the difference in the  $Wu^{20}$  estimates of interfacial stress obtained from the air and from the water velocity profiles for which the water friction velocities are systematically smaller than would be predicted by (3) from the observed air friction velocities (Fig. 9). As noted by Wu,<sup>20</sup> this difference likely indicates that a portion of the stress from the air is absorbed directly by the wave field, which was not in local equilibrium at the mid-tank measurement point, but developed downstream in response to the surface forcing until damped by the downstream baffles.

A second departure is indicated by the difference in the modeled and measured water roughness lengths: for this set of solutions, the former are one to two orders of magnitude larger than the latter [Fig. 8(b)]. With other parameters held fixed, variations in the model water roughness length  $z_0^w$  have a much larger relative effect on the model surface drift velocity  $U_0$  than on the air free-stream velocity  $V_{\infty}$  and friction velocity  $v_*$  (Fig. 10). For  $z_0^w > v_w/(\kappa u_*)$ , the drift velocity decreases linearly with the logarithm of  $z_0^w$ ; for smaller  $z_0^w$ , the dependence is weaker because the viscous sublayer effectively prevents the surface from becoming arbitrarily slippery as  $z_0^w \to 0$ . Note that, perhaps surprisingly, the measured water roughness lengths are smaller than the measured air roughness lengths, rather than larger as in the numerical simulations of Lombardi *et al.*<sup>26</sup>



**FIG. 8**. Observed (solid dots) and model (pluses: solutions with original  $\alpha$  and crosses: modified solutions with  $\alpha' = 0.85\alpha$ ) roughness lengths vs free-stream air velocity  $V_{\infty}$  for (a) air and (b) water. The viscous sublayer cutoffs are indicated in both panels (dotted line); in (b), the cutoff is shown for both the original and modified  $u_*$  values.



**FIG. 9.** Air friction velocities  $v_*$  (solid dots) and water friction velocities scaled by  $\alpha$ ,  $u_*/\alpha$ , (circles), with scaled water friction velocity  $u_*/\alpha'$  for  $\alpha' = 0.85\alpha$  (squares), from the Wu<sup>20</sup> measurements. Values of  $v_* = u_*/\alpha$  from the initial (pluses) and modified (crosses) model solutions are also shown.

For the cases with  $V_{\infty} \leq 9$  m s<sup>-1</sup>, the model fit can be improved by changing the stress condition (2) to reflect an assumption that roughly one-quarter of the stress goes into the wave field rather than the mean current, consistent with the estimates by Wu,<sup>20</sup> and simultaneously reducing the water roughness lengths to  $z_0^{w'} = 0.1 z_0^w$  [Figs. 9 and 8(b)]. These adjustments bring the model and observed water friction velocities and roughness lengths into better agreement while maintaining the model fit to the observed  $V_{\infty}$ ,  $v_*$ , and  $U_0$ . The modified stress condition is imposed in the model by setting the value of  $\alpha$  to  $\alpha' = 0.85 \alpha$  $\approx$  0.029: because the fluid densities appear only in  $\alpha$ , which enters only through (2), this is equivalent to increasing the water density by 40% so that the stress results in a reduced water velocity. For  $\dot{V}_{\infty}$  > 9 m s<sup>-1</sup>, the water roughness lengths are reduced only to  $z_0^{w'} = 0.33 z_0^w$  in these modified solutions, as a greater reduction results in systematic over-prediction of the drift velocity for reasons that have not been determined but may be related to wave effects either on the dynamics or on the measurement error.

An analytical expression for the corresponding leading-order correction to the predicted drift velocity (20) can be computed for the modified model solutions. These have the approximate form (Appendix B),

$$V(z) \approx U_0 + \frac{v_*}{\kappa} \ln \frac{z_0^a + z}{z_0^a}, \quad U(z) \approx U_0 - \frac{u_*}{\kappa} \ln \frac{z_0^w - z}{z_0^w},$$
 (26)

with dimensionless equivalents

$$\frac{V}{v_{*}} \approx \frac{U_{0}}{v_{*}} + \frac{1}{\kappa} \ln(1+\zeta), \quad \frac{U}{u_{*}} \approx \frac{U_{0}}{u_{*}} - \frac{1}{\kappa} \ln(1-\zeta), \quad (27)$$

where  $\zeta = z/z_0^a$  for  $z \ge 0$  and  $\zeta = z/z_0^w$  for  $z \le 0$ . Let  $z_{\infty}^a$  and  $z_{\infty}^w$  be levels at which *V* and *U* approach their respective free-stream values,  $V_{\infty}$  and  $U_{\infty}$ , with  $\zeta_{\infty}^a \gg 1$  and  $\zeta_{\infty}^w \gg 1$  being the corresponding dimensionless levels. Then,

$$\frac{V_{\infty}}{v_*} \approx \frac{U_0}{v_*} + \frac{1}{\kappa} \ln \zeta_{\infty}^a, \quad \frac{U_{\infty}}{u_*} \approx \frac{U_0}{u_*} - \frac{1}{\kappa} \ln |\zeta_{\infty}^w|, \quad (28)$$

and

$$\frac{V_{\infty}}{v_{*}} + \frac{U_{\infty}}{u_{*}} \approx \frac{U_{0}}{v_{*}} + \frac{U_{0}}{u_{*}} + \frac{1}{\kappa} \ln \frac{\zeta_{\infty}^{a}}{|\zeta_{\infty}^{\omega}|}$$
(29)

from which it follows that

$$U_0 \approx \frac{1}{1+\alpha} \left( \alpha V_{\infty} + U_{\infty} + \frac{u_*}{\kappa} \ln \frac{\zeta_{\infty}^a}{|\zeta_{\infty}^w|} \right).$$
(30)

If  $|\zeta_{\infty}^{w}| = \zeta_{\infty}^{a}$  so that the symmetry assumption holds, (30) reproduces (18). If, however, the water roughness length is replaced by a smaller value  $z_{0}^{w'} = \beta z_{0}^{w}$ , then  $|\zeta_{\infty}^{w}|$  is replaced by  $|\zeta_{\infty}^{w'}| = |\zeta_{\infty}^{w}|/\beta = \zeta_{\infty}^{a}/\beta$  and (30) yields the leading-order correction to (18),

$$U_0 \approx \frac{1}{1+\alpha} \left( \alpha V_{\infty} + U_{\infty} - \frac{u_*}{\kappa} \ln \beta \right).$$
(31)

With  $u_*$  held constant at the value obtained for the solution with  $|\zeta_{\infty}^w| = \zeta_{\infty}^a$ , consistent with weak dependence of friction velocity on  $z_0^w$  [Fig. 10(b)], the approximation (31) quantitatively reproduces the leading-order linear dependence of the drift velocity on the logarithm of the water roughness length that was found in the numerical solutions (Fig. 10). The dependence is such that the drift velocity increases with decreasing water roughness length and decreases with increasing water roughness length. Note that the stress is constant in the high-shear near-surface log layers, and consequently, the surface stress remains constant while the drift velocity changes in response to the variations in roughness length.



**FIG. 10**. Model (a)  $V_{\infty}$ , (b)  $v_{*}$ , and (c) drift velocity  $U_{0}$  as a function of  $z_{0}^{w}$ , with other parameters held fixed as for the solution in Figs. 6 and 7, with corresponding observed values (solid dots and circles) from Ref. 20 for the case  $V_{\infty} \approx 7 \text{ m s}^{-1}$ . In (c), the value  $z_{0}^{w} = v_{w}/(\kappa u_{*})$  is also indicated (dotted line).



**FIG. 11**. As in Figs. 7(a) and 7(b), but for the modified solution with  $\alpha' = 0.85\alpha$  and  $z_0^{w'} = 0.1 z_0^{w}$ .

By (28), with  $|V_{\infty}| \gg |U_0|$  for  $\zeta_{\infty}^a \gg 1$ , the ratio  $u_*/\kappa \approx \alpha V_{\infty}/\ln \zeta_{\infty}^a$  so that (31) may be approximated for  $|V_{\infty}| \gg |U_{\infty}|$  as

$$U_0 - U_\infty \approx \tilde{\alpha} V_\infty$$
, where  $\tilde{\alpha} = \alpha \left( 1 - \frac{\ln \beta}{\ln \zeta_\infty^a} \right)$ , (32)

a modified form of (20). The improved prediction (32) for the wind drift, which includes the leading-order correction for asymmetry of the dimensionless velocity profiles, may be compared with (20). For the model solutions with  $\alpha$  replaced by  $\alpha' = 0.85\alpha$  and  $z_0^{w}$  replaced by  $z_0^{w'} = 0.1z_0^{w}$  (Figs. 8 and 11),  $\beta = 0.1$  and  $\zeta_{\infty}^{a} \approx 10^3$  so that (32) gives  $\tilde{\alpha} \approx 1.3\alpha' \approx 1.1\alpha$ . Thus, for these solutions, the effects of the dimensionless asymmetry and of the partial absorption of the stress by the wave field essentially compensate, giving a predicted effective value  $\tilde{\alpha}$  for the proportionality of wind drift to free-stream velocity that happens nearly to coincide with the true value of  $\alpha$ . Both  $\tilde{\alpha}$  and  $\alpha$  then appear to predict accurately the measured wind drift, especially for  $V_{\infty} < 9$  m s<sup>-1</sup> (Fig. 3).

#### V. SUMMARY

The theoretical argument leading to the results (17)–(21) for the drift velocity on the interface relies on a classical assumption of turbulent universality and Reynolds' number similarity, which asserts that the statistically steady and horizontally homogeneous boundary layers on either side of the interface must be controlled by the same universal turbulent processes. This equivalence then implies that the mean velocity deviation profiles, when scaled by the respective friction velocities and intrinsic length scales, will be antisymmetric across the interface. This condition is sufficient to determine the interface drift velocity in terms of the free-stream velocities. The resulting expression (17) provides a simple, physically intuitive, and quantitatively useful first-order prediction of the interface drift velocity for two-fluid systems with general density ratios. The comparisons with classical laboratory experiments on

Among the many possible combinations of fluids to which the result (17) may be relevant, the air-water system is of particular interest, especially in the context of surface wind drift in the ocean-atmosphere system. The classical empirical rule for the ocean wind drift reported already by Keulegan<sup>16</sup> and Van Dorn<sup>17</sup> has remained remarkably consistent over many decades and in a wide range of environmental conditions: Wu<sup>20</sup> stated that it is "commonly accepted" that the total wind drift is about 3% of the wind velocity at long fetches; Madsen<sup>30</sup> and Weber<sup>32</sup> cited a "commonly employed rule of thumb" that oil slicks are advected with a velocity that is 3% of the wind speed and in a direction that is  $\sim 10^{\circ}$  or  $15^{\circ}$  to the right of the wind; Morey et al.<sup>33</sup> characterized the wind drift as a correction to the numerical model surface velocity that is "typically 3%" of the wind speed and directed "to the right of the wind direction," while for the contemporary operational ocean model described by Zelenke et al.,<sup>18</sup> the suggested proportionality is "typically about 3% of the wind speed" but may range from 1% to 4%, depending on conditions. This long-standing empirical rule nonetheless remains without definitive observational confirmation or accepted theoretical explanation.

It is striking that the universality predictions (17)-(20) essentially reproduce this empirical rule, and consequently, it is also tempting to hypothesize that the approximate universality of the turbulent dynamics gives a plausible explanation for its stability and reliability. In this interpretation, the apparent rotation to the right of the surface (10-m) wind in the northern hemisphere would be understood as arising from the rotation of the surface (10-m) winds to the left of the geostrophic wind, the rotating boundary layer analog of the free-stream velocity,<sup>34</sup> which, by (21), would determine the direction of the drift velocity. Arguing against this interpretation, especially, is the ubiquitous role of wave-breaking in ocean surface dynamics at all but the lowest wind speeds, and the expected strong effect of surface waves on near-surface Lagrangian motion, with much recent work on the ocean surface drift problem focused instead on estimating a wavedriven Stokes' drift at or near the surface.<sup>11-15,33</sup> From the point of view of dimensional analysis, the presence of waves introduces an additional length scale either through the ratio of kinematic stress to the gravitational acceleration<sup>35</sup> or through a characteristic wave amplitude, which can cause departures from universality and symmetry.

In addition to its long-standing importance for such practical problems as pollutant dispersal, search and rescue, and navigation, the determination of the surface ocean drift has recently drawn renewed scientific attention. Simultaneous measurement of ocean surface winds and currents has been identified, for example, as a priority for future satellite observations by the 2018 Decadal Review,<sup>36</sup> which cites their importance in a variety of Earth science contexts, including the determination of air-sea momentum exchange, ocean upwelling, upper ocean mixing, and seaice drift. An airborne Doppler scatterometer has been developed that simultaneously measures surface stress and currents in the upper few centimeters of the ocean,<sup>37</sup> and a design for a satellite version of this instrument exists.<sup>38</sup> Novel *in situ* technologies are also in development that promise better direct measurements of currents within centimeters of the ocean surface.<sup>2,33</sup> An improved understanding of the dynamics at the atmosphereocean interface is sure to follow from these developments and from improved theoretical insight into the controlling processes. It is hoped that the present contribution may prove to be useful in this context and help to stimulate further work on these topics.

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#### APPENDIX A: DATA FROM Ref. 20

Data digitized by the author from figures in Ref. 20 are recorded in Table I. The roughness lengths provided in this table use the definitions (B2) and (B11)–(B16), which give values smaller by a factor of  $10^{-(8.5/5.75)} \approx 1/30$  than the definition used by Wu.<sup>20</sup>

**TABLE I.** Values of free-stream air velocity  $V_{\infty}$  (m s<sup>-1</sup>), total surface drift velocity  $U_0$ , estimated Stokes' surface drift velocity  $U_S$ , air and water friction velocities  $v_*$ , and  $u_*$  (cm s<sup>-1</sup>), and air and water roughness lengths  $z_0^a$  and  $z_0^w$  (10<sup>-5</sup> m), digitized from Figs. 2, 6, and 9 of Ref. 20.

I	Figure 2		Figure 6			Figure 9		
$V_{\infty}$	$v_{\star}$	$z_0^a$	$V_{\infty}$	$U_0$	Us	$V_{\infty}$	u*	$z_0^w$
						0.93	0.10	33
1.96	14.2	33				1.92	0.33	8.0
2.96	16.5	17	2.91	9.50	0.43	2.84	0.39	6.7
						2.88	0.45	4.0
3.94	20.7	14	3.87	12.9	1.3	3.85	0.53	4.3
						3.82	0.58	1.0
4.96	28.0	11	4.90	17.0	2.2	4.79	0.75	4.3
						4.79	0.81	1.5
5.87	33.2	8.3	5.86	21.3	2.8	5.76	0.91	3.7
						5.72	1.00	0.50
6.83	39.3	8.0	6.82	26.9	3.4	6.74	1.09	2.7
						6.74	1.12	0.67
7.85	40.2	6.0	7.82	30.4	4.1	7.64	1.12	1.8
						7.64	1.26	1.8
8.81	43.3	4.7	8.77	33.4	5.2	8.61	1.35	0.40
9.75	63.3	28	9.74	37.1	5.2	9.58	1.53	0.37
10.81	78.5	50	10.69	39.0	5.9	10.51	1.90	0.67
11.74	87.9	70	11.69	46.6	5.6	11.53	2.21	1.8
12.71	104.2	110	12.65	49.0	5.8	12.46	2.96	5.3
13.68	116.7	167	13.65	52.7	6.2	13.43	3.26	6.0

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# APPENDIX B: ANALYTICAL MODEL SOLUTIONS FOR WU<sup>20</sup> FLOW

For the interface air roughness length  $z_0^a \ge v_a/(\kappa v_*)$ , where  $v_a = 1.5 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$  is the kinematic viscosity of air at 20 °C, the model equations for air (z > 0) are

$$-\frac{d\tau_a}{dz} = G, \quad \tau_a = -\rho_a A_V(z) \frac{dV}{dz}, \tag{B1}$$

where  $\tau_a$  is the turbulent stress, *G* is the pressure gradient,  $\rho_a$  is the air density, and  $A_V(z)$  is the air eddy viscosity, defined by

$$A_{V}(z) = \begin{cases} \kappa v_{*}(z_{0}^{a} + z), & 0 \le z < z_{1}^{a} \\ \kappa v_{*}(z_{0}^{a} + z_{1}^{a}), & z_{1}^{a} \le z < z_{2}^{a} \\ \kappa v_{*H}(H + z_{0}^{H} - z), & z_{2}^{a} \le z < H. \end{cases}$$
(B2)

In (B2),  $\kappa = 0.4$  is the von Kármán constant,  $v_*$  and  $v_{*H}$  are the wind friction velocities at the interface and the tank top, respectively,  $z_0^a$  and  $z_0^H$  are the roughness lengths associated with the interface and the tank top, respectively,  $z_1^a$  and  $z_2^a$  are constants, and *H* is the height of the tank top. For the interface water roughness length  $z_0^w \ge v_w/(\kappa u_*)$ , where  $v_w = 1.0 \times 10^{-6}$  m<sup>2</sup> s<sup>-1</sup> is the kinematic viscosity of water at 20 °C, the model equations for water (z < 0) are analogous,

$$-\frac{d\tau_w}{dz} = F, \quad \tau_w = -\rho_w A_U(z) \frac{dU}{dz}, \tag{B3}$$

where  $\tau_w$  is the turbulent stress, *F* is the pressure gradient,  $\rho_w$  is the water density, and  $A_U(z)$  is the water eddy viscosity, defined by

$$A_U(z) = \begin{cases} \kappa u_*(z_0^w - z), & z_1^w < z \le 0\\ \kappa u_*(z_0^w - z_1^w), & z_2^w \le z < z_1^w\\ \kappa u_{*D}(D + z_0^D + z), & -D \le z < z_2^w. \end{cases}$$
(B4)

In (B4),  $u_*$  and  $u_{*D}$  are the water friction velocities at the interface and the tank bottom, respectively,  $z_0^w$  and  $z_0^D$  are the roughness lengths associated with the interface and the tank bottom, respectively,  $z_1^w$  and  $z_2^w$  are constants, and *D* is the depth of the tank bottom. In order that the eddy viscosities be continuous, the constants  $z_2^a$  and  $z_2^w$  are computed from the conditions

$$v_{*}(z_{0}^{a}+z_{1}^{a}) = v_{*H}(H+z_{0}^{H}-z_{2}^{a}), \quad u_{*}(z_{0}^{w}-z_{1}^{w}) = u_{*D}(D+z_{0}^{D}+z_{2}^{w}).$$
(B5)

The boundary conditions are that the velocities vanish at the top and bottom of the tank, supplemented by the continuity of the velocities and the stress at the interior fluid points  $z_1^a$ ,  $z_2^a$ ,  $z_1^w$ ,  $z_2^w$  and at the interface z = 0. In addition, the condition of no depth-integrated water flow is imposed,

$$\int_{-D}^{0} U(z) \, dz = 0. \tag{B6}$$

The momentum equations (B1) and (B3), with (B2) and (B4), can be integrated analytically in each subregion, subject to unknown constants to be determined by the boundary and continuity conditions. For the solutions considered here, imposed air pressure gradients G < 0 are chosen so that  $V \ge 0$ , dV/dz < 0 at z = H, dV/dz > 0 and dU/dz > 0 at z = 0, and dU/dz < 0 at z = -D. The friction velocities, which are determined as part of the solution, then satisfy

$$\rho_a v_{*H}^2 = -\rho_a v_{*H} \kappa z_0^H \frac{dV}{dz} \quad \text{at } z = H, \tag{B7}$$

$$\rho_a v_*^2 = \rho_a v_* \kappa z_0^a \frac{dV}{dz} = \rho_w u_*^2 = \rho_w u_* \kappa z_0^w \frac{dU}{dz} \quad \text{at } z = 0, \qquad (B8)$$

$$\rho_w u_{*D}^2 = -\rho_w u_{*D} \kappa z_0^D \frac{dU}{dz} \quad \text{at} \quad z = -D.$$
(B9)

Vertical integrals of the momentum equations (B1) and (B3) yield

$$-GH = v_*^2 + v_{*H}^2, \quad FD = u_*^2 + u_{*D}^2, \tag{B10}$$

while (B8) implies (4) with  $\rho_1 = \rho_a$ ,  $\rho_2 = \rho_w$  so that  $v_{*H}$ ,  $u_*$ , and F can be computed directly from G,  $v_*$ , and  $u_{*D}$ . Solutions are obtained by specifying the air pressure gradient G and the roughness lengths at the interface and the top and bottom boundaries, and solving simultaneously, by numerical iteration, for the water pressure gradient F and the friction velocities.

The model equations may be integrated to find explicit solutions in each region, which satisfies the continuity conditions on velocity and stress at  $z = \{z_1^a, z_1^w\}$ , the continuity conditions on stress at z = 0, and the no-slip boundary conditions at  $z = \{-D, H\}$ , as follows:  $0 \le z \le z_1^a$ :

$$V(z) = U_0 + \frac{v_*}{\kappa} \left( 1 + \frac{Gz_0^a}{v_*^2} \right) \ln \frac{z_0^a + z}{z_0^a} + \frac{G}{\kappa v_*} z, \qquad (B11)$$

 $z_1^a \le z \le z_2^a$ :

$$V(z) = V(z_1^a) + \frac{1}{\kappa v_*(z_0^a + z_1^a)} \left[ \frac{1}{2} G(z^2 - z_1^{a^2}) + v_*^2(z - z_1^a) \right],$$
(B12)

 $z_2^a \leq z \leq H$ :

$$V(z) = -\frac{G}{\kappa v_{*H}} (z - H) + \frac{v_{*H}}{\kappa} \left(1 - \frac{G z_0^H}{v_{*H}^2}\right) \ln \frac{H + z_0^H - z}{z_0^H}, \quad (B13)$$

 $z_1^w \leq z \leq 0$ :

$$U(z) = U_0 - \frac{u_*}{\kappa} \left( 1 + \frac{F z_0^w}{u_*^2} \right) \ln \frac{z_0^w - z}{z_0^w} - \frac{F}{\kappa u_*} z,$$
(B14)

 $z_2^w \leq z \leq z_1^w$ :

$$U(z) = U(z_1^w) + \frac{1}{\kappa u_*(z_0^w - z_1^w)} \bigg[ \frac{1}{2} F(z^2 - z_1^{w^2}) + u_*^2(z - z_1^w) \bigg],$$
(B15)

 $-D \leq z \leq z_2^w$ :

$$U(z) = \frac{F}{\kappa u_{*D}}(z+D) - \frac{u_{*D}}{\kappa} \left(1 + \frac{Fz_0^D}{u_{*D}^2}\right) \ln \frac{D+z_0^D+z}{z_0^D}.$$
 (B16)

For the interface roughness length  $z_0^a < v_a/(\kappa v_*)$  or  $z_0^w < v_w/(\kappa u_*)$ , a corresponding laminar viscous sublayer analogous to that in (C1) is included adjacent to the interface, with the continuity of velocity and stress imposed at the transition points  $z = \{z_{lam}^a, -z_{lam}^w\}$ , where  $z_{lam}^{a,w} = \{\ln[v_a/(\kappa v_* z_0^a)]v_a/(\kappa v_*), \ln[v_w/(\kappa u_* z_0^w)]v_w/(\kappa u_*)\}$ . The log-layer solutions in the regions  $z_{lam}^a < z < z_1^a$  and  $z_1^w < z < -z_{lam}^w$  are also then modified from (B11) and (B14) as in (C1). Note that  $v_w/v_a = 0.067 \approx 2\alpha$ , so these solutions do not satisfy the additional symmetry condition  $v_L = \alpha v_G$  imposed by Lombardi *et al.*<sup>26</sup>

By (B10),  $v_{*H}$ ,  $u_*$ , and F can be computed directly from G,  $v_*$ , and  $u_{*D}$  using the continuity of stress at z = 0 and vertical integrals of the momentum equations. The velocity profiles and roughness lengths at the tank top and bottom were not measured by Wu, <sup>20,28</sup> so the corresponding model roughness lengths were likewise chosen simply to be proportional to the respective interface roughness lengths, with  $z_0^H = 0.1 z_0^a$  and  $z_0^D = 0.2 z_0^D$ . For given G, an iterative numerical method may be used to find the values of  $v_*$  and  $u_{*D}$  for which the solutions (B11)–(B16) are continuous at  $z_2^a$  and  $z_2^w$  and the integral condition (B6) is satisfied. The method was implemented by computing three independent estimates of the unknown drift velocity  $U_0$  in (B11)–(B16), from the integral relation (B6) and the continuity conditions at  $z_2^a$  and at  $z_2^w$ , and then minimizing the squares of the differences of two pairs of these estimates.

An initial set of solutions approximately matching the Wu<sup>20</sup> observations of free-stream velocity and air friction velocity were obtained by iteration, primarily through the adjustment of the air pressure gradients *G* and air roughness lengths  $z_0^a$ , with the first guesses for the air roughness lengths taken from the values reported by Wu.<sup>20</sup> For these solutions, the water roughness lengths were set according to  $z_0^w = 4 z_0^a$  for  $V_\infty \le 9$  m s<sup>-1</sup> and  $z_0^w = 3 z_0^a$  for  $V_\infty > 9$  m s<sup>-1</sup>. Given the experimental uncertainties, the iteration was conducted manually and halted when the approximate agreement of the model free-stream and air friction velocities with the measurements was achieved.

#### APPENDIX C: ANALYTICAL MODEL SOLUTIONS FOR LOMBARDI *et al.*<sup>26</sup> FLOW

For the analytical model of the Lombardi *et al.*<sup>26</sup> flow, the domain is divided into a laminar sublayer region  $0 < |\zeta| \le \zeta_{lam}$  with unit dimensionless molecular viscosity and a turbulent outer region with dimensionless eddy viscosity  $\kappa(\zeta_{\kappa} + \zeta)$ . The transition point  $\zeta_{lam}$  and the roughness–length parameter  $\zeta_{\kappa}$  are related by the requirement that the eddy and molecular viscosities be equal at the transition point, where continuity conditions are imposed on the velocity and stress, i.e.,  $\kappa(\zeta_{\kappa} + \zeta_{lam}) = 1$ . The resulting dimensionless model velocity profiles  $\tilde{V}$  and  $\tilde{U}$  in the gas (G) and liquid (L) are

$$\begin{split} \bar{V}(\zeta) &= \begin{cases} \alpha \bar{U}_0 + \zeta - \frac{1}{2} F \zeta^2, & 0 < \zeta \leq \zeta_{lam}^G \\ \bar{V}(\zeta_{lam}^G) + \frac{1}{\kappa} [(1 + F \zeta_{\kappa}^G) \ln \kappa (\zeta_{\kappa}^G + \zeta) - F(\zeta - \zeta_{lam}^G)], & \zeta_{lam}^G < \zeta \leq \zeta_1, \\ \\ \bar{U}(\zeta) &= \begin{cases} \bar{U}_0 + \zeta + \frac{1}{2} F \zeta^2, & -\zeta_{lam}^L \leq \zeta < 0 \\ \bar{U}(\zeta_{lam}^L) - \frac{1}{\kappa} [(1 + F \zeta_{\kappa}^L) \ln \kappa (\zeta_{\kappa}^L - \zeta) - F(\zeta + \zeta_{lam}^L)], & -\zeta_1 \leq \zeta < -\zeta_{lam}^L, \end{cases} \end{split}$$
(C1)

where  $\zeta = zv_*/v_G = zu_*/v_L$  (for  $v_L = \alpha v_G$ , as assumed),  $\zeta_1 = 2\sqrt{2}$  Re = 170.8, and  $\overline{U}_0$  is the dimensionless interface velocity. The outer boundary conditions  $(dV/d\zeta = dU/d\zeta = 0$  at  $\zeta = \zeta_1$ ) cited by Lombardi *et al.*<sup>26</sup> would suggest  $F = 1/\zeta_1$ , but a value of  $F = 0.1/\zeta_1$  was used to obtain a better fit to the numerical velocity profiles in Fig. 5. For the log-layer only model, these equations are replaced by

$$\begin{split} \bar{V}(\zeta) &= \alpha \bar{U}_0 + \frac{1}{\kappa} \ln \frac{\zeta}{\zeta_0^G}, \quad \zeta \ge \zeta_0^G, \\ \bar{U}(\zeta) &= \bar{U}_0 - \frac{1}{\kappa} \ln \frac{-\zeta}{\zeta_0^L}, \quad \zeta \le -\zeta_0^L, \end{split} \tag{C2}$$

for which the dimensionless velocities relative to the interface,  $\bar{V}' = \bar{V} - \alpha \bar{U}_0$  and  $\bar{U}' = \bar{U} - \bar{U}_0$ , are purely logarithmic with the effective dimensionless roughness lengths  $\zeta_0^G$  and  $\zeta_0^L$ . The liquid-side profiles are reflected about  $\zeta = 0$  in Fig. 5, i.e.,  $|\bar{U}(\zeta)|$  and  $|\bar{U}'(\zeta)|$  are shown vs  $|\zeta|$ .

#### DATA AVAILABILITY

Data sharing is not applicable to this article as no new data were created or analyzed in this study. The numerical codes used to solve the equations described in Appendixes B and C are available from the author upon reasonable request.

#### REFERENCES

<sup>1</sup>S. López Castaño, B. J. Geurts, and V. Armenio, "Energy redistribution dynamics in coupled Couette-Poiseuille flows using large-Eddy simulation," Int. J. Heat Fluid Flow **81**, 108519 (2020).

<sup>2</sup>N. J. M. Laxague and C. J. Zappa, "Observations of mean and wave orbital flows in the ocean's upper centimetres," J. Fluid Mech. **887**, A10 (2020).

<sup>3</sup>F. Ardhuin, B. Chapron, C. Maes, R. Romeiser, C. Gommenginger, S. Cravatte, R. Morrow, C. Donlon, and M. Bourassa, "Satellite Doppler observations for the motions of the oceans," Bull. Am. Meteorol. Soc. **100**, ES215–ES219 (2019).

<sup>4</sup>A. Bender, A. Stroh, B. Frohnapfel, P. Stephan, and T. Gambaryan-Roisman, "Combined direct numerical simulation and long-wave simulation of a liquid film sheared by a turbulent gas flow in a channel," Phys. Fluids **31**, 022103 (2019).

<sup>5</sup>X. Yu, K. Hendrickson, B. K. Campbell, and D. K. P. Yue, "Numerical investigation of shear-flow free-surface turbulence and air entrainment at large Froude and Weber numbers," J. Fluid Mech. 880, 209–238 (2019).

<sup>6</sup>A. A. Ayati, P. Vollestad, and A. Jensen, "Detailed measurements of interfacial dynamics in air-water pipe flow," Proceedia IUTAM 26, 59–69 (2018).

<sup>7</sup>G. Chinello, A. A. Ayati, D. McGlinchey, G. Ooms, and R. Henkes, "Comparison of computational fluid dynamics simulations and experiments for stratified airwater flows in pipes," J. Fluids Eng. 141, 051302 (2018).

<sup>8</sup>N. Pizzo, W. K. Melville, and L. Deike, "Lagrangian transport by nonbreaking and breaking deep-water waves at the ocean surface," J. Phys. Oceanogr. 49, 983– 992 (2019). <sup>9</sup>L. Grare, L. Lenain, and W. K. Melville, "Vertical profiles of the wave-induced airflow above ocean surface waves," J. Phys. Oceanogr. **48**, 2901–2922 (2018).

<sup>10</sup>L. Romero, L. Lenain, and W. K. Melville, "Observations of surface wave-current interaction," J. Phys. Oceanogr. 47, 615–632 (2017).

<sup>11</sup>P. B. Smit, T. T. Janssen, and T. H. C. Herbers, "Nonlinear wave kinematics near the ocean surface," J. Phys. Oceanogr. **47**, 1657–1673 (2017).

<sup>12</sup>Ø. Breivik, J.-R. Bidlot, and P. A. E. M. Janssen, "A Stokes drift approximation based on the Phillips spectrum," Ocean Modell. **100**, 49–56 (2016).

<sup>13</sup> M. Curcic, S. S. Chen, and T. M. Özgökmen, "Hurricane-induced ocean waves and Stokes drift and their impacts on surface transport and dispersion in the Gulf of Mexico," Geophys. Res. Lett. **43**, 2773–2781, https://doi.org/10.1002/2015gl06 7619 (2016).

<sup>14</sup>T. H. C. Herbers and T. T. Janssen, "Lagrangian surface wave motion and Stokes drift fluctuations," J. Phys. Oceanogr. 46, 1009–1021 (2016).

<sup>15</sup>F. Ardhuin, L. Marié, N. Rascle, P. Forget, and A. Roland, "Observation and estimation of Lagrangian, Stokes, and Eulerian currents induced by wind and waves at the sea surface," J. Phys. Oceanogr. **39**, 2820–2838 (2009).

<sup>16</sup>G. H. Keulegan, "Wind tides in small closed channels," J. Res. Natl. Bur. Stand. 46, 358–391 (1951).

<sup>17</sup>W. G. Van Dorn, "Wind stress on an artificial pond," J. Mar. Res. **12**, 249–276 (1953); available at www.journalofmarineresearch.org.

<sup>18</sup>B. Zelenke, C. O'Connor, C. Barker, and C. Beegle-Krause, "General NOAA operational modeling environment (GNOME) technical documentation," Technical Report NOAA Technical Memorandum NOS OR&R 41, National Oceanic and Atmospheric Administration, Office of Response and Restoration, Seattle, WA, USA, 2012.

<sup>19</sup>K. Lofquist, "Flow and stress near an interface between stratified liquids," Phys. Fluids 3, 158–175 (1960).

<sup>20</sup>J. Wu, "Wind-induced drift currents," J. Fluid Mech. 68, 49–70 (1975).

<sup>21</sup>P. Vollestad, A. A. Ayati, L. Angheluta, J. H. LaCasce, and A. Jensen, "Experimental investigation of airflow above waves in a horizontal pipe," Int. J. Multiphase Flow **110**, 37–49 (2019).

<sup>22</sup> M. P. Buckley and F. Veron, "Structure of the airflow above surface waves," J. Phys. Oceanogr. 46, 1377–1397 (2016).

<sup>23</sup>H. Schlichting and K. Gersten, in *Boundary Layer Theory*, 9th ed. (Spinger-Verlag, 2017).

<sup>24</sup>T. K. Cheung and R. L. Street, "The turbulent layer in the water at an air-water interface," J. Fluid Mech. **194**, 133–151 (1988).

<sup>25</sup>L. Thais and J. Magnaudet, "Turbulent structure beneath surface gravity waves sheared by the wind," J. Fluid Mech. **328**, 313–344 (1996).

<sup>26</sup> P. Lombardi, V. De Angelis, and S. Banerjee, "Direct numerical simulation of near-interface turbulence in coupled gas-liquid flow," Phys. Fluids 8, 1643–1665 (1996).

<sup>27</sup>G. K. Batchelor, *An Introduction to Fluid Dynamics* (Cambridge University Press, 1967).

<sup>28</sup>J. Wu, "Laboratory studies of wind-wave interactions," J. Fluid Mech. 34, 91– 111 (1968).

<sup>29</sup>W. D. Baines and D. J. Knapp, "Wind driven water currents," J. Hydraul. Div., Proc. Am. Soc. Civ. Eng. **91**, 205–221 (1965).

<sup>30</sup>O. S. Madsen, "A realistic model of the wind-induced Ekman boundary layer," J. Phys. Oceanogr. 7, 248–255 (1977).

<sup>31</sup>J. B. Edson, V. Jampana, R. A. Weller, S. P. Bigorre, A. J. Plueddemann, C. W. Fairall, S. D. Miller, L. Mahrt, D. Vickers, and H. Hersbach, "On the exchange of momentum over the open ocean," J. Phys. Oceanogr. 43, 1589–1610 (2013).

<sup>32</sup>J. E. Weber, "Steady wind- and wave-induced currents in the open ocean," J. Phys. Oceanogr. 13, 524–530 (1983).

<sup>33</sup>S. L. Morey, N. Wienders, D. S. Dukhovskoy, and M. A. Bourassa, "Measurement characteristics of near-surface currents from ultra-thin drifters, drogued drifters, and HF radar," Remote Sens. **10**, 1633 (2018).

<sup>34</sup>R. M. Samelson, "Wind drift in a homogeneous equilibrium sea," J. Phys. Oceanogr. (submitted).

<sup>35</sup>H. Charnock, "Wind stress on a water surface," Q. J. R. Meteorol. Soc. **81**, 639–640 (1955).

<sup>36</sup>National Academies of Science and Medicine, in *Thriving on Our Changing Planet: A Decadal Strategy for Earth Observation from Space* (The National Academies Press, Washington, DC, 2018).

<sup>37</sup>E. Rodríguez, A. Wineteer, D. Perkovic-Martin, T. Gál, B. Stiles, N. Niamsuwan, and R. Monje, "Estimating ocean vector winds and currents using a Ka-band pencil-beam Doppler scatterometer," Remote Sens. **10**, 576 (2018).

<sup>38</sup>E. Rodriguez, M. Bourassa, D. Chelton, J. T. Farrar, D. Long, D. Perkovic-Martin, and R. Samelson, "The Winds and Currents Mission concept," Front. Mar. Sci. **6**(6), 438 (2019).