Relative Crest Lengths of Sea and Swell

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ABSTRACT

A simple formula is proposed for evaluation of average relative crest length from the directional spectrum. The formula is derived theoretically on the basis of certain reasonable assumptions concerning the shape of the directional spectrum.

It is supposed that the sea surface elevation η relative to local MSL represents a random Gaussian variate characterized by a directional spectrum $\phi(k,\theta)$ in which k is wavenumber magnitude and θ the angle of propagation referred to an appropriate reference, which we will identify as the local mean direction of propagation. If $\phi(k,\theta)$ is symmetric about $\theta=0$ then it can be shown (Longuet-Higgins, 1957) that the average crest length γ, normalized to the average wavelength, is given by

$$\gamma=(m_{20}/m_{02})^{\frac{1}{2}}, \tag{1}$$

where m_{pq} are the moments of the directional spectrum $\phi(k,\theta)$:

$$m_{pq}=\int_{-\pi}^{\pi}\int_{0}^{\infty}\phi(k,\theta)k^{p+q}\cos^{p}\theta\sin^{q}\theta dkd\theta. \tag{2}$$

In particular, m_{00} is the variance of η, while m_{20} and m_{02} are proportional to the square of the expected number of zero crossings of η per unit length along lines parallel and transverse, respectively, to the mean direction of wave propagation, provided that ϕ is symmetric about this direction.

As an approximation, it is assumed that the directional spectrum can be written in the form

$$\phi(k,\theta)=f(k)h(\theta), \tag{3}$$

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where, in particular,

$$h(\theta)=\begin{cases}\cos^{s}\theta, & |\theta|<\dfrac{\pi}{2}\\[2ex] 0, & \dfrac{\pi}{2}<|\theta|<\pi\end{cases} \tag{4}$$

Observations of the directional spectrum show that assumption (3) is not always valid. Usually the energy spreading varies with the frequency, with a maximum in the high-frequency range (cf. Ewing, 1969, or Hasselmann *et al.*, 1973). However, the wave-energy frequency spectrum is usually sharply peaked, i.e., the energy content will mainly be concentrated in a narrow frequency band. The JONSWAP measurements (Hasselmann *et al.*, 1973) on the average showed the peak in the energy spectrum to be about three times higher than in the corresponding Pierson-Moskowitz spectrum. For swell also, most of the energy will usually be concentrated in a narrow frequency band. It is the spreading characteristics within this narrow frequency band that governs the dominant three-dimensional statistical properties of the surface.

Observational data on the directional spreading function $h(\theta)$ in (3) are often fitted to a $\cos^{s}\theta$ function where s is a positive number. The $\cos^{s}\theta$ approximation is satisfactory in most cases (Hasselmann *et al.*, 1973). The exponent s in this relation controls the angular bandwidth of the spreading; the larger the s the smaller the angular bandwidth.

As long as ϕ is separable as in (3), the value of γ is independent of the form of $f(k)$ since the expression for the moments can be represented as the product to an integral over k and an integral over θ. The result is

$$\gamma^2=\frac{\int_0^{\pi/2}\cos^2\theta\cos^s\theta d\theta}{\int_0^{\pi/2}\sin^2\theta\cos^s\theta d\theta}.$$

These are standard integrals and lead to

$$\gamma=(1+s)^{\frac{1}{2}}. \tag{5}$$

This simple relation can be useful in two respects. In calculation of wave forces on structures, the ocean engineer can estimate γ from a knowledge of s. Or from aerial photos, say, one can estimate γ and calculate the corresponding value of the parameter s, assuming that (3) and (4) are valid.

As example applications of (5) three different cases are considered.

1) *Totally confused seas.* Here, the wave energy propagation is distributed equally in all directions at all frequencies. In this case, s in the spreading function equals zero since no direction is preferred. This gives $\gamma=1$, implying that the average crest length equals the average wavelength.

2) *Wave growth.* During the JONSWAP experiment, the directional spectrum for a fetch-limited, growing sea was measured for steady wind conditions. The spreading at the peak frequency fitted a $\cos^2\theta$ distribution fairly well (Hasselmann *et al.*, 1973). This spreading gives for the average crest length $\gamma=1.7$.

3) *Swell.* The directional distribution for swell may be very narrow, with an s factor in the vicinity of 10 (private communication with W. Sell and K. Hasselmann). This gives for the average crest length $\gamma=3.3$.

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