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Verification and Improvement of a Spectral Finite Element Wave model

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Abstract: The **Wind Wave Model** (**WWM**) and the **SWAN** model were implemented at the **Baltic Sea** and the **Sargasso Sea**. Several scenarios were investigated in order to verify the new spectral wave model through comparison with buoy measurements. The newly developed wave model produced very similar results as the **SWAN** model. The flexibility of the **Finite element Method** makes it possible to use the **WWM** more efficiently in comparison to the usually implemented **FDM** methods, when complicated nesting procedures are necessary in order to describe the natural boundary conditions properly. During our study it was found that with the default source term combination, which is implemented within the **SWAN** model, the measured average period is consistently underestimated by the wave models. For the case of the **Baltic Sea** it was found that the reason for this was on one hand the **cut-off frequency** of the measurement buoy and on the other hand an underestimation of low frequency energy or/and an overestimation of high frequency energy by the numerical models. Alternative source term combinations have been implemented in the wave models and the model performance improved significantly for the investigated cases.

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1. INTRODUCTION

The numerical solution of the Wave Action Equation is a popular method for the analysis of the wave climate at our oceans and seas. The numerical implementation of the Wave Action Equation is usually carried out on structured grids with the aid of the **FDM** like done in the models **SWAN**, **WAM** or **WWIII**. The application of structured grids in coastal environments is problematic, because the spatial scales in which the sea state changes due to topography effects becomes small as the waves approach the coasts. This problem can be solved with the use of unstructured grids. More recently spectral wave models have been developed that can work on unstructured grids, like **TOMAWAC** or **CREST**. These models solve the propagation part of the **WAE** with the method of wave **characteristics,** but this **lagrangian** point of view in these models becomes problematic in deep water, in cases with cyclonal wind fields and in areas, where the tidal influence is of great importance. **Hsu, Ou & Liau (2004)** have developed the **WWM** where these problems have been solved with the implementation of the **Taylor-Galerkin Method** for the calculation of the wave energy advection in spatial space. To make this possible the **Fractional Step Method** was utilized to integrate the **WAE**. The integration in spatial space and in time is carried out with an implicit scheme in order to permit a larger time step than prescribed trough the **CFL** (**Courant-Friedrich-Levy**) criterion. The numerical scheme used in this new model is more computation intensive than the **FDM** used in the **SWAN** model. Even if the performance of the **WWM** is comparable to the **SWAN** model in certain cases due to its great flexibility, see e.g. **Hsu, Ou & Liau**, it was our further intention to improve the model performance. In the first model version a standard elimination method from the **LINPACK** library was used to solve the linear equation system. In the first development phase the source term formulation of this new model has been chosen according to the **SWAN** model in order to have a sharp comparison to this state of the art near-shore wave model, in this study also alternative source term combinations have been implemented in both models and investigated.

2. VERIFICATION OF THE WWM

For the validation of the **WWM** and the alternative source term combinations we have investigated several wind events in the **Baltic Sea** and the **Sargasso Sea** with the **WWM** and the **SWAN** model. For the case of the **Baltic Sea** we used for validation the data of two **WAVERIDER®** buoys, which are operated by the **Institute for Coastal Research, Gestacht (IfK, GKSS)** and located nearby the island of Rügen. For the atmospheric boundary condition high resolution wind field data was used, which was obtained from the German Meteorological Service (**DWD**). The bathymetry for the region around Rügen was obtained from the **Baltic Sea Research Institute, Warnemünde** (**IOW).** For the remaining area the foundation for bathymetry was the **ETOPO-2** dataset. The boundary conditions for the **Sargasso Sea** case were taken from ***"The ONR Test bed for Coastal and Oceanic Wave Models"***, Ris et al. (2003)**.** The bathymetry data for the model in the region of the **Sargasso Sea** is based on the **ETOPO-2** dataset courtesy of the **NGDC** (**NOAA**). The wind data used courtesy of **V.J. Cardone** from **Oceanweather Inc.** and wave data used courtesy of **Dr. R.E. Jensen** from the US Army Engineer Research and Development Center (**ERDC**), **USA**.

2.1 THE BALTIC SEA

The Baltic Sea is a nearly enclosed basin located at the north-eastern part of Europe. The area where the measurement buoys are moored is close to the island of Rügen in the western part of the **Baltic Sea**. Two different storm events have been investigated in this study. Both wind events are resulting from depressions which approached the area of interest from north-western and north-eastern directions respectively. Both events have similar wind velocities but the second event has a longer duration and the greatest fetch lengths. The wave heights during this event exceed **3 m** at the location of buoy 1 and are the biggest measured waves in this study for the **Baltic Sea** case. The two buoys are moored within a water depth of **20m** and **8m** respectively (see fig. 1). The buoy 1 is nearly unaffected by shallow water processes, at the location of the buoy 2 bottom friction becomes important.

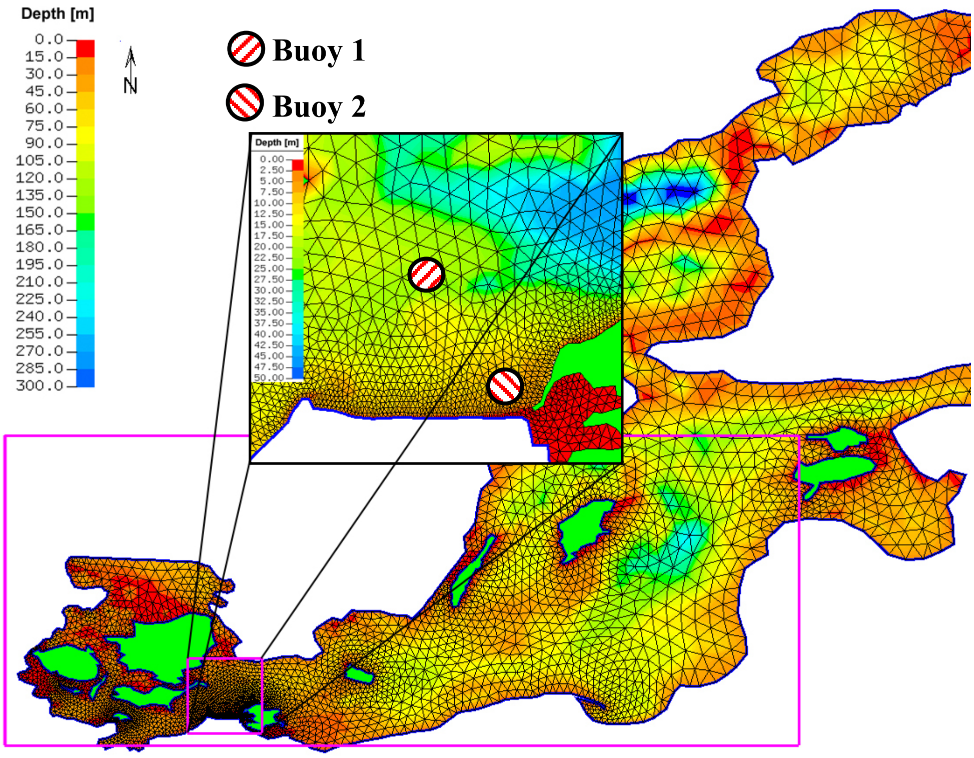

Fig. 1: Bathymetry of the **Baltic Sea** and computational mesh of the **WWM**. The boundaries of the coarse mesh and the nested mesh for the **SWAN** computation are indicated by the magenta frames.

For the **SWAN** model it was necessary to utilize a nested grid in order to describe the depth distribution and the coastline curvature properly. The coarse grid for the **SWAN** simulations (**16400 nodes, 7748 active**) has a resolution of **0.0625°** (approx. 7km) which is the same like the atmospheric model **LM** (Local Model) of the **DWD** from which the atmospheric boundary conditions were obtained. The nested grid (**11200 nodes, 10377 active**), which covers the region around the island of Rügen where the buoys are located, has a resolution of **0.01°**. For **WWM** a grid was used with a resolution from **0.2°** up to **0.01°** (**5948 nodes**) in the region of interest. The spectral resolution was set in both models to **36 direction increments** and **35 frequency increments** with a frequency bandwidth from **0.0625-1.0Hz**. The integral wave results are plotted in **fig. 2**. We found that the zero-down crossing period is consequently underestimated by both models. The reason for this can be the source term formulation of the wave model or the fact that the integration range for the estimation of the integral values is different for the wave model and measurement buoy. The **WAVERIDER®** buoys have cut-off frequency of **0.58 Hz** and in spectral wave simulations the cut-off frequency is usually set to **1.0 Hz**. This has no strong influence on the estimated significant wave heights but it becomes important for the average wave period, as already mentioned by **Dykes et al (2000).**

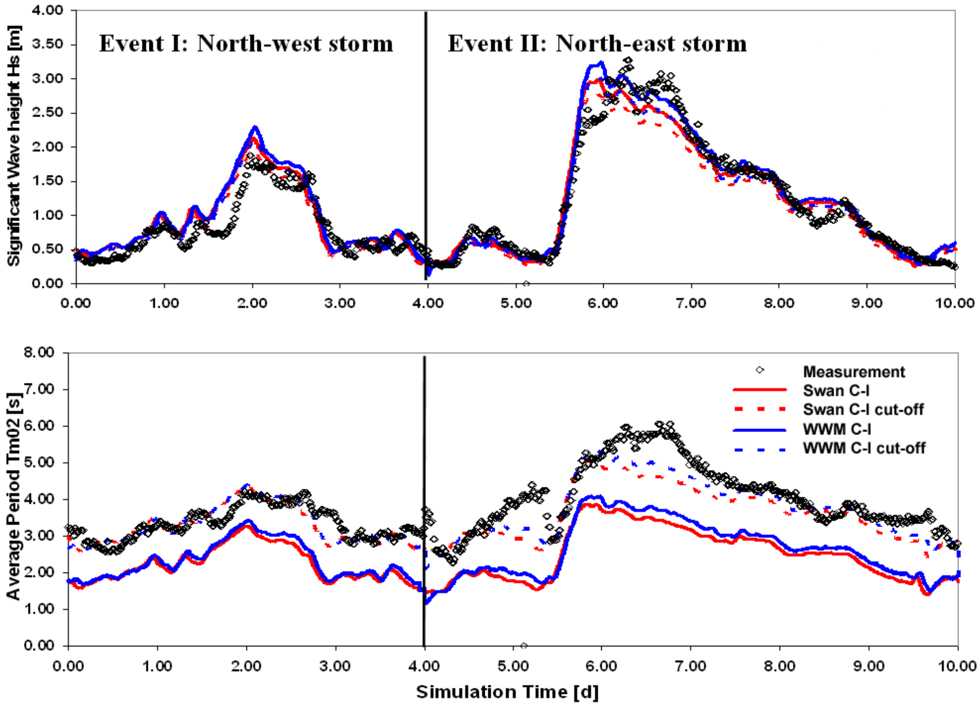

Fig. 2: Simulation results for the Baltic Sea case at the location of buoy I.

Therefore the integral wave parameters of the simulation runs have been recalculated on the basis of the cut-off frequency of the buoy (dashed line fig. below). Both models hindcasted the significant wave height well and produced comparable results. The simulations with the **WWM** resulted in something greater wave heights and periods. After recalculation of the integral wave parameters the model results fit better but at the peak of the second event the average period is still underpredicted by both models. The literature research showed various studies e.g. **Rogers et al (2002), Botema & Bayer (2002)** and **Ou, Hsu & Liau (2003)** where also the average period was underestimated. The study of **Rogers et al (2002)** analyzed the measured and hindcasted wave spectra and found in that the low frequency energy was under predicted and that the high frequency energy was over predicted with the default parameterization in the **SWAN**. We came for the **Baltic Sea** case to similar results. Different wind input and whitecapping functions were implemented in both wave models in order to find out, whether this model behavior is linked to the source term definition or rather to the numerical scheme of the wave model. In a preliminary study (**Roland et al. (2004)**), it was found that the modification of the whitecapping source term had a strong impact on the results and that it can improve the model behavior but it was not clear if this was linked to the numerical implementation of the **WAE** in the

SWAN model. In this study we have utilized two totally different numerical implementation of the **WAE** in order to bring more clearance in this question. **C-I** is the default formulation, same as assessable in the **SWAN** model. **C-II** reflects the findings of **Günther et al. (1992)**, **Janssen (1989)** and **Rogers et al (2002). Komen et al. (1994)** justified the different scaling on the wave number in the **WAM Cycle 4** whitecapping definition that **Hasselmann's** assumption of large separation of the length scales of the waves of origin and the whitecaps may not be existent in the high frequency part of the spectrum. As wind input function in **C-II** the formulation after **Janssen (1986)** which accounts for the gustiness of the wind was utilized, which was parameterized like suggested in **Komen et al (1994)**. **C-III** is based on the work of **Makin & Kudryavtsev (1999)** and **Alves & Banner (2003)**. The whitecapping dissipation function takes into account the onset of wave breaking of the *dominant waves* due to nonlinear wave group modulation. **Michael Banner's** group conducted several studies to identify the dependency of the wave breaking probability of the dominant waves on spectral properties. They found that the wave breaking probability of ocean waves has a strong threshold behavior that can be well described in terms of azimuth integrated **"spectral saturation"**, $\mathbf{B_{(\sigma)}}$.

$$S_{in(\sigma,\Theta)} = \frac{\rho_a}{\rho_w}\cdot \min\left\{A_W, m_\beta \cdot \left(1 - m_c\left[\frac{c_p}{28\cdot u_*}\right]^{n_c}\right)\right\}\cdot\left(\frac{u_*}{c_p}\right)^2 \cdot\cos(\Theta-\Theta_w)\cdot\left|\cos(\Theta-\Theta_w)\right|\cdot S_{(\sigma,\theta)}$$

$$S_{ds(\sigma,\theta)} = -C_{ds}\cdot\left[\frac{B_{(\sigma)}}{Br}\right]^{p/2}\left(\frac{\alpha}{\alpha_{PM}}\right)^m\left(\frac{k}{\bar{k}}\right)^n\cdot\sigma\cdot S_{(\sigma,\theta)},\quad \alpha = \sqrt{E_{tot}\cdot\bar{k}^2}$$

$$B_{(\sigma)} = \int_0^{2\pi}\cdot B_{(\sigma,\theta)}d\Theta,\; B_{(\sigma,\theta)} = k^3\cdot C_g\cdot\sigma^2\cdot S\left(\sigma,\theta\right)$$

$$\begin{cases} p = \dfrac{p_0}{2} + \dfrac{p_0}{2}\tanh\left[10\left(\left(\dfrac{B_{(\sigma)}}{B_r}\right)^{\frac{1}{2}} - 1\right)\right] & , B_{(\sigma)} > B_r \\ p = 0 & , B_{(\sigma)} < B_r \end{cases}$$

Equation 1: Wind input and dissipation functions for **C-III.**

In the above equations $\mathbf{S_{(\sigma,\theta)}}$ is the variance density spectrum [m²/Hz], $\mathbf{S_{in\,(\sigma,\theta)}}$ [m²] and $\mathbf{S_{ds\,(\sigma,\theta)}}$ [m²] are the rate of change in variance density due to the influence of wind and dissipative processes respectively, $\mathbf{B_{(\sigma,\theta)}}$ is the saturation spectrum, $\boldsymbol{\rho_a}$ [kg/m³] and $\boldsymbol{\rho_w}$ [kg/m³] are the wind and the water density respectively, $\mathbf{u_*}$ [m/s] and $\mathbf{c_p}$ [m/s] are the friction velocity of the wind velocity over the sea and the phase velocity of the certain wave respectively, $\boldsymbol{\theta}$ and $\boldsymbol{\theta_W}$ are the wave direction and the wind direction respectively, $\boldsymbol{\sigma}$ is the Doppler shifted (by current in the water column) relative frequency, **k** [1/m] and $\bar{k}$ [1/m] are the certain wave number and the average wavenumber respectively, $\boldsymbol{\alpha}$ [-] and $\boldsymbol{\alpha_{PM}}$ = 4.57E-3 [-] are the average steepness and the **Pierson-Moskowitz** steepness respectively, $\mathbf{A_w}$ = 0.0, $\mathbf{m_\beta}$ = 36, $\mathbf{m_c}$ = 0.3, $\mathbf{n_c}$ = 5.0, **Br** = 4.E-3, **p0** = 6, **m** = 2 and **n** = 1 are parameters (for details see **Alves & Banner** and **Makin & Stam (2003)**).

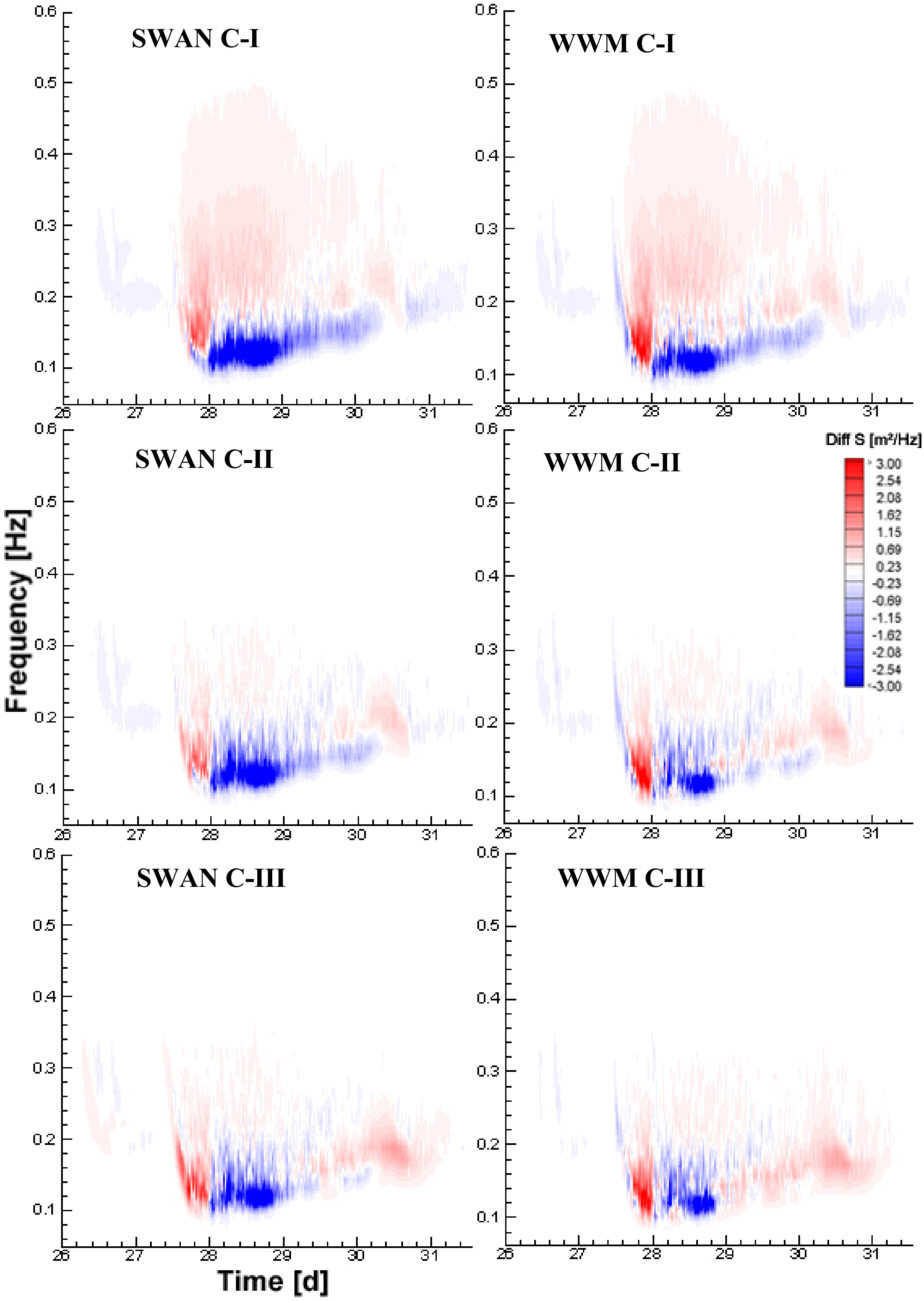

Fig. 3: Differences in variance density between the hindcasted and the measured values for the duration of the storm at the Baltic Sea from the north-east. Left side presents the **SWAN** results and the right side the **WWM** results. From top to bottom the results with the different source term combinations are presented.

From **fig. 3** it can be seen that both models overestimate the high frequency energy and underestimate the low frequency energy with **C-I**. The alternative formulation improved the model behavior with respect to the overestimation of the high frequency energy but the low frequency energy was still underestimated. Both models produced very similar results in spectral space with the implemented source term formulation, which is highly appreciated as the **SWAN** model is validated by a great community over a long time. In **fig. 4** the averaged **RMS** errors in wave energy was estimated for both buoys and events. The results show that the alternative source term formulation improved the model behavior over the whole frequency band and especially at higher frequencies. The **WWM** model results in slightly smaller **RMS** errors than the **SWAN** model. The impact of the source term formulation on the model behavior is similar for both numerical codes.

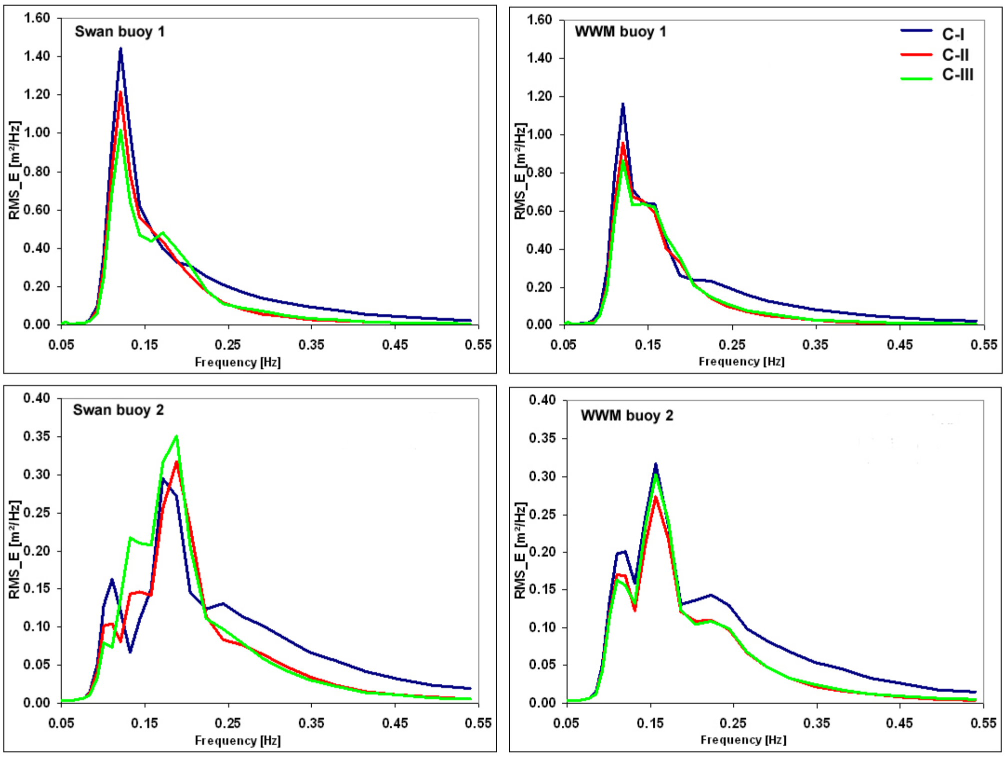

Fig. 4: Time-averaged absolute RMS errors for the two simulations, as a function of frequency for the investigated source term combinations.

2.2 THE SARGASSO SEA

The **Sargasso Sea**, located at the eastern part of **Northern America**, is the playground for extra tropical depressions and strong hurricanes. The first event that was investigated is the “**Halloween storm”,** an extra tropical depression occurred in autumn **1991**. The second event was “**Hurricane Felix”,** occurred in August 1995. In **fig. 5** the computational mesh of the **WWM** is presented. For the **SWAN** model we have used a mesh which covers the same area with a spatial resolution of **0.25°** (**11960 Nodes, 7998 Active**). The spatial resolution of the wind field was same as for the wave simulation. For the **WWM** a mesh was used with a minimum mesh size about **0.05°** and a maximum of **0.35°** (**5964 Nodes**). The calculation time step was set to **20 min** for both models. The directional resolution was set to **72** increments and the frequency resolution was set to **35**. The frequency bandwidth was set from **0.04 – 1.0 Hz**.

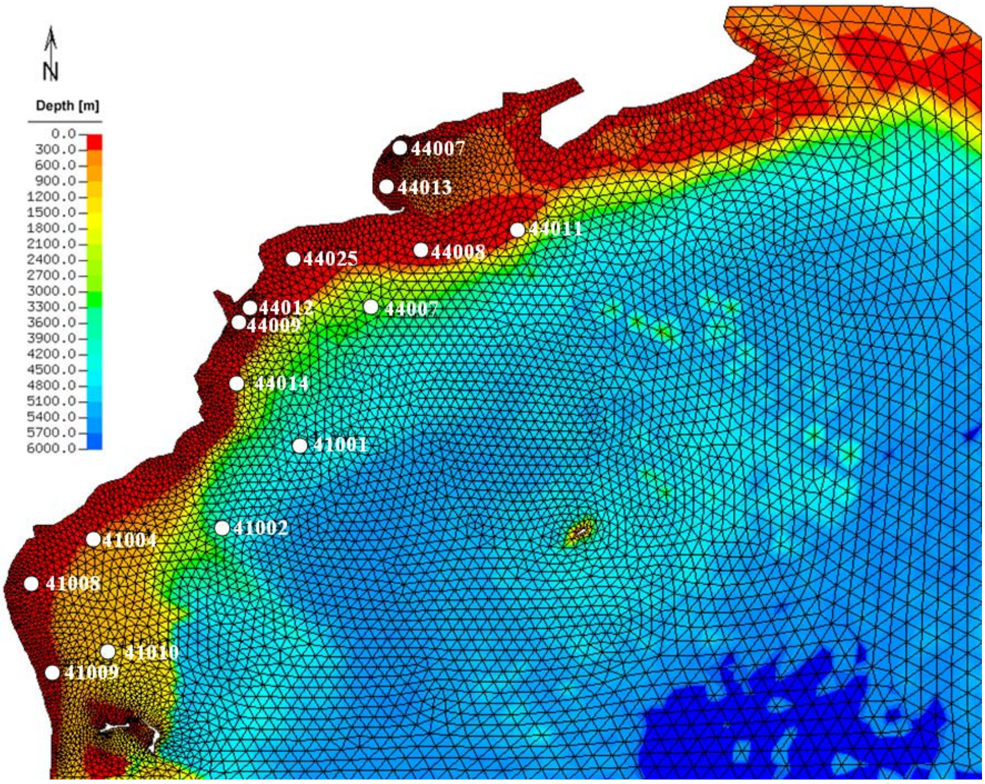

Fig. 5: Bathymetry of the **Sargasso Sea**, finite element mesh and locations of the **NOAA** buoys. Measurements from buoys **41010, 41012, (41004, 44004)** are only accessible during **"Halloween Storm" ("Hurricane Felix")**.

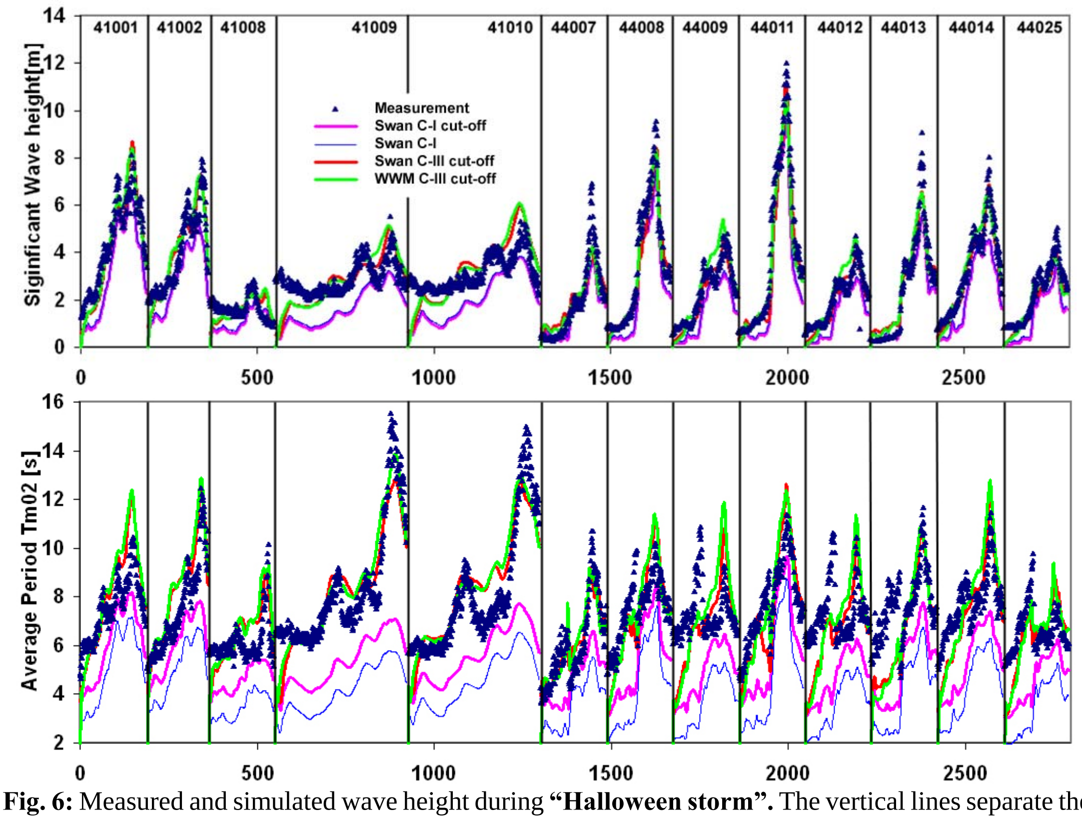

Fig. 6: Measured and simulated wave height during **"Halloween storm".** The vertical lines separate the measurements from each buoy in the plot.

In **fig. 6** the measurement of the buoy and some of the simulation results of both models are presented. The models underpredict the average period when the wave heights reaches its peak dramatically, much stronger than for the **Baltic Sea** case. The results from the simulations with **C-III** are plotted for both models in **fig. 6**. The underprediction of the average period is significantly reduced and the results fit much better to the measurements than with **C-I**. The **WWM** produced also at the **Sargasso Sea** very similar results as the **SWAN** model and the impact of the alternative source term formulation on the model results is also comparable within both models. The statistical analysis of the simulation results are presented in the tables below, the results of the wave models are recalculated on the basis of the cut-off frequency of the **NDBC** buoy (0.35 Hz) and compared with the measurements of the available buoys (see fig.5)

Table 1: Statistical analysis for the **"Halloween storm"** case of all measurements (**HS**: Significant wave height, **TM02**: Zero-down crossing period, **MDIR**: average wave direction, **SCI**: Scatter Index, **MAE**: Mean average Error, **RMS**: Root mean square error, **R²**: Correlation coefficient).

		WWM C-I	WWM C-II	WWM C-III	SWAN C-I	SWAN C-II	SWAN C-III
HS	BIAS [m]	-0.81	-0.58	0.00	-0.92	-0.65	-0.09
	RMS [m]	1.03	0.87	0.72	1.11	0.91	0.67
	SCI [-]	0.31	0.20	0.19	0.35	0.20	0.17
	R² [-]	0.87	0.86	0.84	0.88	0.87	0.86
TM02	BIAS [s]	-1.86	-0.47	0.08	-2.20	-0.70	-0.08
	RMS [s]	2.33	1.50	1.40	2.63	1.51	1.28
	SCI [-]	0.35	0.30	0.25	0.38	0.31	0.23
	R² [-]	0.42	0.64	0.63	0.40	0.65	0.64
MDIR	BIAS [°]	-23.68	-17.47	-8.74	-20.04	-12.82	-14.27
	MAE [°]	33.23	25.18	17.07	32.86	23.65	21.34
	RMS [°]	39.84	32.33	23.88	40.80	29.32	26.61

Table 2: Statistical analysis for the **"Hurricane Felix"** case of all measurements.

		WWM C-I	WWM C-II	WWM C-III	SWAN C-I	SWAN C-II	SWAN C-III
HS	BIAS [m]	-0.46	-0.37	0.12	-0.51	-0.36	0.23
	RMS [m]	0.94	0.85	0.82	0.90	0.81	0.83
	SCI [-]	0.46	0.41	0.40	0.44	0.39	0.41
	R² [-]	0.67	0.69	0.67	0.70	0.70	0.65
TM02	BIAS [s]	-1.63	-0.47	-0.04	-1.85	-0.43	0.16
	RMS [s]	2.30	1.88	1.71	2.39	1.85	1.76
	SCI [-]	0.31	0.25	0.23	0.32	0.25	0.24
	R² [-]	0.32	0.34	0.35	0.34	0.38	0.31
MDIR	BIAS [°]	9.06	11.22	9.30	10.53	15.73	4.16
	MAE [°]	40.14	30.91	23.34	40.27	32.82	21.16
	RMS [°]	59.69	45.51	34.88	59.86	50.52	33.24

3. IMPROVEMENT OF THE WWM MODEL

In the first Version the direct solver of the **LINPACK** library was used for developing purposes. In the new version the **LINPACK** equation solver was replaced by **iterative sparse solvers** like accessible in the **SPARSKIT2 (Saad (1986))** or **ITPACK (Young & Kincaid ())** library. The effective implementation of these methods improved the model performance of the new wave model dramatically. The best performance was achieved with the **ILU-BCGSTAB** method from the **SPARSKIT2** package. The result of a comparison between the different solvers is presented in **fig.7** (left). From **fig.** 7 (right) it can be seen that the **WWM** is slower if compared node wise with the **SWAN** model. The performance enhances significantly, if information's about the sea state conditions are needed at different spatial scales (e.g. large-scale oceanic deep water regions and small-scale shallow water regions), which is mostly the case. Then the **WWM** benefit from its flexible numerical scheme. The conservative **FDM** methods need to utilize at this point grid nesting strategies which complicates the data management, enhances the simulation time and can lead to more human errors during operation.

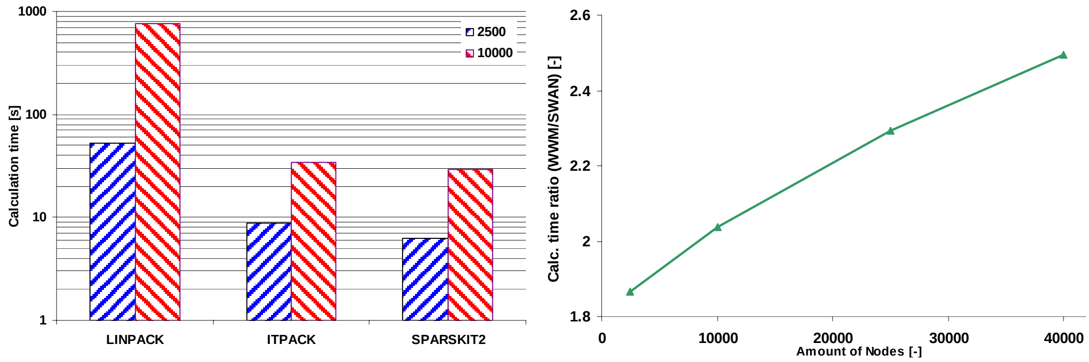

Fig. 7: Left: Performance of the **WWM** with different equation solvers for **2500** and **10000** nodes. **Right:** Ratio between the calculation time for the **WWM** model and the **SWAN** for a certain amount for nodes. (Frequency increments: 24, directional increments: 36).

4. CONCLUSIONS

The **SWAN** model and the newly developed **WWM** were used to simulate certain wind events at the **Baltic Sea** and the **Sargasso Sea**. It was found that both models produced very similar results for the investigated cases. For the **Baltic Sea** it was shown that the reason for the underprediction of the average period was an under prediction of low frequency energy and an over prediction of high frequency energy during the north-east storm. For the **Sargasso Sea** we anticipate from our analysis the same reasons for the worse model behavior with the default source term formulation like for the **Baltic Sea** case. The underprediction of low frequency energy may result from the rigorous tuning of the whitecapping source term which is necessary, as the **DIA** transfer to much energy to higher and lower frequency and distorts so the wave spectrum (**see e.g. VAN VLEDDER (2001)**. The alternative source term formulation improved the results especially concerning the hindcast of the average period. With the source term formulation like suggested by **Makin & Stam** we could achieve the best results in this study.

The implementation of the **SPARSKIT2** and **ITPACK** library in the new version of the **WWM** improved the model performance significantly. Due to the flexibility of the numerical scheme this model can be more efficiently used to simulate the sea state in complicated coastal environments than spectral wave models that utilize the **FDM** or wave-ray method to calculate the wave action advection in spatial space. One of our further intentions is the formulation of a new spectral balance on the foundation of an optimized method for the calculation of the quadruplet wave-wave interactions, like suggested e.g. by **van Vledder (2001), Hashimoto (1998)** or **Tolman (2004),** in order to improve the wave forecast and hindcast in coastal and oceanic regions.

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