Solution of the Wave Action Equation on unstructured meshes using the Residual Distribution (RD) scheme in spatial space

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The solution of the WAE (Wave Action Equation) still remains a formidable task. The advection in spatial space of more than thousands wave packets and their interaction with the atmosphere, the bottom, the currents and themselves is computationally very intensive. The theory for the various processes governing the evolution of wind waves from deep to shallow water, summarized in the “source terms” at the right hand side of the WAE, have been extensively revisited in the last decade (see e.g. The WISE paper). Especially in the context to the ongoing advance in the understanding of the nonlinear processes in deep and shallow water and the development of more sophisticated theories and algorithms to evaluate the nonlinear energy transfer, not only in homogenous moreover in inhomogeneous media (e.g. Stiassnie, 2002) it is from advantage to solve the spatial part of the WAE equation on unstructured meshes in order to optimize the amount of spatial grid points where the source terms must be evaluated without loosing the ability to represent the boundary conditions accurately and retaining acceptable precision of the solution for left hand side part of the WAE.

Hsu et al., 2005 solved the Eulerian wave action balance with the aid of a fractional step approach on triangular spatial meshes with the aid of a Taylor-Galerkin approach in spatial space. Other approaches on unstructured meshes which implement wave ray methods which use Lagrangian approaches to solve the WAE (see e.g. Ardhuin, 2001, Benoit et al., 1997). A first finite volume approach to solve the wave action equation on cell-cantered triangular elements is given by Soerensen et al., 2004. In this paper we will present a robust numerical scheme to solve the unsteady wave action equation on unstructured meshes on the foundation of Residual Distribution schemes for nonlinear conservation laws.

For the solution of the WAE in spatial space we have applied the Residual Distribution (RD) scheme founded by Roe, 1992 and further revised by Abrall, 2001, 2003, Csik et al., 2004, and Ricchuoto et al., 2006. The resulting schemes are conservative, monotone and positive for all investigated cases. The time integration of the spatial derivatives is done with explicit and implicit schemes. The applied schemes are stable, conservative, monotone and of 2nd order accuracy space (CR-PSI scheme) and time (CR-N1 and CR-N2 scheme). The schemes have the benefit that they can operate and arbitrary triangular meshes they are able to capture strong gradients without oscillations and need only the values at the vertices of the triangular mesh for the solution procedure. The compactness of the numerical stencil makes the parallelization of the schemes easy.

The diffusion characteristics of the residual distribution schemes and the Taylor Galerkin schemes have been evaluated in numerical experiments further verification was done within the WWM model (Wind Wave Model, Hsu et al., 2005, Roland et al. 2005, 2006) for ocean and coastal scale applications. The results have been compared to the SWAN model Ris et al., 1998. The newly developed numerical schemes showed to be efficiently applicable and ocean and coastal scale applications. Especially the robustness and efficiency renders the residual distribution scheme as a good alternative to already applied numerical methods on unstructured method. A comparison of all the available advection schemes on unstructured meshes in terms of accuracy, robustness and efficiency is an open challenge for the future. In this comparison with the Taylor Galerkin methods used by Hsu et al., the Residual Distribution schemes are a better choice for near coastal applications because of their robustness, monotonicity and positivity.

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