A draft of "Directional validation of wave predictions"

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Abstract

A methodology for quantitative, directional validation of a long-term wave model hindcast is described and applied. Buoy observations are used as ground truth and the method does not require the application of a statistical model to the observations. Four frequency ranges (relative to the peak frequency) are considered. The validation of the hindcast does not suggest any systematic bias in predictions of directional spreading. Idealized simulations are presented to aid in the interpretation of results. The paper includes a review of literature related to directional validation of wave models.

1. Introduction

[Importance/Relevance]

Wave direction (expressed as mean or peak value) is of obvious importance to wave prediction. Directional distribution (about the mean or peak direction) is also very important for wave modeling. It can have a large impact on the prediction of swells, since it determines how far and wide the swells will disperse. Nonlinear interactions computed by a wave model are sensitive to directional distribution of energy. Further, as wave model dissipation terms with more sophisticated directional dependency are developed, we can expect that directional spreading will have greater influence the modeled source term balance (and thus, total energy).

[Present capability]

Validations of modeled peak (or mean) wave direction in the literature typically show good skill [though the response of a third generation wave model to rapidly turning winds is a concern]. The ability of third generation (3G) models to accurately predict the width of the directional distribution is poorly understood. Indeed, as will be described in Section 2, evaluations in the literature show very little consensus.

1.1 Model description

The so-called "third generation" (3G) of spectral wave models calculate wave spectra without a priori assumption regarding spectral shape. For this investigation, we used the SWAN model ("Simulating WAves Nearshore"; Booij et al. 1999). SWAN is a 3G model designed to address the excessive computational expense of applying predecessor 3G models (such as WAM, WAMDIG 1988) in coastal regions. The governing equation of SWAN and other 3G wave action models is the action balance equation. In Cartesian coordinates, this is:

$$\frac{\partial N}{\partial t}+\frac{\partial C_{g,x}N}{\partial x}+\frac{\partial C_{g,y}N}{\partial y}+\frac{\partial C_{g,\sigma}N}{\partial \sigma}+\frac{\partial C_{g,\theta}N}{\partial \theta}=\frac{S}{\sigma}\,. \quad (36)$$

where σ is the relative (intrinsic) frequency (the wave frequency measured from a frame of reference moving with a current, if a current exists), N is wave action density, equal to energy density divided by relative frequency ($N=E/\sigma$), θ is wave direction, C_g is the wave action propagation speed in (x, y, σ, θ) space, and S is the total of source/sink terms expressed as wave energy density. In deep water, the right hand side of the governing equation is dominated by three terms, $S \approx S_{in}+S_{nl}+S_{ds}$ (input by wind, four wave nonlinear interactions, and dissipation, respectively). [These three deepwater source/sink terms are discussed at several points later in this manuscript.] SWAN also includes physical processes associated with intermediate-depth and shallow water (e.g. bottom friction, depth-limited breaking).

1.2 Objective

[Prologue to objective]

It has become increasingly common for a wave modeler to have at his/her disposal directional wave observations within a model computational domain. This often leads to an expectation—perhaps a naïve expectation—that the wave modeler can readily use these observations to validate the model. Unfortunately, validating a model using directional observations is much less straightforward than traditional validations of wave height or peak period. What if the model in question is a long term simulation with continuous directional observations (perhaps at multiple locations)? How could one perform a meaningful validation that is compact enough to be presented to others? How far can one go in condensing these comparisons? At what point do the comparisons become meaningless or misleading?

[State Objective]

The objectives of this manuscript are as follows:

- To provide an appreciation of the complexity of directional validation.
- To provide an overview of literature relevant to this subject.
- To perform a model validation in which directional characteristics are the *primary* focus. Usually, when directional metrics are used in validation, they are secondary (the primary focus is usually wave height, wave period, and/or frequency spectra). Here, we want to give our (almost) undivided attention to the directional issue. We do this by taking a modeling system which has consistently good skill with regard to non-directional metrics.
- To utilize *long-term* directional validation. Usually, when directional spreading *is* a primary focus, the investigators focus on specific events. This leads to uncertainties with regard to generality ("Is this conclusion specific to the event, or is it a systematic symptom of the model physics?"). We address this limitation using a relatively long simulation.
- To develop and employ a method of quantitative evaluation of model directional skill. Since many comparisons in the literature are short-term comparisons, it is possible to simply present modeled and observed two-dimensional spectra side-by-side, thereby avoiding the necessity of condensing results. With long-term simulations, it is necessary to condense results somehow. ["Quantitative" is used here to indicate a comparison of model and observation value pairings from which

statistics may be calculated; traditionally this is presented as a scatter plot comparison of modeled and observed values.]

- To develop and employ a method of evaluation of model directional skill which *utilizes observational data as they are given,* rather than applying some statistical model to transform the observational data into a (subjective) directional spectrum. This is necessary to avoid the criticism that buoys yield too-broad directional spectra (Young 1994).
- To determine whether a typical 3G model (SWAN, Booij et al. 1999) in a typical implementation, has a systematic tendency to overpredict or underpredict directional spreading. The Discrete Interaction Approximation (DIA) for four-wave nonlinear interactions, S_{nl4}, is the approximation used by all operational third generation (3G) wave models today. It is well-accepted that this approximation leads to broader directional spreading than would be obtained with more rigorous calculations. This can result in an expectation that 3G wave models systematically overpredict directional spreading. This is sometimes observed in the literature, but the reverse has also been observed. One wonders how much this "expectation" has influenced prior comparisons. Long term comparisons can be used to convincingly argue for or against this broadening effect. We know from the literature that in cases of pure wind sea, directional spreading tends to follow a fairly consistent pattern relative to the peak frequency: directional spreading at the peak is relatively narrow, and spreading is broader further from (higher and lower than) the peak. One objective is to determine whether an operationally used wave model (SWAN with the DIA approximation for four-wave interactions) adequately reproduces this pattern in directional spreading.

1.3 Terminology

The two dimensional energy density spectrum is defined as $E(f,\theta) = D(f,\theta)E(f)$, where $D(f,\theta)$ is the normalized directional distribution and $E(f)$ is the one-dimensional energy density spectrum. The function $D(f,\theta)$ is normalized such that $\int_0^{2\pi} D(f,\theta)d\theta = 1$.

"Directional spreading" refers to the degree to which a directional distribution of wave energy is "broad". It does not refer to the normalized directional distribution itself which is sometimes referred to as the "directional spreading function". Notations used in this manuscript are given in Section__.

1.4 Organization of manuscript

In Section 2, a review of prior work on the subjects of directional metrics, directional model validation, and parametric directional distributions is presented. In Section 3, the methodology of this study (general validation strategy and definition of metrics used) is described. In Section 4, an idealized case is examined to isolate the effect of the inaccuracy of the Discrete Interaction Approximation for four-wave nonlinear interactions. In Section 5, an example directional validation is presented (for a hindcast with the SWAN model in Lake Michigan during fall 2002). Results are summarized in Section 6. Discussion is given in Section 7 and Conclusions in Section 8.

2. A review of prior work

To provide appreciation for the development of the state-of-the-art through the years, we present a listing of relevant literature in approximate chronological form:

- Longuet-Higgins et al (1963) developed the methodology for deriving directional properties from heave, pitch, roll buoys which we still use today. This paper, with companion paper Cartwright et al. (1963) proposed the "$\cos^{2s}$ model" for directional distribution, $D(\theta) = \cos^{2s}\left[(\theta - \theta_0)/2\right]$. The "*s*" parameter is a measure of directional spreading. Since the $\cos^{2s}$ model is a model, we choose not to use it herein. However, it has been widely used in the literature.
- Mitsuyasu et al. (1975) were the first of many to propose an empirical, parametric form for the directional distribution $D(f,\theta)$ based on field measurements. Their "*s*" parameter was a function of the frequency relative to the peak, f/f_p, and the wave age U_{10}/C_p. Other parametric forms followed (not all of them based on the $\cos^{2s}$ form). In particular, the dependence on wave age has been questioned. A good review of these forms and related discussions can be found in Young et al. (1996), Ewans (1998), and Young (1999).
- Forristall et al (1978) used the $\cos^{2s}$ model to quantify directional spreading. The study was motivated by the need to calculate forcing on a structure (for the oil industry) without assuming unidirectional waves (which would give incorrect calculations). They compared directional spreading and mean direction from a wave model hindcast for a hurricane event to that of directional spectra estimated from electromagnetic current meter measurements. They did not integrate these two parameters across frequencies. Thus, they did not show time series, but rather comparisons as a function of wave frequency for three specific time periods during the hurricane wave event. They concluded that their early generation wave model compared well with the measurements.
- Komen et al. (1984) is the landmark paper in the early development of third generation wave models. Directional characteristics are of minor concern in this study.
- Hasselmann et al. (1985) introduced the Discrete Interaction Approximation (DIA) for four-wave nonlinear interactions, S_{nl4}. The DIA is the approximation used by all operational third generation wave models today. Using idealized test cases, Hasselmann et al. (1985) and Young et al. (1987) compared two-dimensional spectra obtained from a model using the DIA to those from a model using more rigorous calculations of S_{nl4} (EXACT-NL). The former appear to be considerably more directionally broad than the latter. Comparisons such as this have led to belief by some that third generation wave models tend to overpredict directional spreading, due to the use of the DIA.
- Kuik et al. (1988) provide an excellent discussion and analysis of methods used to interpret buoy directional measurements and suggest four directional metrics that can be calculated from buoy measurements without the use of any model, such as the $\cos^{2s}$ form. These four "model-free" metrics—mean direction, directional width, skewness, and kurtosis—are each calculated as a function of wave frequency. Their calculation of directional width is used in a frequency-integrated

fashion by third generation models such as SWAN (Booij et al. 1999) and WAVEWATCH-III (Tolman 1991, Tolman 2002, denoted “WW3” herein). Kuik et al. (1988) is usually given as the reference for the directional spreading calculation, though the metric is used in earlier articles (e.g. Hasselmann et al 1980, Long 1980, Vlugt et al. 1981). We will refer to the metric as “circular RMS spreading”.

- WAMDIG (1988) is the introduction of the WAM model. There is a directional validation in this paper: modeled mean wave direction (as a function of frequency) is compared to measured values at several instants in time for a hurricane case. Modeled two dimensional spectra are compared to two dimensional spectra derived from Synthetic Aperture Radar (SAR) at a few instants in time in the North Atlantic. The disadvantage of both methods of presentations is that only sample results are possible, rather than long time series. The approach of side-by-side comparisons of sample two dimensional spectra is necessarily rather qualitative.
- Guillaume (1990) compares mean wave direction from second and third generation wave models to buoy data at one location (the “BEATRICE” buoy location) for a time period slightly less than one month. They use one mean wave direction metric that is integrated across all frequencies, a second metric for 0.17 Hz frequency band, and a third metric for the 0.12 Hz frequency band. Mean direction is compared at all frequencies for a shorter (two day) time period. This paper is a good example of a struggle to condense directional comparisons into a readable presentation without rendering the presentation meaningless.
- Tolman (1991) includes a validation of directional spreading using the circular RMS spreading metric. Since it is a validation of refraction in the model using an idealized test case, there is a known, analytical solution that is used as ground truth. The waves are monochromatic, so there is no concern about the presentation of frequency variation of directional spreading.
- Holthuijsen and Tolman (1991) present model vs. model comparisons of directional spreading. That metric is a frequency-integrated version of the circular RMS spreading metric.
- Beal (1991) is a collection of papers, with nine papers dealing with wave models in some fashion pages. Three of the paper make very interesting qualitative comparison (model vs. observations) of times series, with wave height, direction, and frequency indicated for various waves systems. Some of these systems occur simultaneously, see pages 145, 161, and 185.
- Van Vledder and Holthuijsen (1993) make model-data comparisons of the “dimensionless response time scale of the mean wave direction” to shifts in the wind direction. In simulations of idealized cases, a frequency-integrated version of the circular RMS spreading metric is presented. Their model uses exact computations of nonlinear interactions. They observe that simulated time scales of directional response to changes in wind direction are “considerably larger than the observed time scales”.
- Banner and Young (1994) include relatively extensive comparisons of directional distributions and directional spreading (at the peak wavenumber and four times the peak wavenumber). They are able to present in this level of detail because

they remove the temporal dimension by considering only idealized time-independent fetch-limited simulations (and observations). Empirically derived directional distribution functions are used as ground truth. Their formula for directional spreading is described in Section __ (it is the formula used in Young 1999). Young and Van Vledder (1993), Banner and Young (1994), and Young et al. (1995) argue that in the spectral region higher than f/f_p=2 (i.e. frequency greater than twice the peak frequency), spreading is controlled by the four-wave nonlinear interactions S_{nl4}.

- Khandekar et al. (1994) use the method of side-by-side comparison of two-dimensional spectra, similar to that of WAMDIG (1988), to evaluate the performance of a first generation and third generation wave model. Buoy data two-dimensional spectra are used as ground truth (inferred from buoy motion via a statistical model). In the examples shown (four instants in time), directional spreading of the third generation wave model appears to be too broad.
- Jensen et al. (1995) compared directional spreading from a WAM hindcast to that of measurements at three locations near Duck, N.C. during a large synoptic-scale northeaster (dubbed IOP-2). At the one nearshore location (8m depth), agreement is fairly good, while the modeled directional spreading is too narrow at two further from shore (27m and 47m depth). The results were preliminary; it is not known how much of the difference was due to the use of an unusual spreading calculation. However, the results are noteworthy since they are contrary to conventional wisdom (that third generation wave models overpredict directional spreading).
- Heimbach et al. (1998) took a new approach to validating mean wave directions: they compared climatologies of WAM mean wave direction to SAR mean wave direction over the globe (on a 5°×5° grid). The comparisons are made for four separate seasons and sea and swell are presented separately.
- Forristall and Ewans (1998) focus specifically on directional spreading. Similar to Forristall et al (1978), the concern is the reduction (associated with directional spreading) of wave forces on structures. They use a metric associated with this force reduction, as well as the circular RMS spreading metric (both with and without frequency integration). Similar to earlier works, Forristal and Ewans (1998) avoid the problem of presenting time series by using the idealized time-independent fetch-limited scenario as a basis for comparing models to observations[1]. They observe that both the Exact-NL-based third generation wave model and the DIA-based third generation wave model (a WAM variant) "are broadly consistent with the empirical distribution, but the Exact-NL spreading is lower than the [WAM] spreading at higher frequencies." The frequency-integrated circular RMS spreading metric indicates greater spreading with the DIA-based model (approximately 36°) than that from the Exact-NL-based model (approximately 29°).
- Forristall and Greenwood (1998) also focus specifically on directional spreading. Based on idealized test cases, they argue that WAM has a tendency to overpredict

[1] Other cases are studied in the paper, but the fetch-limited case is the only case where the models are applied (our focus is model validation).

directional spreading and that this overprediction is primarily due to the DIA approximation for S_{nl4}. A long-term (multi-year) hindcast with a third generation wave model (with an extended version of the DIA) is compared to measurements at two sites in the North Sea. Frequency-integrated directional spreading is used as the metric. They conclude that there is reasonably good agreement between the hindcast and measured directional spreading. A figure shows the model spreading is underpredicted by the long-term model hindcasts[2] (too narrow) for cases with significant wave heights below 6.5 m, but in discussion of the figures, the authors state that the hindcast spectra are too broad, which if true, would be consistent with the trend observed in their comparisons for idealized cases. The paper also compares hindcasts from two models (a first generation model and the third generation model mentioned above) for Hurricane Opal (Gulf of Mexico, 1995) to directional measurements at an oil platform. Both models greatly overpredict spreading (too broad) during most of the storm duration.

- Krogstad et al. (1999) acknowledge the difficulty of comprehensive directional validation and make qualitative comparisons of two-dimensional spectra.
- Alves and Banner (2002), like Banner and Young (1994), consider the reduced fetch-limited case. Directional spreading is one of several metrics used to evaluate the performance of variations of a third generation wave model. The directional spreading at the peak wave number k_p and $4k_p$ is plotted against wave age, with observation-based relations also shown. With the models used, there is a tendency to overpredict directional spreading at the spectral peak, particularly more mature stages of wave development. Model performance by this metric is poor relative to performance by other metrics (such as total wave energy or peak frequency).
- Moon et al. (2003) compare SRA data to high resolution WAVEWATCH-III hindcast results for a hurricane case. Mean wave direction at the spectral peak is one of the primary metrics for evaluation; the authors report that it is simulated very accurately. They also make side-by-side comparison of collocated measured and modeled directional spectra (18 collocated points), similar to the comparisons made by WAMDIG (1988). Again, excellent agreement is reported, though the authors observe that "the model produces smoother spectra with narrower directional spreading than do the observations when the real spectrum has multiple peaks". One can reasonably expect that this behavior is associated with the complexity of the wind regime, rather than a systematic tendency on the part of the model to generate wave spectra that is too narrow.
- Ardhuin et al. (2003) are specifically concerned about the directional spreading. They validate a wave model developed to simulate shelf-scale processes (refraction, shoaling, bottom friction, Bragg scattering). Their metric is the circular RMS spreading over a narrow frequency range near the spectral peak. Two directional buoys are used as ground truth. They argue that the change in directional spreading across the shelf (in their case, at least) is a balance of the effects of refraction (which tends to narrow spectra) and Bragg scattering (which tends to broaden spectra), and possibly some other unknown physical process(s) which tends to broaden spectra.

[2] Their spreading parameter, inversely related to spreading, is overpredicted by the model.

- Wyatt et al. (2003) made extensive directional comparisons of sub-regional WAM simulations to wave observations by buoy and radar instruments for a duration greater than one month. The metrics they use are 1) time series of one-dimensional directional wave spectra (i.e. two-dimensional spectra integrated across frequencies; a normalized comparison), 2) 7-day time series of circular RMS directional spreading integrated across all frequencies , 3) 7-day time series of a peak direction metric 4) statistics for mean wave direction at the peak frequency, 5) statistics for mean wave direction integrated over four different frequency ranges (which are constant in time), 6) sample side-by-side comparisons of directional spectra (4 instants in time, qualitative). Inspection of comparison (2) suggests a persistent tendency by WAM to overpredict the frequency-integrated spreading during the 7 days shown. The authors report that there is evidence that WAM "responds slowly to changing conditions perhaps due to the coarser resolution in wind forcing".

3. Method

3.1 General strategy

Ground truth

Buoy data are the "ground truth" of this study, specifically, the National Data Buoy Center (NDBC) buoy 45007 in Lake Michigan. Directional buoys are often the most cost-effective method of obtaining directional data outside the surf zone. [In depths shallower than around 150 m, three-elements pressures gage arrays and p-u-v gages can be cost-effective methods of obtaining information essentially the same as a heave-pitch-roll buoy (additional elements in a pressure gage or wave staff array will yield higher resolution directional data, see Young 1994).]

Challenge: Problem complexity

Our objective is to determine the feasibility of conducting a quantitative directional validation of a long-term hindcast. Anticipating that is will be a major challenge even under the most favorable circumstances, we simplify our case study by:

1. Using a lake (Lake Michigan, Fig. 1). Thus, the wave climate is dominated by windsea. Mixed sea/swell states (identifiable as having multiple peaks) do occur (especially when the wind shifts rapidly), but are uncommon. Certainly, old swells do not occur.
2. Using a model (SWAN) which has proven to be skillful predicting non-directional spectra at this scale, in wind sea-dominated cases (Rogers et al. 2003).
3. Making comparisons at only one location (at the location of buoy 45007 in Fig. 1).
4. For model-data comparisons, we use a location near the center of the lake. The depth is 165 m, which is relatively deep water for the typical wave frequencies in the lake. Thus, the impact of finite depth physics is limited.

Challenge: Degrees of freedom

The primary challenge with quantitative directional validation of a long time series is that there exists a different set of low-order moments for every frequency band. That is one dimension. Combine that with the time dimension, and the validation quickly becomes unmanageable. One can make qualitative comparison by plotting these moments as a function of time and frequency, but our objective is to make quantitative comparisons. Thus it is necessary to perform some kind of integration in frequency space. Yet we cannot throw out the frequency-wise variation of these moments altogether, since (as was mentioned in Section 1) one objective of this study is to determine whether an operationally used wave model adequately reproduces the directional spreading as a function of frequency relative to the spectral peak. Thus, there are two competing motivators: 1) the desire to make the problem more manageable via frequency-wise integration of directional metrics, and 2) the desire to describe the frequency-wise variation in directional spreading.

Our approach is a compromise between these two motivators: we retain frequency-wise bins, but use fewer bins than are used in the model computational grid:

1. 0.5 to 0.8 times the relative frequency f/f_p. (“low frequencies”)
2. $0.8f/f_p$ to $1.2f/f_p$ (“frequencies at and near the peak”)
3. $1.2f/f_p$ to $2.0f/f_p$ (“frequencies above the peak”)
4. $2.0f/f_p$ to $3.0f/f_p$ (“highest frequencies”).

Challenge: Defining the peak frequency

In order to quantify the variation of directional spreading as a function of relative frequency, it is obviously necessary to define the peak frequency. Though this may sound simple, it is subject to problems, since even in a region like Lake Michigan, with its typically simple sea states, peak frequency can be a rather unstable quantity, with significant model/data mismatch being not uncommon. Obviously, it is very problematic to compare “model directional spreading as a function of modeled relative frequency” to “observed directional spreading as a function of observed relative frequency” in cases where modeled and observed peak frequency is very dissimilar. Model predictions of mean period tend to be more reliable. To address this, we use a “synthetic peak period” which is a simple function of the mean period, a much more stable quantity. The relation is determined using a simple linear regression of the two metrics for the time period of the hindcast described in Section __. The mean period is calculated over the frequency range of 0.07 to 0.4 Hz.

For the modeled values, the result of the regression is:

$$T_p = 1.2165T_{mean} - 0.72385$$

For the buoy, the regression is:

$$T_p = 1.2325T_{mean} - 0.70509\,.$$

[The discrepancy between the two suggests a small problem in modeled spectral shape, but addressing this is beyond the scope of this study.]

Challenge: Avoiding statistical models

It was mentioned in Section 1 that one objective is to not use any statistical models (e.g. Maximum Likelihood Method) to infer directional characteristics from the buoy data.

The solution is simply to use only quantities that the buoy measures: We transform the model to yield quantities analogous to what the buoy measures. [This is the approach that has been taken in the past by Dr. William C. O'Reilly (unpublished).] The specific calculations are described in Section __, "directional metrics".

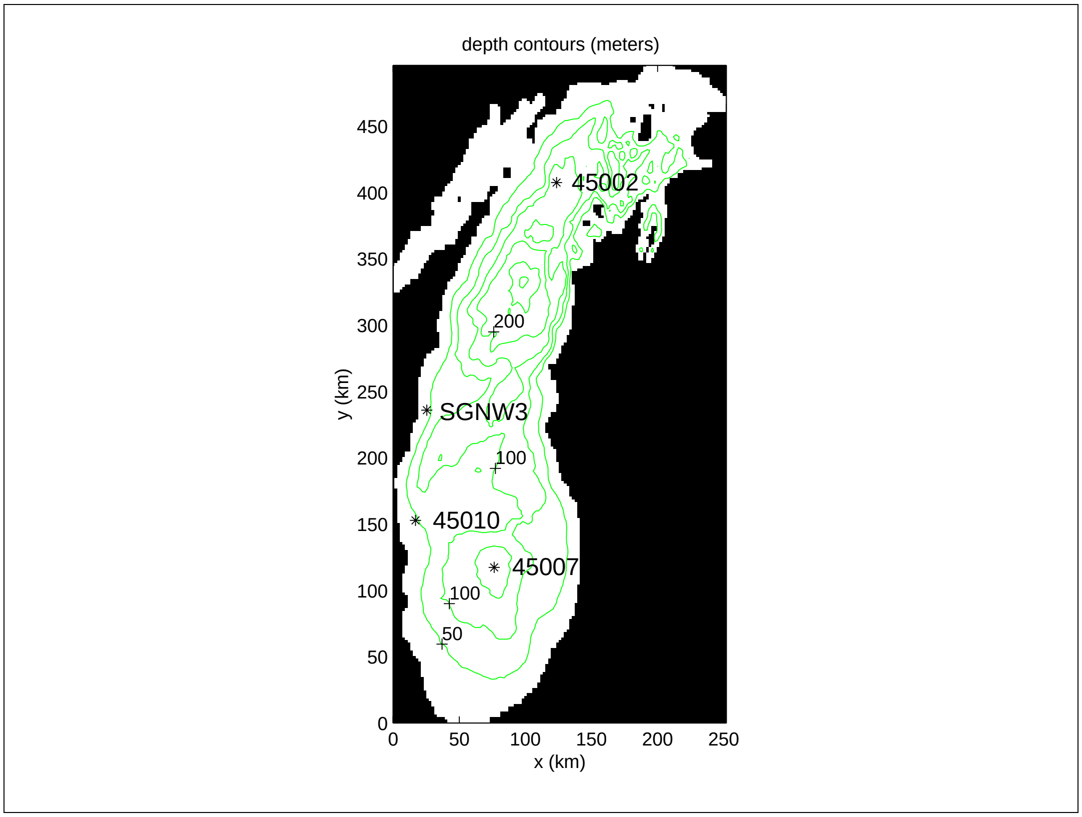

Fig. 1. Lake Michigan, with depth contours (meters) and National Data Buoy Center instrument locations shown.

3.2 Definition of directional metrics

Four Fourier coefficients (a_1, b_1, a_2, b_2) can be inferred from the signals measured by a directional waverider buoy. Those are the four coefficients of a truncated Fourier series describing the nondimensional directional distribution function $D(\theta)$ (Longuet-Higgins et al. (1963), Kuik et al. (1988)):

$$D(\theta) = \frac{1}{\pi}\left[\frac{1}{2} + \sum_{n=1}^{2}\left\{a_n \cos(n\theta) + b_n \sin(n\theta)\right\}\right]. \quad (1)$$

Here $D(\theta)$ is related to the (dimensional) spectral density functions by $E(f,\theta) = D(f,\theta)E(f)$. Here, f denotes radial wave frequency. The function $D(\theta)$ is normalized such that $\int_0^{2\pi} D(\theta)d\theta = 1$. The Fourier coefficients in this expression are in non-dimensional form (they are sometime presented in the literature in dimensional form). There exists a separate function $D(\theta)$ for each frequency component derived from the buoy measurements, so equation (1) might be expressed as

$$D(\theta, f) = \frac{1}{\pi}\left[\frac{1}{2} + \sum_{n=1}^{2}\left\{a_n(f)\cos(n\theta) + b_n(f)\sin(n\theta)\right\}\right]. \quad (2)$$

Unfortunately, equation (2) has limited utility for describing $D(\theta, f)$, since it is only accurate if the unmeasured, higher order Fourier components are very small. Models (such as the $\cos^{2s}$ form) have been developed to yield more natural (and thus presumably more accurate) representations of $D(\theta, f)$ given the measured low order moments, but these models can be misleading, since they give details of $D(\theta, f)$ that are not actually determinable from buoy motion. Kuik et al. (1988) suggest "model-free" expressions for mean wave direction θ_0 and directional width θ_σ.[3] Kuik also suggested two higher order statistics (skewness and kurtosis) that we do not use herein. All four statistics are expressible as functions of the four Fourier coefficients ($a_1(f)$, $b_1(f)$, $a_2(f)$, $b_2(f)$). Mean wave direction is given as $\theta_0(f) = \arctan(b_1(f)/a_1(f))$. Directional width is quantified as the "circular RMS spreading", $\theta_\sigma(f) = \sqrt{2(1-m_1(f))}$, where $m_1(f) = \sqrt{a_1(f)^2 + b_1(f)^2}$. The calculation in reverse is

$a_1 = r_1 \cos\alpha_1$ and $b_1 = r_1 \sin\alpha_1$ (National Data Buoy Center notation)

$a_1 = m_1 \cos\theta_0$ and $b_1 = m_1 \sin\theta_0$ (our notation)

If the full directional spectrum is known (as is the case with a model spectrum), the non-dimensional Fourier components can be calculated as

$$a_1(f) = \frac{\int_0^{2\pi} E(f,\theta)\cos\theta d\theta}{E(f)}, \quad b_1(f) = \frac{\int_0^{2\pi} E(f,\theta)\sin\theta d\theta}{E(f)},$$

[3] Kuik et al. use the symbol σ to denote directional width. Here, we use θ_σ to denote the same quantity, to avoid confusion with frequency.

$$a_2(f) = \frac{\int_0^{2\pi} E(f,\theta)\cos 2\theta d\theta}{E(f)},\ b_2(f) = \frac{\int_0^{2\pi} E(f,\theta)\sin 2\theta d\theta}{E(f)}.$$

Realtime and historical data from directional National Data Buoy Center (NDBC) buoys include estimates of the low order moments θ_0 and m_1 (Steele et al 1985). We will first discuss in detail calculation of mean direction and then do the same for directional spreading.

On second order moments: We experimented with calculation of "α_1" and "α_2" for idealized bimodal distributions $D(\theta)$ and found that whereas "α_1" ["θ_0"] is fairly robust and predictable, "α_2" is unpredictable and unstable (small shifts in $D(\theta)$ led to large changes in α_2). In the literature, Ewing and Laing (1987) find that the second order moments are more reliable in their measurements. However, this is unexpected and is the opposite of what has been found by others (e.g. Forristall et al. 1978).

On notation used elsewhere: NDBC uses the notation "α_1" instead of "θ_0" (used by Kuik et al. and herein) and "r_1" instead of "m_1" (used by Kuik et al. and herein). Further, the NDBC definitions of the Fourier coefficients (a_1, b_1, a_2, b_2) (as used in their literature such as Steele et al. 1985) are dimensional, whereas we use the Kuik convention of nondimensional Fourier coefficients (a_1, b_1, a_2, b_2). The notation "α_1" is useful, as it indicates a relation to (a_1, b_1) and distinguishes from the second order direction, "α_2". However, we do not use the 2nd order directional moments here.

3.2.1 Mean direction

In the literature, mean direction is the most commonly presented directional property of waves (for example, in maps of wave heights with arrows representing mean direction). Since there exists a $D(\theta)$ for each frequency component, $\theta_0 = \theta_0(f)$. A method is needed for calculating the "mean-mean" direction, that is, the mean (over a specified range of frequencies) of $\theta_0(f)$.

Models such as SWAN (Booij et al. 1999) and WAVEWATCH-III (Tolman 1991, Tolman 2002) provide actual two-dimensional spectra $E(f,\theta)$, rather than Fourier coefficients or other low order moments. We adopt the SWAN/WW3 definition of

$$\bar{\theta}_0 = \arctan\left[\frac{\bar{b}_1}{\bar{a}_1}\right]. \tag{3}$$

$$\bar{a}_1 = \int_0^{2\pi}\int_{f_1}^{f_2} E(f,\theta)\cos\theta df d\theta$$

$$\bar{b}_1 = \int_0^{2\pi}\int_{f_1}^{f_2} E(f,\theta)\sin\theta df d\theta$$

[SWAN and WW3 are coded to output $\bar{\theta}_\sigma$ only for f_1 and f_2 equal to 0 and ∞, respectively.]

Now, we want to make the buoy calculation similar to the SWAN/WW3 definitions, so we start with our relations for a_1 and b_1:

$$a_1(f)E(f)=\int_0^{2\pi}E(f,\theta)\cos\theta d\theta,$$

$$b_1(f)E(f)=\int_0^{2\pi}E(f,\theta)\sin\theta d\theta,$$

which makes (3)

$$\overline{\theta}_0=\arctan\left[\frac{\int_{f_1}^{f_2}b_1(f)E(f)df}{\int_{f_1}^{f_2}a_1(f)E(f)df}\right].$$

Note that if we choose f_1 and f_2 as values close to f_p, say $f_1=0.9f_p$ and $f_2=1.1f_p$, this is in practice very similar to the "MWD" reported by NDBC ("Mean wave direction corresponding to energy of the dominant period") to the "D_p" reported by the Coastal Data Information Program (CDIP) ("mean direction from which energy is coming at the peak period"). The use of a band of frequencies makes the metric more stable.

3.2.2 Frequency-Dependent Directional Spreading

Recall that our definitions for frequency-dependent directional spreading are

$$m_1(f)=\sqrt{a_1(f)^2+b_1(f)^2}$$

and

$$\theta_\sigma(f)=\sqrt{2(1-m_1(f))}\ .$$

With a wave model, we have full directional spectra, rather than Fourier coefficients, so (recall from above) we calculate the (dimensionless) a_1, b_1, from $E(f,\theta)$ as :

$$a_1(f)=\frac{\int_0^{2\pi}E(f,\theta)\cos\theta d\theta}{E(f)},\ b_1(f)=\frac{\int_0^{2\pi}E(f,\theta)\sin\theta d\theta}{E(f)}.$$

Thus

$$m_1(f)=\frac{1}{E(f)}\sqrt{\left[\int_0^{2\pi}E(f,\theta)\cos\theta d\theta\right]^2+\left[\int_0^{2\pi}E(f,\theta)\sin\theta d\theta\right]^2}\text{, and}$$

the RMS circular spreading is

$$\theta_\sigma(f)=\sqrt{2\left(1-\frac{1}{E(f)}\sqrt{\left[\int_0^{2\pi}E(f,\theta)\cos\theta d\theta\right]^2+\left[\int_0^{2\pi}E(f,\theta)\sin\theta d\theta\right]^2}\right)}$$

Note that this calculation differs slightly from the method used by Young (1999, pg. 128), which is referred to as the "mean directional width":

$$\theta_\sigma(f) = \frac{\int_0^{2\pi} E(f,\theta)(\theta-\theta_0)\,d\theta}{E(f)}$$

To provide a sense for how the calculations differ, example calculations by the two methods are shown in Fig. 2.

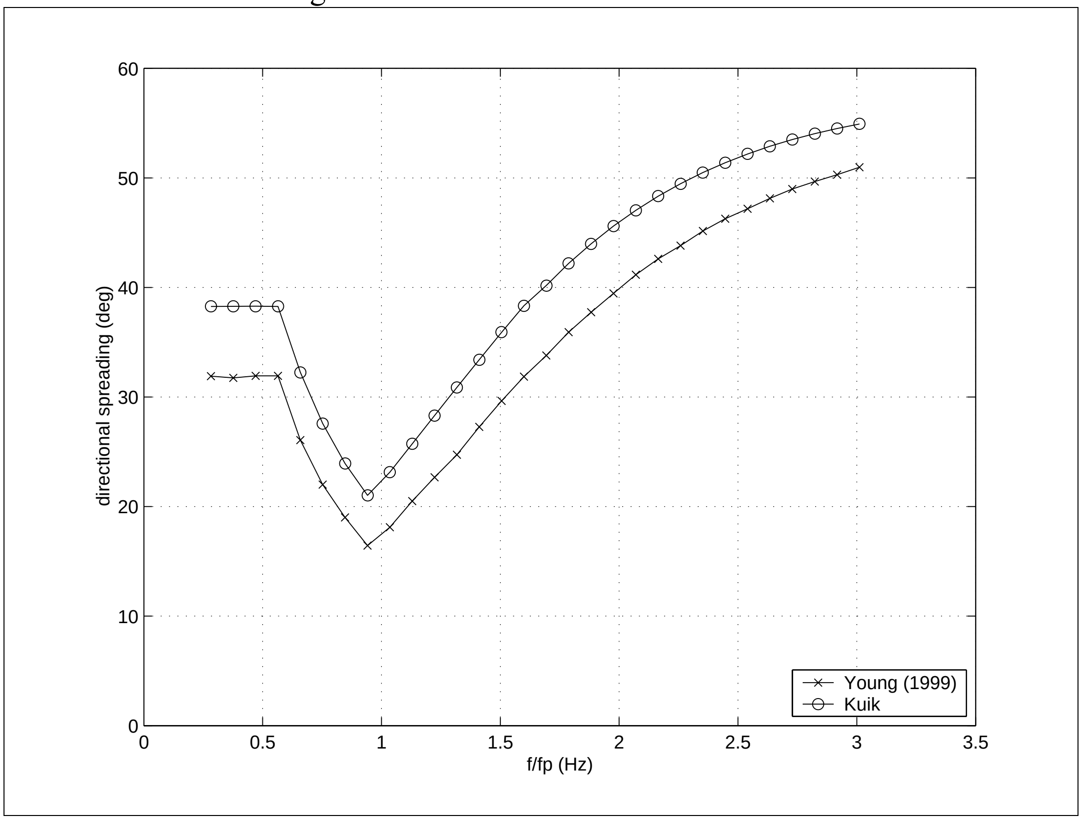

Figure 2. Directional spreading $\theta_\sigma(f)$, Young (1999) method vs. Kuik et al. (1988) method. The Kuik method is used in this study.

3.2.3 Mean Directional Spreading

As with mean direction, the directional spreading $\theta_\sigma = \theta_\sigma(f)$. In this study, we use a weighted mean of θ_σ over particular frequency ranges. We denote this as $\overline{\theta}_\sigma$. Our first inclination was to integrate across frequency bands at the last step,

$\overline{\theta}_\sigma = \dfrac{\int_{f_1}^{f_2} \theta_\sigma(f) E(f) df}{\int_{f_1}^{f_2} E(f) df}$. However, to be more consistent with our calculation of mean direction, and with calculation methods of SWAN and WW3, we take a different approach. WW3 integrates the dimensional Fourier coefficients and energy density across frequencies:

$$\overline{a}_1 = \int_0^{2\pi} \int_{f_1}^{f_2} E(f,\theta) \cos\theta df d\theta$$

$$\overline{b}_1 = \int_0^{2\pi} \int_{f_1}^{f_2} E(f,\theta) \sin\theta df d\theta$$

$$\overline{E} = \int_0^{2\pi} \int_{f_1}^{f_2} E(f,\theta) df d\theta = \int_{f_1}^{f_2} E(f) df$$

[WW3 uses $f_1 = 0$ and $f_2 = \infty$. This approach is similar to that of Forristall and Ewans (1998), in fact. In this paper, we retain the more general form (arbitrary frequency range) here].

The form of calculation of mean directional spread adopted in this paper is:

$$\overline{\theta}_\sigma = \left[2 \left\{ 1 - \left(\frac{\overline{a}_1^{\,2} + \overline{b}_1^{\,2}}{\overline{E}^2} \right)^{1/2} \right\} \right]^{1/2}$$

Since we want to calculate a mean directional spreading from the buoy data consistent with the calculation for the model, we use the relations for the nondimensional Fourier coefficients (provided by NDBC):

$$a_1(f) E(f) = \int_0^{2\pi} E(f,\theta) \cos\theta d\theta$$

and

$$b_1(f) E(f) = \int_0^{2\pi} E(f,\theta) \sin\theta d\theta$$

so

$$\overline{a}_1 = \int_{f_1}^{f_2} \int_0^{2\pi} E(f,\theta) \cos\theta d\theta df = \int_{f_1}^{f_2} a_1(f) E(f) df$$

$$\overline{b}_1 = \int_{f_1}^{f_2}\int_0^{2\pi} E(f,\theta)\sin\theta d\theta df = \int_{f_1}^{f_2} b_1(f)E(f)df$$

<u>4. An idealized case</u>

Rather than move straight to the hindcast simulation, we will first provide an idealized application, since the idealized application is used as a point of discussion when interpreting the hindcast results.

Introduction of nonlinear computation methods

One limitation of the generation mechanisms used by third generation (3G) wave models is the highly simplified Discrete Interaction Approximation (Hasselmann et al. 1985, denoted here "DIA") used to compute four wave nonlinear interactions in both models. A software routine based on the Webb-Resio-Tracy (WRT, see Resio and Perrie 1991 and references therein) has been implemented in the WW3 model. This method is essentially exact, but very time-consuming.

Simulation descriptions

An example application of this WRT subroutine is shown in Fig. 3. The computation is with a "point model", implying either no propagation or infinite fetch. First, the model was run using all three deepwater source terms (DIA for nonlinear interactions, Tolman and Chalikov (1996) for wind input and dissipation), a constant wind speed of U_{10}=18m/s, and a duration of one day. The resulting spectrum (line with plus (+) symbols) was used to initialize a second and third simulation, which are identical except one uses WRT and the other DIA. The latter two simulations, also of one-day duration, include only nonlinear interactions, to lend insight regarding the effect of nonlinear interactions on swell as it leaves its source. Thus, three spectra are presented here:

1. The final condition of the first simulation, which includes all three deepwater source/sink terms, $S = S_{in} + S_{ds} + S_{nl4}$. This spectrum is used as the initial condition for the second and third simulations.
2. The final condition of the second simulation, which includes only four-wave interaction, $S = S_{nl4}$, calculated using the WRT routine.
3. The final condition of the third simulation, which includes only four-wave interaction, $S = S_{nl4}$, calculated using the DIA routine.

Note that since this model does not include propagation, dispersion of the swell is not represented. The effects of dispersion could be significant within one day (it would be expected to reduce nonlinear interactions), depending on the size of the storm. The difference seen here between DIA and WRT models are qualitatively consistent with computations of the nonlinear source term by Hasselmann et al. (1985) (see their Fig. 7, for example).

Discussion of results

The top panel of Fig. 3 shows the nondirectional spectral density of the three spectra and the bottom panel shows the directional spreading of the second and third spectrum. Since only three spectra are being presented (with no time dimension), it is not necessary to integrate in frequency space, and the actual variation with frequency (at the model resolution) is shown. Skewness and kurtosis for the second and third spectra were also compared, but the comparisons were not notable and are not presented here. The directional spectral density distributions for the second and third idealized simulations are shown in Fig. 4. In this figure, both spectra have been normalized by 1.19 $m^2Hz^{-1}deg^{-1}$, which is the maximum of the third simulation. Thus the contours are labeled relative to the peak of the larger spectrum. The following observations can be made:

- Though it is not directly related to the subject matter of this study, the effect of the inaccuracy of the DIA on frequency downshifting is seen clearly in the spectral density plot..
- The inaccuracy of the DIA is leading to (slightly) too narrow spectra at the low frequencies and overly broad spectra at the high frequencies, most noticeable beyond 0.1 Hz.
- The directional spectrum plot gives an immediate visual impression that directional spreading is much greater with the DIA model. However, the higher directional spreading is really apparent only in the lowest energy contour (2% of peak).

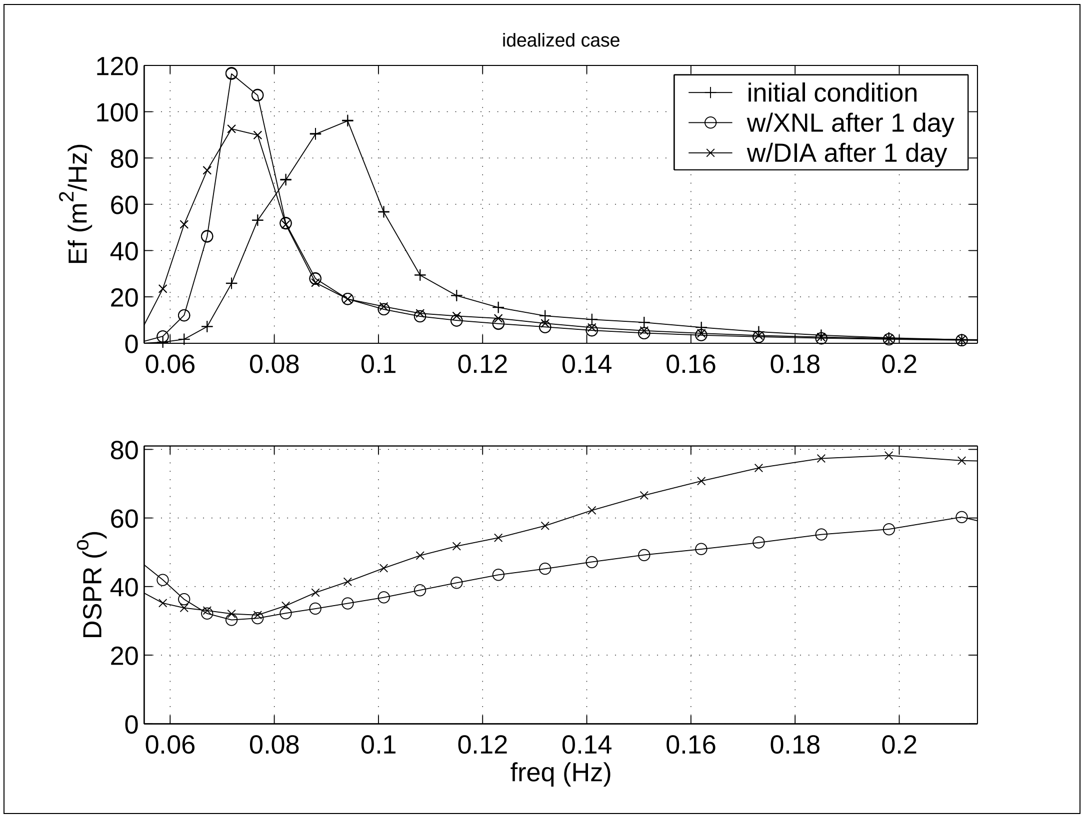

Fig. 3. Non-directional spectral density distributions for the three idealized simulations and circular RMS spreading for the second and third idealized simulations.

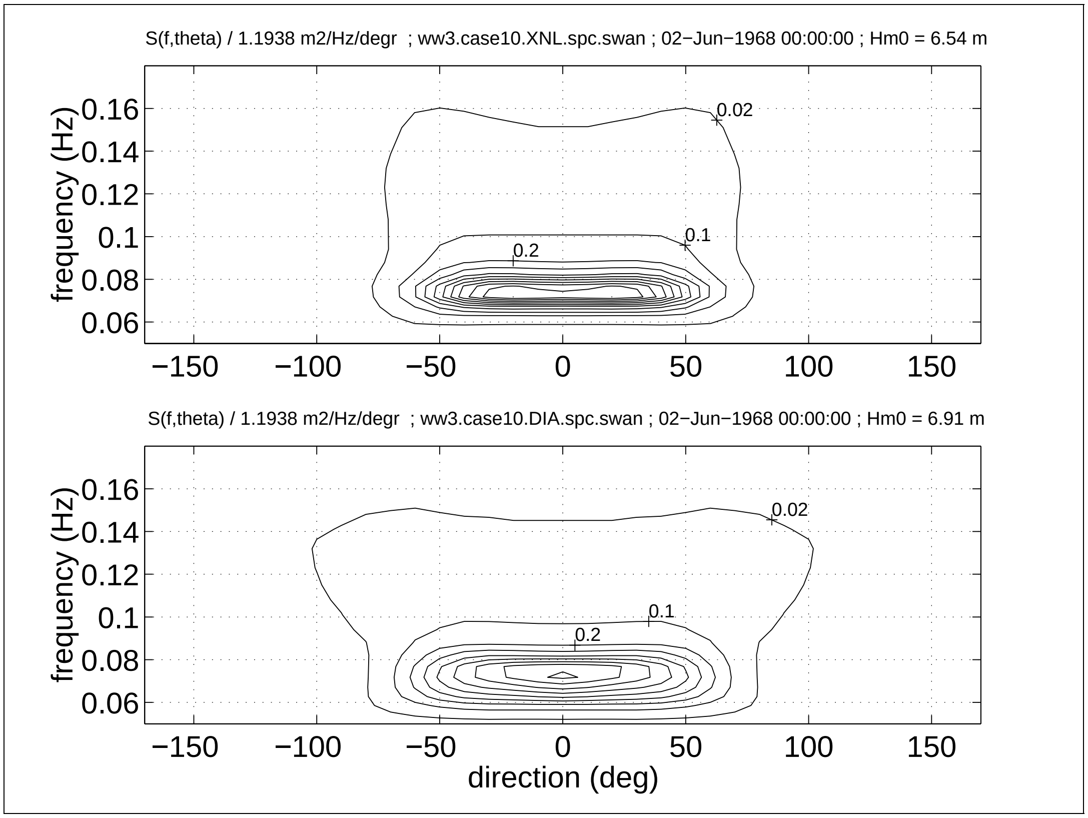

Fig. 4. Directional spectral density distributions for the second and third idealized simulations. The upper panel shows the XNL result (the second simulation) and the lower panel shows the DIA results (third simulation).

5. A model validation

5.1 Simulation description

The grid domain is shown in Fig. 1. The following settings/features were identical to that of Rogers (2003).

- Cartesian coordinates were used, with grid spacing of 2 km.
- The lake bathymetry is provided by the NOAA/Great Lakes Environmental Research Laboratory.
- The directional resolution is 10°. The frequency grid is logarithmic, with 34 frequencies from 0.07 Hz to 1.0 Hz.
- The wind field is created using wind observations from the two open water buoys in Lake Michigan (45002 and 45007), adjusted to 10 m elevation, with linear interpolation in the latitudes (y), and no variation in longitude (x).
- Default parameterizations for S_{in}, S_{ds}, S_{nl4} are used, except that the power on the relative wavenumber (denoted n in Rogers et al. 2003) is set to 2.0. [The default parameterizations in SWAN are that of WAM, Cycle 3, sometimes referred to in the literature as "WAM3 physics".]

The following settings/features are different from that of Rogers (2003).

- Season hindcast: 0000 UTC Sept. 1 – 0500 UTC Nov. 14 2002.
- Since Sept. 1 2002 was relatively calm, only a very short "ramp" time was needed (six hours), so the comparisons to data start at 0600 UTC Sept. 1.
- A time step of 6 minutes is used.

The physical parameterizations used are not tuned for this simulation or for this area; rather they are the same as what are used in SWAN forecasting systems run at NRL for other areas.

5.2 Simulation results

The primary focus of the study is the accuracy of the predictions of directional spreading in the hindcast. However, it only makes sense to investigate the accuracy of directional spreading if the non-directional spectra and mean direction are well predicted. Thus, we first present results other than directional spreading, before making the comparisons of directional spreading.

5.2.1 Results other than directional spreading

To provide a sense of the length of the simulation and how many events are being verified, a time series of zero moment wave height H_{m0}, at location 45007 is shown in Fig.5. These wave heights are also compared to data in scatter-plot form, along with mean period and the mean-mean wave direction in Fig. 6. The wave height and mean period are for the frequency range of 0.07 to 0.4 Hz (essentially the entire spectrum). The mean-mean wave direction is the mean wave direction integrated over 0.8 f_p to 1.2f_p using (3), so it is a stable metric of the mean direction near the peak frequency. By the standards of a wave model which uses only wind forcing, the agreement is very good for all three metrics. The good prediction of wave height and mean period suggests that the non-directional wave spectra $E(f)$ are fairly well predicted. This provides confidence that the hindcast is suitable for detailed study of accuracy of prediction of directional spreading.

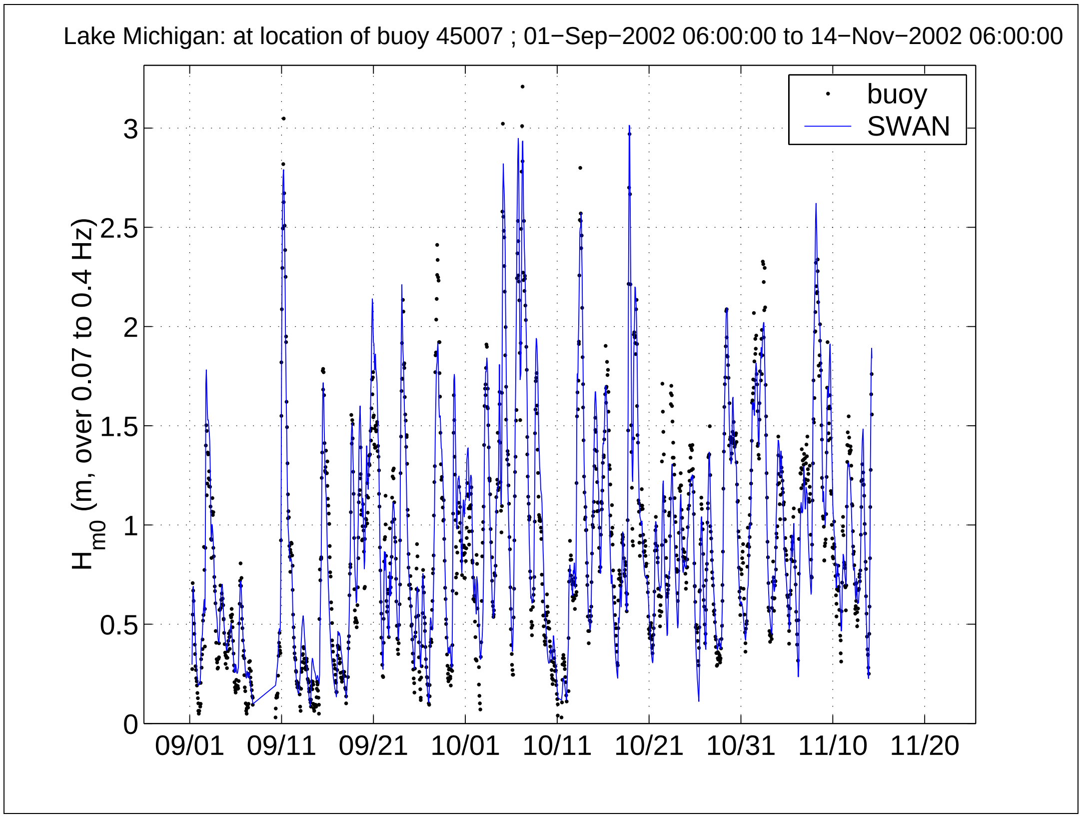

Fig. 5. Time series of zero moment wave height (hindcast vs. observation).

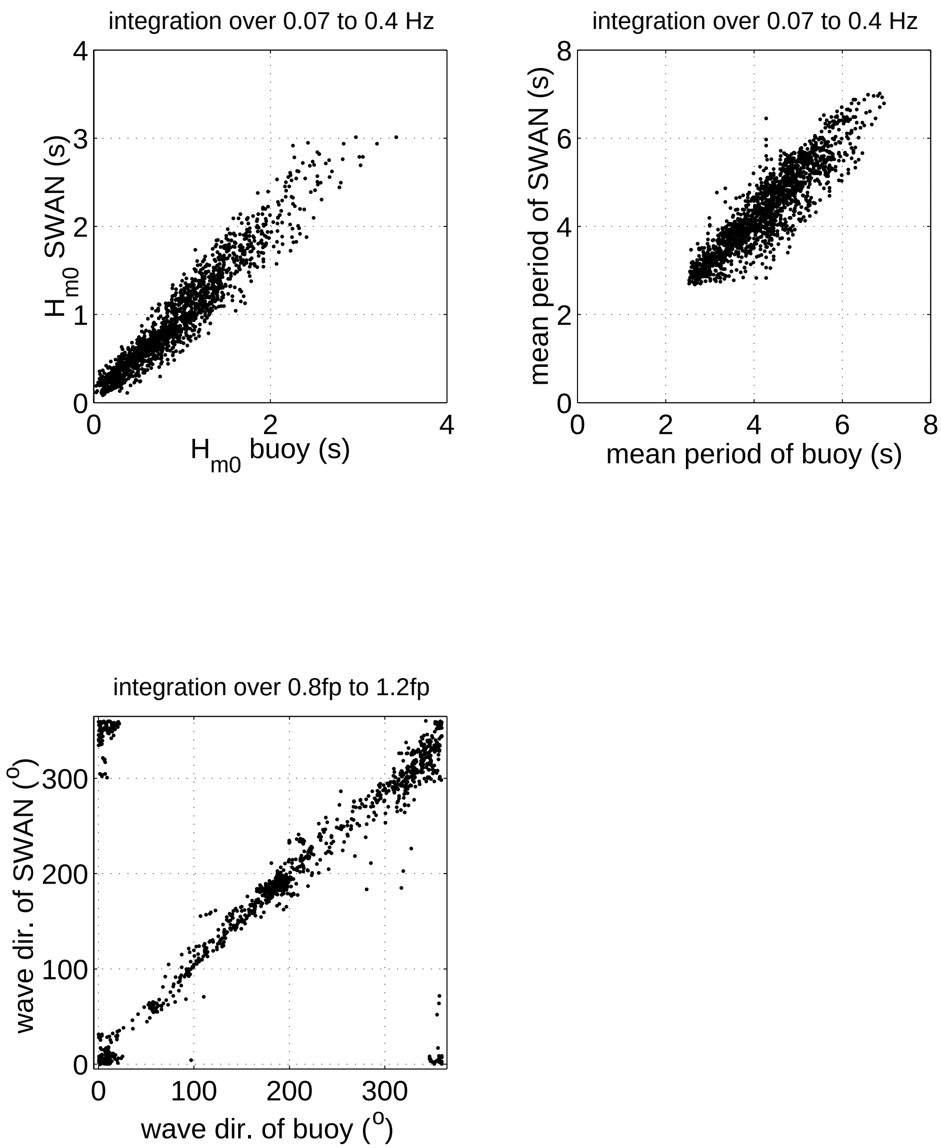

Fig. 6. Scatter plot comparison of wave height, mean period, and mean-mean wave direction for the hindcast.

5.2.2 Directional spreading

Scatter plot comparisons

The scatter plot comparisons of mean directional spreading $\bar{\theta}_\sigma$ are made in Figs. 7a,b. Fig. 7a is a simple scatter plot comparison of $\bar{\theta}_\sigma$. In Fig. 7b, the horizontal axis is the buoy partial wave height for the indicated frequency range, and the vertical axis is the misfit in mean directional spreading, $\bar{\theta}_{\sigma,hc} - \bar{\theta}_{\sigma,obs}$. The "partial wave height" is calculated from the variance (i.e. energy) of the wave spectrum over a frequency range defined by lower and upper bounds f_1 and f_2: $H_{m0,partial} = 4\sqrt{\upsilon_{partial}}$ and $\upsilon_{partial} = \int_{f_1}^{f_2} E(f)df$, the "partial variance". [The fictitious quantity $H_{m0,partial}$ is used rather than variance, since wave height has a more visceral quality.] There are fewer points in the highest frequency comparisons ($2f_p$ to $3f_p$) because the highest frequency in the buoy data is 0.35 Hz; thus often no data are available in this frequency range, depending on the value of f_p.

We make the following observations:

- <u>Low frequencies</u> ($0.6f_p$ to $0.8f_p$): SWAN underpredicts spreading and there is much scatter. However, in this case, the "ground truth" is relatively unreliable, because the buoy has difficulty measuring long, low waves.
- <u>Frequencies near peak</u>, ($0.8f_p$ to $1.2f_p$). Random error is smaller, but still not as good as it is for the other metrics (wave height, etc.). There is not a discernable systematic error. The agreement is especially good agreement for moderate and large waveheights. Note that the buoy data are more reliable for these moderate and large wave heights.
- <u>Frequencies above the peak</u>, ($1.2f_p$ to $2f_p$). SWAN does not do a very good job of following the observations (predicted spreading varies much less than the observed spreading), but error tends to be low, and there is no significant systematic error.
- <u>Highest frequencies</u>, ($2f_p$ to $3f_p$). Like the prior frequency range, SWAN does not do a very good job of following the observations: predicted spreading is consistently close to 40°. However, again the error tends to be low, since the observations, though they show more variation, are also clustered near 40°.

Time-averaged comparisons

To perform time-averaging, hindcast and observed directional spreading is calculated over smaller frequency bins of $0.1f_p$ (so the bins are $0.5f_p$, $0.6f_p$,…$2.7f_p$, $2.8f_p$). To enhance stability, the integration to calculate $\bar{\theta}_\sigma$ is performed over a $\pm 0.1 f_p$ range, so points are used more than once, similar to a moving average comparison. A simple time-averaging is used (i.e. the values are not weighted). The resulting distributions are shown in Fig. 8, along with the empirical, parametric model of Donelan et al. (1985), extended by Banner (1990) (see also Young 1999, equation (5.66)).

At the lower frequencies, directional spreading of the buoy is higher than of both the parametric model and the numerical model. At the highest frequencies, the directional spreading of the parametric model is higher than that of the buoy and the numerical model.

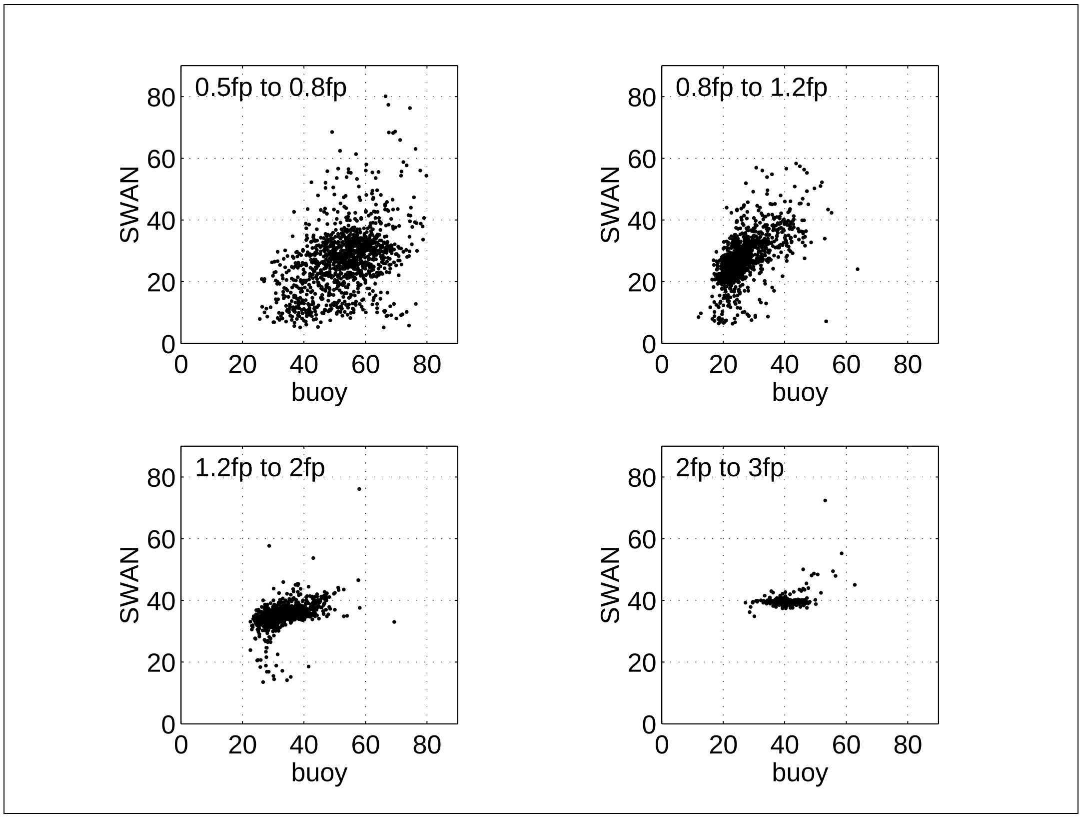

Fig. 7a. Scatter plot comparisons (hindcast vs. observation) of mean directional spreading over four frequency ranges.

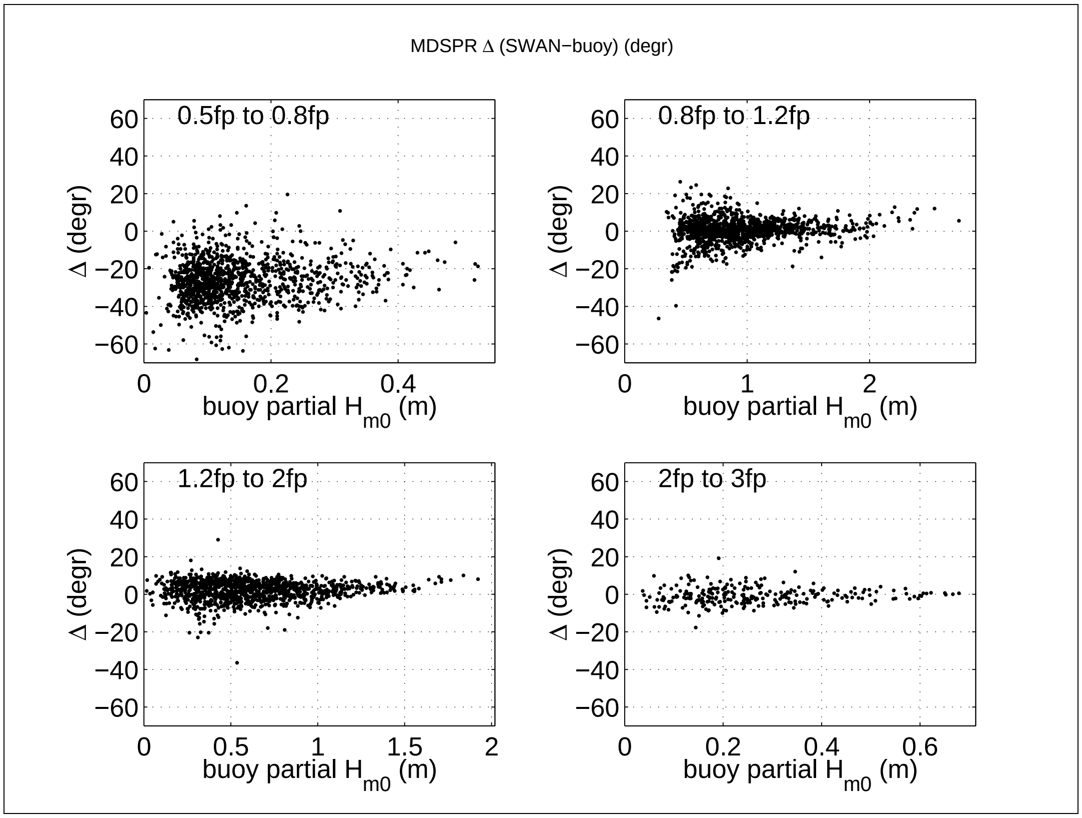

Fig. 7b. Scatter plot comparisons (hindcast vs. observation) for four frequency ranges. In this figure the horizontal axis is the buoy partial wave height for the indicated frequency range, and the vertical axis is the misfit in mean directional spreading (hindcast minus observed).

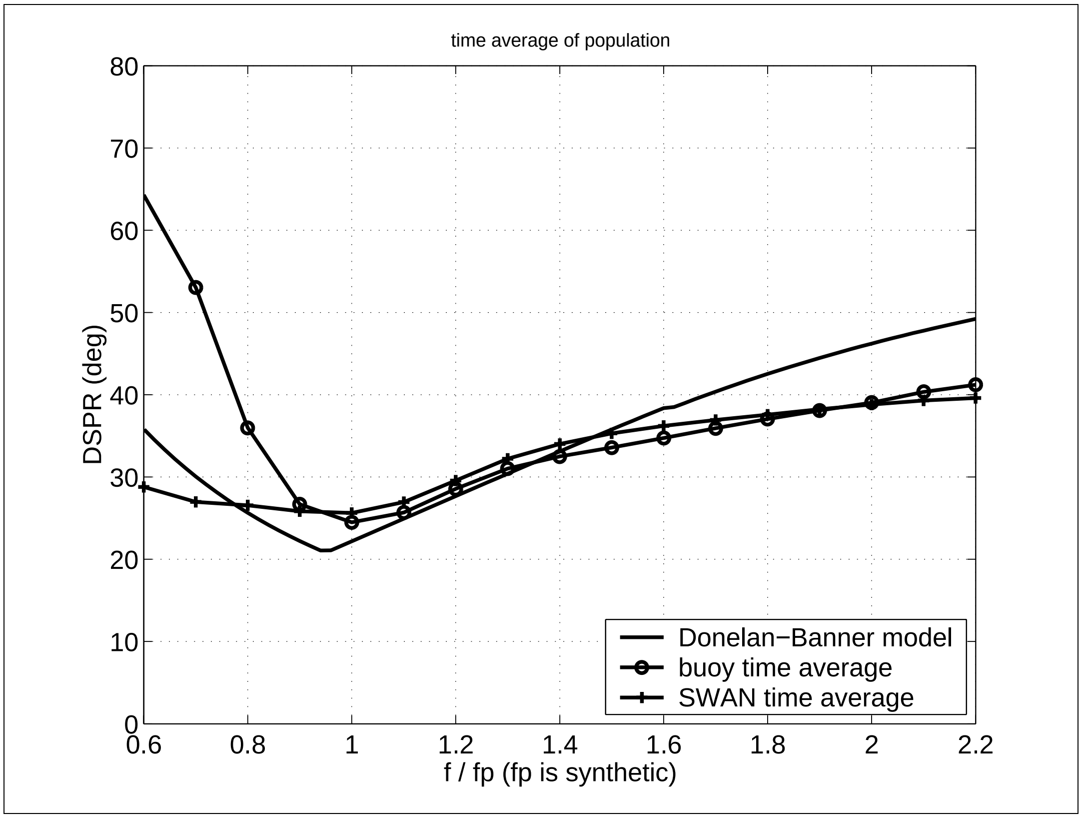

Fig. 8. Comparison of time-averaged results (model and observation) with the parametric model of Donelan et al. (1985) and Banner (1990).

<u>6. Summary of results regarding directional spreading</u>

In this section we consider the results of the hindcast relative to the idealized simulations and conventional wisdom. For the idealized simulations, a model with exact calculations of nonlinear interactions is taken as “ground truth” and for the hindcast, buoy data are taken as “ground truth”.

Conventional wisdom

Based on our review of the literature, we feel that the there is a belief (or perhaps just a suspicion) within the wave modeling community that inaccuracy associated with the DIA approximation of nonlinear interactions used in third generation wave models (such as SWAN) lead to an overprediction of directional spreading by these models (e.g. Forristall and Greenwood 1998; Cardone and Resio 1998). Though this belief (or suspicion) is by no means universal and may be nuanced in many cases, for the sake of discussion we will refer to this as the “conventional wisdom”. There is little or no information in the literature regarding the frequency-variation of the asserted bias, so we will presume here (again for the sake of discussion) that the conventional wisdom is the same for any frequency.

The comparison

Low frequencies: Conventional wisdom expects the model to be too broad, but in both the idealized case and the long hindcast, the model directional spreading is narrow relative to the ground truth.

Near the peak: Conventional wisdom expects the model to be too broad, but in both the idealized case and the long hindcast, the model directional spreading is quite close to the ground truth.

High frequencies: Conventional wisdom expects the model to be too broad. The idealized simulation supports this, but in the long hindcast, the model directional spreading is quite close to the ground truth.

7. Discussion

Accuracy of mean direction in turning winds

The response of a third generation (3G) wave model to rapidly turning winds is a concern. We do not specifically address this problem here (comparisons of mean wave directions show rather good accuracy overall), but we do not mean to imply that it is not an area in which the models may bear significant improvement.

The challenge of mixed seas and swells

In the case of mixes seas and swells, the challenge of directional validation is much greater. In this case, the peak frequency is rather useless. Thus, the type of comparison made here may not be made in the more general case of mixed sea state. Further, frequency-wise integration introduces the danger of mixing multiple components (e.g. seas and swells). A frequency integrated metric (e.g. mean direction, or directional spreading) in a mixed sea/swell scenario is meaningless and misleading. If different components are sufficiently separated, then the binning procedure addresses this problem, but obviously they may not always be sufficiently separated.

It is possible to identify specific sea/swell components in observations and compare them individually (e.g. Beal 1991) to data. Unfortunately, in the case of buoy observations, this requires application of a statistical model such as the Maximum Likelihood Method (MLM). Further, it is not uncommon to have a swell system that exists in observations but not in model spectra, or vice versa. In this case, validation of directional spreading is obviously not possible.

Based on our experiences, we do not expect that a validation such as was performed here would be feasible for an exposed coastline, with frequent mixed sea/swell conditions. In such a case, some compromise is probably necessary. By way of summary, two compromises are to either:

1. Consider a shorter time period, so that qualitative comparisons can be made, for example by graphing $\theta_\sigma = \theta_\sigma(f,t)$ and $E = E(f,t)$, or
2. Utilize a statistical model such as MLM.

The impact of the nonlinear solver in a "live" simulation

Though we apply the WRT nonlinear solver in an idealized scenario, it would be possible to apply it in a shortened version of our Lake Michigan hindcast to specifically study the impact of the inaccuracy of the DIA. In fact, a study of this sort has been conducted recently (Ardhuin et al., manuscript in review), which suggests that DIA does lead to broader spectra in the higher frequencies (compared to a model with exact nonlinear computations). Further, that the authors of that study conclude that the wind input parameterization of WAM4 is narrower than the actual input.

8. Conclusions

In an enclosed basin such as Lake Michigan, it is demonstrated herein that it is possible to quantitatively validate directional characteristics (mean direction and directional spreading) of a long (i.e. more than one month) wave model hindcast. Further, buoy observations can be used in such a validation without applying a statistical model (such as the Maximum Likelihood Method) to the observations. Populations of model/observation pairs such as the scatter plot comparisons herein are readily condensed to statistics (such as root-mean-square error, bias, and standard deviation of error), so it is feasible to present directional validations for multiple locations within limited space (such as a journal article). Due to the considerable added complexity associated with mixed sea/swell conditions, it is probably not feasible to perform a validation in this manner on an exposed coastline.

In addition to the hindcast validation, a pair of idealized simulations are presented herein. The two simulations differ in their methods of calculating nonlinear interaction. Considering both the hindcast validation and the idealized simulations, we find the following about the “third generation” wave model which uses traditional, operational methods of approximating four-wave nonlinear interactions:

- At frequencies below the spectral peak, in both the idealized case and the long hindcast, model directional spreading is narrow relative to the ground truth.
- Near the peak frequency, in both the idealized case and the long hindcast, the model directional spreading is quite close to the ground truth.
- At frequencies above the spectral peak, the model in the idealized simulation is too broad relative to the ground truth, but in the long hindcast, the model directional spreading is quite close to the ground truth.

Summary of Notations

f	frequency, T^{-1}
σ	the relative (intrinsic) radial frequency, $2\pi T^{-1}$
θ	direction of wave propagation
$\theta_\sigma(f)$	rms (root mean square) circular spreading (note σ here is unrelated to frequency)
$\bar{\theta}_\sigma$	mean rms circular spreading. The word "mean" here refers to some integration over frequencies
E	spectral density, either two-dimensional spectrum $E(f,\theta)$ or one-dimensional spectrum $E(f)$
$D(\theta)$	dimensionless directional distribution at a particular frequency; integrates to unity
f_1 and f_2	lower and upper bounds of a frequency integration
$\theta_0(f)$	mean wave direction. Taken as the circular centroid of $D(\theta)$. Denoted $\alpha_1(f)$ in NDBC notation
$\theta_p(f)$	peak wave direction, the peak of $D(\theta)$. Generally not known, except in context of a model of some sort.
$\bar{\theta}_0$	mean/mean wave direction. The mean wave directions have been integrated across some frequency range.
$m_1(f)$	a parameter related to directional spreading. Denoted r_1 in NDBC notation.
a_1, b_1, a_2, b_2	Fourier coefficients

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SUPPLEMENTARY MATERIAL

THE FOLLOWING PAGES CONTAIN EXTRA PLOTS WHICH WILL NOT GO INTO THE PAPER. I AM KEEPING THEM HERE AS A REFERENCE, SINCE SOME WILL BE DESCRIBED "IN WORDS"

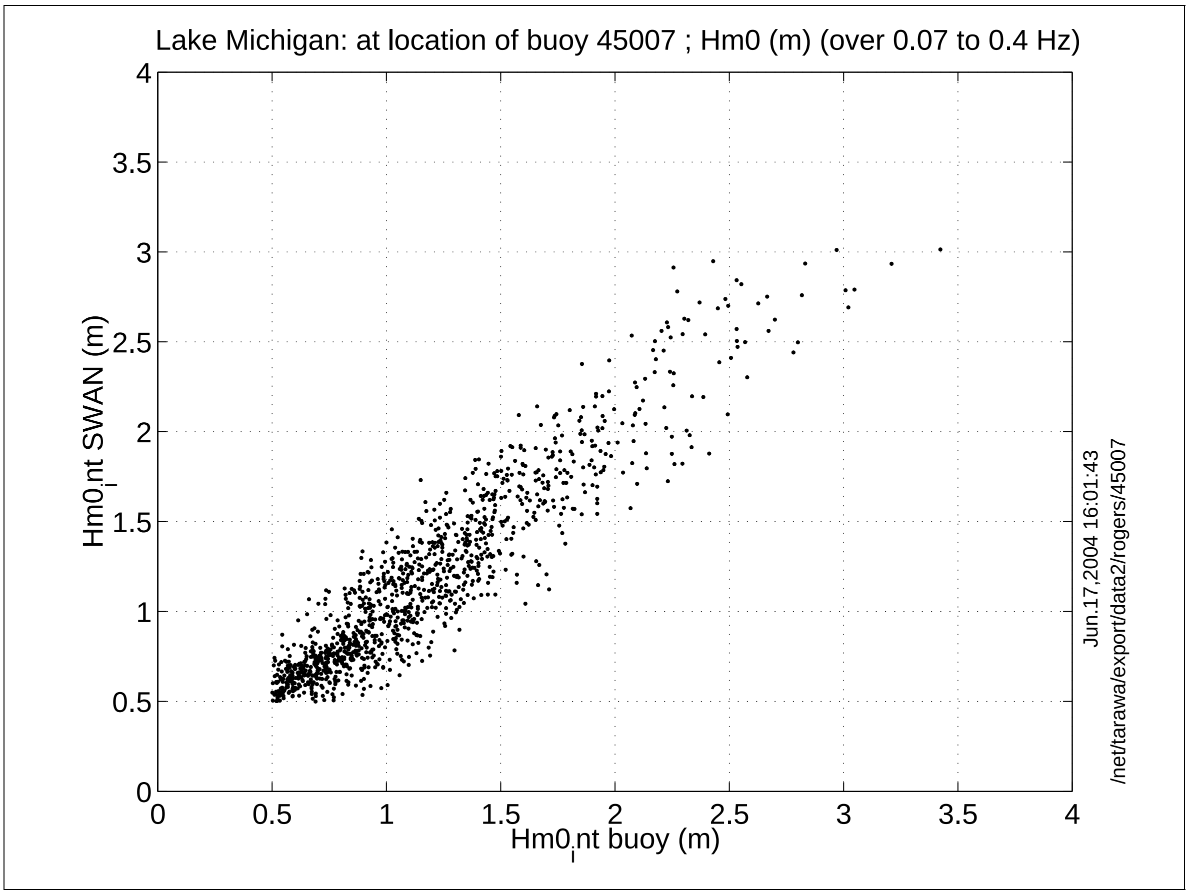

Figure.

These are the points actually included in the DSPR comparison (H>0.5m).

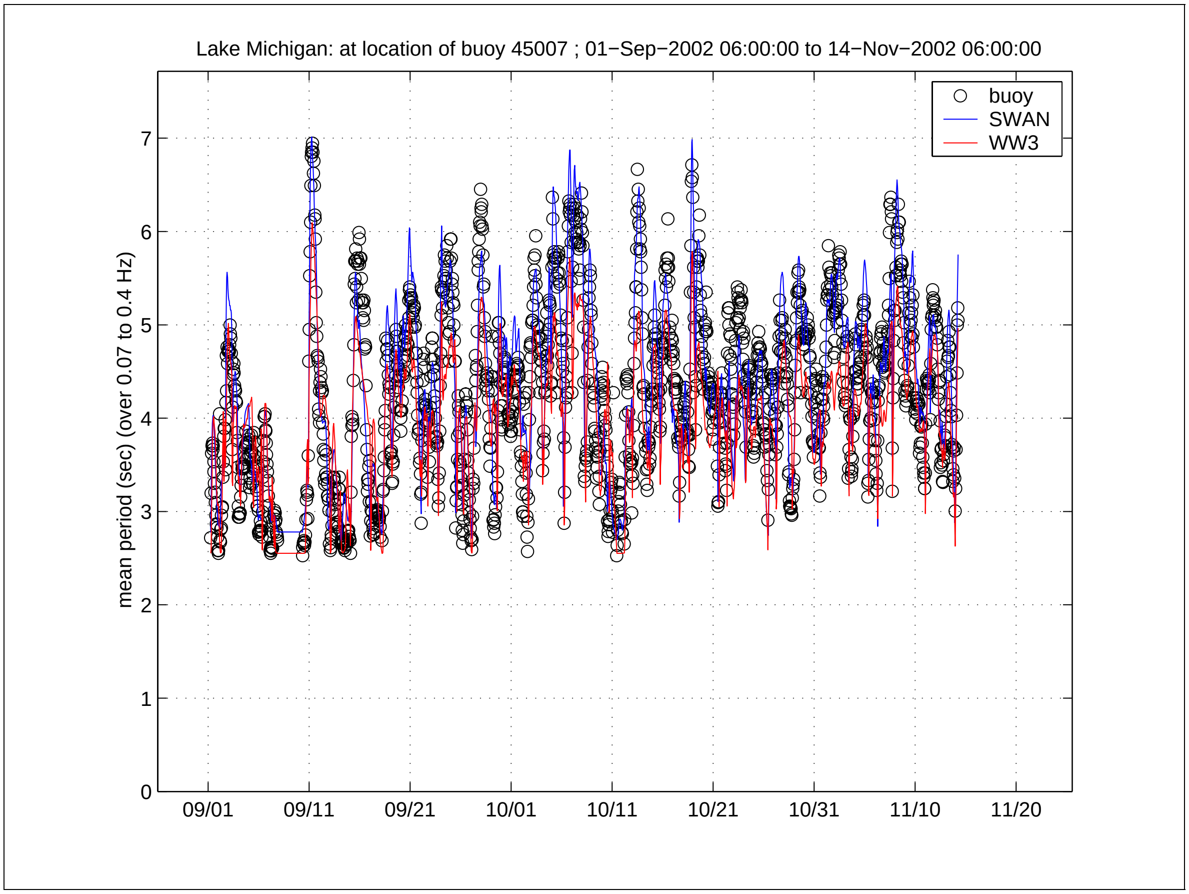

Figure.
Time series of mean period.

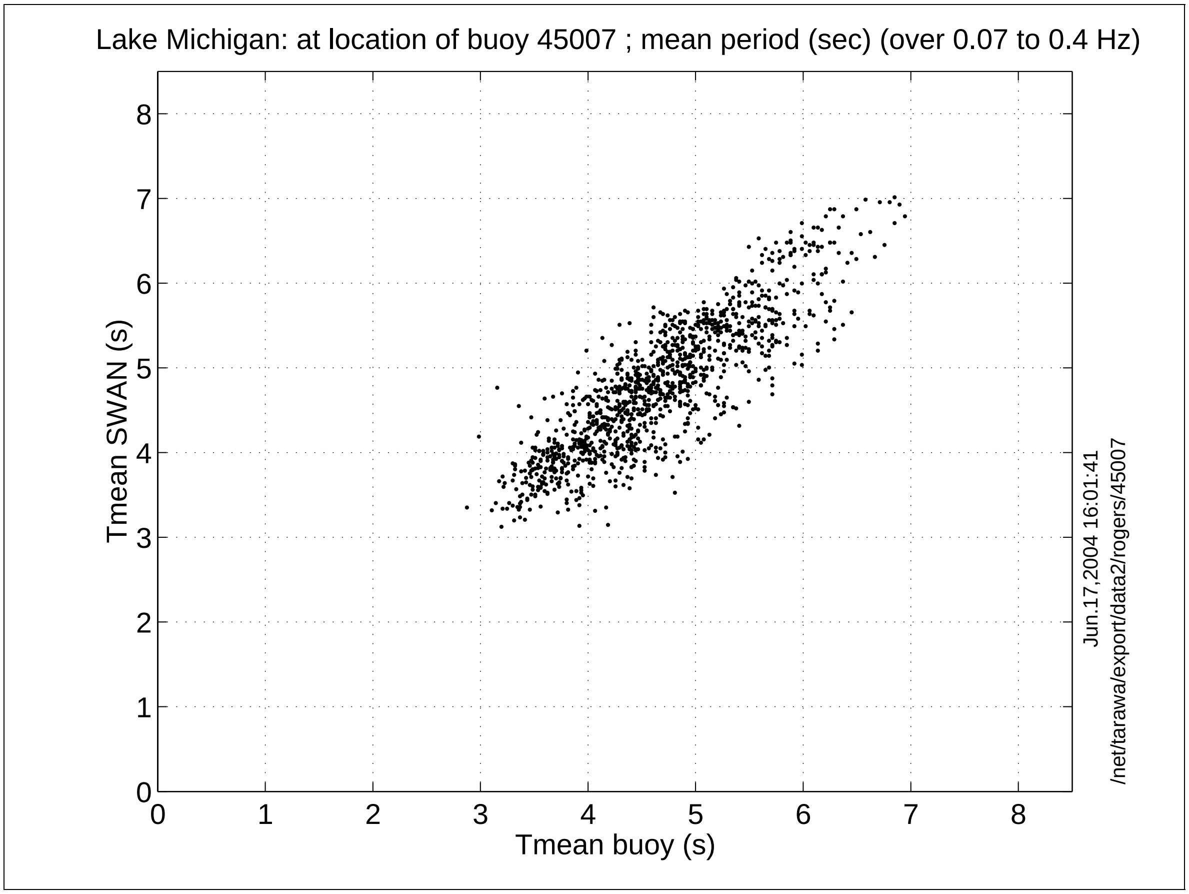

Figure.

These are the points actually included in the DSPR comparison (H>0.5m).

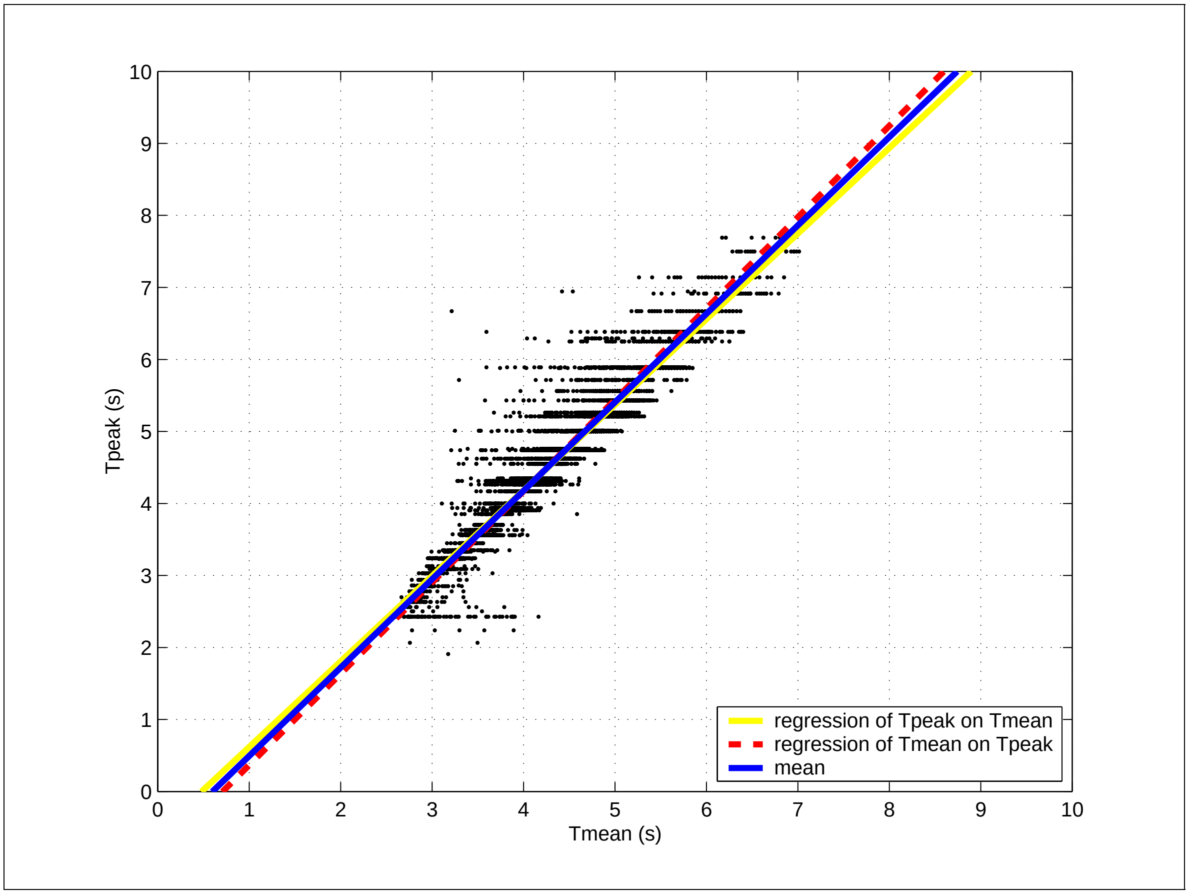

Figure.

This is to show how I create a “synthetic peak period” using mean period. It is more stable than the actual peak period.

Here, both buoy and SWAN values are used in the regression.

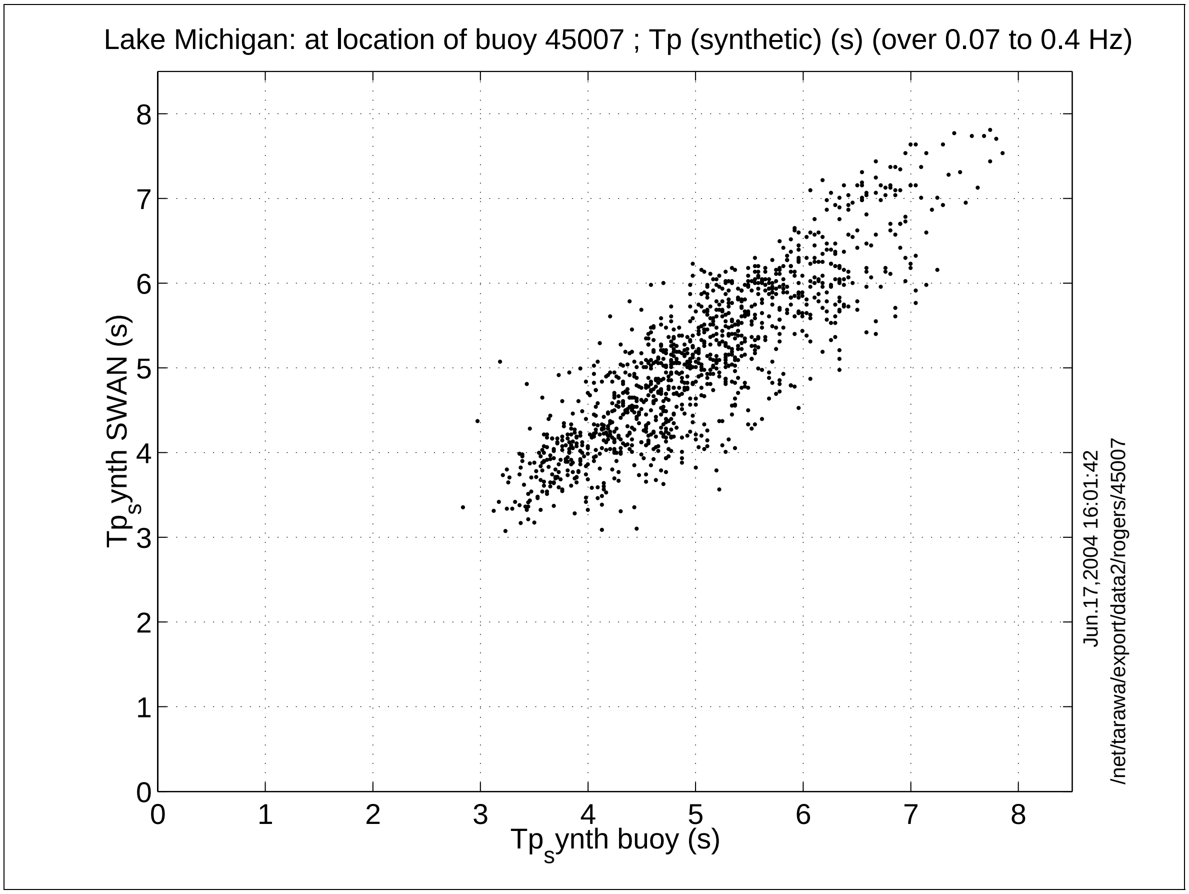

Figure.

This is the scatter plot for the synthetic peak period. It is redundant with the scatter plot of mean period.

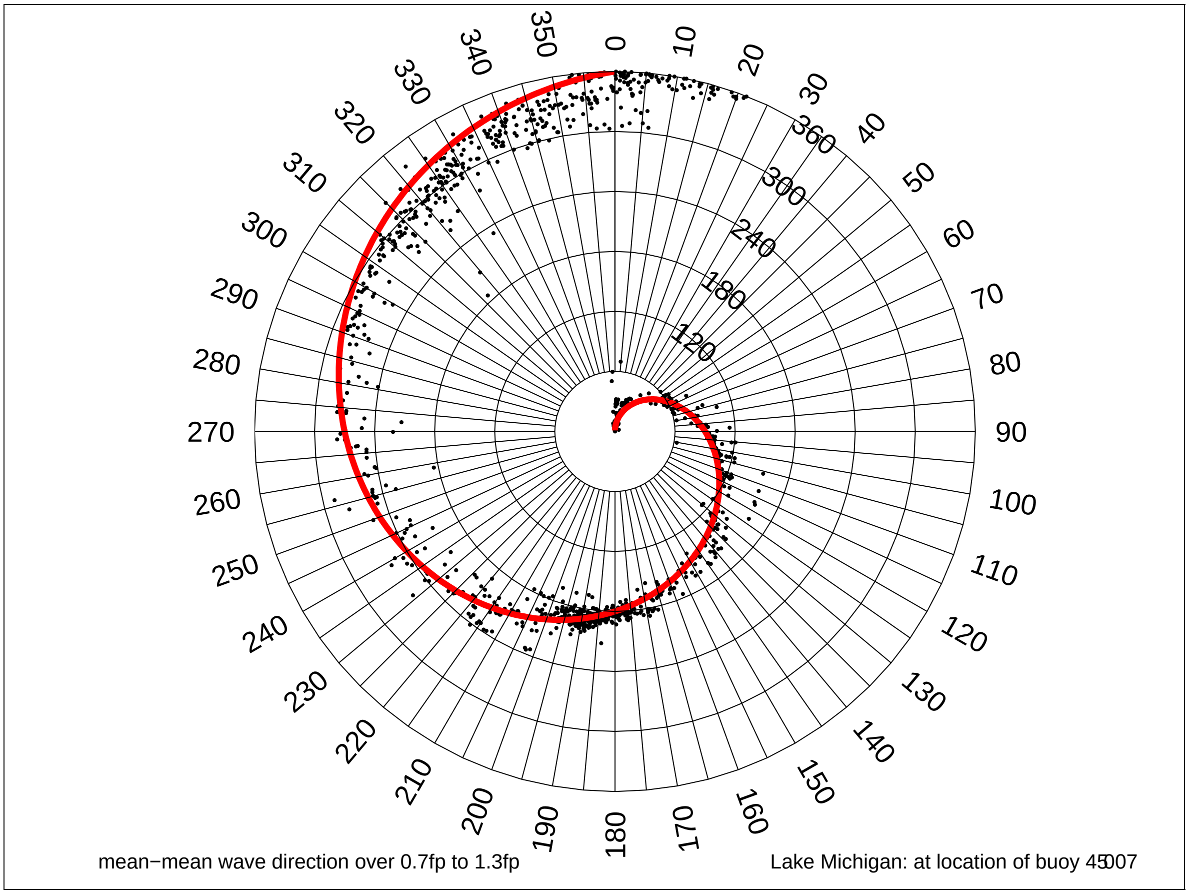

Figure. Mean-Mean direction validation of SWAN results (0.7fp to 1.3fp).

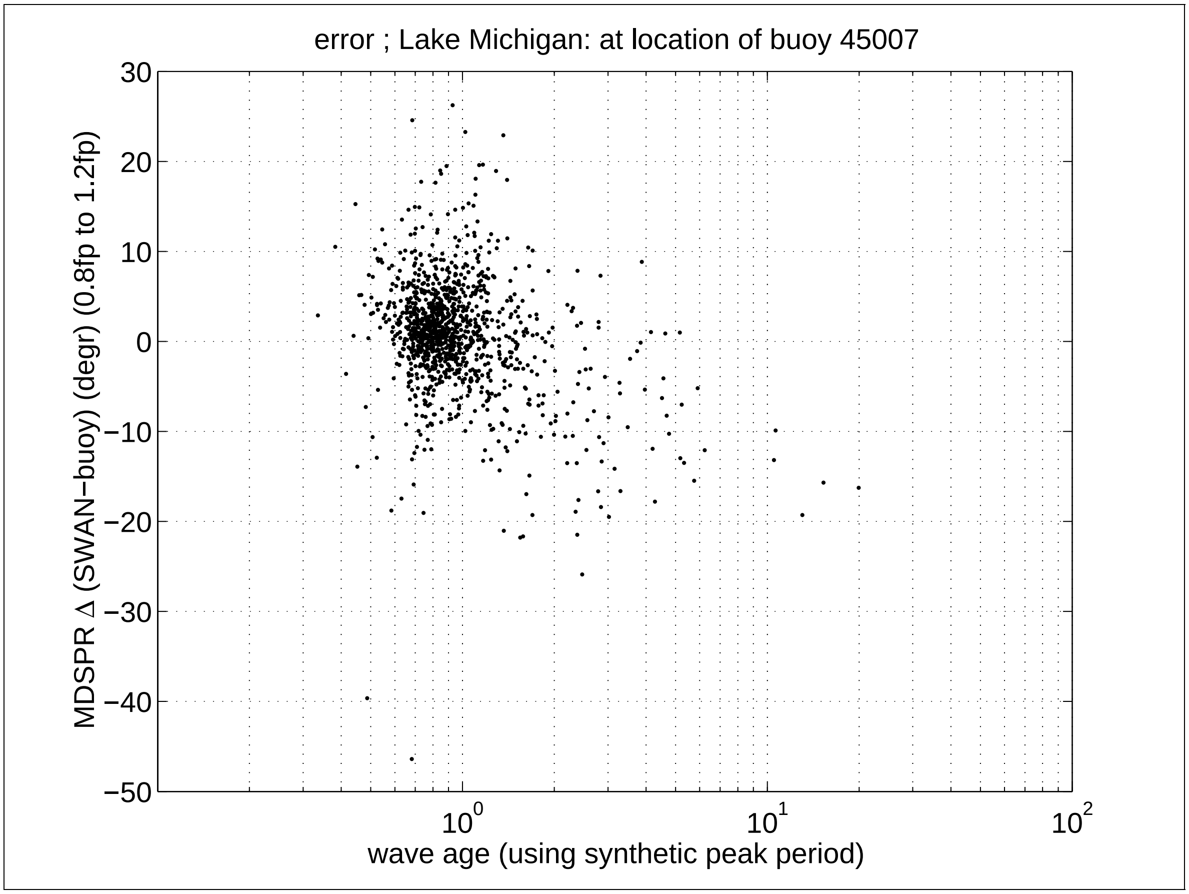

Figure.
Frequencies near the peak
error vs.wave age.

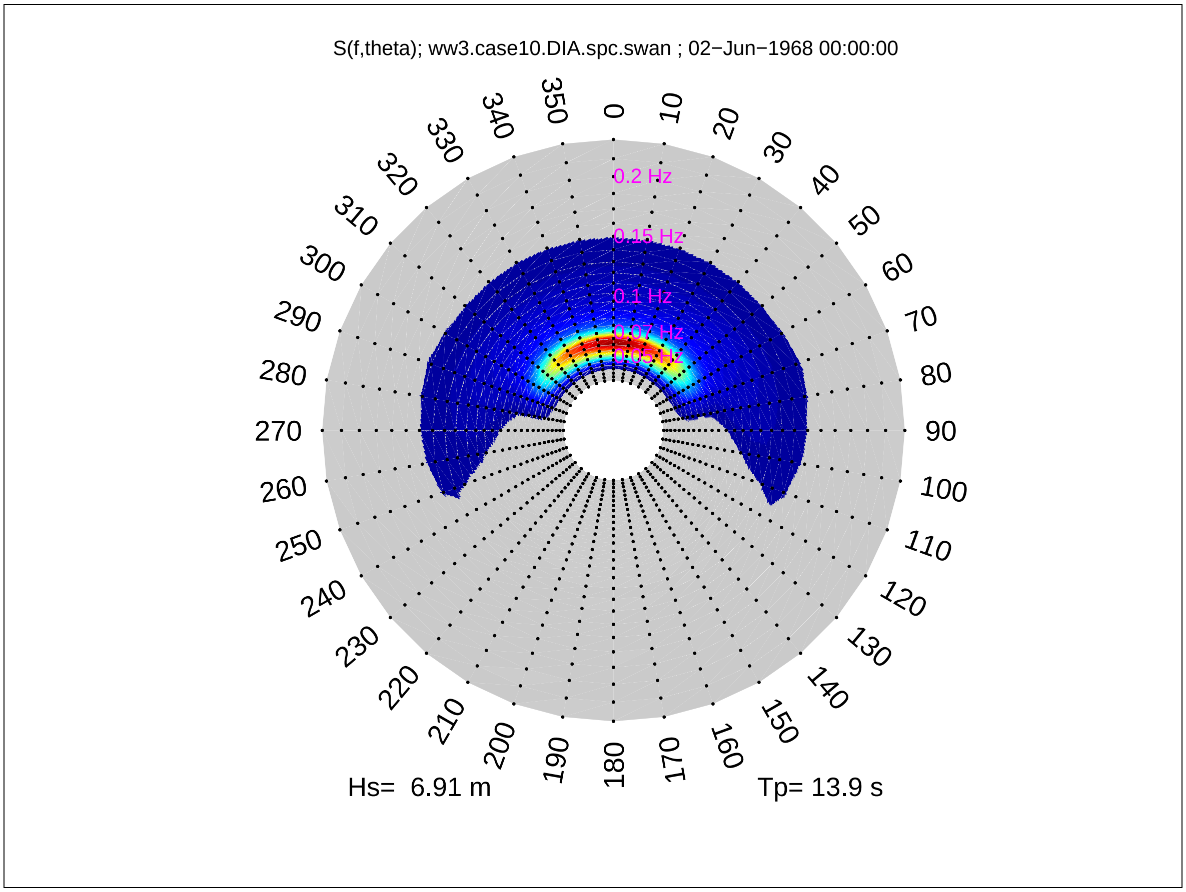

Figure.
Here, DIA result is shown.

Discussion (of this and following figure):
This is a test case that I created to demonstrate the effect of DIA on frequency downshifting. I just use it in this presentation because I had it handy. In this presentation, I am demonstrating the effect of DIA on directional spreading. Both are created using WW3. They are initialized with the same spectra, which is a steep equilibrium-type spectrum. In these two simulations, however, there is no wind forcing or dissipation. The duration of the simulation is 24 hours. These spectra correspond to the end of the simulations. To my eyes, these plots suggest that directional spreading is greater using DIA. We see plots like this in several journal articles. This has led to an expectation that 3GWAMS overpredict directional spreading. My concern is this: Has this expectation inappropriately influenced our interpretation of results? Are we too quick to blame DIA when spreading is overpredicted? Are we oversimplifying the situation?

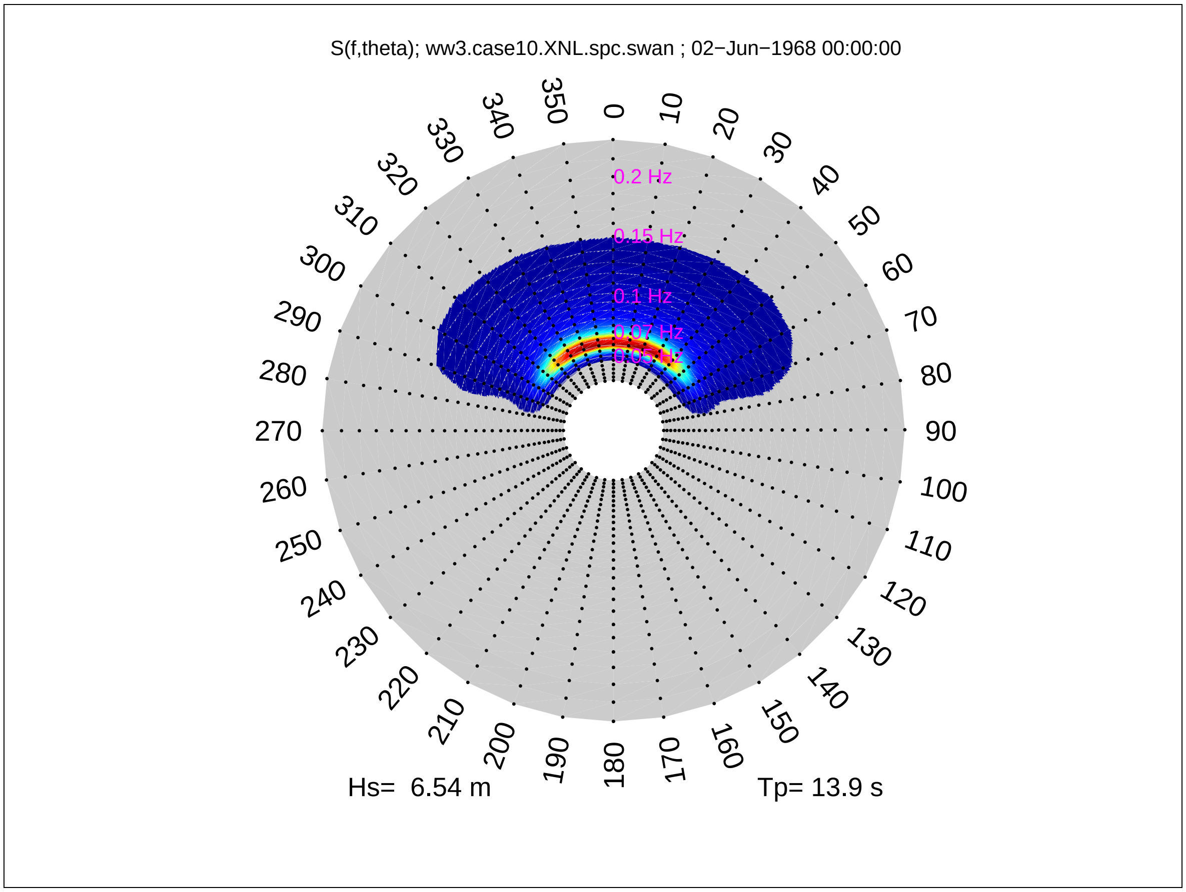

Figure.
XNL result

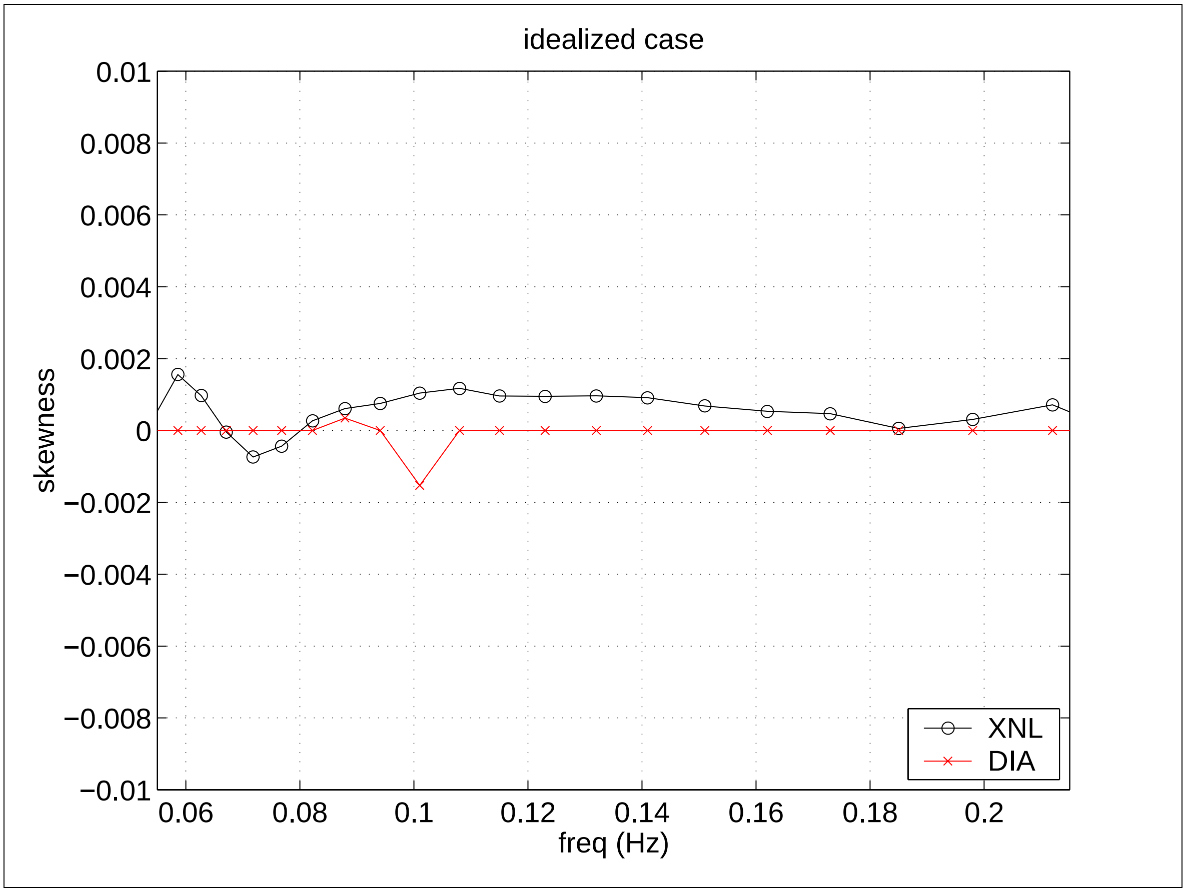

Figure.

Without a priori knowledge of what skewness is (in this context), I am surmising from this plot that it tends to be small when the spectrum is symmetrical.

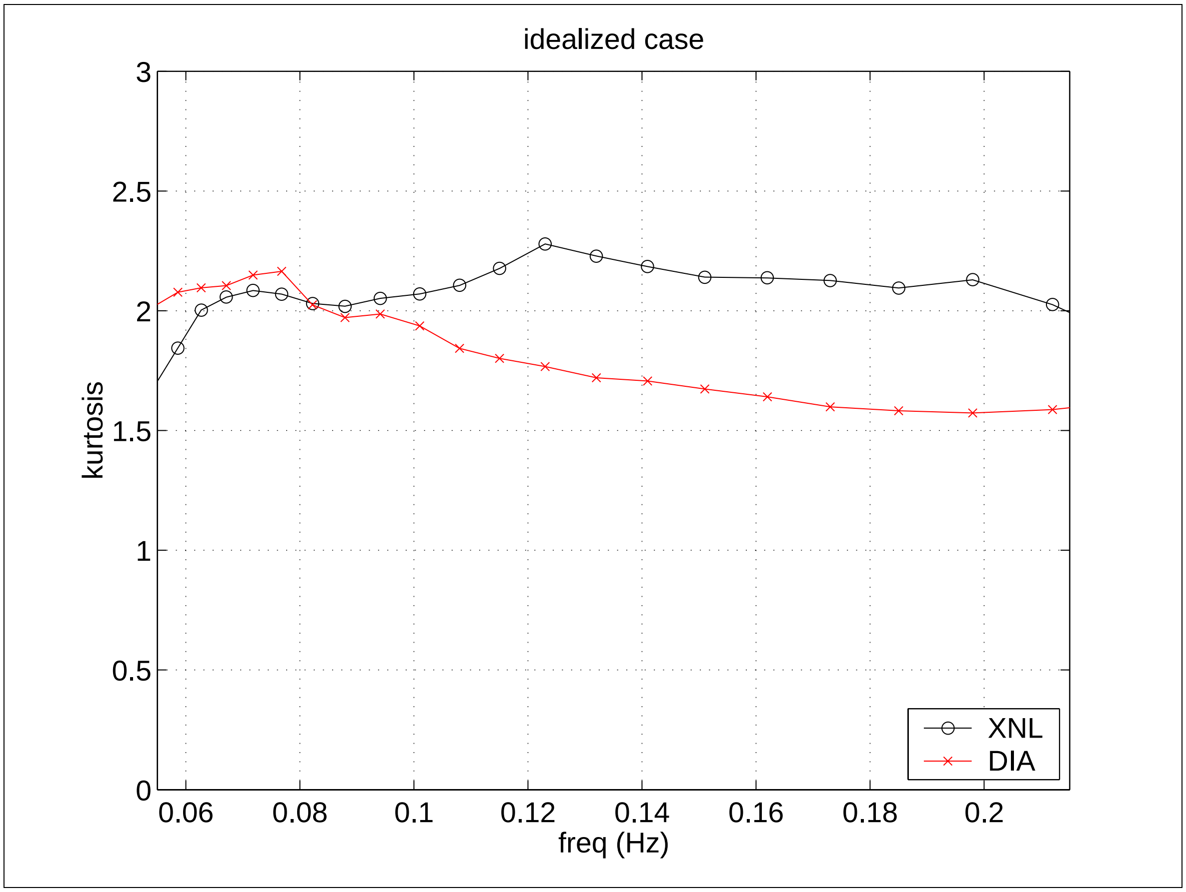

Figure.
Some deviation in kurtosis, but not any more than there is for directional spreading. This suggests that deviation (difference between XNL-based and DIA-based models) in 2nd order moments are comparable to deviations in first order moments.