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Uncertainty of the sea state parameters resulting from the methods of spectral estimation

G. Rodríguez[a,*], C. Guedes Soares[b], U. Machado[b]

[a]Department of Physics, University Las Palmas de Gran Canaria, Las Palmas, Spain
[b]Unit of Marine Technology and Engineering, Instituto Superior Técnico, Av. Rovisco Pais, 1096 Lisboa, Portugal

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Abstract

The uncertainty of some commonly used spectral wave parameters resulting from the spectral estimation procedure is assessed. It is observed that the methods of spectral estimation produce a significant uncertainty for all parameters examined, but this is of considerable importance only for the peak period, which is one of the most important parameters to model the wave climate. © 1999 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Since the spectral estimations obtained from measured wave records are random variables, the wave parameters obtained from spectra will also be statistical estimates with associated uncertainty and confidence limits. In particular, the confidence intervals for those parameters obtained by integration of spectrum are dependent on the spectral shape because they are a function of the effective degrees of freedom of the total spectrum (Donelan and Pierson, 1983; Medina et al., 1985; Young, 1986; Elgar, 1987).

On the other hand, the stability and accuracy of wave parameters obtained from

* Corresponding author.

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the spectral density function can be affected by several factors which, have a different nature than the intrinsic statistical variability of the spectral estimates, and thus may be considered as model uncertainties. Some of these are: (1) definitions used for parameter estimation, (2) numerical procedures for integration, (3) high frequency cut-off selection, (4) frequency resolution, or the degrees of freedom, of the spectral estimations, and (5) the wave record length.

A number of studies have been undertaken to determine the effect of these factors on the reliability and stability of some commonly used spectral wave parameters.

Rye (1977) has studied the variability of the derived sea state parameters, in particular as a function of the cut-off frequency in the integration of the spectral density function. He found that, while the significant wave height was very stable, mean periods were too sensitive. Chakrabarti and Cooley (1977) analysed the importance of the choice of degrees of freedom, or the frequency resolution, of the calculated spectrum as well as the cut-off frequency selection. They observed that spectral parameters depending on spectral moments of order higher than zero, such as average periods, show considerable variations with different values of the high frequency cut-off and of the spectral resolution. However, the significant wave height remained reasonably stable when varying these parameters.

Peña (1983) has also studied the effect of the frequency cut-off and the spectral resolution when the wave spectrum is estimated by means of the maximum entropy method, by considering the spectral resolution in terms of the length of the autoregressive and moving average filter adopted to model numerically simulated wave records. He concluded that the use of this spectral estimation methodology does not improve the wave parameter stability as the frequency cut-off changes but, in general, it produces a large degree of stability as a function of spectral resolution. Similar results have been obtained by Rodríguez et al. (1992) by analysing measured wave records to examine the variability of various spectral bandwidth and nonlinearity wave parameters. Also Gomes and Guedes Soares (1997) have confirmed that using the maximum entropy method yield a more stable spectral density function.

Arhan (1979) and Cavanié (1979) have studied the stability of wave parameters as a function of the length of the record analysed and gave indication of the uncertainty involved. Mansard and Funke (1986) have also analysed the effect of the wave records length, noting that doubling the record length has no appreciable affect on the peak frequency.

Mansard and Funke (1986) considered the effect of the definition, or the algorithm, used to estimate the peak frequency. This study has been extended by Young (1995), who applied a Monte-Carlo simulation procedure to obtain confidence limits for the different estimates of the peak frequency.

Another factor that influences the accuracy of the spectral wave parameters is the numerical procedure used to integrate the spectral density function. Appropriate results have been obtained by using a three-point Simpson's method (Nath and Yeh, 1987) and the Romberg method (Rodríguez, 1995).

The present work considers one aspect not WITH dealt earlier, namely the uncertainty associated with the spectral parameters, as a result of the method of estimating the spectral density function, $S(f)$, of wave records. Some parameters currently used have been selected and their definition follows the recommendation of the working group on wave generation and analysis of the International Association for Hydraulic Research (IAHR, 1989).

For this study, 20 wave records measured in coastal deep waters of Figueira da Foz in Portugal were used. The spectral density for these time series was estimated by different numerical approaches, maintaining fixed the above-referred factors (1–4). A Romberg method was used to estimate the spectral integrals. This method gives excellent results, as reported by Rodríguez (1995). The low and high frequency cut-off are fixed as $f_{\mathrm{low}} = 0.03$ (Hz) and $f_{\mathrm{high}} = 0.5$ (Hz). The frequency resolution is fixed by using the same number of degrees of freedom, ν, for the spectral estimations. The effect of parameter definition is eliminated by using the referred conditions in all cases. Thus, the variability of these parameters should only be affected by the spectral computational methodology.

2. Spectral wave parameters

The wave parameters selected to examine the effect of the spectral estimation procedures on their stability are: the *significant wave height* which, assuming a Rayleigh distribution for wave heights, is given by

$$H_{mo} = 4\sqrt{m_o} \tag{1}$$

The *peak period*, T_p, defined as the inverse of the frequency associated to the maximum of the wave energy spectrum, or *peak frequency*, f_p. That is,

$$S(f_p) = \frac{\mathrm{d}S(F)}{\mathrm{d}f} = 0 \quad T_{\mathrm{p}} = \frac{1}{f_p} \tag{2}$$

The *Delft peak period*, defined as the inverse of the peak frequency estimated by the Delft method. That is, the centroid of the spectral band bounded by the frequencies corresponding to those values of $S(f) = 0.8\,S(f_{\mathrm{p}})$ at both sides of f_{p}. It is named *Delft peak frequency* and is expressed as

$$f_{p_D} = \frac{\int_{f_1}^{f_2} f\,S(f)\mathrm{d}f}{\int_{f_1}^{f_2} S(f)\mathrm{d}f} \quad T_{p_D} = \frac{1}{f_{p_D}} \tag{3}$$

The mean period, T_{01}, the average zero up-crossing period, T_{02}, and the average crests period, T_{24}, defined respectively as,

$$T_{01} = \frac{m_0}{m_1} \quad T_{02} = \left(\frac{m_0}{m_2}\right)^{\frac{1}{2}} \quad T_{24} = \left(\frac{m_2}{m_4}\right)^{\frac{1}{2}} \tag{4}$$

where m_n represents the *nth order spectral moment*, given by

$$m_n = \int_0^\infty f^n S(f) \mathrm{d}f \quad n = 0, 1, 2, 4$$

3. Methods of spectral estimation

The objective of this study is to present evidence of the uncertainty of some spectral parameters as a result of the method used to compute the wave spectrum, rather than a comparative evaluation of various methods of spectral estimation. Hence, only the relevant attributes of the different numerical approaches used to estimate wave spectra are outlined. Specifically, the methods used are two classical, or non-parametric, and one parametric method. They are the indirect non-parametric methods or covariance method, usually named as the Blackman–Tukey (BT) method and the direct Fourier transform, often named as the Fast Fourier Transform (FFT) approach and the maximum entropy (ME) method, as a parametric procedure.

3.1. The Blackman–Tukey method

Consider a time series, $x(t)$, with N data measured with a sampling period Δt during a time interval T. Blackman and Tukey (1958) proposed to estimate the spectral density function of a random time series by using the Wiener–Kintchine theorem. That is, by computing the Fourier transform of the estimated autocovariance function. The estimated spectrum is given by

$$\hat{S}_{\mathrm{BT}}(f) = 4\int_0^\infty \hat{C}(\tau)\cos(2\pi f\tau)\mathrm{d}\tau$$

where $\hat{C}(\tau)$ is an estimate of the autocovariance function and τ is the lag between time series values. Unfortunately, the spectral density function estimated by this procedure cannot be considered as a true estimate of the spectral density, due to the finite length of the time series and, as a consequence, the finite value in the upper limit of the above integral. This fact makes the maximum lag, $\tau_{\max}$, the critical parameter controlling the resolution bandwidth and the variance of the spectral estimations. It can be shown that the raw spectral estimations $\hat{S}_{\mathrm{BT}}$ follow a chi-square distribution with two degrees of freedom (see, e.g. Priestley, 1981). Furthermore, added inaccuracies are caused by considering a zero value of $\hat{C}(\tau)$ for lags greater than $\tau_{\max}$. Some of these drawbacks may be partially alleviated by introducing a time lag window $\lambda(\tau)$ (or its Fourier transform, spectral window), which modifies the autocovariance function values in such a way that they are smoothly decreased to zero as τ increases. Thus, the BT estimator can be rewritten as

$$\tilde{S}_{\mathrm{BT}}(F) = 4\int_0^{\infty} \lambda(\tau)\hat{C}(\tau)\cos(2\pi f\tau)\mathrm{d}\tau \tag{5}$$

Numerous expressions for $\lambda(\tau)$ have been proposed. However, there is not a criterion to select the adequate window for each case. In this work, three well-known spectral windows have been used. These are the Parzen, Hamming and Hanning windows. Details on these and other windows can be found in many textbooks (e.g. Priestley, 1981). Depending on the applied spectral window, the BT method is denoted as BT1, BT2, and BT3, for the Parzen, Hamming and Hanning windows, respectively.

3.2. The direct Fourier transform method

It is possible to estimate the spectral density function by a direct Fourier transformation of the observed time series. By definition, the power spectral function is

$$S(f) = \lim_{T\to\infty} \frac{2}{T}|X(f)|^2$$

where $X(f)$ is the Fourier transform of the observed sequence, that is

$$X(f) = \int_{-\infty}^{\infty} x(t)\exp(-i2\pi ft)\mathrm{d}t$$

which can be efficiently estimated by means of the Fast Fourier Transform (FFT) algorithm. A natural estimator of the spectral density function is

$$\hat{S}_{\mathrm{FFT}}(f) = \frac{2\Delta t}{N}\left|\sum_{t=1}^{N} x(t)\exp(-i2\pi ft)\right|^2$$

This estimator is known as the periodogram. It can be shown (see, e.g. Priestley, 1981) that the spectral estimations obtained with this procedure have a large variance and follow a chi-square distribution with two degrees of freedom (dof). That is, the periodogram is not a consistent estimate of the spectral density function. Thus, it becomes necessary to apply some smoothing technique to reduce the variance of these raw estimates. The most popular procedures used to improve the variance properties of the periodogram are outlined below.

3.2.1. Frequency averaging

Daniell (1946), suggested to smooth the spectrum by averaging adjacent raw spectral estimations over a band of $2m + 1$ frequencies, to reduce the variability of the periodogram estimates. The smoothed estimates are computed as

$$\tilde{S}_{\mathrm{FFTD}}(f_k) = \frac{1}{2m + 1} \sum_{j=-m}^{m} \hat{S}(f_{k+j}) \tag{6}$$

Thus, the smoothed estimations have a chi-square distribution with $2(2m + 1)$ degrees of freedom. This approach is often known as Daniell's method and will be abbreviated as FFTD.

3.2.2. *Segment averaging*

A significantly different method to reduce the variance of the raw periodogram estimations was suggested by Bartlett (1950). In this method the time series are broken into K segments with an equal number M of observations each one, verifying that $N = MK$. Then, the periodogram is estimated for each segment and finally the individual spectral estimations are averaged. The smoothed periodogram may be expressed as

$$\tilde{S}(f_i) = \frac{1}{K} \sum_{J=0}^{K-1} \hat{S}_J(f_i) \tag{7}$$

Note that in this method the average is applied on several spectral estimations associated to the same frequency component. Thus, the degrees of freedom for the smoothed estimations are $2K$. This procedure, known as Bartlett's method, will be denoted by FFTB.

3.2.3. *Overlapped windowed segment averaging*

Welch (1967) proposed a more refined method by improving the idea of segment averaging by including two important modifications. First, the original time series is segmented into K blocks of equal length M, which can be overlapped in a given fraction S, usually close to or lower than 50% of the segment length. Second, a data window is applied to each segment to reduce the bias due to the leakage effect. The spectral estimations of the Jth overlapped and weighted segment are defined by

$$\hat{S}_J(f) = \frac{2}{UM\Delta t} |X_J(f)|^2$$

where U is a correction factor to overcome the energy reduction caused by the data window. The Welch spectral estimator is defined by

$$\tilde{S}_{\mathrm{FFTW}}(f) = \frac{1}{K} \sum_{J=0}^{K-1} \hat{S}_J(f) \tag{8}$$

The estimations computed with this procedure have a chi-square distribution with $\nu = 2(N - S)/(M - S)$ degrees of freedom. This approach is usually referenced as Welch's method and will be denoted here as FFTW.

The data windows used to taper the segmented time series are: The Rectangular

(W1), Bartlett (W2), Parzen (W3), Hamming (W4), Hanning (W5), Welch (W6), Cosine taper (W7), Trapezoidal (W8) and Blackman–Harris (W9) windows. In reference to the FFTD method, we use the notation in brackets to denote data windows applied to reduce spectral estimations variance.

3.3. The maximum entropy method

In contrast with the above methods, which estimate the spectral density by Fourier transforming the data or the estimated autocovariance function, the maximum entropy method (Burg, 1967) use the data to estimate the parameters of an autoregressive model of order P which adequately fits the observed data. The spectral density of this model is accepted as the spectrum of the analysed time series.

In this method the autocovariance function is not assumed zero outside the maximum lag used but it is extrapolated by using the statistical concept of entropy, which is maximised subject to certain restrictions (see, e.g. Priestley, 1981). The maximisation problem gives as a solution the following spectral density estimation

$$S_{\mathrm{ME}}(f) = \frac{\sigma_w^2 \Delta t}{\left| 1 + \sum_{n=1}^{P} a_n \exp(-i 2\pi f \Delta t) \right|^2} \tag{9}$$

where the a_n are the parameters of the fitted autoregressive model and σ_w^2 is the variance of a mean zero white noise sequence added to the deterministic part of the AR model.

The maximum entropy spectral method presents several advantages over the above outlined (classical) methods. It does not need the use of smoothing procedures and has a very high frequency resolution, even for short data records. However, it also has some drawbacks. One of the most important problems in this procedure is the selection of the order of the AR model. There is not a universal criterion to select an adequate order model for a given process. Furthermore, a practical method does not exist to estimate the degrees of freedom corresponding to the spectral estimations.

4. Results and discussion

The analysis of the statistical variability of the spectral wave parameters given by Eqs. (1–4), as a result of the applied spectral estimation methods, has been performed by using 20 measured wave records with $T = 20$ min and $\Delta t = 0.78125$ s. Thus, the total number of data for each time series is $N = 1536$. The spectral density for each record has been estimated by means of the estimators given by Eqs. (5)–(9). In the particular cases of the BT and FFTW methods, each one of the spectral and data windows mentioned above has been applied. Furthermore, the MEM procedure was applied to each time series, varying the order of the AR model from 20 to 30. Then, a total of 25 spectral estimations were obtained for each wave record.

The values of the wave parameters obtained by the different computational approaches, are shown in Fig. 1 for one of the analysed time series. In this picture the black dots represent the values of the wave parameters given in the vertical axis, computed by using the procedures indicated in the horizontal axis. All spectral estimations have been obtained with a number of degrees of freedom very close to 14, which should be considered a low degree of smoothness. The choice of this low number of degrees of freedom is due to limitations in the FFTB and FFTW methods to obtain spectral estimations with a larger number of dof. Thus, it is necessary to divide wave records into very short segments, which might not be enough to extract the spectral structure of the process. Due to this, these methods are often applied jointly with the Daniell's method.

Furthermore, together with these data, the figure shows the values of the corresponding parameter estimated by the ME method with the AR model order, P, increasing from 20 (left side) to 30 (right side).

Inspection of Fig. 1 reveals a clear variability of the spectral parameters as a function of the methods spectrum estimation. The range of variability showed by the significant wave height (Fig. 1(A)) is of the order of a few tens of centimetres, which is close to the accuracy of the popular *waverider* buoys. It can also be seen that for the average periods (Fig. 1(B–D)) the variability is always smaller than the sampling period, while for the peak periods the variability reaches differences of three and four seconds among different methods.

However, it is worth noting that while for different spectral estimators the peak periods suffer considerable modifications, the effect of the different spectral and data windows in the BT and FFTW methods, respectively, is totally insignificant. On the another hand, while for FFTW the effect of the different windows on the values of the significant wave height and the average periods is small but can be identified, in BT it is non-existent. Furthermore, in BT this is true for all the parameters examined.

It may seem paradoxical that the effect of tapering the time series prior to the periodogram computation may be detected in the average wave height and periods but not in the peak periods. However, a detailed exam of Fig. 1(E) and (F) reveals a slight variability in the Delft period which does not exists for the peak period. This fact makes clear that tapering data affect the overall spectral structure, but not the location of the spectral components. Naturally, this is reflected in the integration procedures applied to compute the spectral moments and the Delft peak frequency. Note that this effect is more substantial for the value of wave parameters that depend on lower spectral moments and decreases as the order of the spectral moment increases. Moreover, it should be noted that the largest deviations for these parameters are obtained with the FFTW method without tapering time series, that is, with the rectangular window.

Another interesting feature observed in Fig. 1 is the large stability of the ME estimations, which remain practically constant over the whole range of orders used. This reinforces previous results suggesting a model order close to 25 to compute the maximum entropy spectral density for stationary wave records (see, e.g. Rodríguez, 1995 or Gomes and Guedes Soares, 1997). Furthermore, note that the value of the wave parameters given by the ME method coincide with those of the BT model,

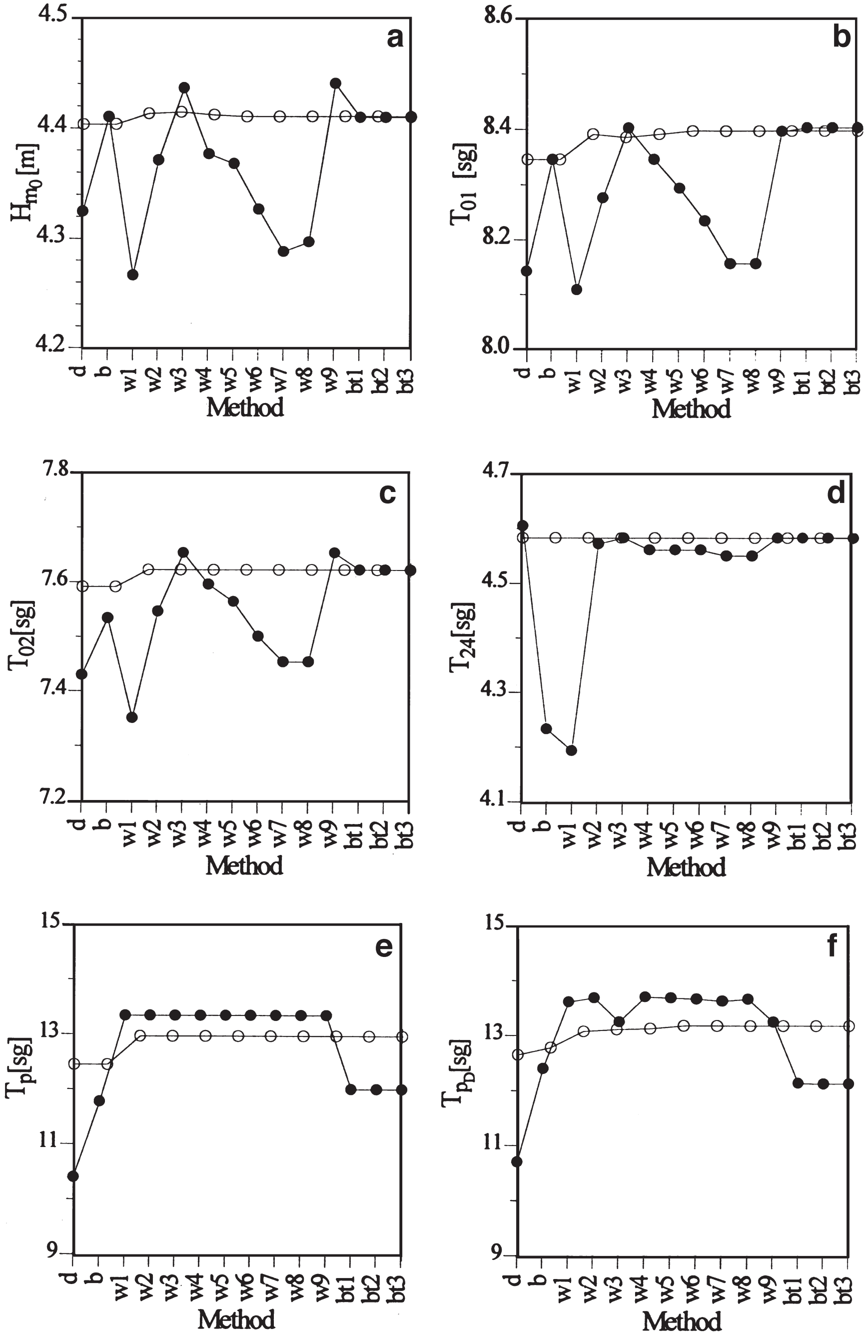

Fig. 1. Variability of the spectral wave parameters as a function of the spectral estimation method used for its computation (•, parameter value estimated by means of conventional spectral methods; ○, parameter value computed with the ME technique for AR order varying from 20 to 30).

Table 1
Statistics for spectral parameters computed by different spectral methods with the same degrees of freedom, for the 20 wave records analysed, as illustrated in Fig. 1

	Mean value	Standard deviation	Minimum value	Maximum value
H_{mo} (m)	4.37	0.0587	4.27	4.44
T_P (s)	12.72	0.9323	10.40	13.33
T_{PD} (s)	12.97	0.9250	10.70	13.70
T_{01} (s)	8.29	0.1155	8.11	8.40
T_{02} (s)	7.54	0.0938	7.35	7.65
T_{24} (s)	4.52	0.1317	4.19	4.60

except for the peak periods, where they take an intermediate value between the BT and the FFTW ones.

The basic statistics for the spectral parameters shown in Fig. 1 are given in Table 1. It can be observed that the deviations from the mean values are considerable only for the peak periods, which have the larger standard deviations. This effect can be observed in the various spectra corresponding to the wave record, shown in Fig. 2.

Note that for the degree of smoothness chosen, the overall structure of the spectra changes slightly according to the estimation method, but the peak frequency presents a significant variation. Then, a larger variability is observed for the peak periods as a result of the inverse relationship with the peak frequencies.

Thus, the larger uncertainty, resulting from the methods used to compute the wave spectrum, is observed for the wave parameters that depend on one or few spectral estimates, that is, for the peak periods, while it is practically insignificant for the wave parameters that depend on the overall spectral structure.

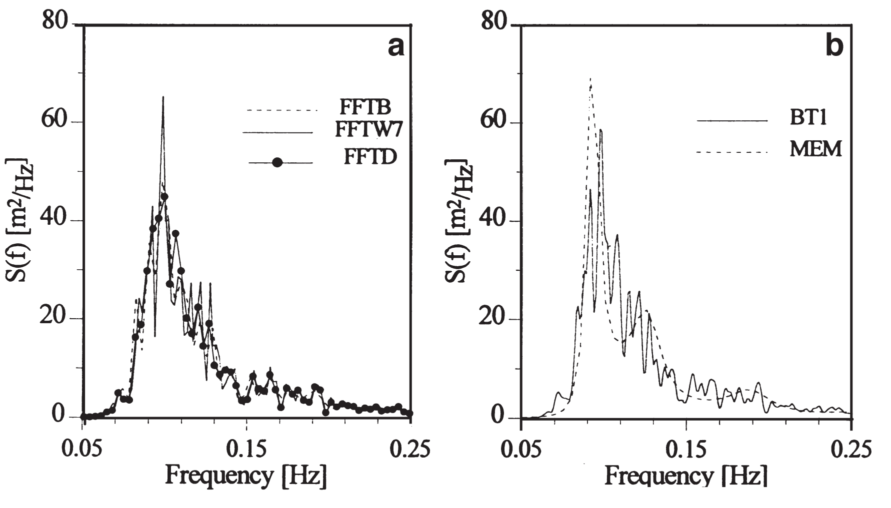

Fig. 2. Spectral density functions associated to time series used to estimate the wave parameters shown in Fig. 1.

It is important to emphasise that the results obtained for several other time series studied are very similar to those shown in Fig. 1.

5. Conclusions

It can be concluded that the use of different methods of spectral estimation does not have a significant effect on the variability of the spectral wave parameters whose magnitude depend on the overall spectrum. However, the parameters depending only on few spectral estimations, such as the peak period, show great differences as a function of the spectral method adopted. This effect is enhanced by the inverse relationship between the spectral peak frequencies and the peak periods.

Furthermore, it can be concluded that tapering data slightly modifies the values of the parameters whose estimation involves an integration procedure over the spectrum. However, this effect is not caused by the application of a spectral window.

Finally, the results show an excellent statistical behaviour of the spectral wave parameters estimated by means of the maximum entropy method with an autoregressive model of order close to 25.

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References

Arhan, M.F., 1979. Analysis of the variability of spectral estimatives using long continuous wave records. In: Proceedings of the International Conference on Sea Climatology, Paris, pp. 27–47.

Bartlett, M.S., 1950. Periodogram analysis and continuous spectra. Biometrika 37, 1–16.

Blackman, R.B., Tukey, J.W., 1958. The measurement of power spectra. Dover Publications, New York.

Burg, J.P., 1967. Maximum entropy spectral analysis. In: Proceedings of the 37th meeting of the society of exploration geophysicists.

Cavanié, A.G., 1979. Evaluation of the standard error in the estimation of mean and significant wave heights as well as mean period from records of finite length. In: Proceedings of the International Conference on Sea Climatology, Paris, pp. 73–85.

Chakrabarti, S.K., Cooley, R.P., 1977. The stability of some currently used wave parameters—a discussion. Coastal Engineering 1, 359–365.

Daniell, P.J., 1946. Discussion on ‘Symposium on autocorrelation in time series’. J. Roy. Statist. Soc. 8 (Suppl.), 88–90.

Donelan, M., Pierson, W.J., 1983. The sampling variability of estimates of spectra of wind-generated waves. J. Geophysical Research 88, 4381–4392.

Elgar, S., 1987. Bias of effective degrees of freedom of a spectrum. J. Waterways, Port, Coastal and Ocean Eng. 113, 77–82.

Gomes, D., Guedes Soares, C., 1997. Estimation of directional spectrum by maximum entropy method. In: Proceedings Offshore Mechanics and Arctic Engineering, vol. II, pp. 9–17.

IAHR working group on wave generation and analysis, 1989. List of sea-state parameters. J. Waterways, Port, Coastal and Ocean Eng. 115, 793–808.

Mansard, E.P.D., Funke, E.R., 1986. On the statistical variability of wave parameters. Technical Report, TR-HY-015, NRCC, Hydraulics Laboratory.

Medina, J.R., Aguilar, J., Diez, J.J., 1985. Distorsions associated with random sea simulators. J. Waterways, Port, Coastal and Ocean Eng. 111, 603–628.

Nath, J.H., Yeh, R.C., 1987. Some time and frequency relations in random waves. J. Waterways, Port, Coastal and Ocean Eng. 113, 672–683.

Peña, H.G., 1983. On the stability of the moments of the maximum entropy wind wave spectrum. Ocean Engineering 10, 79–96.

Priestley, M.B. 1981. Spectral analysis and time series. Academic Press, New York.

Rye, H., 1977. The stability of some currently used wave parameters. Coastal Engineering 1, 17–30.

Rodríguez, G.R., 1995. Analysis of the wind-generated gravity waves in deep waters. Ph.D. Thesis, University of Las Palmas de G.C. (in Spanish).

Rodríguez, G.R., Grisolía, D., Díaz, M., 1992. On the statistical variability of some spectral bandwidth and nonlinearity parameters of wind generated gravity waves. In: Proceedings of the 6th International Symposium on Stochastic Hydraulics, Taipei, pp. 353–360.

Welch, P.D., 1967. The use of the Fast Fourier Transform for the estimation of power spectra: a method based on time averaging over short modified periodograms. IEEE Trans. on Audio and Electroacoustics 15, 70–73.

Young, I.R., 1986. Probability distribution of spectral integrals. J. Waterways, Port, Coastal and Ocean Eng. 112, 338–341.

Young, I.R., 1995. The determination of confidence limits associated with estimates of the spectral peak frequency. Ocean Engineering 22, 669–686.