WAVE PROPAGATION THROUGH FIELDS OF PACK ICE

By G. de Q. Robin
Scott Polar Research Institute, Cambridge

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[Plates 3 and 4]

CONTENTS

Experimental studies of penetration of sea waves and swell into fields of loose pack ice were carried out by means of a ship-borne wave recorder, during a voyage into the Weddell Sea in R.R.S. *John Biscoe* in 1959–60. This reconnaissance study has provided the first systematic data within an ice field of the variation of wave amplitudes and period over the normal wave spectrum of 4 to 24 s. Although observations were confined to a single ship, a reasonably constant background of swell, together with varied ice conditions, has made it possible to draw certain conclusions for waves and swell of relatively small amplitudes. The penetration of long ocean swell, of periods from 11 to 23 s, into ice fields consisting of large floes of more than half a wavelength across takes place by bending of the floes. The results suggest that the fraction of the wave energy penetrating such an ice field is proportional to λ^4/h^3, where h is the thickness of the ice floes and λ the wavelength of the swell. For periods of less than 10 s, floes of around 1·5 m thick and 40 m or less in diameter approximate to rigid floating plates. For these periods, the main energy cut-off took place when floe diameters were about one-third of the wavelength; little loss of energy occurred when floes were less than one-sixth of the wavelength across, while no detectable penetration took place when the floes were half a wavelength or more in diameter.

Consideration of the results, together with limited evidence available from tide and gravimeter observations, shows that most long waves penetrate polar ice fields with little loss of energy.

Discussion of the energy required to bend large ice floes indicates that long-period swell is propagated through regions covered by pack ice with little loss of energy only when the energy required to bend the floes is at least an order of magnitude smaller than the total energy of the waves.

1. Introduction

The rapid attenuation of sea waves and swell on entering pack ice is a phenomenon well known to ice navigators, but, owing to the difficulties of obtaining wave records at sea, very few measurements of the amplitude of waves within pack ice have been made. A theoretical study of the effect of pack ice on wave amplitudes in water of infinite depth has been made by Peters (1950), and for finite depth by Weitz & Keller (1950). This work has been interpreted numerically by Shapiro & Simpson (1953) for the deep sea, and a simplified theory for shallow water has been developed by Keller & Goldstein (1953). However, the theoretical studies deal only with the case of a non-rigid floating mat of ice, and it is seldom that field conditions approximate to this picture; a satisfactory theoretical approach considering the ice field as a series of rigid or elastic plates has not yet been made. An experimental approach to this problem therefore appeared to be particularly desirable. This paper records such a study made on board R.R.S. *John Biscoe* during the 1959–60 relief voyage to the Falkland Islands Dependencies Survey base at Halley Bay (lat. 75° 31′ S, long. 26° 36′ W), formerly the Royal Society station. In spite of observations being limited to a single ship, the investigation throws some light on the mechanisms of wave and swell penetration into fields of loose pack ice. Possible effects of pressure in fields of pack ice are also mentioned.

2. Equipment

A ship-borne wave recorder, designed at the National Institute of Oceanography, England (Tucker 1956) was used to obtain wave records inside the pack-ice belt as well as on the open ocean. In order to cope with the small wave amplitudes expected within the pack ice, an additional d.c. amplifier which provided an amplitude gain of 2, 4 or 8 was used within the pack ice.

The standard ship-borne wave recorder has a sensitivity of 30·5 cm per scale division, but the additional amplifier increased this to 3·8 cm per division (figure 1). Chart speeds of 2·5 or 7·6 cm per min were provided. The instrument is intended to cover the ocean wave spectrum for periods between 4 and 24 s, but periods down to 2 s were present on some records from within the shore lead. The calibration of the instrument was re-checked at the National Institute of Oceanography after the return of the ship to England, and no significant change had taken place. Calibration of the d.c. amplifier was checked periodically, and the gain did not vary from the stated figures by more than 5 % during the voyage. The siting of the accelerometer and pressure units was most satisfactory, the pressure inlet holes being amidships on the vertical ships sides about 9 ft. below the mean water level. Nominal depths of the pressure hole of 6·75 ft. (2·1 m) for the ship lightly laden, or 9·25 ft. (2·8 m) for the ship fully laden, were used in calculating corrections to the results. Actual depths should not vary from the assumed depths by more than 0·5 m.

The instrument response has been calculated from the formula

$$\frac{\text{true height}}{\text{observed height}} = 0{\cdot}83[1+(8{\cdot}8\times 2\pi f)^{-2}]^{\frac{3}{2}}\exp\left(\frac{4\pi^2 f^2 kd}{g}\right),$$

where f is the frequency, g the value of gravity, d the depth and k is a constant. The exponential term in this expression is due to the falling-off of wave motion with increasing depth. In the absence of interference from the ship, $k = 1$; but the ship has an effect which is difficult to allow for theoretically. Practical measurements by Cartwright and Darbyshire (Darbyshire 1961) indicate that the presence of the ship increases k to a value between 2 and 3, with 2·25 as the most probable value, which has therefore been used in our calculations. When allowances are made for possible error in the value of k, for errors in our assumed depths, and for instrumental errors, we find that wave amplitudes may be in error by factors of 1·1 at 16 s period, 1·3 at 8 s period and 2·0 at 4 s period.

Apart from errors in amplitude, any variation in the speed of the chart drive will result in proportional errors in derived wave periods. The chart was driven by a synchronous motor, and errors in speed will be due to errors in the frequency of the 50 c/s power supply. Initially this was drawn from a rotary converter run off the ship's d.c. mains, but chart speeds varied from 0·5 to 4·5 % slow under these conditions. Shortly before entering the ice, a vibrator power supply run off 24 V accumulators was installed and the chart speed then varied between correct and 2·2 % fast. No corrections for chart speeds have been applied during analyses, since the errors involved will not affect the conclusions drawn.

Although the accuracy of the instrumental observations was not high, statistical errors arising from the limited duration of each recording are larger. Furthermore, the attenuation being studied was so large that the possible errors do not seriously affect the conclusions. Future studies will require improved methods of recording sea-ice characteristics rather than improved wave-measuring techniques, if the conclusions drawn from this work are to be subject to a more rigorous test.

3. Observations

(a) *Waves and swell*

Throughout the voyage from Port Stanley to Halley Bay and return (30 December 1959 to 12 January 1960) the ship was stopped at 6 h intervals for a 10 min period so that the instrument would record the true wave period as well as amplitude. Three typical wave records are shown in figure 1 (*a*) (open ocean), figure 1 (*b*) (pack ice, large floes) and figure 1 (*c*) (within shore lead), together with the analyses of the spectral energy densities of the respective records.

(b) *Ice observations*

In addition to the normal ice log kept by the ship's officers, a more detailed ice log was kept as part of the wave-recording programme. Estimates of ice thickness, the mean diameter of ice floes and the total density of pack-ice cover in tenths, were included, in addition to any special comments entered under remarks. The intervals between entries in this log depended on the rapidity with which ice conditions were changing, and varied from less than $\frac{3}{4}$ h to 4 or 6 h.

(c) *Density of ice cover*

There was general agreement between estimates of ice cover made by ship's officers and by the wave observer. Since analysis indicates that this is not an important parameter, errors in estimating the ice cover will not affect the conclusions.

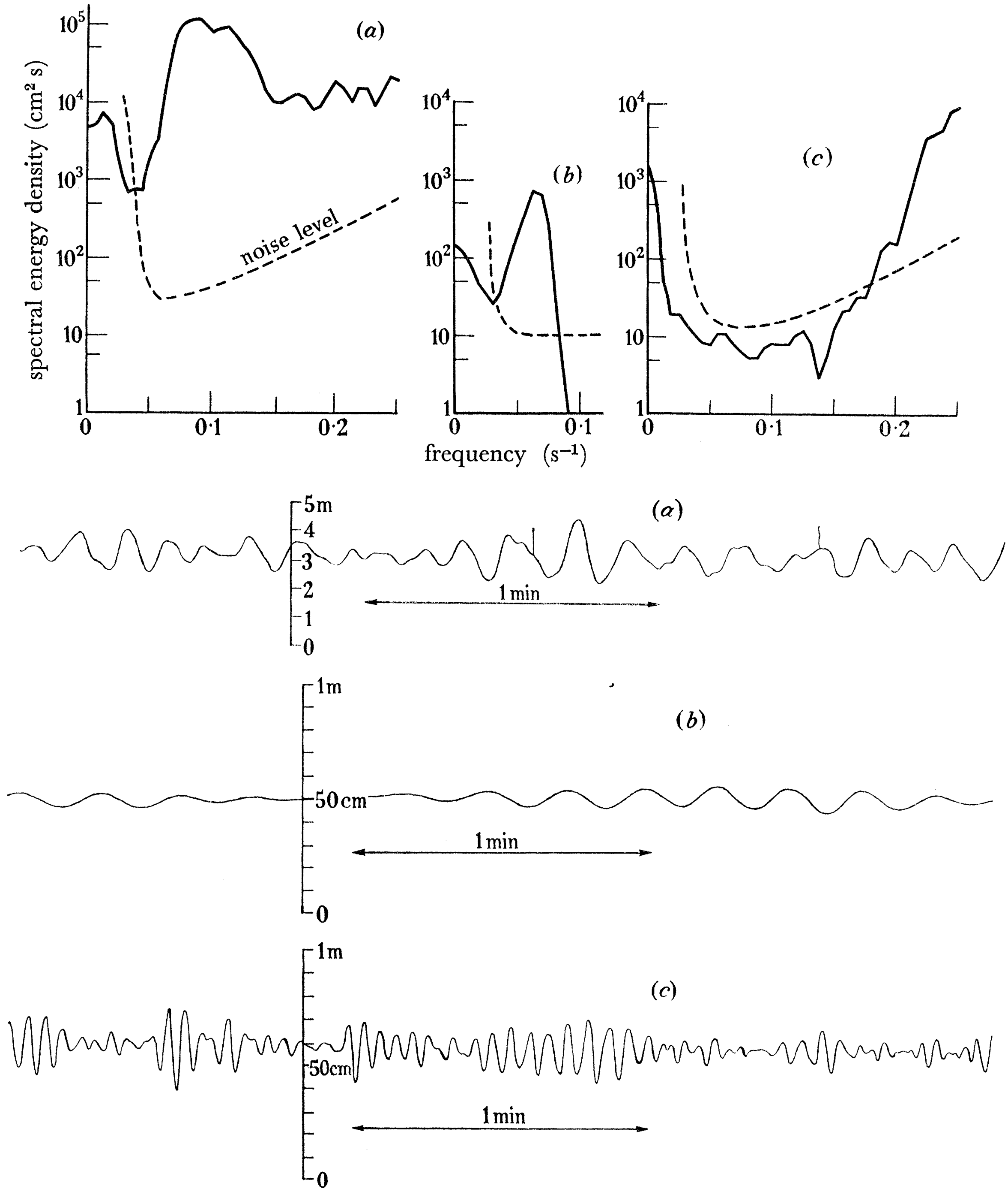


FIGURE 1. Wave records and wave spectra. (*a*) Open ocean, lat. 55·6° S. Outward voyage. (*b*) In large ice floes, lat. 69·1° S. Outward voyage. (*c*) In shore lead, lat. 75·5° S (Halley Bay).

(*d*) *Thickness of ice floes*

This is the most difficult estimate to make. Whenever possible, the thickness was estimated from looking at pieces of ice broken off large floes by the ship. Pieces of ice tilted over so that their whole thickness was exposed above the water level, made it possible to avoid refraction effects. Errors of estimates should not exceed one-third of the total thickness for uniform flat floes of a thickness up to 2·5 m. However, older ice which was 2 to 3 m thick or

FIGURE 2. Small ice floes from navigating bridge at lat. 67·4° S, on return voyage, estimated at 0·5 to 0·75 m thick, 5 to 20 m diameter (see table 1).

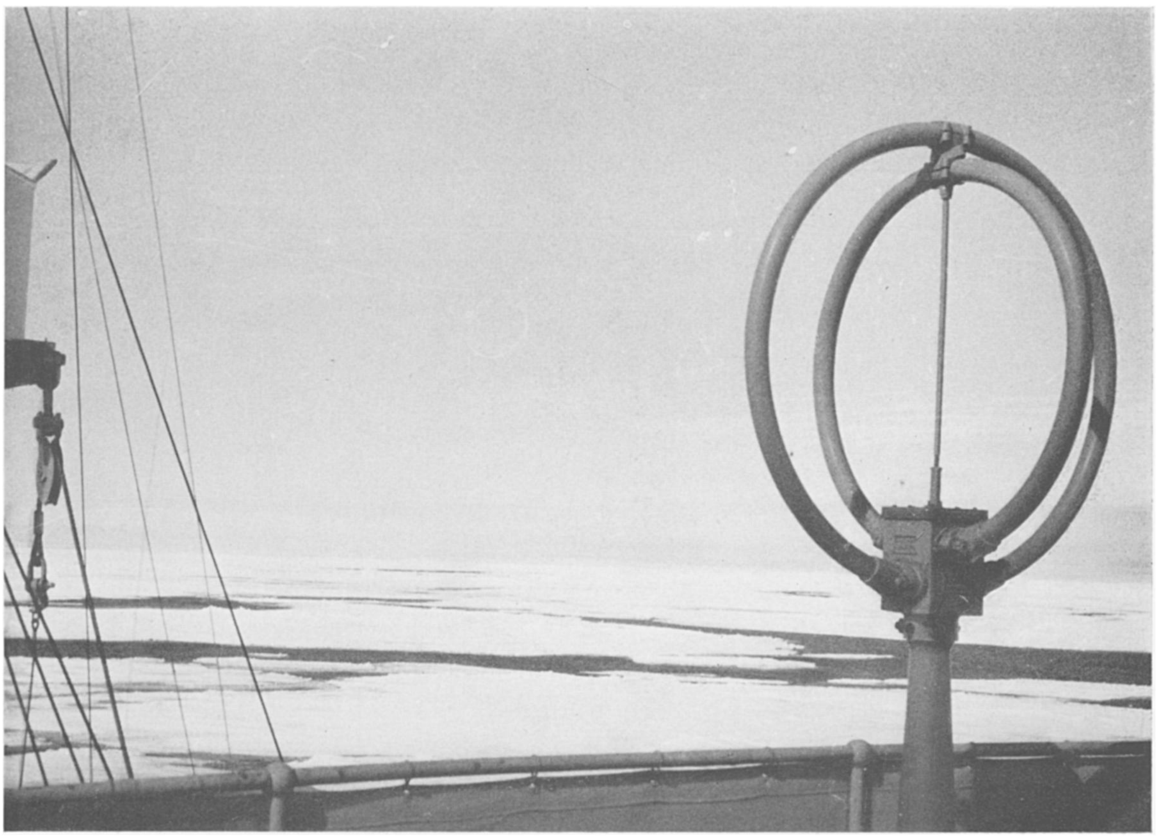


FIGURE 3. Ice floes from navigating bridge at lat. 68·7° S on outward voyage, estimated at 1 to 1·5 m thick, mean diameter 100 m (see table 1).

(*Facing p.* 316)

FIGURE 4. Heavy ice floes from radar platform at lat. 71·1° S on outward voyage, estimated at 3 to 10 m thick, 50 m to 1 km in diameter.

more, was considerably rafted and distorted, and not at all uniform in thickness. Plates of ice were sometimes seen to protrude downwards an estimated 10 m below water level, and it is quite likely that such plates or ridges might have protruded down to 20 m in places, since such protruberances were found beneath similar heavy polar pack ice in the Arctic by U.S.S. *Nautilus*.

(*e*) *Diameter of ice floes*

These were estimated by eye, the dimensions of the ship providing a useful measure for comparison. When lighting conditions were suitable, photographs were taken from the navigating bridge (see figures 2 to 4, plates 3 and 4). These photographs have been used to check the accuracy of visual estimates of the size of the smaller floes. From the known height of the camera, the focal length of the lens, the horizontal angle between the ends of an ice floe, and the vertical angle from the horizon to the centre of the flow one can calculate its width normal to the line of sight. A transparent grid based on these factors was superimposed on the photograph, and approximate floe diameters could be read off rapidly for floes within 300 m of the ship. Comparison of such photographic measurements with entries in the ice log are shown in table 1. The comparison for 11.30 G.M.T. on 2 February indicates that the estimates tend to be too small by about one-third. Statistics for the two other comparisons are less satisfactory but also show a tendency to underestimate floe sizes.

TABLE 1. COMPARISON OF ESTIMATES AND PHOTOGRAPHIC MEASUREMENTS OF DIAMETERS OF ICE FLOES

	photographic measurements			estimated diameters from ice log	
		ice floes			
date	time (G.M.T.)	no.	size (m)	time (G.M.T.)	diameter (m)
9. i. 60	17.45	3	0 to 15	16.30 to 18.00	5 to 20
		6	16 to 25	—	—
		2	26 to 35	18.00 to 24.00	5 to 40
		2	36 to 45	—	—
10. i. 60	11.00	2	83		
		1	93		
		1	114		
		1	135 mean 124	11.00	mean 100
		1	155		
		1	208		
2. ii. 60	11.30	11	0 to 5	—	—
		5	6 to 10	—	—
		29	11 to 15	11.30	5 to 20
		26	16 to 20	—	—
		8	21 to 25	—	—
		7	25 to 30	—	—

4. ANALYSIS

During the voyage, the height of the water level was read off the wave records at 1 s intervals and recorded. On return to Cambridge, these heights were punched on tape for subsequent analysis by EDSAC in the University Mathematical Laboratory. The system of

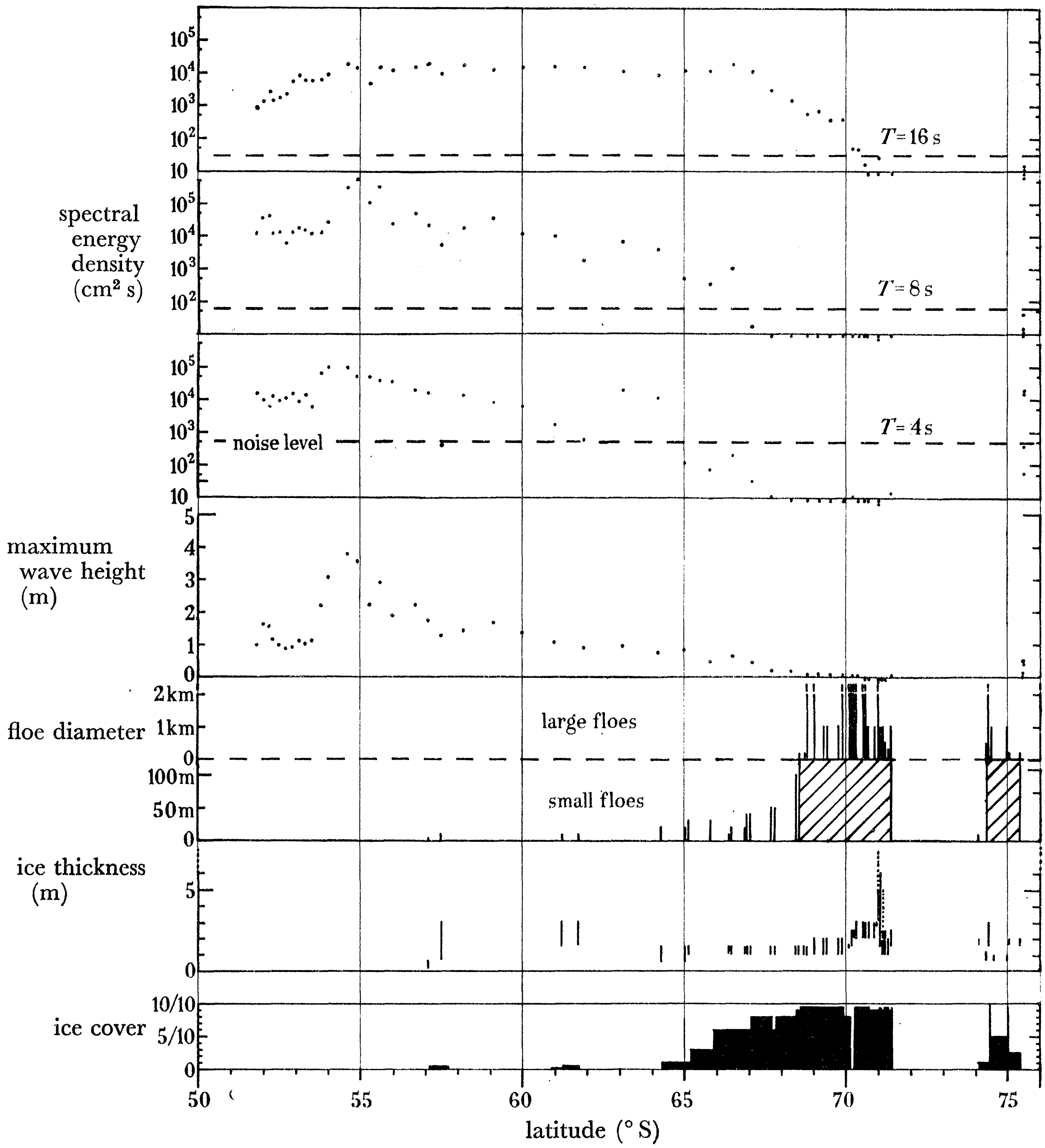


FIGURE 5(*a*). For legend see facing page.

power spectra analysis used was that given by Pierson & Marks (1952). As one feature of the results to be studied was the sharpness of the energy cut off at shorter periods, analysis was carried out using a narrow frequency band to give high resolution (80 intervals of $0{\cdot}00625\,s^{-1}$). Although the analysis made it possible to detect waves down to a period of 2 s results presented here are confined to periods of 4 s and longer owing to uncertainty about the instrument response at shorter periods. The statistical errors with a 10 min wave record are such that there is 90 % chance that the true power spectrum lies within a range of 0·60 to 2·1 times the results given by the analyses shown in figures 1, 5 and 7 to 9, and within these

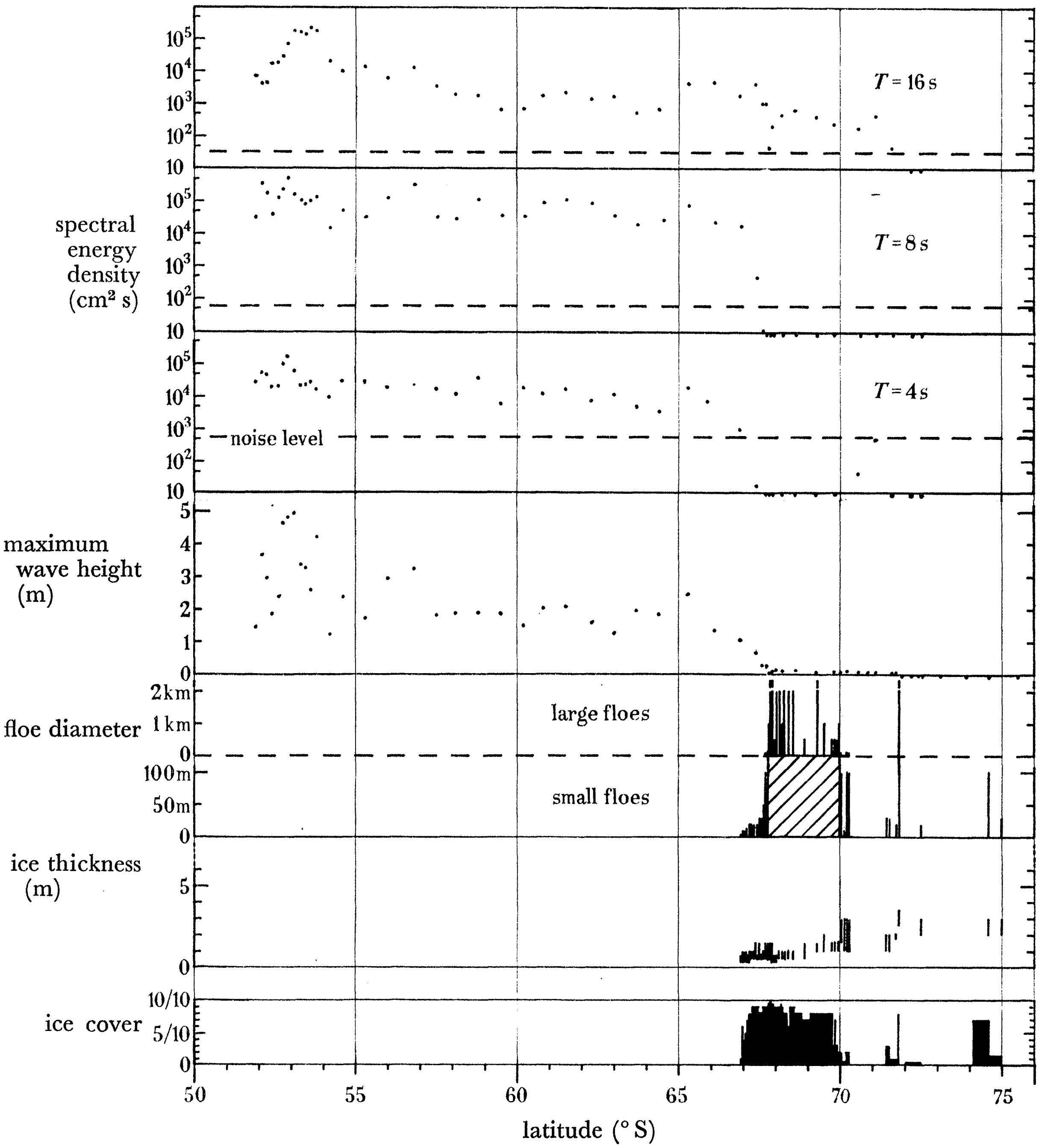


FIGURE 5 (*b*)

FIGURE 5. Wave and ice data. (*a*) Outward voyage. (*b*) Return voyage. Wave heights are the maximum peak to trough heights recorded during 10 min. When diameters of large floes exceed 2 km this is indicated by a short addition to the line beyond the 2 km point.

limits moderate fluctuations of the power spectrum may be obscured. Consideration of the uniform nature of the power density of low frequency over several days (e.g. figure 5 (*a*): 16 s swell, lat. 56° S to lat. 67° S, outward voyage) when conditions appeared to be constant, gives confidence that the statistical errors do not exceed the above limits.

5. Results

(a) General presentation

Wave heights and spectral densities throughout the voyage are shown in figure 5 for waves of 4, 8 and 16 s period together with the relevant sea-ice data, while the noon positions shown in figure 6 give a general guide to the location of measurements. Results are plotted against latitude and, since there is a general tendency for the edge of pack-ice belts in this region to run in an east–west direction, the latitude scale serves as a better guide to the distance from open water than would the distance along the ship's track. Latitudes were estimated by dead reckoning between fixes. This is an inaccurate method when continually altering course through fields of ice. Relative errors between adjacent observations should not exceed 0·2° of latitude, and will normally be less than 0·1°, which corresponds to an error of 1 knot in estimating the northerly component of the mean speed of the ship over a 6 h period. The total latitude error should not exceed 0·2° on the outward voyage, or 0·4° on the return voyage. The standard time interval between recordings was 6 h when wave activity was present, although the interval was sometimes decreased within the pack-ice belt.

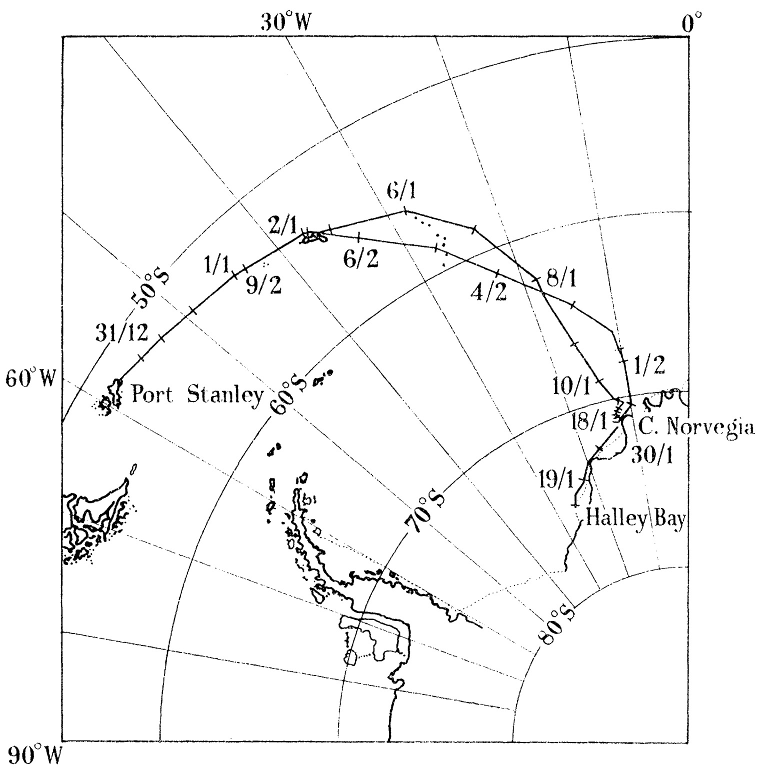


Figure 6. Noon positions.

(b) Noise level

The noise level on the records appears to be due to three separate causes: (i) At low frequencies, variations in the supply voltage of the ship's mains were the dominant cause of noise during the 1959–60 season. A considerable decrease in this noise the following season, when the instrument was operated off a battery power supply, confirms this cause. (ii) At higher frequencies, errors in reading off water levels at 1 s time intervals from steeply sloping traces such as those in figure 1 (*c*) must contribute to the noise. (iii) Instrumental effects such as those caused by slight rolling of the ship will produce some noise.

In heavy pack ice, to the south of lat. 71° S, the fluctuations of the instrument record lacked any regular wave form, and when the inputs to the amplifiers were switched off similar fluctuations continued on the record. Since the apparent spectral energy densities for frequencies less than $0{\cdot}04\,s^{-1}$ on these records were similar to those of other records obtained with considerable wave activity present, we conclude that records obtained south of lat. 71° S give a satisfactory guide to noise levels at low frequencies. It is difficult to specify noise levels at higher frequencies which result from the presence of lower frequency waves. It can, however, be seen in figures 7 and 8 that irregular fluctuations of spectral energy density occur on the high-frequency side of cut-offs owing to the presence of ice floes. Since these irregular fluctuations are about 0·1 to 1·0 % of the magnitude of the maximum spectral energy densities present, the most satisfactory explanation appears to be that they represent noise. The principal reason for the rise in noise level with frequency, shown in figures 7 and 8, is due to the decreased sensitivity of the instrument at higher frequencies. Before corrections for instrumental response are applied, noise levels of individual records between 0·1 and $0{\cdot}25\,s^{-1}$ show little variation with frequency. It is still more difficult to estimate noise levels at intermediate frequencies, but in one case, shown in figure 9, curve 1, the arguments used to estimate high-frequency noise can be applied on the low-frequency side of the wave spectrum obtained in the shore lead since long-period waves from the open ocean are absent.

The maximum 'noise' levels noted in the 16, 8 and 4 s bands on the basis of this discussion were 28, 40 and 208 cm^2 s, respectively. However, in the 4 s band, it is uncertain whether values of 423 and 558 cm^2 s at lat. 57·5° S and 61·9° S on the outward voyage should be attributed to noise. For presentation of diagrams, noise levels have been shown conservatively as 30, 60 and 500 cm^2 in the 16, 8 and 4 s bands. Since wave activity at other frequencies influences the noise level at shorter periods, our arbitrary figures will be unduly high for cases when little or no wave activity is present.

Once all the ocean swell of less than 10 s in period had disappeared, the apparent spectral energy density in the 4 s band dropped below 10 cm^2 s for twenty analyses made when the ship was surrounded by pack ice, but rose above this figure on seven occasions when the ship was clear of pack ice within the shore lead, and on one occasion when the ship was in an exceptionally large pool of over 30 km in length within the pack ice. It therefore appears that the noise level in the 4 s band fell to around 10 cm^2 s at times when south of lat. 71° S, and that spectral energy densities of greater than this value within the shore lead may show that wave activity is present as indicated in figure 9.

(*c*) *Waves of* 4 *s period*

On the open ocean, waves of 4 s period did not fall below a spectral energy density (E) of 3000 cm^2 s; they did not rise above $E = 600$ cm^2 s when within the pack-ice belt, except within the shore lead. When a small amount of brash was present on two occasions, values of 1700 and 800 cm^2 s were measured. In the discussion of noise level in 5 (*b*) it was seen that, within the shore lead and in a pool some 30 km long, a level of E greater than 10 cm^2 s appeared to be significant.

Within the limits set by noise level, it appears that the ice floes were sufficient in size to stop appreciable transmission of 4 s waves at all times. The presence of slight activity of waves of this period within the shore lead can only be due to wave generation within the lead. The presence of small amounts of brash ice caused some decrease of wave energy.

(*d*) *Swell of* 8 *s period*

On the open ocean, the spectral energy density did not fall below 4000 cm^2 s in the 8 s band, but the presence of a partial cover of ice floes, estimated to be 10 m in diameter, and up to 3 m in thickness, caused a decrease in E to around one-third of this value. In the presence of floes of estimated sizes of 20 or 30 m in diameter and 0·5 to 1·5 m thick, some wave energy was still present, but no wave energy of 8 s period was detected when estimated floe diameters were 40 m or more. No significant wave energy in this band was detected in the shore lead, but no winds of sufficient strength to generate energy in this band were experienced when within the lead.

(*e*) *Swell of* 16 *s period*

On the open ocean, the spectral energy density did not fall much below 500 cm^2 s at any time. The level was reasonably constant at around 10^4 cm^2 s for 3 days (lat. 56 to 67° S) before entering the main ice fields on the outward journey, but it was lower and more variable after leaving the ice fields on the return journey. When within fields of ice floes of 3 m or more in thickness, and of diameters of 1 km or more, the swell dropped below the noise level. Appreciable penetration occurred with floes of similar diameter, but 2 m or less in thickness. When floes were 40 m or less in diameter, no appreciable loss of energy in this band was evident.

Swell in this band was detected in the shore lead only in the northern section traversed during the return journey, and this swell had clearly penetrated from the open ocean. It had probably not encountered any floes of large diameter with a thickness of greater than 2 m judging by the ice conditions seen from the ship during the return journey, when the only thicker floes encountered did not exceed 100 m diameter.

(*f*) *Source of swell*

The results from the three frequency bands described present a general picture of the variation of wave energy with time when passing through the pack-ice belt. In general, the source of the wave energy in each band could not be readily identified, except when related to local wind conditions. Ship reports and weather analyses for the Atlantic south of 50° latitude were inspected, and it was clear that most of the long period swell originated outside this region. Normal methods of relating ocean swell studies to the storms causing the swell were not applicable, hence the necessity for this presentation of results.

(*g*) *Variation of wave spectra within pack ice*

In figure 7 are shown wave spectra from 4 to 20 s corresponding to each set of ice conditions when proceeding from the open ocean into large heavy pack ice on the outward journey. Some wave spectra have been omitted in order to preserve the clarity of the diagram, but those omitted are consistent with the ones shown in figure 7, as can be seen for the 16 and 8 s wave data presented from all analyses in figure 5 (*a*). Similar spectra are presented in figure 8 for the return passage through the pack ice, from the time at which the ship was near the centre of the belt of large floes until it reached the open ocean. Both sets of observations show that the change in the wave spectra, as one moves in from the open open ocean, first consists in a gradual elimination of the energy of shorter period waves with little change in the energy of longer period waves, then the latter suffer a general decrease

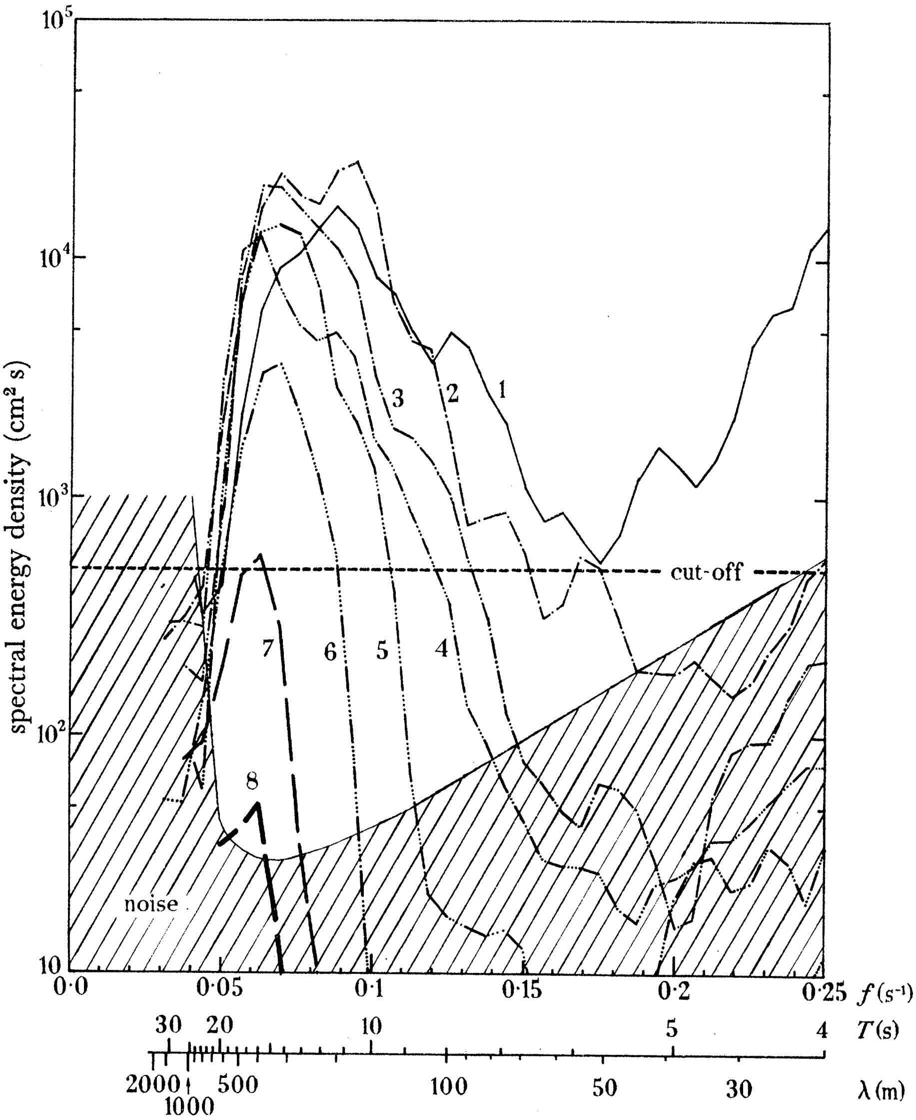


FIGURE 7. Wave spectra during outward voyage. 1, Open ocean (64·2° S); 2, floes 10 m diam. (61·9° S); 3, floes 20 m diam. (66·5° S); 4, floes 30 m diam. (65·8° S); 5, floes 40 m diam. (67·1° S) 6, floes 50 m diam. (67·7° S); 7, floes 3 km diam., 1 to $1\frac{1}{2}$ m thick (68·8° S); 8, floes 5 km diam., 2 to 3 m thick (70·3° S).

as one moves through very large floes. In general, the thicker the ice the lower is the remaining spectral energy density at all frequencies when within fields of floes of more than 100 m across.

The wave spectra obtained within the shore lead have a different energy distribution to most wave spectra, since they result from any swell of long period that passes through the pack-ice belt, as mentioned in § 5 (e), together with shorter period waves generated within the shore lead, as outlined in § 5 (*c*). In figure 9, curve 1 shows the wave spectra being generated by a 20-knot wind across the shore lead at Halley Bay at the time of observation, while curves 2 and 3 show weaker activity of similar periods which has remained from earlier storms, or come from storms some distance away within the shore lead. Such activity during winter months may well be greater if prolonged gales open up a wide shore lead. Only curve 2, obtained in the northern section of the shore lead, shows the presence of a long-period swell penetrating from the open ocean.

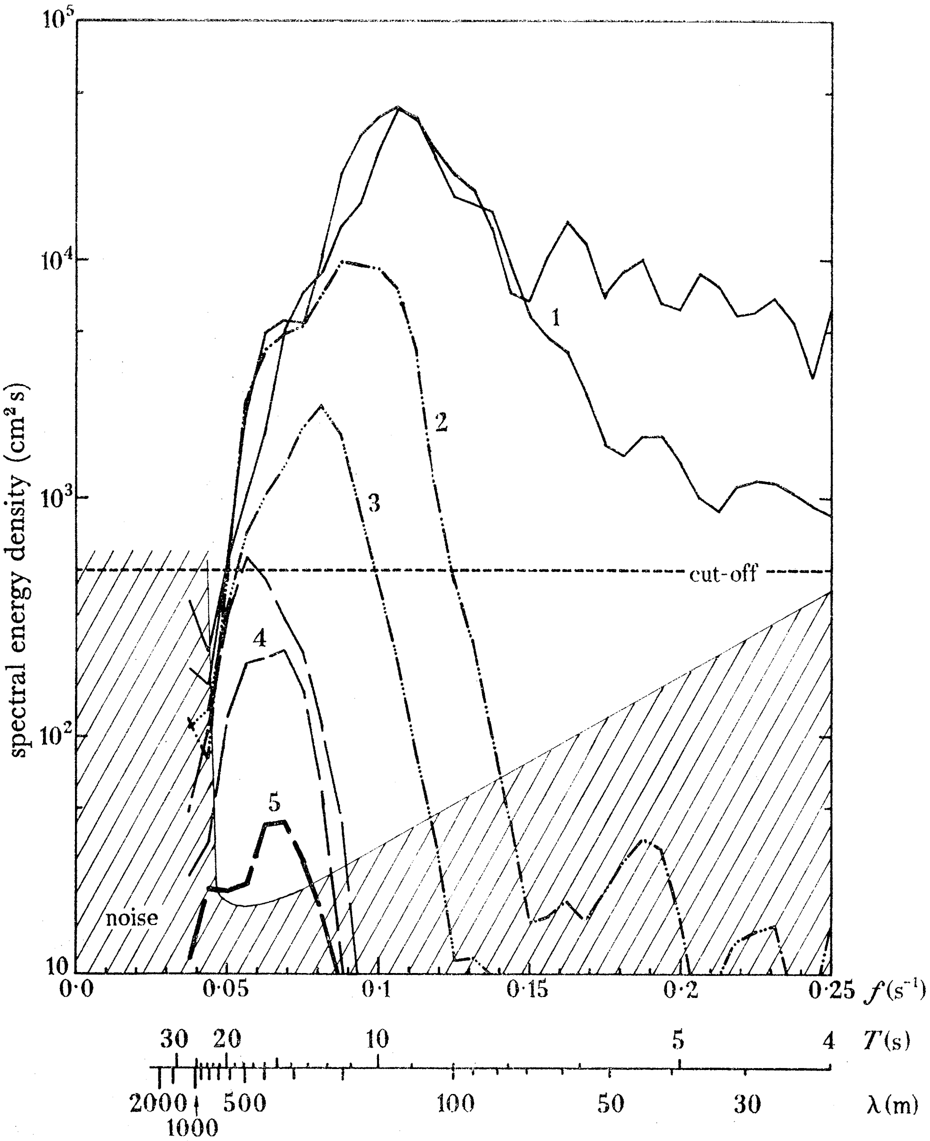


FIGURE 8. Wave spectra during return voyage. 1, Open ocean (66·1 and 66·9° S); 2, floes 20 m diam. (67·4° S); 3, floes 30 m diam. (67·6° S); 4, floes 1 to 5 km diam., $\frac{1}{4}$ to 1 m thick (67·9 and 68·2° S); 5, floes 1 km diam., $\frac{1}{2}$ to $1\frac{1}{2}$ m thick (67·8° S).

(*h*) *Continuous recording of waves with ship under way*

In addition to the regular 6 h results presented so far, a more detailed study of the variation of wave amplitude with time was made for the last 9 h that the ship spent traversing pack ice, consisting of small floes, during the return journey. The results of this study are presented in figure 10. Owing to the ship's movement through the water while these records were being made, analyses of spectral energy densities are not practicable. Instead, the parameters presented are the maximum wave heights (peak to trough) and number of zero crossings. Relevant sea ice data are presented on an expanded scale compared to figure 5.

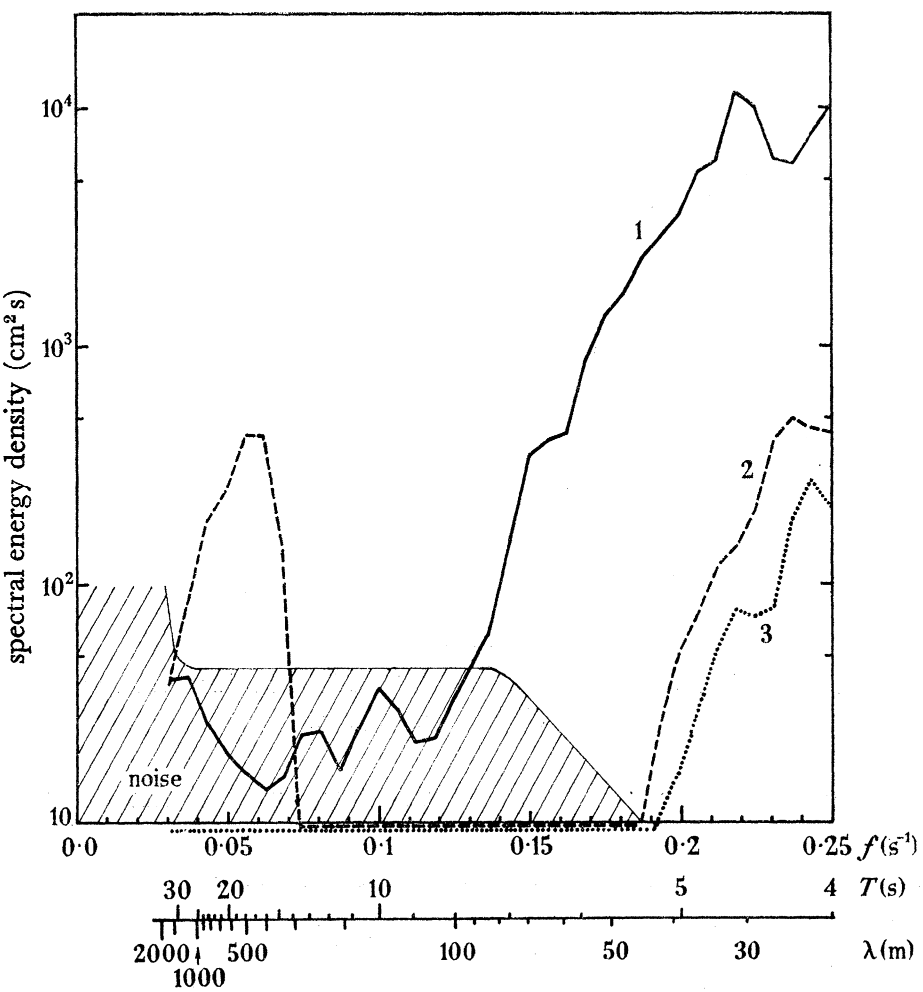


FIGURE 9. Wave spectra in shore lead. 1, 75·5° S, 20 knot wind; 2, 71·1° S, calm; 3, 75·5° S, calm.

The tendency for the number of zero crossings to be lower with the ship stopped confirms that the ship was steaming into a swell coming from the open sea. The rough correspondence of the increase of wave height with increase in the number of zero crossings is due to the wider spectrum of waves present when approaching the edge of the pack, as shown in figure 8.

These observations show clearly that large changes in wave characteristics can occur without any appreciable change in the density of pack-ice cover taking place. Changes in diameter of the ice floes appear to be related to the changing wave heights and number of zero crossings more closely than do changes of ice thickness or of the extent of pack-ice cover. A similar conclusion can be drawn from figure 5 for waves of 8 s period.

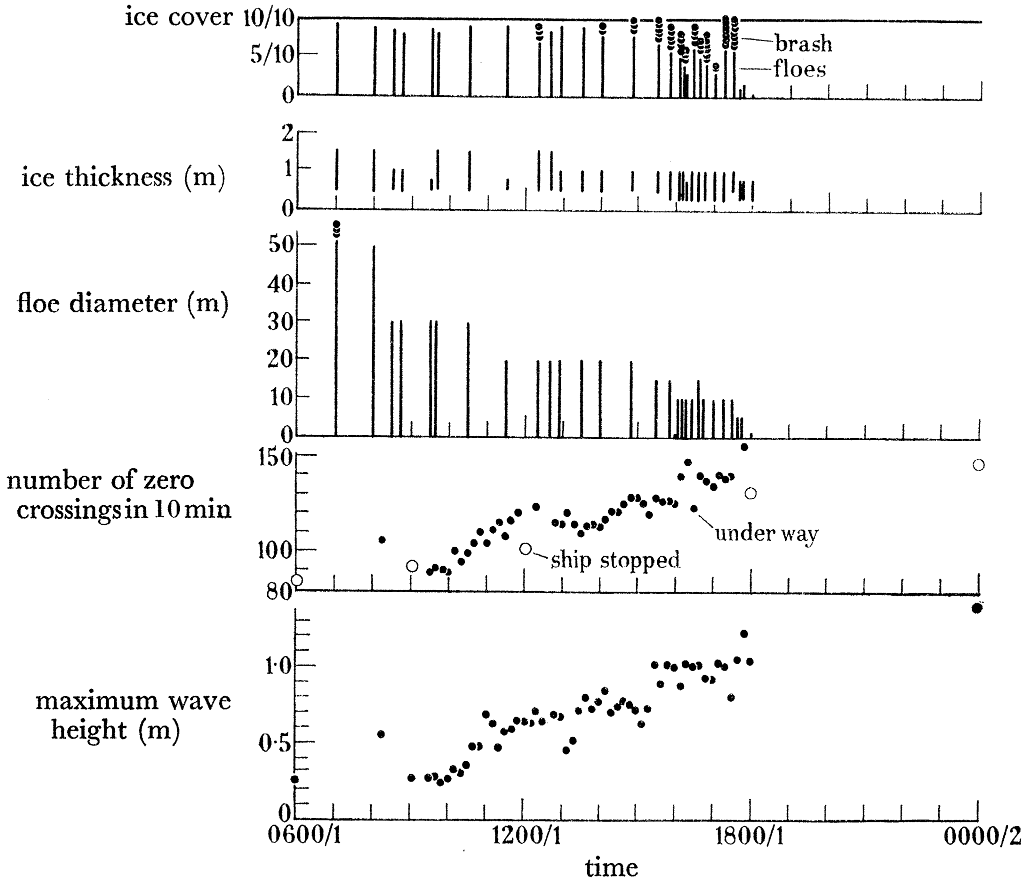


Figure 10. Wave and ice data from continuous record made with ship under way, lat. 67·7° S to lat. 66·1° S, return voyage.

6. Discussion: floe diameters small compared to wavelength

(a) *Wavelengths*

The general discussion of the results obtained so far has shown that the horizontal dimensions of the ice floes are a major factor controlling wave penetration at shorter wavelengths, while with longer wavelengths which penetrate regions of large ice floes the thickness of the floes may be of importance. The discussion may therefore be divided into two sections depending on whether the floe size is small or large compared to the wavelength.

Discussion of certain of the results in terms of wavelength, instead of the frequency or period given by our analysis, provides a better picture of the processes involved. Although the presence of ice floes will have some effect on the wavelength (λ), this is neglected and the value for open water $\lambda = gT^2/2\pi$ (where g is the value of gravity and T the period) is used throughout. This appears to be the correct procedure if the ice covers only a small proportion of the sea. However, the figures of Shapiro & Simpson (1953) indicate that a broken ice field covering the whole surface will reduce the wavelength on the open sea by 50 % for ice 10 ft. (3 m) thick for waves of 5 s period, or by 33 % for 6 s waves. Corresponding figures for ice 4 ft. (1·2 m) thick are 20 and 15 %, respectively. These results are due only to the mass effect of the ice, while bending of the ice will have the opposite effect and tend to increase the wavelength. Under the conditions experienced, such increases would be of the order of

1 % for floes large compared to the wavelength. The effect would be smaller when the floes are broken. After taking the above factors into account, it is concluded that errors in the calculated wavelengths due to the presence of ice and to errors in the chart speed should not exceed 6 % while the ship was within the pack ice.

(*b*) *Wave attenuation in small floes*

Figure 2, plate 3, shows R.R.S. *John Biscoe* proceeding through such an icefield at 11.30 G.M.T. on 2 February. Although most of the sea surface was covered by ice, individual floes were floating in loose contact only.

It is of interest to compare the numerical predictions of Shapiro & Simpson (1953) with our results for 'broken' ice fields. In order to determine the ratios of the wave energies within each frequency band inside the ice fields to those outside the ice fields for presentation in figure 11, we assume that wave conditions do not vary over a 6 h period. The extent to which this assumption is justified is shown by the spectral energy density curves in figure 5.

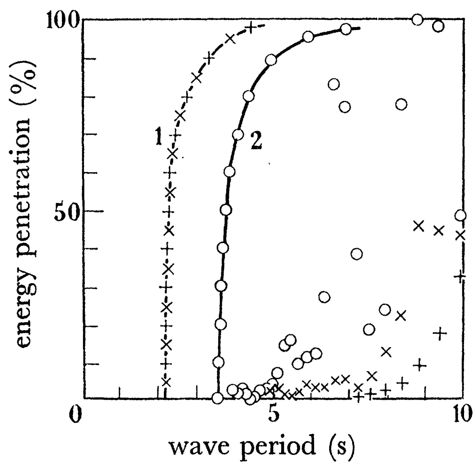


FIGURE 11. Energy penetration into a broken ice field. Some theoretical estimates and observed values with smaller floes. Theoretical curves (Shapiro & Simpson): 1, ice 1·2 m thick; 2, ice 3 m thick. Observations: ○, 3 m thick, 10 m diam. (57·5° S); ×, 1 m thick, 20 m diam. (65·0° S); +, 1 m thick, 20 m diam (67·4° S).

Figure 8 shows a large discrepancy between the theoretical curves of Shapiro & Simpson and the observations. This might be explained in part by the fact that the observations apply to waves that have traversed numbers of small patches of pack ice, rather than the single continuous ice field assumed as the basis of the theoretical calculations. However, the dominant effect of the diameter of the ice floes as compared to thickness, already mentioned in § 5 (*f*), is again apparent in that the energy losses are greater with floes of larger diameter, even though these were thinner than the smaller floes. If the thickness of small floes is unimportant as a parameter, it is apparent that bending of the floes can be neglected and we may visualize the floes as rigid floating plates. A field of loosely floating rigid plates, very small compared to the wavelengths involved, would not be expected to affect wave propagation except for the mass effect, but if they are an appreciable fraction of a wavelength one would expect them to cause attenuation.

In order to determine the frequency cut-off due to the size of the floes, we define the frequency of cut-off arbitrarily for the purpose of this study as the frequency at which the spectral energy density falls below 500 cm^2 s. This cut-off equals the approximate noise level at 4 s period, and exceeds the noise level for longer periods and is lower than the spectral energy densities found on the open sea at all periods. The relationship between the wavelengths corresponding to the cut-off frequencies and the floe diameters is shown in figure 12. Since our floe diameters are visual estimates only, some scatter of the results is to be expected.

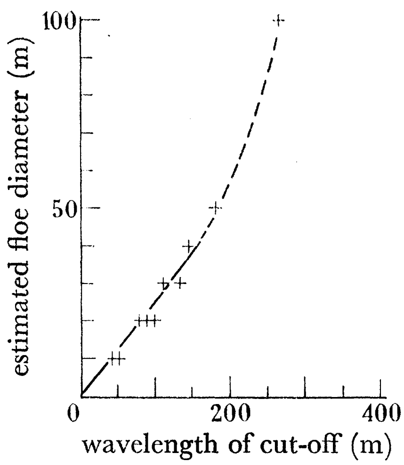


Figure 12. Floe diameters and wavelength of cut-off with smaller floes.

The results indicate that an approximately linear relationship exists between floe size and the wavelengths at which the cut-off of wave transmission occurs. This cut-off takes place when the estimated floe diameter is about one-quarter of a wavelength, but if we allow for a general tendency, shown in table 1, for floe sizes to be one-third larger than their estimated diameters, we should modify our conclusion to say that the cut-off in wave transmission occurs for wavelengths less than three times the floe diameters.

Inspection of the wavelength scales on figures 7 and 8 shows that the spectral energy densities have fallen below the noise level by the time the wavelengths are one-half the corrected floe diameters. It is more difficult to decide the point at which attenuation starts to become appreciable but this appears to be at roughly double the wavelength of the 500 cm^2 s cut-off, or six times the floe diameter, except for floes of 50 m or more in size for which bending of the floe is significant. This result suggests that the same criterion applies in the case of ocean waves and small ice floes as in the case of elastic and electromagnetic waves, where scattering becomes appreciable when the diameter of the scatterer is greater than $\lambda/2\pi$.

7. Discussion: floe diameters large compared to wavelength

(a) General

Results presented in § 5 (*c*) to (*g*) show that, within the limits of the sensitivity of our instrument, no penetration of wave energy occurred for waves of less than 11 s in period when floe sizes were over 100 m in diameter. The wavelengths recorded within fields of large floes run from around 200 m up to 1000 m, at which point the instrument sensitivity is decreasing rapidly. These wavelengths were large compared to the widths of open water leads found between the ice floes, and very large compared to the thickness of the floes. Under these circumstances, it is clear that the wave energy could not be confined to the open water leads and that bending of the ice floes must take place as a normal process of wave propagation.

Another feature of the results from regions covered by large floes, is that between lat. 68·8 and 69·9° S on the outward voyage, and between lat. 70·0 and 68·2° S on the return, the maximum wave height (peak to trough) noted during the 10 min recordings varied only from 7 to 11 cm from 14 separate recordings. It dropped to 3 cm in tighter pack ice at lat. 67·8° S for one recording on the return journey. This constancy of level may have been fortuitous, but it could indicate that the ice floes of around 2 m in thickness readily transmit swell only up to a certain amplitude, beyond which additional forms of loss control the energy penetrating ice field. More observations are needed before such a hypothesis could be justified.

(b) Wave attenuation in large floes

We must now consider the possible causes of attenuation of waves passing through fields of large ice floes. Types of energy loss which may occur are:

(i) mechanical friction between floes and turbulent flow of water around floe boundaries;

(ii) hysteresis losses involved in repeated bending of floes;

(iii) wave reflexion, or other energy losses, at the boundary of large ice floes corresponding to energy losses in passing from one media to another in acoustical theory.

More detailed consideration of changes of wave energy in the 16 s band (400 m wavelength) than was presented in § 5 (*e*) helps to show which of the above losses are important.

Mechanical friction between floes and turbulence are likely to cause most loss when the floes are relatively small, and there are numerous floe boundaries around which friction and turbulence may occur. In figure 5 (*a*), the constant level of wave energy in the 16 s band when floes were smaller than 40 m (one-tenth of a wavelength) indicates that this loss is unlikely to be important when floe sizes are large compared to the wavelength.

The energy in the 16 s band within fields of large floes remained within the range of 170 to 700 $cm^2 s^{-1}$ between lat. 68·8 and 69·9° S on the outward journey, and between lat. 70·0 and 68·2° S on the return journey. This relative constancy of energy over distances of some hundreds of wavelengths indicates that losses due to hysteresis, turbulence or friction do not rapidly absorb a major part of the wave energy. However, a possible limitation to the amplitude of waves penetrating this region mentioned earlier could result from some of these losses increasing rapidly as wave amplitudes increase.

On both outward and return journeys, the northernmost regions in which energy losses in the 16 s band (400 m wavelength) occurred correspond to the zones in which the floe sizes change from a fraction of the wavelength to several wavelengths across. On the outward journey this zone started when floes 1 to 1·5 m in thickness were more than 40 m across, while on the return journey the zone may have started with floes 0·5 to 1 m thick and 30 m in diameter. On the outward journey, the second zone in which a major decrease in energy took place was that in which the ship left relatively large floes up to 2 m thick and entered older ice up to 3 m thick, with floes still large in diameter. The same factors probably caused the change in energy between lat. 72 and 71° S on the return journey, when the energy level in the 16 s band became significant only after the ship emerged from a gap of 1 km in width between heavy Weddell Sea pack ice on one side and fast ice on the other, both being of 3 m estimated thickness. The rise in energy between lat. 71·6 and 71·1° S is probably due to decreased 'shadowing' from the heavy Weddell Sea pack ice, as the ship moved north-east around Kap Norvegia.

The sharp decrease in energy which took place during the return journey at lat. 67·8° S does not fit the above pattern. It occurred at a time when the ship was almost stopped because of lack of open leads, and because channels partly broken open by the ship were closing up before the ship could reverse and take a second run through. In other words, there appeared to be some horizontal pressure building up between the floes which limited the energy of the 16 s wave band. This was the only time within the fields of large floes of 2 m or less in thickness when the ship's progress was held up to this extent, although similar delays between lat. 70·7 and 71·4° S took place on the outward voyage when the wave energy was too small to detect.

Our evidence therefore leads to the conclusion that, provided the fields of floes are floating loosely without pressure between the floes, major energy changes in the 16 s band occur at the seaward boundary of fields of large floes, and at places where the thickness of large floes increases markedly. This is the third of the three suggested effects, and appears to be analogous to change of acoustical impedance.

The analogy with acoustics cannot be taken far, however, since acoustical impedance is given by the product of density and compressional wave velocity. With ice floes, the change of density of around 10 % and the change of wave velocity of the order of 1 % when 16 s waves enter large ice floes of 2 m in thickness (calculated from Greenhill 1887) would be much too small to account for the small amount of energy transmitted through fields of large floes on the basis of acoustical theory.

(c) *Bending of ice floes*

Before we go into detailed hypotheses about the effect of a floating ice sheet on wave transmission, it is necessary to show that the ice will behave as an elastic medium if we are to use the theory of elasticity. We can check this point roughly by considering the stresses involved in bending an ice floe of thickness h into a form given by

$$y = A\sin(2\pi x/\lambda). \tag{1}$$

Provided that elastic bending takes place about a neutral plane in the centre of the ice floe, the surface stress σ_{xs} is given by

$$\sigma_{xs} = Mh/2I, \tag{2}$$

where M is the bending moment at any point and $I = \int_{-\frac{1}{2}h}^{\frac{1}{2}h} y^2\,\mathrm{d}y$; now

$$M = \frac{-EI}{(1-\nu^2)}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}, \tag{3}$$

where E is Young's modulus for sea ice and ν Poisson's ratio for sea ice.

The maximum peak to trough wave height experienced within the fields of large floes was about 10 cm with the period centred around 16 s. The greatest floe thickness for such waves was estimated at 1 to 2 m. We shall therefore calculate the maximum stress in the ice for such waves, for the following values:

$$\left.\begin{aligned} \lambda &= 400\,\mathrm{m}, \\ A &= 5\,\mathrm{cm}\ \text{(peak to trough wave height 10 cm)}, \\ \left.\begin{aligned} E &= 5\times 10^{10}\,\mathrm{dyn\,cm^2} \\ \nu &= 0{\cdot}3 \end{aligned}\right\}&\ \text{approximate value for sea ice,} \\ h &= 1{\cdot}5\,\mathrm{m}. \end{aligned}\right\} \tag{4}$$

From (1), (2) and (3) we have

$$\sigma_{xs} = \frac{E}{(1-\nu^2)}\frac{h}{2}A\frac{4\pi^2}{\lambda^2}\sin\frac{2\pi x}{\lambda}. \tag{5}$$

Using (4) we get

$$\sigma_{xs}(\max) = 5{\cdot}1\times 10^5\,\mathrm{dyn\,cm^{-2}}. \tag{6}$$

This value is the maximum stress which we would expect to be present in ice floes during our observations.

Butkovich (1956) found that, when beams of sea ice were bent 'in place', fracture took place at stresses of $2{\cdot}2$ to $3{\cdot}9\times 10^6$ dyne cm^2 under conditions similar to those during our studies. Experiments on the visco-elastic constants of sea ice by Tabata (1955) indicate that the major part of the deformation of sea ice, under stresses lasting a few seconds, will be elastic in character. We therefore conclude that for the wave amplitudes we encountered the bending of the floes is mainly elastic. This conclusion may not hold for waves an order of magnitude greater in amplitude when inelastic bending or fracture may be significant. At such amplitudes increased turbulence and other forms of energy loss may also be more significant.

(*d*) *Forces necessary for elastic bending*

The vertical pressures (σ_y) necessary to bend an elastic plate of thickness h are given by

$$\sigma_y = \frac{-Eh^3}{12(1-\nu^2)}\frac{\mathrm{d}^4y}{\mathrm{d}x^4}. \tag{7}$$

If the plate is bent about a central (neutral) plane into the form of equation (1),

$$\sigma_y = \frac{-4Eh^3}{3(1-\nu^2)}\frac{\pi^4}{\lambda^4}A\sin\frac{2\pi x}{\lambda}. \tag{8}$$

For the parameters given in (4), we find that

$$\sigma_y = 46{\cdot}8\sin\left(\frac{2\pi x}{400}\right)\mathrm{dyn\,cm^{-2}}. \tag{9}$$

This is equivalent to a maximum pressure 'head' of sea water of about $0{\cdot}05$ cm, compared to the wave amplitude $A = 5$ cm. This change in wave amplitude of about 1 % appears to be within reasonable limits for the suggested process to take place.

Although direct observations of the relative change in surface levels of ice and water were not made during 1959–60, observations recorded in the diary of F. Worsley, the Master of Shackleton's *Endurance*, throw some light on this point. The observations were made when the party was camped on an ice floe, initially 400 m across, which was still of the order of a wavelength in diameter when the following was recorded:

'9 March 1916. Noon position 63° 59′ S, 53° 12′ W.

'All thro' the day the pack is working and creaking rhythmically to a very slight but undoubted swell. The cycle is from 16 to 20 s. The loose pieces of ice run to and fro from 4 to 6 in. (10 to 15 cm) from a direction between NNW and NNE. The rise and fall on the large floes shows to no more than $\frac{1}{4}$ in. to 1 in. At times the swell seems to die away for a few minutes and then gradually increases to a maximum. A seiche has been suggested, but we are too far out for that to be likely and continuing all day proves to our joy it must be a swell from some open stretch of sea, probably within 30 miles or less as the period of the swell (16 to 20 s) is so short.'

Since we have shown that the floes bend to the swell, Worsley's rise and fall of 0·6 to 2·5 cm should correspond to double the calculated pressure head for our observation (i.e. 0·1 cm). The run to and fro of the loose pieces of ice 10 to 15 cm relative to the ice floe should indicate the approximate wave amplitude if the ice floe does not move horizontally. This assumption gives us $A = 5$ to 7·5 cm for Worsley's observation, compared to 5 cm used in our calculations. Worsley's observations thus indicate a vertical pressure head small in comparison with the wave amplitude, as indicated by our calculations. However, the pressure head he indicates appears to be an order of magnitude larger than that calculated, but this could be partly due to differences of ice thickness ($\propto h^3$) or wavelengths ($\propto \lambda^4$) involved, horizontal movement of the ice floe (possibly less than one wavelength across) giving too small a value of A, or to other causes.

Worsley's diary notes, on 10 March 1916, that:

'Not the faintest sign of yesterday's swell is to be seen, this is probably accounted for by the fact that the pack is closer'. From then until they left the pack on 13/14 April, there are several references to swell. Little swell was present when the pack closed up, while the swell varied from 'nil' to a 'large rolling swell' with loose pack ice. These observations give support to the tentative hypothesis advanced to explain the low wave amplitude measured at lat. 67·8° S on the return journey. This could be explained in terms of elastic theory by adding an additional compressive stress in the horizontal direction, so that the 'neutral' plane is no longer taken to be at the centre of the ice floe. The energy required to bend the floe then increases rapidly with increase of the additional stress.

(e) *Energy of elastic bending*

The amount of energy per wavelength (W_e) necessary for the elastic bending of a strip of an ice floe of unit width to the form of (1) is

$$W_e = \frac{2E\pi^4 h^3 A^2}{3(1-\nu^2)\,\lambda^3}, \tag{10}$$

whereas the total energy of an ocean gravity wave (W_g) of the same amplitude is

$$W_g = \tfrac{1}{2}\rho g A^2 \lambda, \tag{11}$$

where g is the value of gravity and ρ the density of sea water.

Again, using the value of various parameters in (4), we get

$$W_e = 4{\cdot}7 \times 10^6 \text{ ergs per 400 m wavelength}$$

and

$$W_g = 5{\cdot}0 \times 10^8 \text{ ergs per 400 m wavelength.}$$

Thus about 1 % of the wave energy goes into bending the floes for our typical case.

It is also of interest to calculate the total kinetic energy per wavelength of the top 135 cm of sea water (the mass of water displaced by 150 cm of ice) for the same wavelength and amplitude on the open ocean. This is approximately $1{\cdot}0 \times 10^7$ ergs for 16 s period, or about 2 % of the total wave energy. Since the ice floe moves vertically with approximately the same amplitude as that of the rotation of the water particles, it will account for a kinetic energy of half that of the water displaced, that is about 1 % of the total wave energy. It therefore appears that in the 16 s energy band, the sum of the elastic and kinetic energy involved in bending the ice floe approximately equals the total kinetic energy which would have been possessed by the water mass displaced. In such a case, the motion of the water beneath the depth of the ice will be the same whether the ice is present or absent, except close to the ice-water interface where relative horizontal movement will cause some losses due to viscosity or turbulence. This appears to explain why wave energy in the 16 s band can be transmitted over long distances with little loss.

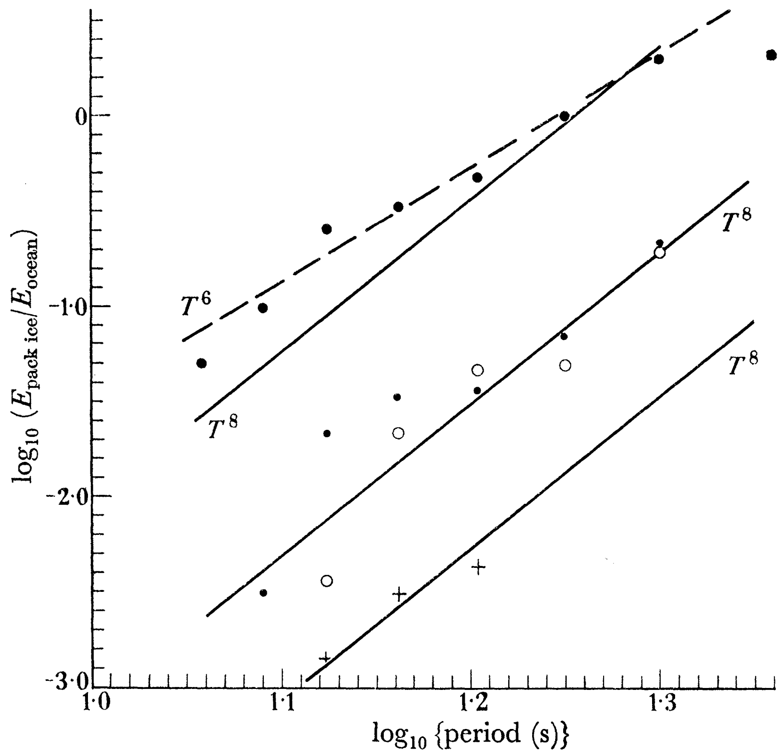
Since no effective transmission of wave energy took place for wavelengths less than 200 m in fields of large floes, we conclude that appreciable transmission of wave energy will only take place when the energy involved in the elastic bending of floes is an order of magnitude, or more, smaller than the total wave energy.

(*f*) *Variation of wave transmission with wavelength and ice thickness*

Since on our elementary theory, both the ratio of the elastic to gravitational wave energies (W_e/W_g) and the pressures necessary to bend the ice floe to a sinusoidal form (σ_y) vary in proportion to h^3/λ^4, the results have been analyzed to see if the variation of energy transmission appears to fit these characteristics. A series of double logarithmic plots in figures 13 to 15 show the proportion of wave energy transmitted through the ice field in each frequency band as a function of wave period (figures 13 and 14) and as a function of thickness for the 16 s band in figure 15.

In figures 13 and 14 are plotted the ratio of spectral energy densities inside and outside the fields of large floes for different time intervals, which range from 6 to $47\frac{1}{2}$ h, each time interval being roughly centred on the time at which the boundary of the fields of large ice floes was crossed. Any variation of wave spectra with time will then tend to cause variations with time of the slope and form of curve shown in figures 13 and 14.

The boundary of the field of large ice floes was not well defined on the outward journey, when some bending of 50 m floes was evident at 00.00/10 January in figure 7, and floes had only reached a little over 100 m in size by 08.00/10. The energy ratios for 06.00/10 to 00.00/10 which show a slope of T^6 are therefore less typical of large floes than the T^8 result from the energy ratios for 12.00/10 to 18.30/9 (floe sizes of 3 km and 40 m, respectively). On the return journey, the boundary of the largest ice floes was more clearly defined and was passed around 01.30/2 February. The energy ratios over the interval 00.00/2 to 06.00/2 fit the T^8 relationship quite closely as do those for 12.00/2 to 12.00/1. Although not as conclusive as one would wish, the results indicate that the fraction of wave energy transmitted into fields of large ice floes is proportional to approximately the fourth power of the wavelength. This again confirms that elastic bending of the floes is the main parameter which determines the energy transmission at these amplitudes, rather than frictional or hysteresis losses which one would expect to be in linear proportion to the wavelength.



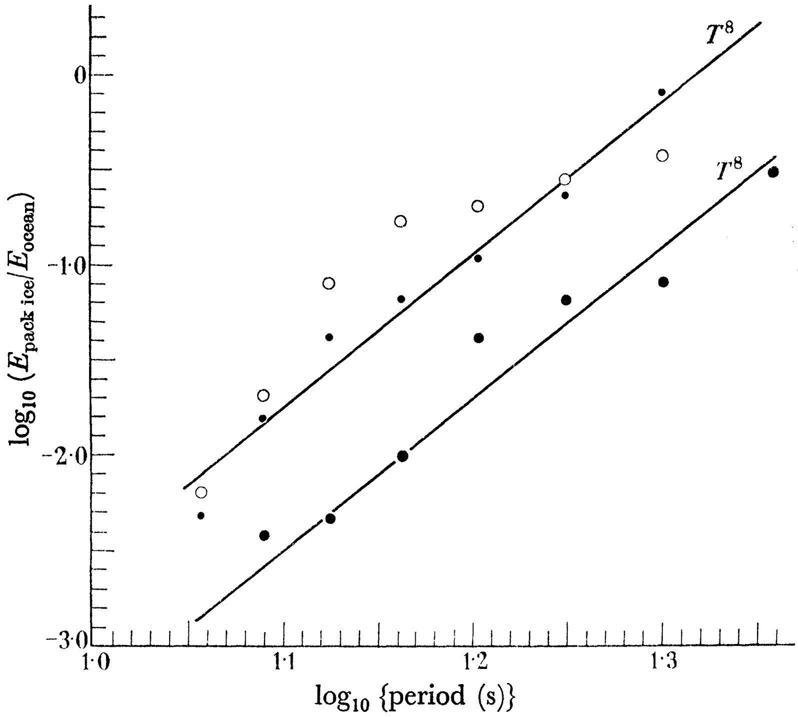
	lat. (° S)	thickness (m)	diam.
● $\dfrac{E_{06.00/10}}{E_{00.00/10}}$	68·3 67·7	1 to $1\frac{1}{2}$ 1 to $1\frac{1}{2}$	100 m 50 m
○ $\dfrac{E_{12.00/10}}{E_{18.30/9}}$	68·8 67·1	1 to $1\frac{1}{2}$ 1 to $1\frac{1}{2}$	3 km 40 m
• $\dfrac{E_{18.00/10}}{E_{13.30/9}}$	69·1 66·5	1 to 2 1 to $1\frac{1}{2}$	3 km 20 m
+ $\dfrac{E_{18.00/11}}{E_{18.30/9}}$	70·3 67·1	2 to 3 1 to $1\frac{1}{2}$	5 km 40 m

FIGURE 13. Energy penetration into fields of large ice floes as a function of wave period. Outward voyage.

It is more difficult to show the influence on wave penetration of changing ice thicknesses than of changing wavelengths, since thicknesses have been estimated by eye only, whereas wavelengths have been derived from recorded wave data. Figure 15 shows the plot of ice thickness against the energy ratios for all records from regions of large floes where significant wave energies were measured in the 16 s band. For these calculations the energy outside the pack ice has been taken from figure 5 as constant at $1{\cdot}5 \times 10^4\,\text{cm}^2\,\text{s}$ for the outward voyage, and as $2 \times 10^3\,\text{cm}^2\,\text{s}$ for the return voyage. A line of slope $h^{\frac{1}{3}}$ shows rough agreement with the observations. A similar trend is apparent from the shift of the lines in figures 13 and 14 between results from ice floes of different thicknesses.

Our results there suggest that the amount of wave energy penetrating fields of large ice floes for periods of 11 to 23 s is proportional to λ^4/h^3. We can write this as

$$\frac{E_{\text{icefield}}}{E_{\text{ocean}}} = C\lambda^4/h^3, \tag{12}$$



	lat. (° S)	thickness (m)	diam.
● $E_{00.00/2}$	67·8	$\frac{1}{2}$ to $1\frac{1}{2}$	1 km
$E_{06.00/2}$	67·7	$\frac{1}{2}$ to $1\frac{1}{2}$	100 m
○ $E_{18.00/1}$	67·9	$\frac{1}{4}$ to $\frac{3}{4}$	4 km
$E_{09.00/2}$	67·6	$\frac{1}{2}$ to 1	30 m
• $E_{12.00/1}$	68·2	$\frac{1}{2}$ to 1	1 km
$E_{12.00/2}$	67·4	$\frac{1}{2}$ to $1\frac{1}{2}$	20 m

FIGURE 14. Energy penetration into fields of large ice floes as a function of wave period. Return voyage.

where C is a constant which we can determine from each line of slope drawn in figures 13 and 14. The values we obtain for C are 3·6, 2·7, 4·5, 1·8 and $0{\cdot}8 \times 10^{-14}\,\text{cm}^{-1}$ when λ and h are measured in centimetres. The low value of $0{\cdot}8 \times 10^{-14}\,\text{cm}^{-1}$ corresponds to the presence of pressure between large floes at 67·8° S on the return journey, as mentioned in § 7 (*b*). Apart from this figure the values of C vary over a range of 2·5:1. Relative errors in estimating thickness of 1·35:1 which may be present, would explain these variations.

As an approximate guide, we write (12) as

$$\frac{E_{\text{icefield}}}{E_{\text{ocean}}} = 3 \times 10^{-14}\frac{\lambda^4}{h^3}\,\text{cm}^{-1}. \tag{13}$$

Under steady conditions, the wave energy in the icefield cannot exceed that in the open ocean, therefore the above ratio should not exceed unity. The point at which the above empirical relationship equals unity should, however, serve as a rough guide to the minimum wavelengths and periods which will penetrate fields of large ice floes with little attenuation. Values calculated in this way from (13) are shown in table 2, which includes an extrapolation to 5 m thickness. Extrapolation to thinner ice is not justified.

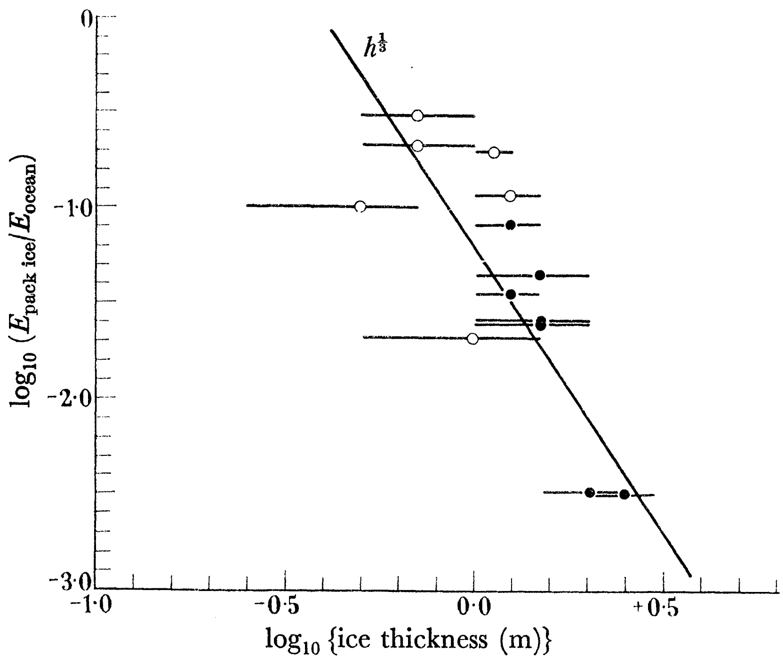


FIGURE 15. Energy penetration into fields of large ice floes as a function of ice thickness. ●, Outward journey; ○, return journey.

TABLE 2. WAVELENGTHS AND PERIOD AT WHICH OCEAN SWELL PENETRATES LOOSE FIELDS OF LARGE ICE FLOES WITH LITTLE ATTENUATION

ice thickness (m)	wavelength (m)	period (s)
0·5	450	17
1·0	760	22
2·0	1280	27·5
3·0	1730	33
5·0	2540	40

The constant C in (12) is not a dimensionless constant, but will depend on the elastic constants of the sea ice and possibly other factors. The elastic constants of sea ice are a complex function of age and temperature which affect the amount of brine in the ice. While these may cause some variation of the values given in table 2, the resultant variations will not be large since only the first power of Young's modulus of elasticity is involved in the bending equation.

8. Wind generation of swell within pack ice

During the outward voyage, at lat. 67·1 and 70·6° S, winds of 18 knots blowing over pack ice produced no detectable wave action. Worsley also records that a southerly gale on 14 March 1916 produced no swell. In view of the poor transmission of all but the longer period waves through fields of pack ice, these observations are to be expected.

9. Long waves

The figures derived in table 2 on the basis of these results indicate that long waves of a minute or more in period should be propagated with little loss of energy over most ice-covered seas. The presence of waves as short as 3 min in period has been noted on tide-gauge records obtained in Terre Adélie during the Antarctic winter in Imbert (1954), and similar fluctuations are visible on winter tide records of the British Graham Land Expedition, 1934–37.

Another check on our conclusions is possible from gravimeter observations made on floating ice. An extensive series of observations of gravimeter oscillations on Arctic Ice Island T. 3 has been published and roughly analyzed by Crary & Goldstein (1959). The mean period of oscillation ranged from 23 to 61 s, with 70 % of occurrences falling between 33 and 43 s. If the oscillations were primarily controlled by long-period swell through the pack ice, the latter periods, together with table 2, would suggest that the mean ice thickness over the Arctic Ocean was from 3 to 5 m, which is the correct order of magnitude. However, such a conclusion should be treated with caution, since although swell is the most likely cause, the natural oscillations of a floating ice island 50 m thick and a few kilometres across may obscure the true picture. In the Weddell Sea, Pratt (1960) made some estimates of the period and amplitude of movement on large ice floes from gravimeter observations some 200 to 300 km in from the edge of the pack. Half amplitudes of around 3 and 5 cm at a period of 30 s and of 83 and 66 cm at periods of 90 s were present.

An attempt to measure wave amplitudes by noting the tilt of a theodolite bubble relative to the horizon was made off 'Syowa' station by Murauchi & Yoshida (1959). They detected waves of 20 s in period with half amplitudes of 0·7 cm. The ship was then 30 km south of the ice edge, with 10/10 sea-ice cover, of which 2/10 consisted of large floes and 8/10 of small floes. The ice was about 4 m thick. This amplitude is similar to that measured by R.R.S. *John Biscoe* when heavy ice was estimated to be 3 m thick at lat. 70·3° S, on the outward voyage.

Thus limited evidence from other investigations is in general agreement with the observations and conclusions of this paper.

10. Microseisms

Both Imbert (1954) and Hatherton (1960) base their discussion of microseisms on the assumption that waves and swell do not penetrate the pack-ice belt. Hatherton concludes that short-period microseisms (1 to 3·5 s) recorded at 'Scott base' originate in the open-water areas of the Ross Sea. This is consistent with the findings of this paper which indicate that the necessary wave periods of 2 to 7 s are largely confined to open water.

Long-period microseisms (4 to 7 s) at 'Scott base' which would be caused by waves of period 8 to 14 s are thought to be due to swell moving on to the continental shelf in the Ross Sea. Their decrease in April and early May may be due as much to the thickening of the sea ice limiting wave transmission at these periods as to the growing extent of the pack ice.

Hatherton's conclusion that ultra long-period microseisms (7 to 10 s) are produced at or near storm centres and are transmitted through both oceanic and continental crusts to 'Scott base' appears unnecessary, since swell of the 14 to 20 s period necessary to cause such microseisms would appear capable of penetrating through the pack ice for long distances, and might well produce the microseisms over the continental shelf as before. If wave energy is reflected from the boundaries of fields of large ice floes, or from regions where the ice floes increase in thickness, the pattern of standing waves believed necessary for the generation of microseisms would be present.

Hatherton finds that the amplitude of his shorter microseisms are proportional to the fourth power of their period, which is the same as our relationship for swell in fields of large ice floes. However, the microseisms are of too short a period for ice floes to exercise any control, but control of swell by ice floes may well contribute to the higher power of the period (T^6) which he finds proportional to the amplitude at long periods.

The programme has been made possible through the continued co-operation of the National Institute of Oceanography and the British Antarctic Survey (formerly the Falkland Islands Dependencies Survey). The National Institute of Oceanography made the wave recording instrument available on loan at short notice when the investigation was first proposed, and installation and servicing of the equipment has been done by L. Draper of their staff, while M. J. Tucker has given general support to the work and has made valuable criticisms of this paper. The British Antarctic Survey covered the cost of engineering work on R.R.S. *John Biscoe* necessary for the installation of the wave recorder, and subsequently purchased the equipment to make a longer term project possible. Thanks are due especially to the Master of R.R.S. *John Biscoe*, Captain W. Johnstone, and to the officers and crew for their ready co-operation in stopping the ship at frequent intervals and for suffering the heavy rolling which set in when the ship was stopped. C. Dean and other members of the survey helped to operate the equipment both before and after the author joined the ship.

Analysis of records has been carried out by the staff of the Mathematical Laboratory in the University of Cambridge, and thanks are due especially to Mr P. Swinnerton-Dyer who looked after the programming and computation on EDSAC.

The programme of recording sea and swell within the pack ice is being continued in the hope of gaining further information on aspects of the work not fully covered by this paper.

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