A New Method for Measuring the Directional Wave Spectrum. Part II. Measurement of the Directional Spectrum and Phase Velocity of Laboratory Wind Waves

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A new method for measuring the directional spectrum, introduced in the preceding paper (Rikiishi, 1978), has been applied to actual wind waves in a large experimental tank 70 m × 8 m with the water 3 m deep, and in a wind-wave tunnel 850 cm × 60 cm with the water 35 cm deep. Measurements of the directional spectrum have shown that the mean wave direction of propagation agrees generally with the wind direction, that a bimodal distribution in the spectrum is not generally seen, and that the angular width of the directional spectrum is not correlated consistently with the wave frequency. These results conflict with the existence of Phillips' resonant angle. Measurements of the phase velocity have shown that the phase velocity of the spectral-peak component wave is larger than that obtained from linear small-amplitude wave theory, that the measured phase velocity shows a roughly constant value among frequencies near the dominant frequency, and that the deviation of the constant value from the theoretical varies with fetch in proportion to the wind speed over the water surface. Based on these observational facts, it has been stressed that wind waves under the direct action of wind stress should not be regarded as the linear superposition of free Airy waves.

1. Introduction

The determination of the directional distribution of wind-wave energy is necessary for studies of the growth, decay, diffraction and refraction of wind waves, and for the prediction of the response of such floating structures as ships and buoys to sea conditions. A number of observations of the directional spectrum made by many authors have been used successfully in these geophysical and engineering studies. However, most of these observations are based on the use of an assumed linear dispersion relation $\omega^2 = gk$. As we can see in another paper (Rikiishi, 1977), the directional spectrum is highly dependent on this assumption. Considering that the real dispersion relation for wind waves differs markedly from the linear dispersion relation as suggested by recent experimental studies (Yefimov *et al.*, 1972; Ramamonjiarisoa, 1974; Kato and Tsuruya, 1974), it is quite likely that the directional spectra reported thus far by many others may contain significant estimation errors. On the other hand, the dispersion relation (or phase velocity) of spectral component waves reported by the above authors may also have significant errors, because they did not take the effect of the directional energy distribution into account. In general, the directional spectrum and the dispersion relation are interrelated, so that neither can be determined independently.

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However, a new method introduced by the present author in an earlier paper (Rikiishi, 1978; hereafter referred to as I) overcomes these difficulties. In this method, the use of an incorrect dispersion relation for wind waves is the only possible factor in methodological error, and most of the unreasonable measurements can be ascribed to the incorrect dispersion relation used in the analysis. Therefore, as has been discussed in detail in I, it is possible to determine both the directional spectrum and the dispersion relation simultaneously by the condition that the calculated directional spectrum should give the minimum spurious estimate for the direction opposite to the mean wave direction.

Now we apply the method to actual laboratory wind waves and determine both their directional spectrum and phase velocity. Based on the experimental results, we discuss the nature of the physical process involved in laboratory wind waves, in particular the reality of the spectral component wave.

2. Experimental setup and procedure

Two sets of experiments were performed at the Tsuyazaki Sea Safety Research Laboratory of Kyushu University. The first series of runs, referred to as Experiment A, were made at a large experimental tank 70 m long by 8 m wide, with the water 3 m deep (Fig. 1a). Two wind blowers were mounted on a carrier at one end of the tank, with the air stream guided by six passages (see Fig. 1b). The

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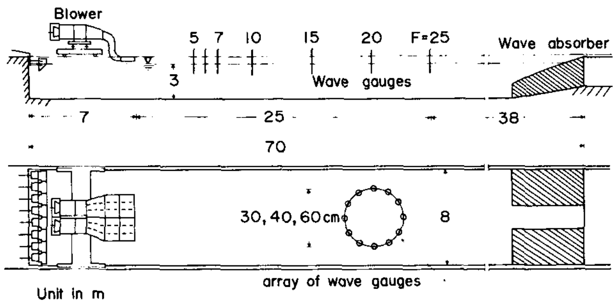

FIG. 1a. Schematic sketch of experimental tank layout for Experiment A.

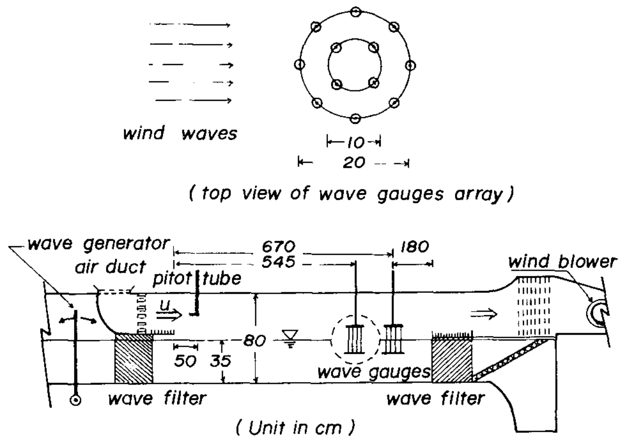

FIG. 1b. Schematic sketch of wind-wave tunnel layout for Experiment B.

clearance between the water surface and the bottom of the passages was 0.4 m. The wind-generated waves were measured at several stations along the center line of the tank by changing the location of the "parallel wire" resistance wave gages. The fetches F of the stations were 5, 6, 7, 10, 15, 20 and 25 m.

The second series of runs, referred to as Experiment B, were conducted at a wind-wave tunnel 850 cm long by 60 cm wide, with the water 35 cm deep (Fig. 1b). The winds were generated by drawing air over the water with an exhaust fan, which was equipped with honeycombs and a number of fine-mesh screens. The wind-generated waves were measured by means of resistance wave gages at two stations (F = 545 cm and F = 670 cm). Most of the wave energy was dissipated when the waves passed through a wave filter.

The arrangement of the wave gages—Arrays A and B (see I) for experiments A and B, respectively—are shown in the figures. The diameter D of the wave-gage array for Experiment A was varied from 30 cm (for fetches of 5, 6, and 7 m) to 40 cm (for fetches of 10 and 15 m) to 60 cm (for fetches of 20 and 25 m), while that for Experiment B remained unchanged (D = 20 cm).

The wind field for Experiment A was quite different in spatial structure from that of a typical wind-wave tunnel. Since there were no side walls and no top, the wind decreased in speed with increasing distance from the wind blower and from the center line of the tank. In addition, the wind tended to go through the two outer passages. Thus, in a region of smaller fetch, the wind field showed a bimodal distribution of wind speed with respect to distance from the center line. These features of the wind field are well represented by the detailed observations made by Mitsuyasu (1967) under the same experimental conditions (Fig. 2). The wind speed was kept at U = 20 cm s^{-1} at the mouth of the passages.

The wind field for Experiment B, on the other hand, was typical in that wind speed profiles were logarithmic with height, and that the speed did not decrease very much with increasing fetch. The wind speed U was set to be approximately 10 or 12 m s^{-1} at the pitot tube station.

3. Simultaneous determination of the directional spectrum and phase velocity

So far, three techniques have been used to determine the phase velocity of wind waves. The first one, used by Hamada *et al.* (1953) and Plate and Trawle (1970), determines the phase velocity of individual wave by measuring the time required for each wave crest to pass through a given distance. The technique of Hidy and Plate (1966) using succesive frames of a movie film for wave movement belongs to this category. The second one determines the phase velocity by measuring both the wavelength and frequency simultaneously. (Phase velocity is given by wavelength multiplied by frequency.) For the determination of the wavelength, Shemdin (1972) used coherent signals for the case of mechanically generated regular waves, and Francis (1951) used a number of photographs for the case of wind-generated waves.

The last one, the theory of which is described below, has been used recently by many authors[2] (Yefimov *et al.*, 1972; Ramamonjiarisoa, 1974; Kato and Tsuruya, 1974; Mitsuyasu and Kuo, 1976). Consider a long-crested wave field expressed by

$$\eta(x,t) = \sum_n a_n \cos(k_n x - \omega_n t + \epsilon_n), \qquad (1)$$

and suppose that two wave recorders are set up with separation X along the wave direction. Then the phase shift of component waves between the two wave records $\eta(0,t)$ and $\eta(X,t)$ is obtained from the cross spectrum as

[2] It should be noted that the first two techniques cannot be applied to the spectral component wave, and that all these techniques ignore the effect of the directional spectrum.

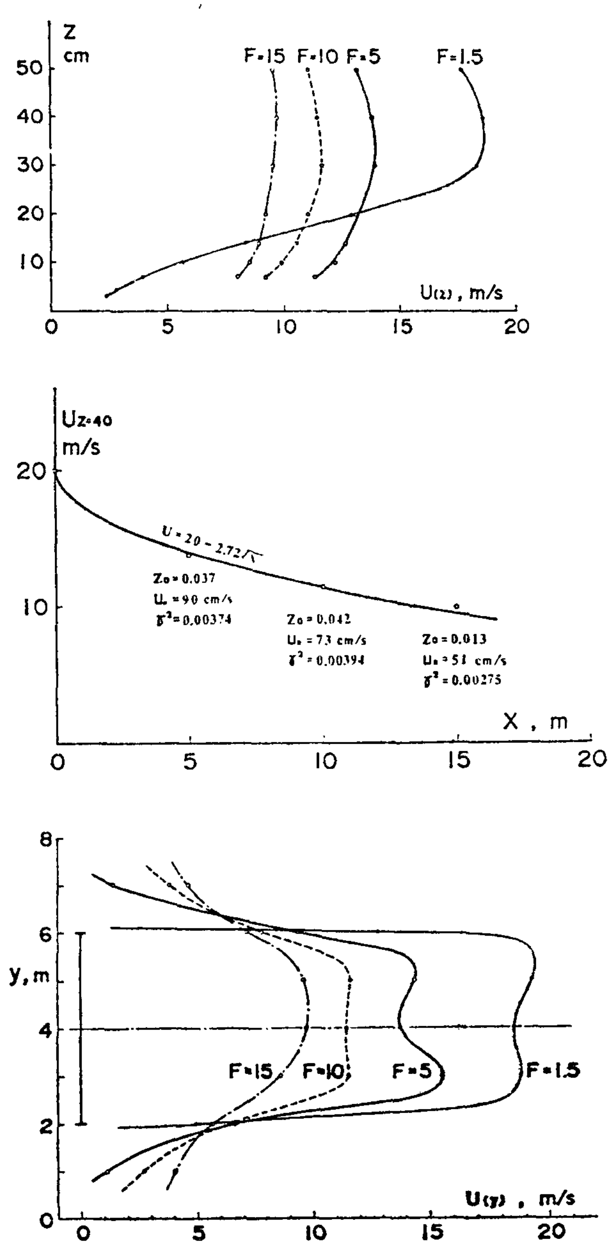

FIG. 2. The structure of wind field for Experiment A. The figures represent, from top to bottom, the vertical, longitudinal and lateral structure of wind speed as a function of fetch. (Reproduced from Mitsuyasu, 1967.)

$$\theta_n = \tan^{-1}\left(\frac{\text{quadrature spectrum}}{\text{cospectrum}}\right)$$

$$= \tan^{-1}\left(\frac{\sin k_n X}{\cos k_n X}\right) = k_n X. \quad (2)$$

Thus the phase velocity is given by

$$C_{\mathrm{I}}(\omega_n) = \frac{\omega_n}{k_n} = \frac{\omega_n X}{\theta_n}. \quad (3)$$

We now introduce a new method of measuring phase velocity. In this method both the directional spectrum and phase velocity are determined simultaneously so that the calculated directional spectrum may give reasonable estimates (for detailed discussion, see I). To define a reasonable directional spectrum, we assume only that the spectral estimate is nearly zero for the direction opposite to the wind. This requirement may not be unreasonable, because the directional spectra reported by other authors showed such a distribution. In I we demonstrated by numerical analysis of artificial wave fields that spurious spectral estimates for the direction opposite to the wind increase in proportion to the deviation of the dispersion relation from the true one. Based on this fact, we can determine both the directional spectrum and the phase velocity (dispersion relation) simultaneously.

The detailed procedure of analysis is as follows: First, five sets of twelve time series are sampled from the records measured by twelve wave detectors. The number N of data points of each time series is 256. Then the time series of each set are Fourier-transformed and processed along the line of theory described in I to obtain the raw directional spectrum. For the data of Experiment A the direct method is applied and for Experiment B, the least-squares method. The raw directional spectra are smoothed by averaging over the five sets.

This procedure is repeated for various dispersion relations ($\omega^2 = \alpha g k$, $\alpha = 0.6-1.6$ at intervals of 0.05). Finally, the most reasonable dispersion relation and corresponding directional spectrum are determined by the condition mentioned above.

Fig. 3 is an example showing how a reasonable dispersion relation is determined. In the figure it can be seen that the directional spectrum at 0°, i.e., opposite to the wind, becomes minimal when the value α is 1.05, and 1.15 and 1.25 for frequencies of 1.60, 1.72 and 1.88 Hz, respectively. In practice the determination is made more carefully by inspecting

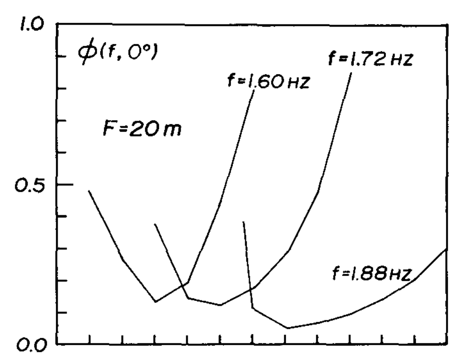

FIG. 3. The α-dependency of the directional spectrum for the direction opposite to the wind. The figure indicates that α values of 1.05, 1.15 and 1.25 give the most reasonable directional spectra for frequencies of 1.60, 1.72 and 1.88 Hz, respectively.

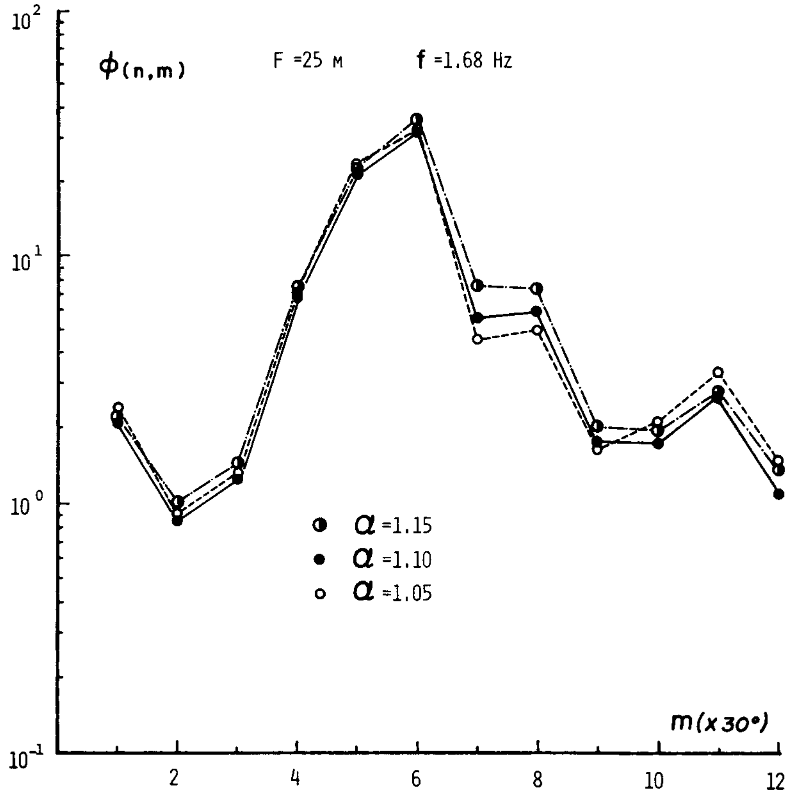

FIG. 4. The α-dependency of the directional spectrum near $\alpha = 1.10$ for the case of $F = 25$ m and $f = 1.68$ Hz. The figure shows that an inaccuracy of α within $\pm 5\%$ gives only a small estimation error in the directional spectrum.

the α-dependencies for two or more directions near the direction opposite to the wind.

The value of α determined in this manner should allow for an inaccuracy of ± 0.05, because the value for α was searched at intervals of 0.05, and because there were some cases where two or three numbers qualified as a reasonable α value. Therefore, it is necessary to determine the extent to which possible inaccuracies in α may yield significant error in the directional spectrum. For this purpose, the α-dependency of the directional spectrum near the reasonable α value ($=1.10$) has been examined and is shown in Fig. 4 for the case of $F = 25$ m and $f = 1.68$ Hz. The results indicate that $a \pm 5\%$ error on α results in only a small estimation error in directional spectrum. [The determined α and corresponding phase velocity are referred to hereinafter as α_{II} and $C_{II}(=\alpha_{II} g/\omega)$, respectively, in comparison with the C_I and corresponding $\alpha_I(=C_I \omega/g)$.]

Before applying the new technique to actual wind waves, a preliminary experiment was carried out in order to check its validity: monochromatic waves of permanent type having a wave period of 0.5 s were generated mechanically in the wind-wave tunnel (Fig. 1b). The wave fields were measured by using Array B, and the phase velocities C_I and C_{II} for the frequency component of 2 Hz were determined through the procedure described above. The mean velocity of individual wave crests C_{ob} was also calculated following the technique of Hamada *et al.* (1953). The results were: $C_I = 83.6$ cm s^{-1}, $C_{II} = 74.1$ cm s^{-1}, $C_{ob} = 75.4$ cm s^{-1}. Since C_{ob}[3] is considered to be the most reliable value in the case of a monochromatic wave field, we may conclude that the new technique does, indeed, give reasonable phase velocities. On the other hand, the apparent error of C_I of no less than 10% may be ascribed to the fact that the method is sensitive in this case to the undesirable effects of leakage and the spectral window in estimating the cross spectrum.

4. Directional spectrum of laboratory wind waves

Two series of experiments were conducted in the experimental tank and the wind-wave tunnel shown in Figs. 1a and 1b. In these experiments, laboratory wind waves of stationary state were measured at several stations. The frequency energy spectra for

[3] While the measured C_{ob} was 75.4 cm s^{-1}, the phase velocity of linear theory is 78.0 cm s)1 for component waves of 2 *Hz*. Possibly some defects in our plunger-type wave-maker system may be responsible for the discrepancy.

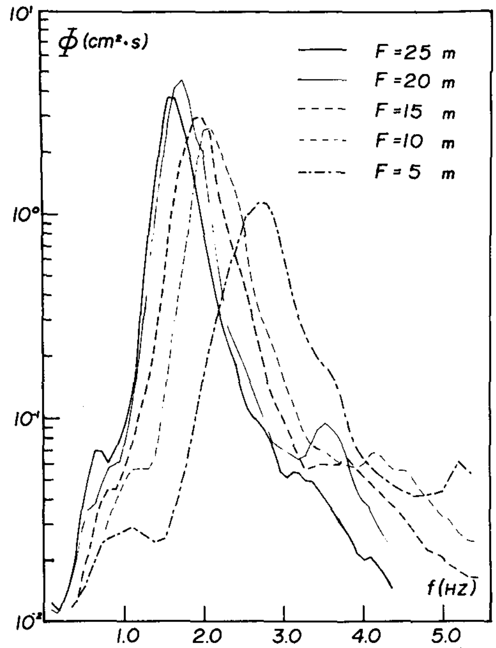

FIG. 5a. Power spectra for the five cases of Experiment A. The spectral estimates are the averages over the 12 wave records.

these wave fields are shown in Figs. 5a and 5b in order to represent the basic situations (the cases of $F = 6$ m and $F = 7$ m are not presented in Fig. 5a). These results were obtained by averaging over the 12 wave records measured simultaneously with the 12 wave detectors. The power spectra for Experiment A are unique in that they show secondary

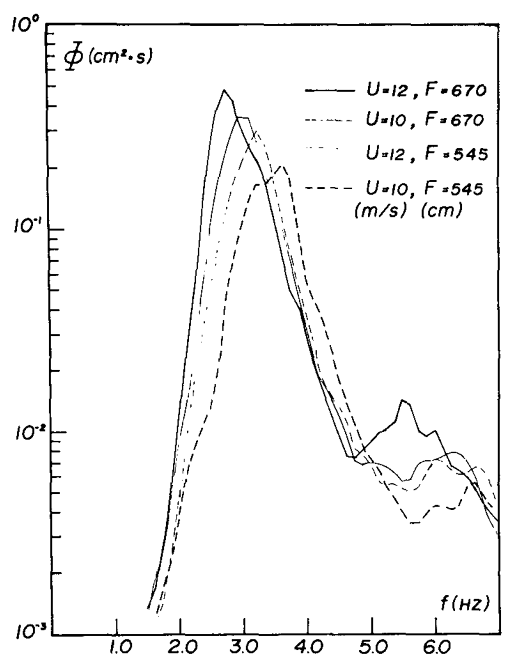

FIG. 5b. As in Fig. 5a except for the four cases of Experiment B.

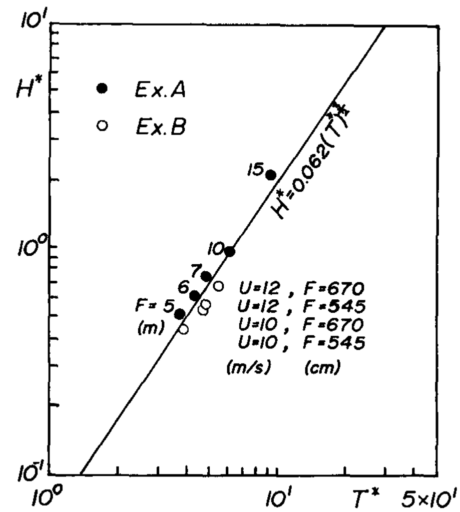

FIG. 6. The dimensionless significant wave height versus the dimensionless significant wave period. The solid line indicates Toba's (1972) proposed relation $H^* = 0.062(T^*)^{3/2}$.

peaks for frequencies lower than the principal peak frequencies. The fact that the wave energy for the case $F = 25$ m is lower than that for the case $F = 20$ m may suggest that the wind waves at $F = 25$ m were in the decay area where the wind speed had decreased.

In order to check the 3/2 power law (Toba, 1972, 1974a)—one of the most important characteristics of wind waves—the dimensionless significant wave height H^* $(=gH_{1/3}/U_*^2)$ has been plotted versus the dimensionless significant wave period T^* $(=gT_{1/3}/U_*)$ in Fig. 6.[4] The experimental results turn out to be in general agreement with the relation $H^* = 0.062(T)^{3/2}$ proposed by Toba.

The directional spectra for Experiments A and B, normalized as

$$\phi_{nm} = \Phi_{nm} / \sum_{m=1}^{M} \Phi_{nm}$$

($M = 12$ for Experiment A and $M = 8$ for Experiment B), are shown in Figs. 7 and 8, respectively. In general, the forms of the angular distribution of wave energy resemble those reported by other authors.

In the figures, the best-fit curves of the form

$$\phi = \phi_{max} \cos^{2S} \tfrac{1}{2}(\theta - \theta_{max}) \qquad (4)$$

have also been indicated. Here ϕ_{max} and θ_{max} denote the maximum directional spectrum and the corresponding angle, and S is a parameter representing a measure of concentration of wave energy in

[4] The significant wave height and period were determined as $T_{1/3} = 1/(1.05f_m)$ and $H_{1/3} = 4[\Phi(f)df]^{1/2}$, respectively, and the values for U_* of Experiment A have been quoted from Mitsuyasu (1967).

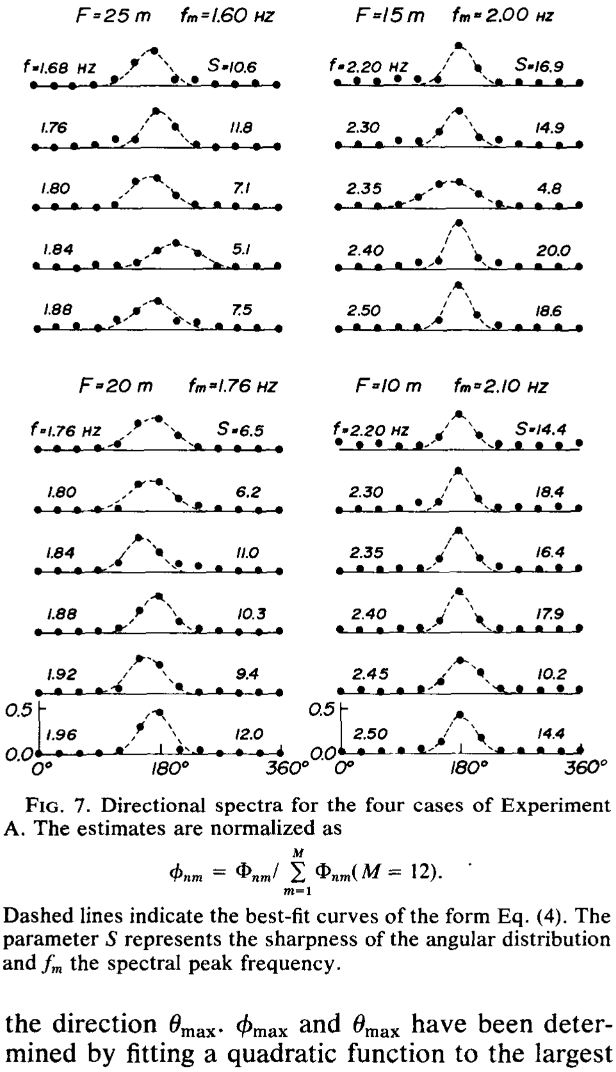

FIG. 7. Directional spectra for the four cases of Experiment A. The estimates are normalized as

$$\phi_{nm} = \Phi_{nm} / \sum_{m=1}^{M} \Phi_{nm} (M = 12).$$

Dashed lines indicate the best-fit curves of the form Eq. (4). The parameter S represents the sharpness of the angular distribution and f_m the spectral peak frequency.

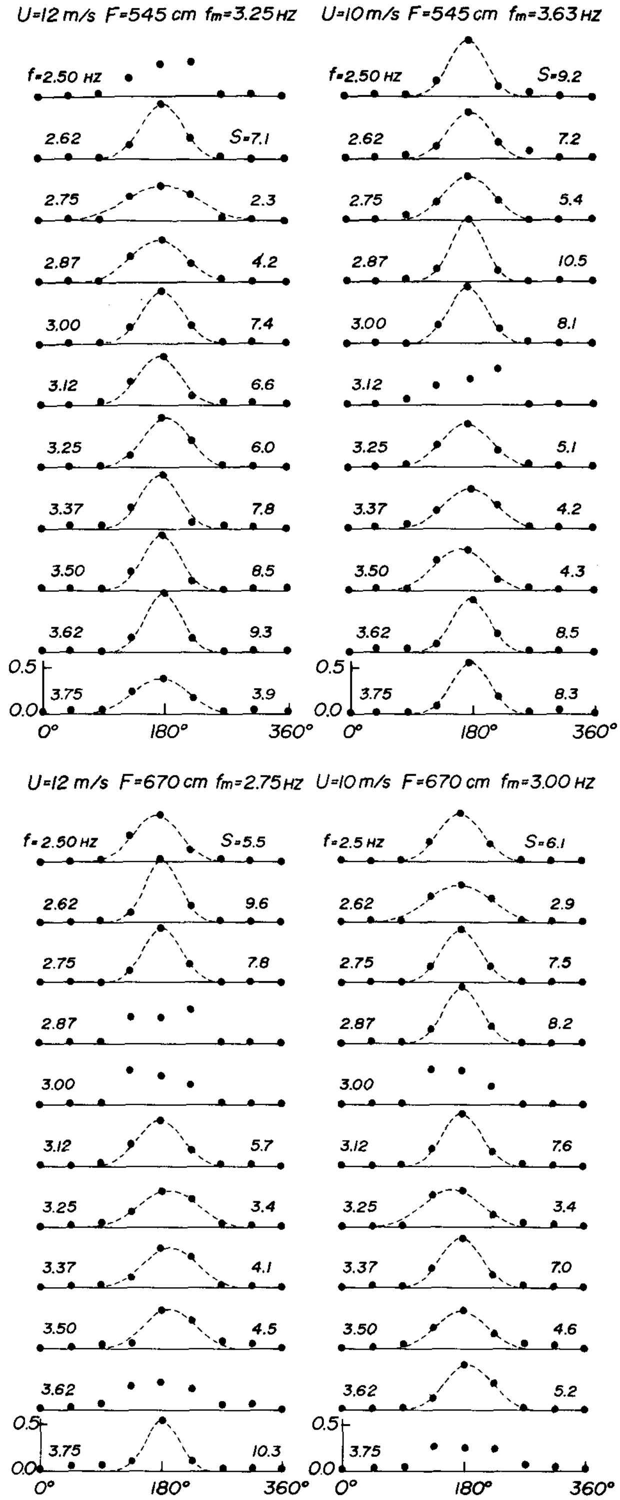

FIG. 8. As in Fig. 7 except for the four cases of Experiment B ($M = 8$).

the direction θ_{max}. ϕ_{max} and θ_{max} have been determined by fitting a quadratic function to the largest three values of each directional spectrum. The parameter S has been determined so that the fitted function minimizes the integration of weighted squares of residuals

$$\epsilon = \sum_{m=1}^{M} [\phi_{nm} - \phi_{max} \cos^{2S} \tfrac{1}{2}(\theta_{nm} - \theta_{max})]^2 \phi_{nm}. \quad (5)$$

The parameter S is indicated in Figs. 7 and 8, and also in Figs. 9a and 9b as a function of θ_{max}. (Some cases of Experiment B have not been fitted a curve and some cases of Experiment A where the measured directional spectrum showed an unreasonable scatter in a wide angular distribution have been omitted.)

According to the results, the value S for laboratory wind waves is large in general as compared with the measurements by other authors: roughly 10 to 20 for Experiment A and 5 to 10 for Experiment B. The larger values for Experiment A may be attributed to the fact that Array A has a higher directional resolution than Array B. This may suggest that, if we use a wave-gage array having a higher resolution than Array A, we can expect to obtain still larger values for the parameter S. In Experiment A

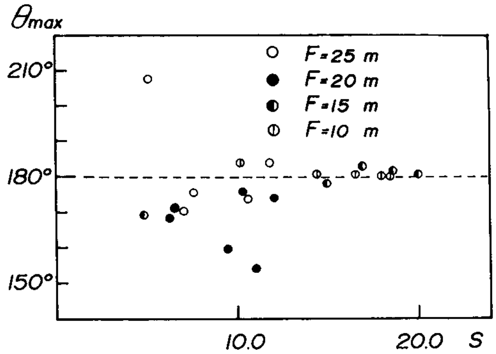

FIG. 9a. θ_{max} versus the parameter S for the four cases of Experiment A. θ_{max}, an angle corresponding to the maximum directional spectrum, has been determined by fitting a quadratic function to the largest three values of directional spectrum.

the value S is larger for F = 10 m and F = 15 m than for F = 20 m and F = 25 m. This may be related to the decrease in wind speed in the regions of larger fetch. Regarding the relation between the parameter S and the wave frequency, we cannot find any consistent correlation for both series of experiments. (Although the value S of Experiment B shows a tendency to take larger values at frequencies near the spectral peak frequency, exceptions can be seen as well.)

In general, θ_{max} = 180°, i.e., the wind direction. Although deviations are found in some cases, they are fairly small and do not show a definite correlation with wave frequency. From Figs. 9a and 9b, we can point out that the deviation of θ_{max} from the wind direction is generally accompanied by a decrease in the S value. The rather large deviations in the cases of F = 20 and 25 m correspond to small values of the parameter S.

As described in Section 2, the wind field of Experiment A had several peculiarities compared with that of usual wind-wave facilities. One of them is the bimodal distribution of wind speed for regions of smaller fetch (see the bottom of Fig. 2). Owing to the peculiar wind field, the wave field also showed a bimodal direction of propagation at smaller fetches. Fig. 10 (courtesy of Prof. H. Mitsuyasu) illustrates the situation very well. In our observational results, the bimodal angular distribution of wave energy is seen for F = 5 m over a wide frequency range. Two examples have been shown in Fig. 11, together with two other examples. For the latter two cases, the bimodal angular distribution does not extend over neighboring frequencies, and has no counterpart in other cases. It is probable, therefore, that they have been caused by some kind of statistical error.

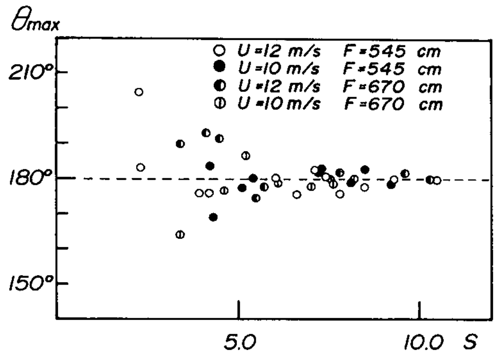

FIG. 9b. As in Fig. 9a except for the four cases of Experiment B.

To summarize the observations, we conclude that the mean wave direction of propagation agrees well with the wind direction, that the bimodal spectral distribution is not seen in general, and that the parameter S is not correlated consistently with the frequency. These conclusions are in contradiction to the existence of the resonant angle [$\pm \sin\theta^{-1}(C/U)$] suggested by Phillips (1957), and also do not agree with works by many authors reporting that the parameter S decreased (or the angular width of the directional spectrum increased) with increasing fre-

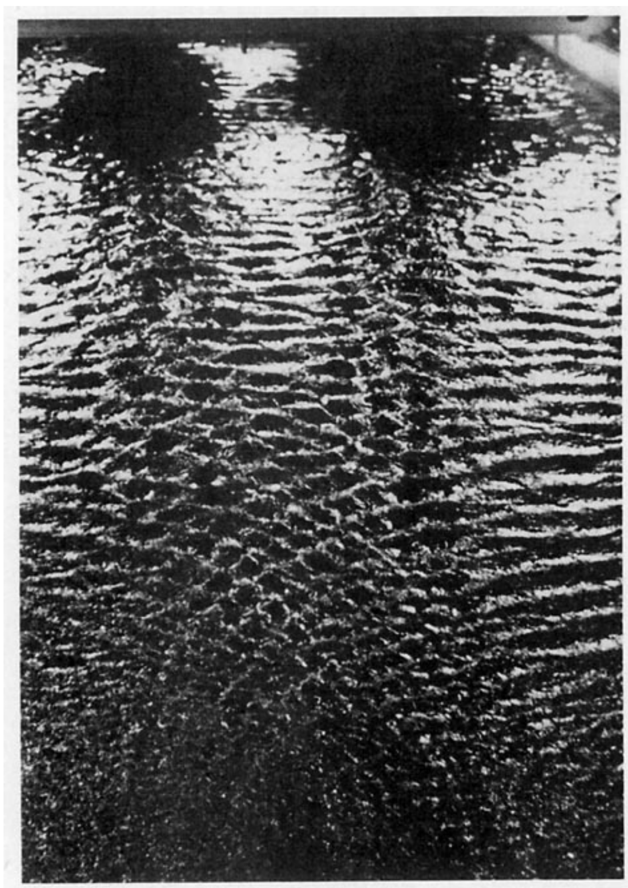

FIG. 10. Wind-wave field peculiar to our wind blower facility. The photograph (courtesy of Prof. H. Mitsuyasu) illustrates clearly the bimodal angular distribution of wave energy near F = 5 m.

quency (Longuet-Higgins *et al.*, 1963; Mobarek, 1965; Gilchrist, 1966; Fujinawa, 1975; Mitsuyasu *et al.*, 1975).

Here we note that the observations by Longuet-Higgins *et al.* (1963) and Mitsuyasu *et al.* (1975) were made in the open ocean, and those by Gilchrist (1966) and Fujinawa (1975) at sea near the coast, while the present investigation was carried out in an experimental tank and a wind-wave tunnel. It is possible, therefore, that differences in some geophysical conditions are responsible for the disagreement.

However, we must point out that the measurements by the above authors are based on the use of an assumed linear dispersion relation $\omega^2 = gk$. In reality, as will be shown in the following section, the dispersion relation of a spectral component of laboratory wind waves in the generating area deviates linearly with frequency from the assumed linear dispersion relation. If the same is true for sea waves, some of their conclusions must be checked by considering the real phase velocity. Indeed we have shown by numerical experiments (Rikiishi, 1977) that the improper use of the linear dispersion relation is expected to yield an apparent increase in the angular spread of the spectrum with frequency if the real dispersion relation deviates linearly with frequency from the linear dispersion relation.

5. Phase velocity of laboratory wind waves

In Section 3 we mentioned a new technique for determining phase velocity from 12 simultaneous wave records. The technique has an advantage over other conventional methods in that it can be applied to a multi-directional wave field. The phase velocities of component waves measured by the method are described below.

In Fig. 12 we show the factor α_{II} ($\omega^2 = \alpha_{II} gk$) and corresponding C_{II} ($C_{II} = \alpha_{II} g/\omega$) for the case $F = 20$ m of Experiment A, together with the phase velocity C_I determined by Eq. (3) and the corresponding α_I. We note that α_{II} increases with increasing frequency, and that the phase velocities of component waves are almost a constant (which is larger than the phase velocity of small-amplitude wave theory for the spectral peak component). This fact may be interpreted as a linear outcome of a nonlinear process. The feature of uniform phase velocity among spectral components has turned out to be common to all other cases. Although the result differs considerably from that of linear small-amplitude wave theory, it agrees well with the observational experience that a given wave crest does not generally seem to get ahead of other crests.

The phase velocity of laboratory wind waves

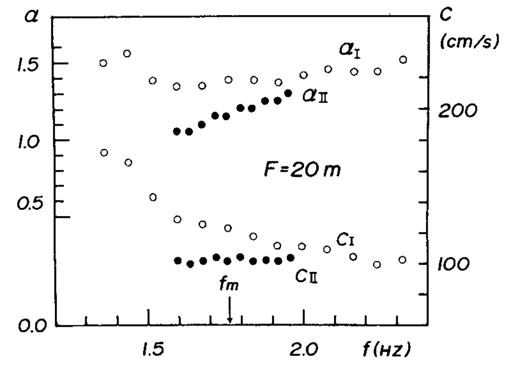

FIG. 12. The phase velocity C_{II} and the factor α_{II} of spectral components for the case $F = 20$ m of Experiment A. For comparison, the phase velocity C_I and the factor α_I determined by Eq. (3) also have been presented.

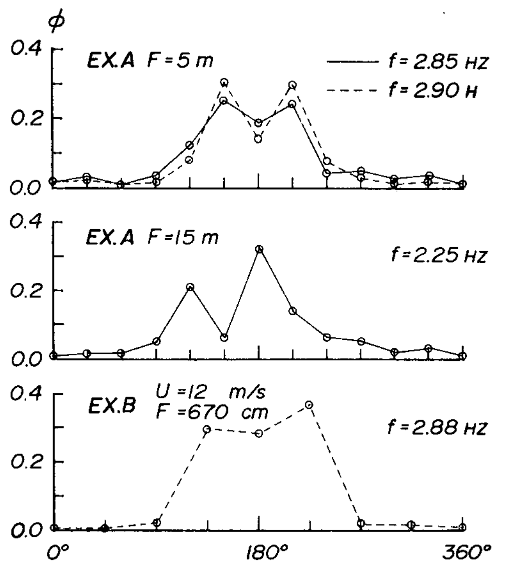

FIG. 11. Some examples of the bimodal angular distribution of wave energy. In the case $F = 5$ m of Experiment A, the bimodal distribution is seen over a wide frequency range, but not in the other two cases.

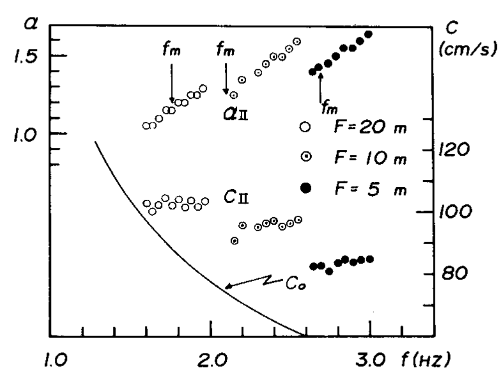

FIG. 13. The phase velocity C_{II} and the factor α_{II} of spectral components as a function of fetch. The line denoted by C_0 represents the phase velocity of the linear small-amplitude wave theory.

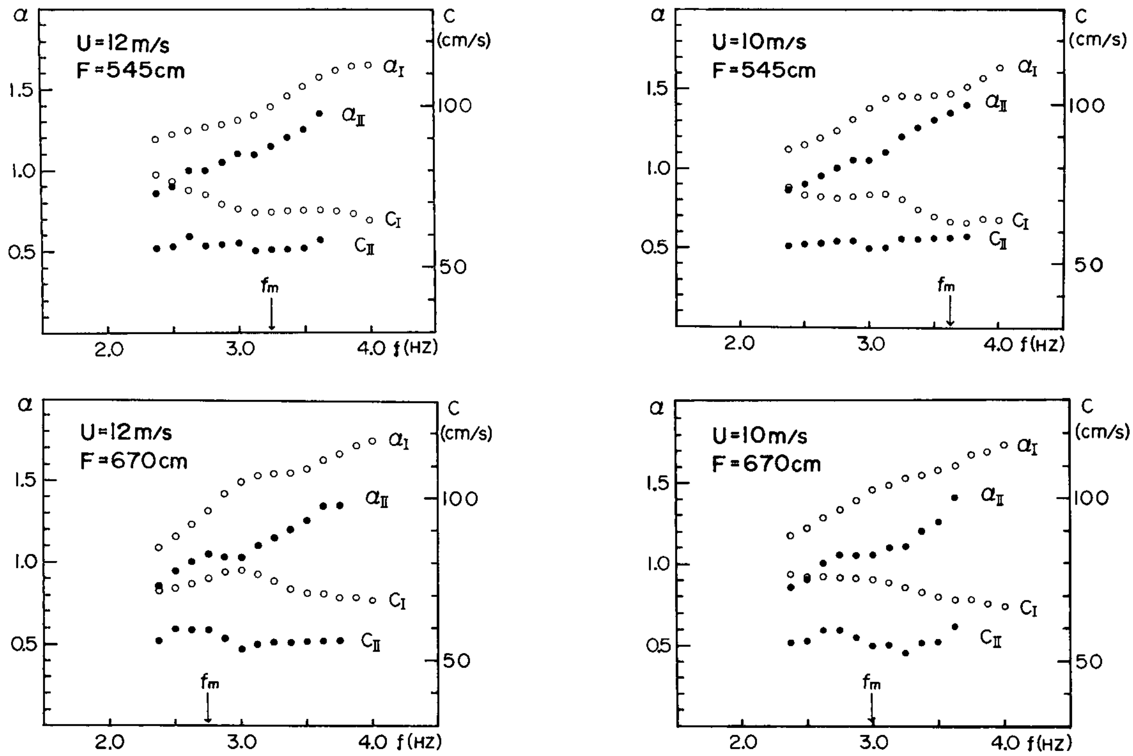

FIG. 14. As in Fig. 12 escept for the four cases of Experiment B.

varies with fetch as well as frequency. To see the variation with fetch, we have shown the measured phase velocities for the cases F = 5, 10, 20 m in Fig. 13. The figure indicates clearly that the uniform phase velocity increases with fetch. This feature is also seen in the distribution of C_I for the spectral peak components (results not presented). (If the wind speed does not decrease with fetch, the increase will be more conspicuous.) In the figure it can be seen that the measured phase velocities approach linear theory with increasing fetch or with decreasing wind speed. This result corresponds well to the measurements by Francis (1951), and to the theoretical conclusion of Lilly (1966) and others that the wind speed or wind-induced drift currently is mainly responsible for the excess phase velocity.

For reference, the values α_I and C_I for the case F = 20 m are also presented in Fig. 12. Because these values are determined by Eq. (3), ignoring the effect of the directional energy distribution, the α_I and C_I may contain significant errors. Indeed Yefimov *et al.* (1972) have shown that neglecting the effect of the directional spectrum yields an apparent increase in measured phase velocity. Therefore the differences between α_I and α_{II}, C_I and C_{II} may be attributed to these errors.

We next examine the results of Experiment B. In Fig. 14 the values α_{II} and C_{II} for the four cases of this experiment are shown. These results also indicate that α_{II} increases linearly with frequency, and that C_{II} does not vary significantly with frequency. On closer inspection of the figure, however, some differences are found between the cases F = 545 and 670 cm: in the cases F = 545 cm the above features of α_{II} and C_{II} are clear, while for F = 670 cm some irregularities can be seen. This may suggest that the wind waves at F = 670 cm were disturbed by the reflected waves from the wave absorber 180 cm to the leeward (see Fig. 1b).

These observational determinations of phase velocity can be summarized in the following three conclusions. First, the phase velocity of the spectral peak component wave is larger, in general, than that determined by the linear theory of small-amplitude wave. The excess phase velocity seems to be proportional to the wind speed. Second, the factor α_{II} increases with frequency and, correspondingly, the phase velocity C_{II} is approximately constant with frequency near the dominant wave frequency. Third, the value of the uniform phase velocity increases with fetch.

The first conclusion has been also reported by many authors, including Francis (1951), Hamada *et al.* (1953), Hidy and Plate (1966), Plate and Trawle (1970) and Shemdin (1972). Lilly (1966) first gave a theoretical explanation for the excess phase velocity by considering the effect of the vertical profile of wind-induced drift current. Kato (1972) also carried out a similar calculation for a more realistic profile than Lilly's. Shemdin (1972) and Mizuno and Mitsuyasu (1973) considered the effect of the wave-induced aerodynamic pressure as well. In general the agreement between theory and experiment is favorable.

The second conclusion is the most conspicuous result of the present investigation. Although other authors including Yefimov *et al.* (1972) and Ramamonjiarisoa (1974) also reported similar characteristics of phase velocity, their results were for component waves of higher frequencies, where the energy level was low and the phase velocities determined by Eq. (3) were subjected to much noise. The uniform phase velocity among various spectral components near the spectral-peak frequency may be one of the most important characteristics of laboratory wind waves in the generating area. Because the field of wind waves has a "strong" nonlinearity as suggested by the uniform phase velocity, theoretical explanations by Lilly (1966) and others for sinusoidal waves may not be applied to the field of nonlinear wind waves. Indeed, the consideration of drift current alone does not seem to account for the fact that α_{II} becomes smaller than 1.0 in the lower frequencies. Huang and Tung (1977) estimated the influence of the directional energy distribution on the dispersion relation by considering weak nonlinear wave-wave interaction in a random gravity wave field, but their theoretical results also do not explain our experimental results.

The third conclusion is illustrated schematically in Fig. 15, together with the first and second conclusions. The figure indicates that the phase velocity of the spectral-peak component is larger than that predicted by the linear theory, that the phase velocity of wind waves is uniform among various spectral components near the spectral-peak frequency, and that the value of uniform phase velocity increases with increasing fetch, i.e., as the wind waves develop. These conclusions state that the phase velocity of a component wave does not remain unchanged, but increases with fetch. This is of great importance in understanding the substance of spectral component waves of laboratory wind waves.

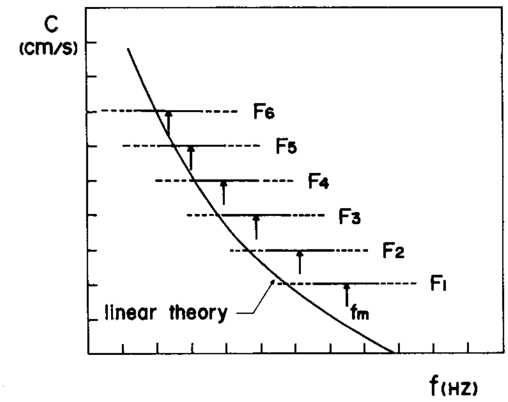

FIG. 15. Schematic representation of phase velocity of laboratory wind waves in the generating area.

6. Discussion

According to the linear small-amplitude wave theory, sinusoidal disturbances on a still water surface propagate along the surface as "free waves" characterized by three variables—wavenumber, frequency and amplitude. Generally the wavenumber is related to the frequency by the dispersion relation. The sinusoidal waves with specific amplitude and specific frequency are often called spectral component waves. The term "component waves" implies that the waves are free waves and independent of each other.

The irregular and complicated pattern of surface elevation of the sea has been assumed to be representable by the linear superposition of a number of such component waves. The introduction of the concept of component waves to the study of wind waves has made the mathematical treatment considerably simpler and easier. Until the present, however, it has not been examined experimentally to see whether or not such a free component wave does exist in actual wind waves.

In general the free component wave should conserve its specific wavenumber, frequency and amplitude. As for the amplitude, however, we allow it to vary with time because the component wave may receive momentum from the air or from two or more other component waves through nonlinear wave-wave interaction. Therefore, the conservation of the wavenumber and frequency is sufficient for the component wave to propagate independently of other components. Accordingly it follows that the phase velocity (or dispersion relation) of the component wave should be conserved, that is, the phase velocity should not vary with both time and fetch during propagation.

As has been seen in the preceding section, however, the phase velocities of component waves near the dominant wave frequency are uniform, and the value of the uniform phase velocity increases with fetch. This means that the phase velocity of a component wave is not independent of other components, but is variable with time and fetch as the wind waves develop.

Thus, measurements of the phase velocity of actual component waves have tended to deny the existence of free component waves. In other words, the observational results may indicate that the physical phenomena of laboratory wind waves is governed by strong nonlinear processes, and that laboratory wind waves may not be expressed well by the linear superposition of free component waves.[5]

The uniform phase velocity of spectral components near the peak frequency means that the individual component wave advances together with other component waves to build up the dominant wave field. The dominant (or significant) wave may change its amplitude, frequency and phase velocity with fetch.[6] On the other hand, it is a fact that the undulation of the water surface with higher frequency can be seen on the surface of the dominant waves. Although these high-frequency undulations may also have gravity wave properties, they do not seem to be persistent in space and time owing to the effect of the wave motion of the dominant waves. As a result the high-frequency undulations may be considered to be turbulence of small scale compared with the dominant wave. The field of laboratory wind waves might, in fact, be regarded as that of the dominant waves plus turbulence of smaller scale.

We have so far discussed the nature of laboratory wind waves in the generating area, and concluded that the wind waves under the direct action of wind stress may not be represented by the linear superposition of free Airy waves. Although it is quite obvious that the winds over the waves are largely responsible for those nonlinear phenomena, we have no theoretical explanation for the dynamics.[7] To determine the theoretical phase velocity, one must take into account the nonlinear effect of finite amplitude, assuming the power spectrum and directional spectrum structure. In addition, one must introduce the effect of wind stress into the mathematical model. Considering that winds over the fluctuating surface may have complicated structure, we doubt whether one can overcome these difficulties successfully.

There have been a number of studies which are concerned with the growth of wind waves on the basis of the superposition of free component waves and the use of the linear dispersion relation $\omega^2 = gk$. The theory of nonlinear wave-wave interaction provides a good example. According to our experimental results, the dispersion relation of laboratory wind waves may be represented by $\omega = \alpha_0 gk$ for frequencies near the spectral peak frequency (α_0 is a constant depending on fetch and wind speed). Then the resonant condition for nonlinear interaction can be satisfied within the framework of a second-order interaction. Thus the works by Phillips (1960) and Hasselmann (1962) may not apply to laboratory wind waves in the generating areas.

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[5] Similar views of the nonlinear aspect of wind waves have been discussed recently by Toba *et al.* (1975) and Okuda *et al.* (1976) based on the detailed observation of wind-induced drift currents. Toba (1974b) also discussed the dual aspects of wind waves—gravity and turbulence—in connection with the 3/2 power law of the significant wave.

[6] Concerning the growth of significant waves, we already know a great deal through the intensive and successful work by many authors including Sverdrup and Munk (1947), Wilson (1965), Mitsuyasu (1968, 1973), Mitsuyasu *et al.* (1975), Toba (1972, 1974a, 1978) and others.

[7] We note here that Mitsuyasu and Kuo (1976) suggested by experimental study that the phase velocity of wind waves in the decay area, being free from the effect of wind stress, roughly followed linear theory.

REFERENCES

Francis, J. R. D., 1951: The aerodynamic drag of a free water surface. *Proc. Roy. Soc. London*, **A206**, 387–408.

Fujinawa, Y., 1975: Measurement of directional spectrum of wind waves using an array of wave detectors. Part II. Field observation. *J. Oceanogr. Soc. Japan*, **31**, 25–42.

Gilchrist, A. W. R., 1966: The directional spectrum of ocean waves: an experimental investigation of certain prediction of the Miles-Phillips theory of wave generation. *J. Fluid Mech.*, **25**, 795–816.

Hamada, T., H. Mitsuyasu, and N. Hase, 1953: Experimental study of wind effects on water surface. *Rep. Trans. Tech. Res. Inst.*, Ministry of Transportation, Japan, No. 8, 1–22.

Hasselmann, K., 1962: On the non-linear energy transfer in a gravity wave spectrum. Part I. *J. Fluid Mech.*, **12**, 481–500.

Hidy, G. M., and E. J. Plate, 1966: Wind action on water standing in a laboratory wave channel. *J. Fluid Mech.*, **26**, 651–687.

Huang, N. E., and C.-C. Tung, 1977: The influence of the directional energy distribution on the nonlinear dispersion relation in a random gravity wave field. *J. Phys. Oceanogr.*, **7**, 403–414.

Kato, H., 1972: The calculation of wave speed for a parabolic current profile (in Japanese). *Proc. 19th Japanese Conf. Coastal Engineering*, Japan, Japanese Soc. Civil Eng., 113–118.

——, and K. Tsuruya, 1974: On the phase velocity of component waves of wind waves. *Proc. 21st Japanese Conf. Coastal Engineering*, Japan, Japanese Soc. Civil Eng., 255–259.

Lilly, D. K., 1966: On the speed of surface gravity waves propagating on a moving fluid. *J. Fluid Mech.*, **26**, 683–686.

Longuet-Higgins, M. S., D. E. Cartwrite, and N. D. Smith, 1963: Observations of the directional spectrum of sea waves using the motions of a floating buoy. *Proc. Conf. Ocean Wave Spectra*, Prentice-Hall, 111–132.

Mitsuyasu, H., 1967: A study on the interaction between water waves and winds (2) (in Japanese). *Proc. 14th Japanese Conf. Coastal Engineering*, Japan, Japanese Soc. Civil Eng., 39–44.

——, 1968: On ghe growth of the spectrum of wind-generated waves (I). *Rep. Res. Inst. Appl. Mech., Kyushu Univ.*, **16**, No. 55, 459–482.

Mitsuyasu, H., 1973: The one-dimensional wave spectra at limited fetch. *Rep. Res. Inst. Appl. Mech., Kyushu Univ.*, **20**, No. 66, 37–53.

——, and Y. Y. Kuo, 1976: A study of wind waves in the decay area—Phase velocity of component waves. *Proc. 23rd Conf. Coastal Engineering*, Japan, Japanese Soc. Civil Eng., 591–596.

——, and K. Rikiishi, 1975: On the growth of duration-limited wave spectra. *Rep. Res. Inst. Appl. Mech., Kyushu Univ.*, **23**, No. 72, 31–60.

——, F. Tasai, T. Suhara, S. Mizuno, M. Ohkusu, T. Honda, and K. Rikiishi, 1975: Observations of the directional spectrum of ocean waves using a cloverleaf buoy. *J. Phys. Oceanogr.*, **5**, 750–760.

Mizuno, S., and H. Mitsuyasu, 1973: Effects of adverse wind on the phase velocity of mechanically generated water waves. *Rep. Res. Inst. Appl. Mech., Kyushu Univ.*, **21**, No. 68, 33–52.

Mobarek, I. E. S., 1965: Directional spectra of laboratory wind waves. *Proc. A.S.C.E.*, **91**, No. WW3, 91–116.

Okuda, J., S. Kawai, M. Tokuda and Y. Toba, 1976: Detailed observation of the wind-exerted surface flow by use of flow visualization methods. *J. Oceanogr. Soc. Japan*, **32**, 51–62.

Phillips, O. M., 1957: On the generation of waves by turbulent wind. *J. Fluid Mech.*, **2**, 417–445.

——, 1960: On the dynamics of unsteady gravity waves of finite amplitude. Part I. *J. Fluid Mech.*, **9**, 193–217.

Plate, E., and M. Trawle, 1970: A note on the celerity of wind waves on a water current. *J. Geophys. Res.*, **75**, 3537–3544.

Ramamonjiarisoa, A., 1974: Contribution à l'étude de la structure statistique et des mecanismes de génération des vagues de vent. Thèse à L'Université de Provence Le Grade de Docteur ès Sciences, 160 pp.

Rikiishi, K., 1977: The role of dispersion relation in measuring the directional wave spectrum. *Rep. Res. Inst. Appl. Mech. Kyushu Univ.*, **25**, No. 79, 103–123.

——, 1978: A new method for measuring the directional wave spectrum. Part I. Description. **8**, 508–517.

Sverdrup, H. Y., and W. H. Munk, 1947: Wind, sea and swell. Theory of relations for forecasting. U. S. Hydrogr. Office, Wash., Publ. No. 601, 44pp.

Shemdin, O. H., 1972: Wind-generated current and phase speed of wind waves. *J. Phys. Oceanogr.*, **2**, 411–419.

Toba, Y., 1972: Local balance in the air-sea boundary processes, I. On the growth process of wind waves. *J. Oceanogr. Soc. Japan*, **28**, 109–120.

——, 1974a: Macroscopic principles on the growth of wind waves. *Sci. Rep. Tohoku Univ.*, Ser. 5, *Geophys.*, **22**, 61–73.

——, 1974b: Duality of turbulence and wave in wind waves. *J. Oceanogr. Soc. Japan*, **30**, 241–242.

——, 1978: Stochastic form of the growth of wind waves in a single-parameter representation and physical implications. *J. Phys. Oceanogr.*, **8**, $$$–$$$.

Toba, Y., M. Tokuda, K. Okuda, S. Kawai, 1975: Forced convection accompanying wind waves. *J. Oceanogr. Soc. Japan*, **31**, 192–198.

Wilson, B. W., 1965: Numerical prediction of ocean waves in the North Atlantic for December, 1959. *Deut. Hydrogr. Z.*, **18**, 114–130.

Yefimov, V. V., Yu. P. Soloviyev and G. N. Khristoforov, 1972: Observational determination of the phase velocities of spectral components of wind waves. *Izv. Atmos. Ocean. Phys.*, **8**, 246–251.