The Transfer of Energy and Momentum by the Wind to the Surface Mixed Layer

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(Manuscript received 15 April 1977, in revised form 18 July 1977)

ABSTRACT

During the initial stages of the deepening of the surface mixed layer, the rate of increase of potential energy is proportional to the input of energy to the mixed layer by the wind. In an attempt to reconcile an apparent discrepancy between the rate of deepening in laboratory experiments (Kato and Phillips, 1969) and in the ocean (Denman and Miyake, 1973), a simple model for the momentum and energy transfer by the wind to surface waves and the mixed layer is suggested. The net transfer of momentum τ_{ml} is the wind stress τ less the local growth of surface wave momentum and the divergence of the surface wave momentum flux, and the net energy transfer $\dot{E}_{ml}$ is the work $\dot{E}$ done on the waves by the wind less the local growth of surface wave energy, the divergence of the surface wave energy flux and the viscous dissipation of the waves. Using the JONSWAP wave observations, the net momentum transfer is 0.97τ (Hasselmann *et al.*, 1973). Using a simple momentum transfer function, allowing direct generation of long gravity waves and capillary-gravity waves, to estimate work done on the waves, the energy actually transferred to the mixed layer is a few percent of τU_{10}, where U_{10} is the 10 m wind speed. The oceanic and laboratory rates of deepening of the mixed layer appear roughly consistent. In addition, the flow in the mixed layer apparently adjusts itself so that the surface flow is $\dot{E}_{ml}/\tau_{ml}$.

1. Introduction

The upper few tens of meters of the ocean are generally well-mixed in temperature and salinity. The surface mixed layer may deepen or stratify under the action of the wind or a buoyancy flux. The rate of deepening of the mixed layer is assumed to be controlled by the production of turbulent kinetic energy. Three primary sources of turbulent kinetic energy are 1) free convection induced by a surface buoyancy flux, 2) the energy input by the wind stress through breaking waves, and 3) shear instability of the flow in and at the base of the mixed layer. Turner (1969) suggested that the rate of change of the potential energy of the layer is a fixed fraction of the energy input by the wind. By dimensional arguments, the energy lost by the wind blowing over the sea is proportional to τU_{10}, where τ is the wind stress and U_{10} the wind speed at a height of 10 m, or U_{10}^3 using a quadratic drag law for the wind stress. Denman and Miyake (1973) calculated a "mixing efficiency," the ratio of the rate of change of potential energy of the layer to τU_{10}, from observations in the northeast Pacific, as 1.2×10^{-3}. Only a very small fraction of the apparent energy input by the wind is used to mix the layer.

In the initial deepening experiment of Kato and Phillips (1969), a mechanical stress was applied to the water; hence the momentum and energy inputs to the mixed layer are known. Kato and Phillips found that the rate of deepening of the layer is inversely proportional to the bulk Richardson number $\mathrm{Ri}=g\Delta\rho H/\tau$, where $\Delta\rho$ is the density jump at the base of the layer with thickness H and applied stress τ, and the rate of increase of potential energy is proportional to τu_*, where $u_*=(\tau/\rho)^{\frac{1}{2}}$ is the friction velocity. The actual input of energy to the mixed layer is τU_s, where U_s is the velocity of the screen applying the stress to the water. The energy input for a given stress depends on the flow in the layer and varies with the stratification below the mixed layer. Using the figure in Kato and Phillips for the screen velocity as a function of the stress, we find that approximately 6–8% of the energy input to the layer is used to increase the potential energy of the layer.

The mixing efficiency of Kato and Phillips is 50–70 times greater than the efficiency estimated by Denman and Miyake. Two obvious sources of the discrepancy come to mind:

1) The energy input by the wind to the ocean is equal to τU_0 only if the wind stress goes entirely into the generation of surface waves with phase speed equal to U_{10}.

2) Not all of the energy input by the wind at the sea surface is actually transferred to the mixed layer. For example, in a developing wave field some of the energy imparted to the waves is advected away by the waves.

The purpose of this paper is to explore these two factors, and show that, with reasonable estimates of wave generation, advection and dissipation, the laboratory experiments of Kato and Phillips may be consistent with oceanic observations of the initial deepening of the surface mixed layer.

2. Exchange of energy and momentum between the wind, surface waves and mixed layer

To determine the momentum and energy input by the wind to the surface mixed layer, we consider the model depicted schematically in Fig. 1. In this model, the momentum supplied by the atmosphere to the ocean across the air-sea interface goes entirely into surface wave generation. The surface waves act as the roughness elements for the atmospheric boundary layer. Thus form drag on the roughness elements will cause surface wave propagation and generation. None of the momentum is used for direct viscous driving of mean currents. The momentum τ_{ml} transferred to the mixed layer is then the total momentum τ transferred by the wind to the surface waves less the local temporal growth of surface wave momentum $\partial\mathbf{M}/\partial t$ and less the momentum advected away by the surface waves $\nabla\cdot\mathbf{S}$, where $\mathbf{M}$ is the total surface wave momentum and $\mathbf{S}$ the wave momentum flux tensor. We do not concern ourselves with the mechanisms to redistribute the momentum within the wave zone, only with the momentum balance integrated vertically over the wave zone and over all wave frequencies. As an example to estimate the terms in the momentum balance, we shall consider a steady wind blowing off a straight coastline where the surface wave spectrum is locally stationary in time. The momentum balance than simplifies to

$$\tau_{ml}=\tau-dS_{11}/dx, \tag{1}$$

where S_{11} is the downwind flux of the downwind wave momentum. In this case, the growth of the wave field is spatial. With the reasonable assumption that a temporally changing wind field may be modeled by changing the effective fetch of the wind, then the example may represent the general oceanic case. Similarly, the energy $\dot{E}_{ml}$ transferred to the mixed layer is the work $\dot{E}$ done on the waves by the wind less the local temporal growth of surface wave energy dE_w/dt, less the energy advected away by the waves $\nabla\cdot\mathbf{F}$ and less the viscous dissipation D of the waves,

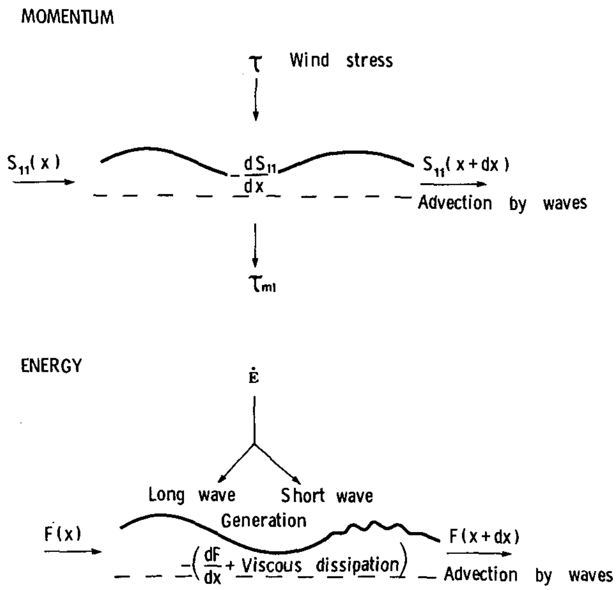

FIG. 1. Schematic models of the momentum and energy transfer from the wind to surface waves and the mixed layer.

where E_w is the total surface wave energy and $\mathbf{F}$ the wave energy flux. For the case considered the energy balance is

$$\dot{E}_{ml}=\dot{E}-dF/dx-D, \tag{2}$$

where F is the downwind component of the wave energy flux. As with the momentum balance, the details of the actual transfer of energy to the mixed layer, such as white capping of long waves to generate turbulence which penetrates into the layer, will not be considered.

Using a quadratic drag law the momentum transfer to the surface waves is

$$\tau=\rho_a C_D U_{10}^2, \tag{3}$$

where the drag coefficient C_D for the purposes of these calculations is assumed to be 1.5×10^{-3} [see Smith and Banke (1975) for a discussion of the variation of the drag coefficient with wind speed] and ρ_a is the density of air (1.22 kg m^{-3} at 15°C). The momentum flux tensor can be determined from the energy spectrum of the waves, i.e.,

$$S_{11}=\int_0^{\Omega}d\omega\int_{-\pi/2}^{\pi/2}E(\omega,\theta)k\omega^{-1}c_g\cos^2\theta d\theta, \tag{4}$$

where c_g is the group velocity, k the wavenumber, ω the frequency, θ the angle from downwind, Ω a high-frequency cutoff and $E(\omega,\theta)$ the energy spectrum. For the example, the surface wave field develops offshore and we shall use the JONSWAP observations of Hasselmann *et al.* (1973). For gravity waves, the

TABLE 1. Estimates of the energy flux and dissipation integrals for various ranges of the JONSWAP spectrum, where the divergence of the energy flux $dF/dx=K\tau U_{10}\tilde{x}^{0.3}$ and the dissipation $D_{GW}=4D'\rho\nu\overline{\zeta^2}\omega_m^4/g$.

	Peak enhancement parameters γ	σ_r	σ_l	$\int_0^\Omega E(\omega)\omega_m/\omega d\omega \Big/ \int_0^\Omega E(\omega)d\omega$	K	$D'=\int_0^\Omega E(\omega)(\omega/\omega_m)^4 d\omega \Big/ \int_0^\Omega E(\omega)d\omega$
Pierson Moskowitz	1	—	—	0.86	1.88×10^{-3}	7.06
Mean JONSWAP	3.3	0.07	0.07	0.92	2.39×10^{-3}	5.15
Sharp JONSWAP	4.2	0.05	0.07	0.92	2.38×10^{-3}	5.17
Very sharp JONSWAP	7.0	0.068	0.123	0.94	6.34×10^{-3}	3.69

energy spectrum is

$$E(\omega,\theta)=E(\omega)S(\theta)=\rho g^3\alpha\omega^{-5} \times\exp\{-5/4(\omega_m/\omega)^4+(\ln\gamma) \times\exp[-(\omega/\omega_m-1)^2/2\sigma^2]\}(2/\pi\cos^2\theta), \quad (5)$$

where the directional spectrum $S(\theta)=(2/\pi)\cos^2\theta$ for $-\pi/2<\theta<\pi/2$ and zero otherwise, ω_m is the peak frequency, α the Phillips constant, γ the peak enhancement and σ the half-width of the enhanced peak which may take different values σ_l and σ_r on the left- and right-hand sides of the peak. The divergence of the momentum flux is

$$dS_{11}/dx=\tfrac{3}{8}\rho g d(\overline{\zeta^2})/dx. \quad (6)$$

where $\overline{\zeta^2}$ is the variance of the sea surface displacement. From JONSWAP the displacement variance increases linearly with fetch, i.e.,

$$\overline{\zeta^2}=1.6\times10^{-7}g^{-2}U_{10}^4\tilde{x}, \quad (7)$$

where the nondimensional fetch $\tilde{x}$ is gxU_{10}^{-2}. For the example, substituting the displacement variance into the momentum flux divergence, the net momentum advection by the wave is a constant independent of fetch and equals 3% of the total momentum input to the waves by the wind, as previously calculated by Hasselmann *et al.* (1973). Long and Huang (1976) observed that except for very small fetches the capillary-gravity wave spectrum is independent of fetch. Thus, the momentum advection by capillary-gravity waves is negligible. For an offshore wind, 3% of the momentum input by the wind is advected away by the wind and the remaining 97% is transferred eventually to mixed layer.

For the energy balance, the downwind gravity wave energy is

$$F=\int_0^\Omega d\omega\int_{-\pi/2}^{\pi/2}E(\omega,\theta)c_g\cos\theta d\theta$$
$$=(4/3\pi)\rho g^2(\overline{\zeta^2}/\omega_m) \times\int_0^\Omega E(\omega)(\omega_m/\omega)d\omega\Big/\int_0^\Omega E(\omega)d\omega. \quad (8)$$

The ratios of the integrals are given for various values of the peak enhancement in Table 1. The peak frequency decreases with increasing fetch (Hasselmann *et al.*, 1973), such that

$$\omega_m \doteq (22g/U_{10})\tilde{x}^{-0.3}. \quad (9)$$

Thus the energy advected away by the surface waves will increase with increasing fetch and is proportional to τU_{10}, i.e.,

$$dF/dx=K\tau U_{10}\tilde{x}^{0.3}, \quad (10)$$

where the proportionality constant is listed in Table 1.[1] Since the capillary-gravity wave spectrum is effectively independent of fetch, the energy advection from capillary-gravity waves is negligible.

The energy transferred by the wind to the waves can be calculated from the momentum transfer function $G(\omega)$ and the phase speed of the waves, assuming that the waves receiving momentum from the wind will also receive energy, with the constraint that all

[1] From the JONSWAP observations, no systematic variation with fetch occurs for the peak enhancement parameters γ and σ. Thus, assuming similarity of the form of the spectrum requires the variance of the wave height $\overline{\zeta^2}$ to be proportional to $\alpha\omega_m^{-4}$. The actual variations with fetch of the wave height variance, peak frequency and Phillips constant given by Hasselmann *et al.* (1973) are not consistent with the mean JONSWAP peak enhancement parameters. We assume the spectral form similarity is valid and associate the above inconsistency with uncertainty in the determination of the Phillips constant.

of the momentum transferred goes into the waves or

$$\int_0^\infty G(\omega)d\omega = \tau.$$

The momentum transfer function will vary with sea state, but we propose a simple form to estimate the energy transfer. Dobson (1971) observed that the momentum transfer to gravity waves often occurred in a broad peak at slightly higher frequencies than the peak in the wave energy with the total momentum transfer to gravity waves approaching the total momentum transferred by the wind. Larson and Wright (1975) in a laboratory experiment found that 75% of the wind momentum transferred by the wind was given to capillary-gravity waves near the minimum phase speed, $c_{\min}=0.23$ m s^{-1}, at a frequency of 84.2 s^{-1}. We shall approximate the actual momentum transfer function by a constant in the band of ω_m to $5\omega_m$ with 40% of the total momentum transferred in this band near the peak frequency of the wave energy and the remaining 60% input at 84.2 s^{-1}, i.e.,

$$G(\omega)=\begin{cases} 0.1\tau/\omega_m, & \omega_m<\omega<5\omega_m \\ 0.6\tau\delta(\omega-\omega_c), & \omega=\omega_c=84.2\ \text{s}^{-1} \\ 0, & \text{elsewhere.} \end{cases} \quad (11)$$

The energy input by the wind to the waves is

$$\dot{E}=\int_0^\infty G(\omega)c(\omega)d\omega = 7.3\times10^{-3}\tau U_{10}\tilde{x}^{0.3}+0.6\tau c_{\min}. \quad (12)$$

We note that the energy input to gravity waves has a form similar to the energy advected away by the gravity waves. Increasing the fraction of the momentum transferred to gravity waves or decreasing the width of the frequency band will increase the energy transferred. The actual difference between the energy transferred to the gravity waves and that advected away in uncertain, but for moderate fetches, $\tilde{x}\sim10^3$, the difference is likely to be a few percent of τU_{10}. The amount of energy input to the waves is only a small fraction of the total energy lost by the atmospheric boundary layer.

In (12) we have ignored a contribution to $c(\omega)$ from the orbital velocity of the rest of the spectrum. In particular, following Garrett and Smith (1976), we note that any variation in the momentum transfer from the wind to the sea correlated with the orbital velocity of long waves can contribute to the growth of long waves. To obtain an upper bound on the long-wave growth due to short waves, suppose the momentum transfer τ_{sw} to the short waves varies linearly from crest to trough of the long wave like $\tau_{sw}=\tau(1-\cos\theta)$, where θ is the phase of the long wave, and all the sea level variance is at the gravity wave spectral peak. Then the growth of the long-wave momentum M_{lw} and energy E_{lw} is

$$dM_{lw}/dt=\tfrac{1}{2}k_l a_l \tau = 0.14\tilde{x}^{-0.1}\tau, \quad (13)$$

$$dE_{lw}/dt=\tfrac{1}{2}k_l a_l \tau c_l = 6.2\times10^{-3}\tilde{x}^{0.2}\tau U_{10}. \quad (14)$$

Thus, for moderate fetches, $\tilde{x}\sim10^3$, say, using the JONSWAP observations, up to about 7% of the wind stress and 2% of τU_{10} are transferred to the long waves from the short waves.

Viscous dissipation of the waves is the remaining term in the energy balance, given by Phillips (1966) as

$$\epsilon=-4\mu(\omega^2/k)|\nabla\zeta|^2. \quad (15)$$

The total dissipation D is the integral overall wave frequencies, but we shall consider separately the contributions from gravity waves and capillary-gravity waves. The former is estimated by integrating over the JONSWAP spectrum. The dissipation integral is logarithmic at high frequencies with the estimates for a cutoff at $5\omega_m$ given in Table 1. The gravity wave dissipation is proportional to the Phillips constant which has a weak inverse fetch dependence. Comparing the dissipation to the advection of energy,

$$D_{GW}/(dF/dx)=(2.7\times10^3\tilde{x}^{-0.5}\ \text{m}^3\ \text{s}^{-3})U_{10}^{-3}, \quad (16)$$

we see that dissipation is small for moderate fetches and wind speeds $\gtrsim10$ m s^{-1}. The capillary-gravity wave dissipation is estimated from the slope spectrum $|\nabla\zeta|^2$ which is inversely proportional to the frequency from approximately 30 to 500 s^{-1} with little dependence on wind speed or fetch (Long and Huang, 1976). The dissipation is comparable to the energy input to capillary-gravity waves for wind speeds less than about 10 m s^{-1}, i.e.,

$$D_{CGW}/0.6\tau c_{\min}\approx(43\ \text{m}^2\ \text{s}^{-2})U_{10}^{-2}. \quad (17)$$

3. Discussion

The energy input to the mixed layer is the net of two contributions, the residual of direct gravity wave generation and dissipation, long-wave modification of short-wave generation and divergence of long-wave energy flux proportional to τU_{10} and the residual of capillary wave generation and dissipation proportional to τ. As the wind speed increases, the relative importance of the gravity wave input will increase. At low wind speeds, viscous dissipation of the waves plays an important role in limiting the energy input to the mixed layer. Thus, a reasonable estimate of the surface energy input to the mixed layer is a few percent of τU_{10}.

The laboratory experiments of Kato and Phillips (1969) show an increase in potential energy of the mixed layer of approximately 6–8% of the surface energy input. In these experiments, for a given stress, the speed of the screen applying the stress and, hence, the surface energy input vary slightly with the flow in the layer and the stratification below the layer. Hence, the actual dynamics of the mixed layer are more complicated than simple penetration of turbulence from the surface. The deepening in the experiments of Kato and Phillips and of the oceanic surface mixed layer during the observations of Denman and Miyake (1973) would be consistent, if the net oceanic surface energy input is 0.02 τU_{10}. Our model yields a surface energy input of 0.04 τU_{10} to 0.09 τU_{10}, approximately two to four times greater depending upon the wind speed and fetch.

In the energy transfer to the mixed layer, we include the energy that goes into wave breaking rather than subtracting this energy as part of the dissipation. During wave breaking, turbulence is generated and viscous dissipation occurs. The breaking-wave-generated turbulence may deepen the mixed layer directly by penetrating to the bottom of the layer to entrain fluid or by increasing the mean flow and shear across the bottom of the layer through Reynolds stress interactions. Thus, dissipation within the layer plays an important role in determining the rate of deepening of the layer. For the observations of Denman and Miyake, dissipation within the mixed layer leads to a decrease in the apparent surface energy input. Without discussing in detail the mechanics of wave breaking or the dynamics of the mixed layer, the rates of deepening in the laboratory and the oceanic mixed layers appear roughly consistent.

For the wind-mixed layer the surface momentum and energy inputs are determined by the wind speed and sea state properties. The ratio of the energy input to the momentum input is a velocity

$$U_s = \dot{E}_{ml}/\tau_{ml}. \tag{18}$$

Kullenberg (1976) and Turner (1969), using observations of mixed layer deepening, estimate the surface energy input as $\tau U_{\omega d}$, where $U_{\omega d}$ is the surface wind drift. The apparent surface velocity from our model is roughly consistent with wind drift observations. Thus, the flow in the layer adjusts itself to the surface inputs rather than responding to the wind stress working on the mean surface flow.

We can use the model to consider other observations that are apparently inconsistent with Kato and Phillips. Wu (1973) investigated in the laboratory the wind-induced mixing in a two-layer system. The rate of deepening of the upper layer was only a tenth of the rate observed by Kato and Phillips for the same stress. The rate of increase of potential energy was only 5×10^{-5} τU_{10} or 2.4×10^{-3} τU_s. The maximum fetch of Wu's experiments was only 2.4 m. Hence we expect that considerable wave growth was occurring downwind. Thus the momentum and energy transfer from the wind to the mixed layer should be corrected for this wave growth. For simplicity we assume that all the wave growth occurs at 84.2 s^{-1}, the minimum phase speed of capillary-gravity waves. Due to the short fetch we do not expect any long waves to be present, although a line spectrum is a crude approximation to the actual wave spectrum. According to Wu, 53% of the wind stress is advected downwind by the waves. We estimate the work done by the wind on the waves as $\tau c_{\min}$, of which 53% is advected downwind and 15% dissipated by viscosity. The observed rate of increase of potential energy is 1% of the energy input to the mixed layer. While this estimate is still smaller than the mixing efficiency of Kato and Phillips, it is encouragingly closer than the estimate of Wu despite the very crude wave spectrum assumption. In addition, as suggested by Wu, the mean flow forced by the wind setup in the tank may modify the shear across the base of the mixed layer and decrease the entrainment. The expected surface current $\dot{E}_{ml}/\tau_{ml}$ is 0.16 m s^{-1} compared to Wu's observation of 0.27 m s^{-1}, within a factor of 2. We should note that the line spectrum assumption limits the surface current to a maximum of $c_{\min}=0.23$ m s^{-1}, which indicates that knowledge of the actual wave spectrum is required for a more accurate calculation of the surface current and the momentum and energy transfers.

We have not discussed the dynamics of the mixed layer nor the mechanisms for mixing and the penetration of energy from the surface to the mixed layer. During the initial deepening of the mixed layer, the mean flow in the layer is in the direction of the wind. For longer time scales, comparable to an inertial period, the earth's rotation modifies the direction of the mean flow relative to the wind. Price *et al.* (1977) find little evidence for deepening proportional to τU_{10} over times comparable to an inertial period. In addition, Langmuir circulations may enhance the mixing within the layer in a fashion only indirectly related to local wind and sea state. Thus, the surface momentum and energy inputs are not necessarily directly related to the mixing in the layer as suggested by the initial deepening experiments of Kato and Phillips. However, when modeling the dynamics and energetics of the mixed layer, the actual momentum and energy transfers from the wind to the layer should be calculated from the wind and sea state in a manner similar to that proposed here.

Acknowledgments. We thank Ken Denman, Fred Dobson and Jim Elliott of the Bedford Institute of Oceanography for helpful comments, Klaus Hasselmann for discussions which lead to the discovery of an error in an earlier version of the paper, and the National Research Council of Canada for financial support.

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