Implications of an f^{-4} Equilibrium Range for Wind-Generated Waves

D. RESIO

Offshore & Coastal Technologies, Inc., Vicksburg, Mississippi

W. PERRIE

Department of Fisheries and Oceans, Bedford Institute of Oceanography, Dartmouth, Nova Scotia, Canada

(Manuscript received 10 August 1987, in final form 23 August 1988)

ABSTRACT

The existence of an f^{-5} equilibrium range was hypothesized for middle to high frequencies for a well-developed sea generated by the physical parameters of gross sea state in the pioneering work of Phillips. Various experimental studies since then, notably JONSWAP, have shown that if the power law is −5, then the proportionality constant is frequency dependent. Recent Lake Ontario data has shown an f^{-4} variation, which agrees with models of the equilibrium range as a Kolmogorov cascade. From this, the JONSWAP fetch relations, and appropriate assumptions about momentum transfer are shown to imply an important new spectral form for energy transfer from wind to wave, which differs slightly from other recent attempts. With suitable parameter relations, the midrange spectral energy can be shown to be essentially the same as its well known f^{-5} counterpart.

1. Introduction

In recent years research has focused on several areas related to wind-wave generation. Better field data has become available and improved understanding of some of the basic nonlinear terms has been achieved. Also, new wave models have been, and are continuing to be, developed and concepts of remote sensing of the ocean surface have been fostered. In light of this activity, it seems appropriate to examine some basic theoretical concepts relative to the implications of some recent empirical findings. In particular, in this paper we shall investigate some consequences of momentum fluxes within the spectrum, related equilibrium range characteristics, and the general growth rates of wind-driven waves.

It is generally evident that wind-wave spectra have a very sharp cutoff at frequencies below the peak, i.e., on the forward face, and a somewhat more gently sloped rear face. In his pioneering work, Phillips (1958) hypothesized the existence of an equilibrium range in high frequencies for a wind-generated sea. The limiting shape geometry of the sharp crests was assumed to be constrained by breaking of their steepest members; and dimensional analysis resulted in spectral energy densities for high frequencies of the rear face, given by

$$\phi(\omega) \sim \alpha g^2 \omega^{-5} \tag{1.1}$$

where α was assumed a universal constant, g is the gravitational acceleration and ω the angular frequency. Under this hypothesis, the energy levels in the equilibrium range were viewed as independent of coincident wind speeds.

The first systematic attempts at looking for an equilibrium rear face for the ω^{-5} type spectrum were by Burling (1959) and Kinsman (1960). Subsequently, Longuet-Higgins (1969) related α to wave age; and Hasselmann et al. (1973) related α to dimensionless fetch. These results demonstrated that α was not a universal constant but rather varied as a function of certain wave generation parameters. Studies by Garrett (1969), Ramamonjiarisoa (1973), Toba (1973), Kitaigorodskii et al. (1975), Mitsuyasu et al. (1975), Forristall (1981) and Kahma (1981), have since shown that the rear face might not follow a strict ω^{-5} law. Furthermore, Zakharov and Filonenko (1967) presented theoretical arguments which suggested that the power law for the rear slope of a spectrum should be −4, not −5. Thus, at the beginning of the 1980s substantial uncertainty existed regarding both the power law of the rear slope of the spectrum and the response of this portion of the spectrum to various external mechanisms.

This uncertainty may have important consequences relative to wave growth rates. Nonlinear wave–wave interactions, regarded as primarily responsible for the transfer of energy to the forward face of the spectrum, are highly dependent on spectral shape, as are most wind input functions. Toba (1973) demonstrates that a consequence of an ω^{-4} power law is that wave growth in time follows a different power law than for an ω^{-5} power law.

Corresponding author address: Dr. Donald T. Resio, Offshore and Coastal Technologies, Inc., 911 Clay Street, Vicksburg, MS 39180.

Donelan et al. (1985) present carefully taken deep-water wave data from Lake Ontario (Fig. 1). In this figure, spectral energies are multiplied by ω^4 and normalized by the average level of the spectral estimate multiplied by ω^4 in the region from $1.5\omega_p$ to $3.0\omega_p$, where ω_p is the peak frequency. The results shown in Fig. 1 are consistent with recent field studies of the equilibrium range (Mitsuyasu et al. 1975; Forristall 1981; Kahma 1981). Consequently, empirical support for the -4 power law is becoming quite convincing, while support for the -5 power law is diminishing.

The ω^{-4} variation observed in recent studies is consistent with Zakharov and Filonenko's (1967) theoretical result, derived as an exact analog of a Kolmogorov spectrum, which assumes that energy cascades through a spectrum from low to high frequencies. This cascade process was first introduced by Kitaigorodskii (1962) who in a later work (Kitaigorodskii 1983) starts with the stationary form of Hasselmann's (1962, 1963a,b) Boltzmann integrals for nonlinear transfer due to wave–wave interactions and obtains an estimate of an equilibrium range of the form

$$\phi(\omega) \sim \epsilon_0{}^{1/3} g \omega^{-4} \tag{1.2}$$

where ϵ_0 is the energy flux via this cascade process from a region near $\omega = 0$ toward a region near $\omega = \infty$. In this process it is presumed that the spectrum adjusts to a form such that the energy flux is a constant.

Resio (1987) rederived Kitaigorodskii's (1983) results with generalizations to finite depth, using the inherent geometry of the Boltzmann integral to explicitly scale the nonlinear transfer due to wave–wave interactions. Expressions for energy flux were also presented, and were found to be compatible with an ω^{-4} equilibrium range.

At present, most spectral wave models are still formulated on the basis of an ω^{-5} equilibrium range. In this paper, we shall investigate some of the implications of an ω^{-4} equilibrium range on wave generation and spectral characteristics in actively growing seas. Our approach here will be not to investigate detailed balances of various source terms, such as have been reported by Komen et al. (1984), Hasselmann et al. (1985) and Hasselmann and Hasselmann (1985), but, rather, to focus on establishing a general idea of wave growth and spectral dynamics that is consistent with the ω^{-4} equilibrium range and observed growth rates along a fetch.

2. Theoretical considerations

a. Background

In deep water with small ambient current velocities, the equation for energy transfer into and out of an element in a directional spectrum of surface waves can be written as

$$\frac{\partial E(f, \theta)}{\partial t} = \vec{c}_g(f) \cdot \nabla E(f, \theta) + \sum_{i=1}^{N} S_i \tag{2.1}$$

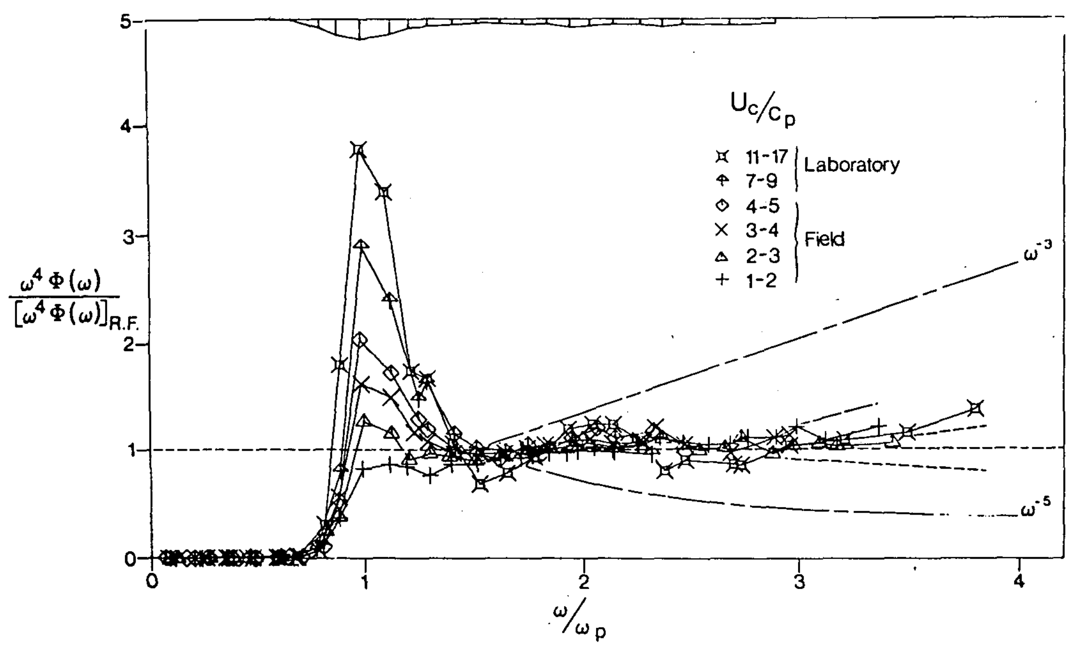

FIG. 1. Frequency spectra times ω^4 normalized by the rear face $\omega^4\Phi(\omega)_{r.f.}$ which is the average of $\omega^4\Phi(\omega)$ in the region $1.5\omega_p < \omega < 3\omega_p$. The lines corresponding to ω^{-5} and ω^{-3} are also shown (long–medium dashed). The effect of a 10 cm s^{-1} ambient current with or against the waves is also shown (medium dashed) as is the effect of wind drift in a 10 cm s^{-1} wind (short dashed). The spectra are grouped in classes of U_c/c_p (from Donelan et al. 1985).

where $E(f, \theta)$ is the energy density at frequency f and propagation direction θ, $\vec{c}_g$ is the group velocity vector for that frequency-direction element and S_i is a source/sink mechanism. Several recent papers examine energy balances among various source/sink mechanisms under certain conditions (Komen et al. 1984; Hasselmann et al. 1985; Hasselmann and Hasselmann 1985). Rather than emphasize that particular approach, we shall examine only certain gross characteristics of the wave generation process. In this context, let us examine only the rate of gain of total energy at a particular point on the sea surface. In other words, let us examine $\partial E_0/\partial t$ where E_0 is given by

$$E_0 = \int_0^{2\pi} \int_0^{\infty} E(f, \theta) df d\theta. \quad (2.2)$$

In the absence of swell, total wave energy must be related to the shape of the spectrum and the location of the spectral peak. For an f^{-5} equilibrium range, we have a dimensionally consistent spectrum of the form

$$E(f, \theta) = \frac{\alpha_1 g^2 f^{-5}}{(2\pi)^4} \psi_1\left(\frac{f}{f_m}\right) \Lambda_1\left(\theta - \theta_0, \frac{f}{f_m}\right) \quad (2.3)$$

where α_1 is a dimensionless equilibrium range coefficient, g is the acceleration due to gravity, f_m is the frequency of the spectral peak, ψ_1 is a dimensionless shape function, which depends on the "overshoot" behavior and the drop-off of energy on the forward face and Λ_1 describes the angular distribution of energy around the mean angle θ. Similarly, for an f^{-4} equilibrium range we have a dimensionally consistent spectrum of the form

$$E(f, \theta) = \frac{\alpha_2 V g f^{-4}}{(2\pi)^3} \psi_2\left(\frac{f}{f_m}\right) \Lambda_2\left(\theta - \theta_0, \frac{f}{f_m}\right) \quad (2.4)$$

where V is a velocity scaling parameter and α_2, ψ_2 and Λ_2 are analogous to α_1, ψ_1 and Λ_1 in Eq. (2.3).

Since ψ_1 and Λ_1 are dimensionless, if we assume that they are not strongly affected by some external parameter, the total energy in an f^{-5} spectrum can be represented as

$$E_{0_5} = \lambda_1 \alpha_1 g^2 f_m^{-4} \quad (2.5)$$

where the subscript "5" refers to the f^{-5} equilibrium range characteristics and λ_1, is a dimensionless constant. Under the same constraints, for an f^{-4} spectrum we have

$$E_{0_4} = \lambda_2 \alpha_2 g V f_m^{-3} \quad (2.6)$$

where the subscript "4" refers to the f^{-4} equilibrium range characteristics and λ_2 is a dimensionless constant.

Hasselmann (1962) established a strong theoretical foundation for the existence of weak nonlinear interactions among various waves in a wave spectrum. The governing equation for this process as derived by Hasselmann can be written as

$$\frac{\partial N(\mathbf{k}_1)}{\partial t} = \int\int\int\int\int\int D(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) \times C(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) \delta(\omega_1 + \omega_2 - \omega_3 - \omega_4) \times \delta(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_3 - \mathbf{k}_4) d\mathbf{k}_2 d\mathbf{k}_3 d\mathbf{k}_4 \quad (2.7)$$

where $\mathbf{k}_1$, $\mathbf{k}_2$, $\mathbf{k}_3$ and $\mathbf{k}_4$ are the vector wavenumbers of the four interacting waves, D is a function of the energy densities at these wavenumbers, C is an algebraically complex coupling coefficient and $\delta(\cdot)$ is the Kroneker delta function. Webb (1978) showed that a simpler form of this integral, at least in terms of numerical integration considerations, could be achieved by removing the delta functions from inside the integral and re-writing the equation in terms of action density, $n(k)$, related to energy density by the relationships

$$N(\mathbf{k}) = \frac{F(\mathbf{k})}{\omega}.$$

Webb's form of the wave–wave interaction or collision integral can be written as

$$\frac{\partial N(\mathbf{k}_1)}{\partial t} = \int_{\mathbf{k}_3} \int \oint C(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) \times D(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) \left|\frac{\partial W}{\partial n}\right|^{-1} ds d\mathbf{k}_3 \quad (2.8)$$

where D is a function of the action densities at $\mathbf{k}_1$, $\mathbf{k}_2$, $\mathbf{k}_3$ and $\mathbf{k}_4$ given by

$$D(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) = N(k_1)N(k_3)N(k_4) + N(k_2)N(k_3)N(k_4) - N(k_1)N(k_3)N(k_2) - N(k_1)N(k_2)N(k_4) \quad (2.9)$$

s and **n** define alongcontour and normal-to-contour directions and $W = \omega_1 + \omega_2 - \omega_3 - \omega_4$.

Tracy and Resio (1982) and Resio (1987) have shown that Eq. (2.8) can be further simplified for numerical integration by implementing it on a polar grid with geometrically spaced increments along the radials. As shown by Resio (1987) this form of equation allows us to examine certain inherent dimensional and geometric properties of the collision integral. Consistent with the findings of Kitaigorodskii (1983), Resio (1987) showed that a constant energy flux condition, as required in a dynamic equilibrium, could only exist in deep water for the case of an f^{-4} spectrum.

Kitaigorodskii and Resio's theoretical results appear to be consistent with Toba's (1973) form for wave spectra, but not with the generally accepted JONSWAP form for equilibrium range wave spectra, or its related shallow-water spectral form which has been termed the TMA spectrum.

Recent studies of deep-water wave spectra (Donelan et al. 1985; Forristall 1981; Kahma 1981) have supported the existence of an f^{-4} equilibrium range. Resio (1987) has presented data which support the existence of a generalized finite-depth form of the f^{-4} spectral shape (Fig. 2). Thus, it now appears that an f^{-4} equilibrium range may be more justifiable on theoretical grounds than an f^{-5} equilibrium range.

If, in fact, an f^{-4} spectral shape is more appropriate than an f^{-5} spectral shape for wind generated waves, what differences exist in the predicted wave growth patterns? The remainder of this section will examine this question by first developing some concepts of wave growth for an f^{-4} based spectrum. Then, wave growth with fetch and duration for an f^{-4} spectrum is estimated and compared with growth patterns for an f^{-5} spectrum.

In a number of wave growth experiments (Mitsuyasu 1968; Hasselmann et al. 1973; Toba 1973; Donelan et al. 1985), it has been found that the dimensionless energy, defined as

$$\hat{E}_0 = \frac{g^2 E_0}{u_*^4} \tag{2.10}$$

where u_* is the friction velocity for the wind, is linearly proportional to the dimensionless fetch,

$$\hat{x} = \frac{gx}{u_*^2} \tag{2.11}$$

where x is the fetch. In other words, we have

$$\hat{E}_0 = M_1 \hat{x} \tag{2.12}$$

where M_1 is a dimensionless empirical constant.

Combining Eqs. (2.10) and (2.11) with (2.12) yields a relationship for the total energy as a function of fetch

$$E_0 = M_1 \frac{u_*^2}{g} x \tag{2.13}$$

or for the rate of change of total energy along a fetch

$$\frac{\partial E_0}{\partial x} = \frac{M_1 u_*^2}{g}. \tag{2.14}$$

Another result of a large number of wave studies has shown that the evolution of wave spectra along a fetch tends to follow a self-similar pattern (Kitaigorodskii 1962; Mitsuyasu 1968; Toba 1973; Hasselmann et al. 1973; to mention a few). For a self-similar spectrum, any ensemble average of a spectral function can be written as a dimensionless coefficient times the value at the spectral peak, i.e.,

$$\langle c_g \rangle = B_1 c_{g_m}$$

$$\langle c_m \rangle = B_2 c_m.$$

where B_1 and B_2 are dimensionless coefficients and c_{g_m} and c_m are group and phase velocities at the peak. Consequently, from Eq. (2.14) the time-rate of change of wave growth can be written as

$$\frac{\partial E_0}{\partial t} = B_1 c_{g_m} \frac{\partial E_0}{\partial x} = B_1 c_{g_m} \frac{M_1 u_*^2}{g}. \tag{2.15}$$

Equations (2.14) and (2.15) provide fundamental constraints on wave growth, since they control the fetch limited and duration-limited growth rates, respectively.

If we convert Eq. (2.15) into a form for the rate of change of total wave momentum, M_0, we see that

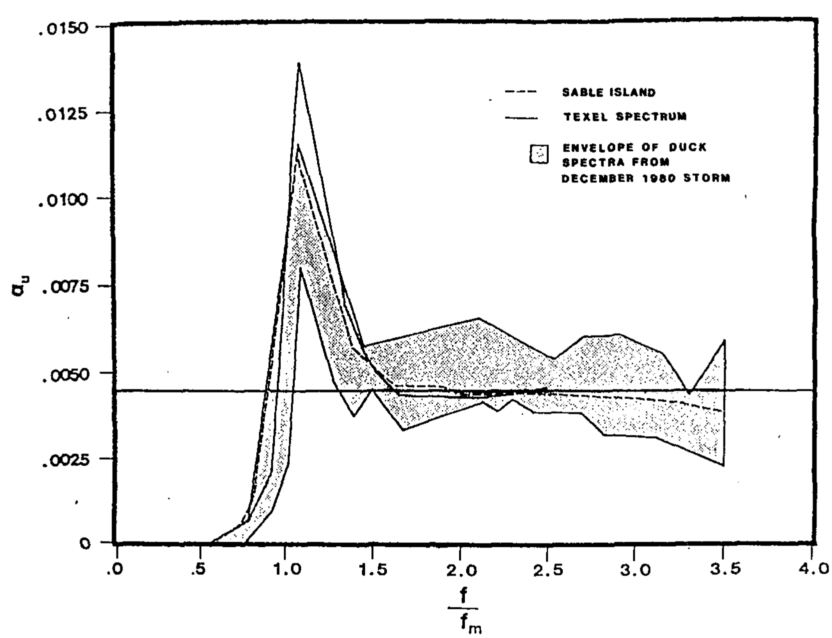

FIG. 2. Similarity shape of TEXEL spectrum, Sable Island data and Duck Pier, North Carolina, spectra from December 1980 storm. $\alpha_u = 2gF(k)k^3/(U\omega)$.

$$\frac{\partial M_0}{\partial t} = \frac{1}{\langle c_m \rangle}\frac{\partial E_0}{\partial t} = \frac{B_1 c_{g_m} M_1}{B_2 c_m}\frac{u_*^2}{g} \approx \frac{M_1}{2}\frac{u_*^2}{g} \quad (2.16)$$

which implies that a constant proportion of the momentum transfer from the wind to the water is retained by the wave field, independent of fetch and peak frequency.

Let us now formulate a simple conceptual model that interprets Eq. (2.16) in terms of some basic constraints on the wave generation process. In this context, let us partition an actively growing wave spectrum into three primary regions (Fig. 3): a "forward-face" region, a "midrange" region, and a "high-frequency" region. Of course this partition is only approximate and represents an idealized situation; but, it still provides a useful framework for discussing important processes in each of these regions.

Detailed numerical calculations of the wave–wave interaction integral have shown that, for most single-peaked spectra, the net energy transfer due to wave–wave interactions is characterized by three lobes (Fig. 4). Those three lobes can be visualized as being approximately coincident with the three spectral regions shown in Fig. 3. In this context, we see that the nonlinear energy transfers will tend to force a loss of energy in the midrange and a gain of energy in the front-face and high-frequency regions. Since these transfers are conservative, the energy gained in the front-face and high-frequency regions must be equal to the energy lost from the midrange region.

Let us assume, in a fashion consistent with the observed self-similar growth patterns of wave spectra, that the main processes governing wave growth are strongly related to the location of the spectral peak. Detailed numerical calculations of the wave–wave interaction integral have shown that transfers of momentum from midrange frequencies tend to preserve a constant proportion of fluxes to lower frequencies (where it is retained) and to higher frequencies (where it is presumed lost) (Hasselmann et al. 1973). In a wave spectrum the

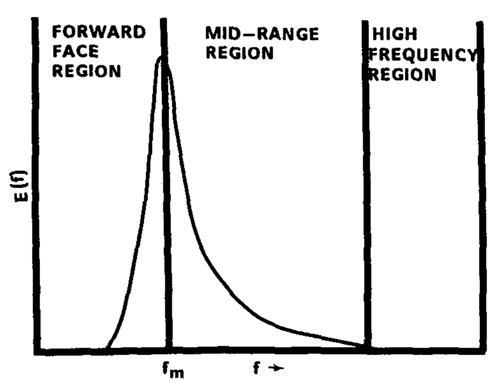

FIG. 3. Transfers of momentum from midrange frequencies to higher and lower frequencies.

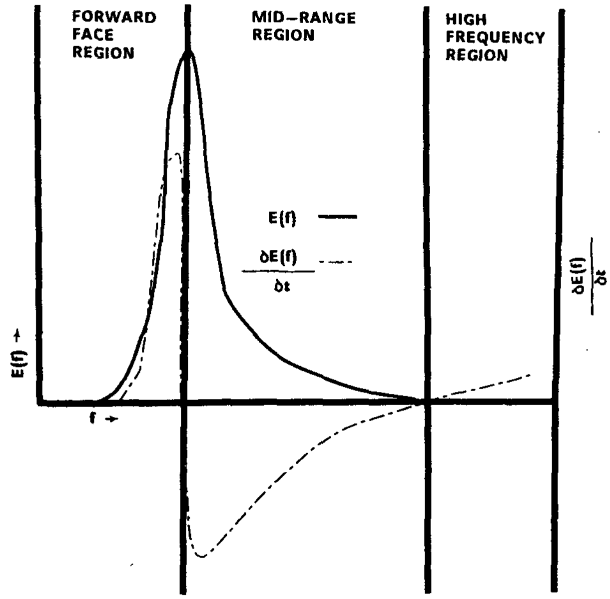

FIG. 4. The nonlinear energy transfer, $\partial E(f)/\partial t$, and the input energy, $E(f)$, as a function of frequency for two JONSWAP frequency spectra with $\gamma = 3.3$ and different shape parameters.

energy levels at frequencies above the spectral peak remain approximately constant after passing through an overshoot–undershoot phase; therefore, the permanent retention of wave energy must occur at frequencies less than that of the spectral peak. Thus, we can assume that the net gain of momentum on the forward face of a spectrum is approximately equal to the net gain of momentum for the entire spectrum. Since the momentum flux onto the forward face of the spectrum represents an approximately constant proportion of the momentum flux out of the midrange frequencies, we can estimate the rate of momentum flux out of the midrange frequencies as

$$\frac{\partial M_m}{\partial t} = \frac{1}{p_0}\frac{\partial M_0}{\partial t} \quad (2.17)$$

where the subscript "m" denotes an integrated quantity over the midrange frequencies, p_0 is a constant "partitioning" coefficient which represents the ratio of the momentum transferred into frequencies less than the frequency of the spectral peak to the total momentum flux out of the midrange frequencies. In order for a spectrum to preserve an approximate self-similar form, this momentum flux must be balanced by an equivalent momentum flux coming into this region from external sources such as the wind.

b. The wind source function

Since the wind is the only likely source of momentum to balance the net flux of momentum out of the midrange frequencies, a reasonable approximation to the wind momentum source in the midrange frequencies is

$$\int_{f_m}^{zf_m} S'_w(f)df = \frac{1}{\rho_0}\frac{\partial M_0}{\partial t} = \frac{M_1}{2\rho_0}\frac{u_*^2}{g} \quad (2.18)$$

where $S'_w(f)$ is the source function for wind-to-wave momentum transfer and z is a dimensionless constant. For a wind source function in terms of rates of energy transfer, rather than momentum transfers as above, Eq. (2.18) can be transformed into

$$\int_{f_m}^{zf_m} S_w(f)df = \frac{B_1 M_1}{2\rho_0}\frac{u_*^2 c_m}{g} \quad (2.19)$$

where $S_w(f)$ is the energy source function and B_1 is a dimensionless constant. Equation (2.19) should provide some guidance for examining integrated wind source functions in wave models.

Wind source terms have conventionally been of the form

$$S_w(f) = BE(f) \quad (2.20)$$

where B has dimensions $(\text{time})^{-1}$. In this form B is usually represented by the dimensionless form (Snyder and Cox 1966; Barnett 1968; Lazanoff and Stevenson 1975; Dobson and Elliott 1978; Snyder et al. 1981).

$$\frac{B}{f} = \phi\left(\frac{u}{c}\right) \quad (2.21)$$

where u is the wind speed at a reference level. Most representations for B up to the point where the spectral peak approaches its fully developed limit can be approximated as

$$B \sim \frac{fu_*}{c} = \frac{f^2 u_*}{g}. \quad (2.22)$$

This suggests that, in this range,

$$\int_{f_m}^{zf_m} BE(f)df \sim \alpha g f_m^{-2} u_*. \quad (2.23)$$

In f^{-5} spectral parameterizations, α has been found empirically (Hasselmann et al. 1973) to vary as a function of dimensionless peak frequency as

$$\alpha \sim \hat{f}_m^{2/3} \quad (2.24)$$

where $\hat{f}_m$ is the dimensionless peak frequency defined as

$$\hat{f}_m = \frac{u_* f_m}{g}. \quad (2.25)$$

Combining Eqs. (2.23), (2.24) and (2.25) yields

$$\int_{f_m}^{zf_m} BE(f)df \sim g^{1/3} u_*^{5/3} f_m^{-4/3} \quad (2.26)$$

which is not equivalent to Eq. (2.19) since it does not retain the same powers of u_* and f_m. Equations (2.19) and (2.26) might be somewhat difficult to distinguish from each other over a typical range of parameters observed in the field. However, Eq. (2.26) does not produce a constant proportion of the total air-to-water momentum flux entering the wave field consistent with Eqs. (2.16) and (2.17). Also, over a large range of conditions, including laboratory data, differences between the two forms should be discernable.

Recently, Phillips (1985) has suggested a new form for the wind source function intended to be consistent with spectrum with an f^{-4} equilibrium range. Although he wrote his source function in terms of the rate of gain of action density, it can be rewritten in an equivalent form for rate of gain of energy

$$S_w(f) \sim \left[\frac{u_*}{c}\right]^2 fE(f). \quad (2.27)$$

Unfortunately, when one integrates Eq. (2.27) for an f^{-4} self-similar spectral shape, one obtains

$$\int_{f_m}^{zf_m} S_w(f)df \sim \frac{u_*^3}{g}\ln(z) \quad (2.28)$$

which would become infinite as the z tends toward infinity. Although this might be remedied by a judicious selection of z, a somewhat different modification yields a result that agrees more satisfactorily with the integrated form for the wind source given in Eq. (2.19). If we multiply Phillips' form for $S_w(f)$ by c/u_*, we obtained a modified wind source function of the form

$$S_w(f) \sim \frac{u_*}{c} fE(f) \quad (2.29)$$

which agrees with Snyder et al. (1981). Evidence from the Bight of Abacoo experiment suggests that this is valid up to $u_5/c_m \approx 2.5$ where u_5 is the wind at 5-m height and beyond this value there is no concensus on wind input behavior (F. W. Dobson, personal communication). Integration of (2.29) yields

$$\int S_w(f)df \sim \frac{\alpha_2 u_*^2}{g} c_m \quad (2.30)$$

which is consistent with Eq. (2.19). Hence, in this paper, the wind source term will be considered reasonably represented by the form given in Eq. (2.29).

c. Wave–wave interaction energy fluxes and the total energy balance in a spectrum: Implications on spectral shape

For an f^{-4} spectrum, Resio (1987) shows that the total flux from the central portion of the spectrum can be parameterized in terms of certain integral properties of the spectrum and the location of the spectral peak. The representation given by Resio is for arbitrary depth and, therefore, is written in terms of wavenumber parameters. Since we are dealing with a deep-water spectral balance in this paper, we convert Resio's form for the energy flux to high frequencies from a wavenumber parameterization to a frequency parameterization,

$$\Gamma_E = d_1 \frac{E_0^3 f_m^9}{g^4} \quad (2.31)$$

where Γ_E is the total energy flux from the midrange frequencies into high frequencies and d_1 is a dimensionless constant. In terms of a momentum flux, we have

$$\Gamma_m = d_2 \frac{E_0^3 f_m^9}{g^4 c_m} \quad (2.32)$$

where Γ_m is the total momentum flux from midrange frequencies to high frequencies and d_2 is a dimensionless constant.

Equation (2.32) provides an explicit estimate for the momentum flux to high frequencies due to nonlinear wave–wave interactions. We can examine the consequences of this momentum flux for the spectral balance in the midrange frequencies, where the spectrum is expected to tend toward an equilibrium range. Assuming that the majority (80%–90%) of the wind input enters the spectrum in the midrange region, say in the range $0.95 f_m$ to $1.5 f_m$ as found in Snyder et al. (1981), an approximate balance between the integrated wind momentum source function and the momentum fluxes to higher and lower frequencies is of the form

$$\tau_{in} - \tau_{ff} - \tau_{hf} \approx 0 \quad (2.33)$$

where τ_{in} is the rate of momentum transfer from the atmosphere into the wave field, τ_{ff} is the rate transfer of momentum from the midrange frequencies onto the forward face of the spectrum and τ_{hf} is the rate of transfer of momentum from the midrange frequencies into the high frequency portion of the spectrum. In order for these to sum to zero, they all must have the same algebraic form in terms of wind and wave parameters.

As pointed out previously, from considerations of the behavior of the complete Boltzmann integral (Hasselmann et al. 1973), it has been found that, independent of the details of the spectral shape, a fixed proportion of the total momentum flux from the midrange frequencies is transferred to the front face and high frequency regions of the spectrum. Although this may not be applicable in the near-saturated-wave condition, it should be applicable in growing sea conditions. Thus, we have

$$\tau_{ff} = p_1 \tau_{hf} \quad (2.34)$$

where p_1 is a dimensionless partitioning constant. Since wave–wave interactions are conservative,

$$\tau_{ff} + \tau_{hf} = (1 + p_1)\tau_{hf} = -\tau_{mr} \approx \tau_{in} \quad (2.35)$$

where τ_{mr} is the rate of momentum transfer due to wave–wave interactions out of the midrange frequencies.

From Eq. (2.35), using similar arguments to those of Kitaigorodskii (1983), we see that an equilibrium will be established when

$$\tau_{in} = (1 + p_1)\tau_{hf} \quad (2.36)$$

which from Eqs. (2.16) and (2.32) is equivalent to having

$$\frac{M_1}{2}\frac{u_*^2}{g} = (1 + p_1) d_2 \frac{E_0^3 f_m^9}{g^4 c_m}. \quad (2.37)$$

If we write the spectral density in the equilibrium range as

$$E(f) = qgf^{-4} \quad (2.38)$$

where q is a constant factor with dimensions of velocity then the total energy in the spectrum can be approximated as

$$E_0 = \lambda_3 qg f_m^{-3} \quad (2.39)$$

which when combined with Eq. (2.37) yields

$$q^3 = \frac{M_1 u_*^2 c_m}{2 d_2 (1 + p_1) \lambda_3^3} \quad (2.40)$$

or

$$q \sim (u_*^2 c_m)^{1/3}. \quad (2.41)$$

This is slightly different from the form used by Toba (1973) and Kitaigorodskii (1983); however, these differences would probably not be very apparent in data from most field measurements. If we examine Kitaigorodskii's form for the equilibrium range

$$E(f) = \frac{\alpha u g f^{-4}}{(2\pi)^3} \quad (2.42)$$

and transform it into a friction velocity form we have

$$E(f) = \frac{\alpha_* u_* g f^{-4}}{(2\pi)^3} \quad (2.43)$$

where $\alpha_* = \alpha/\sqrt{C_D}$ where C_D is the coefficient of drag. From Eqs. (2.38) and (2.40) we can write our form for the equilibrium range as

$$E(f) = \frac{\alpha'_* (u_*^2 c_m)^{1/3} g f^{-4}}{(2\pi)^3} \quad (2.44)$$

where α'_* is expected to be a universal constant. In this context, the value of α_* is expected to vary as

$$\alpha'_* \sim \left(\frac{c_m}{u_*}\right)^{1/3} \alpha_* \quad (2.45)$$

which suggests a slight fetch dependence and a rather large difference between laboratory- and open-ocean-scale data. The type of relationship that should arise from Eq. (2.45) is of the form

$$\alpha'_* \sim \hat{f}_m^{-1/3} \alpha_* \quad (2.46)$$

whereas Mitsuyasu et al. (1980) report that their data suggest a relationship of the form

$$\alpha'_* \sim \hat{f}_m^{-1/7}. \quad (2.47)$$

Table 1 from Phillips (1985) presents a number of different evaluations of α_* (referenced as α in Phillips' original table) by different investigators. Examining differences expected between laboratory- and prototype-scale data, we note that typical values for $\hat{f}_m$ in laboratory experiments lie in the range of 0.5 to 1.5, whereas for field experiments they tend to fall in the range 0.01 to 0.10. The average values of α_* in the five sets of field data shown in Table 1 range from 6 to 11 for values of $\hat{f}_m$ from 0.015 to 0.090. Toba's (1973) results indicate that a mean value of α_* of 2 might be associated with an average $\hat{f}_m$ value of about 1. Phillips (1985) argues that this difference might be due purely to wind-drift effects; however, it is not evident that the wind-drift effects should be quite this severe. The average values of α_* from the field data are about 3 to 5 times larger than those found in the laboratory. This ratio is roughly consistent with the expected ratio of $\hat{f}_m^{1/3}$ in the laboratory to $\hat{f}_m^{1/3}$ in field experiments, which from Table 1 are seen to be in the neighborhood of $(1/0.04)^{1/3}$ or 2.9.

From the preceding arguments, we can see that a consequence of using momentum conservation constraints on the balance of source terms rather than energy conservation constraints leads to a slightly different form for the proportionality factor, α_* versus α'_*, but retains the f^{-4} equilibrium range.

As will be seen in the next section, the difference between having $q \sim u_*$ as in Toba (1973) and Kitaigorodskii (1983) and $q \sim (u_*^2 c_m)^{1/3}$ has significant implications relative to wave growth rates through time.

d. Modeling the evolution of an f^{-4} spectrum

Since much effort is currently being devoted toward a better capacity to predict waves via numerical models, it is perhaps appropriate to examine wave growth rates within the context of wave modeling. Hence in this section, we shall endeavor to examine both the characteristic evolution of wave spectra and some related concepts important to the numerical modeling of this evolution.

An important question in the modeling of the nonlinear wave–wave interaction source term is what time step is required in a model in order to retain the detailed evolutionary characteristics of the spectrum? Figure 4 showed a sample source term calculated from the complete Boltzmann integral. The highly nonlinear shape of this source function near the spectral peaks makes it very difficult to represent over a long time step, even if one obtains an exact evaluation of the Boltzmann integral at the beginning of the time step. If we represent the energy at the $(i + 1)$th time step as

$$E(f)^{i+1} = E(f)^i + S_{nl}^i(f)^i \Delta t \quad (2.48)$$

where the superscript i refers to values at the ith time step Δt is the time step and $S_{nl}(f)$ is the nonlinear wave–wave interaction source term, it is obvious that different results will be obtained for the evolution of a spectrum over fixed time interval, depending on the time step used in the integration. To avoid this distortion, one must allow the location of the nonlinear source pattern to shift over the time interval.

Hasselmann et al. (1976) showed that the characteristic relaxation time for spectral components in the equilibrium range is about 15 minutes. Thus, to model perturbation of equilibrium range energies away from their equilibrium values would require time steps considerably less than 15 minutes in duration. In fact, it is not uncommon for time steps of 4 minutes to be needed for accurately representing spectral evolution in models that attempt to compute this source term explicitly.

Since wind information is available only for time intervals greater than the characteristic relaxation time in the equilibrium range, the best treatment of energy densities in this range may be to restrict them to be equal to their equilibrium values. On the other hand, spectral energies on the forward face of the spectrum are highly transient. Over any finite time step the peak frequency will shift and with it the location of the net wave–wave interaction energy transfers. Neglecting swell decay for now and recognizing that the wave growth process is approximately self-similar, we can

TABLE 1. A summary of measurement values of Toba's constant α, with the ranges of dimensionless fetch gx/U_{10}^2, dimensionless frequency of the dominant wave $\sigma_0 u_*/g$ and 'significant slope' s of the dominant waves (from Phillips 1985)

Author	Number of spectra	α ($\times 10^{-2}$)	gx/U_{10}^2 ($\times 10^{-3}$)	$\sigma_0 u_*/g$ ($\times 10^{-2}$)	s ($\times 10^{-2}$)
Laboratory					
Toba (1973)	—	2	$c\,(2 \times 10^{-4})$	70–130	5
Field					
Kondo et al. (1973)	2	6	—	2–5	—
Kawai et al. (1977)	54	6.2 ± 0.1	0.2–2	2–9	1–2
Mitsuyasu et al. (1980)	14	8.7	5–100	2–7	0.8–2
Kahma (1981)	ca. 50	11	0.5–6	1.5–6	—
Forristall (1981)	Many	11.0	—	2–6	0.8–2

parameterize the source function by estimating the evolution of the spectral peak over a time step

$$E(f)^{i+1} = E(f)^i + \int_{t_i}^{t_i+\Delta t} S^i_{nl}(f) + S_{in}(f)dt \quad (2.49)$$

where t_i is the time at time step i.

In the context of Eq. (2.49), let us investigate the primary characteristics of the net source term $S^i_{nl}(f) + S_{in}(f)$, integrated over time. Since the equilibrium range is taken as fixed, the only variations which can be exhibited are in the energy densities for frequencies in the vicinity of the spectral peak and in the location of the spectral peak. Recognizing that it is not the details of the wave–wave interaction fluxes but rather the net momentum balance constraints derived in section 2b that should control the total wave growth, we can examine the evolution of f_m in that context.

Returning to Eq. (2.15) for the net rate of change of total wave energy and making use of Eqs. (2.39) and (2.41), we can obtain an estimate for the net rate of change of f_m as

$$\frac{\partial f_m^{-3}}{\partial t} = \Lambda \frac{u_*^{4/3} c_m^{2/3}}{g^2} \quad (2.50)$$

where Λ is a dimensionless constant. Integration of 2.50 over a time step, Δt, yields

$$f_m^{i+1} = \left[f_m^{i^{7/3}} + \frac{9}{5}\Lambda u_*^{4/3} g^{-4/3}\Delta t\right]^{-3/7} \quad (2.51)$$

which is similar to the results of Hasselmann et al. (1976) for an f^{-5}-based model. If we had used an equilibrium range without the $c_m{}^{1/3}$ dependence, such as Kitaigorodskii's form for the equilibrium range, we would have obtained

$$f_m^{i+1} = \left[(f_m^i)^{-2} + \frac{9}{5}\Lambda' \frac{u}{g}\Delta t\right]^{-1/2} \quad (2.52)$$

where Λ' is a dimensionless constant. Equation (2.52) can be shown to be similar to Toba's (1978) results for the time rate of change of f_m.

The functional form of Eq. (2.51) is very stable computationally, since no dependence on f_m is found in the second term on the right-hand side of this equation. This means that the same computational result can be obtained independent of the time step chosen. It is somewhat straightforward to see how Eq. (2.51) can be used to infer a net one-dimensional (nondirectional) spectrum; however, it is not so obvious how the directional distributions of energy can be handled. For the one-dimensional energy density we obtain a parametric representation of the form

$$E(f)^i = \alpha'_*(u_*^2 c_m)^{1/3} g f^{-4} \psi\left(\frac{f^i}{f_m}, \frac{u}{c_m}\right) \quad (2.53)$$

where ψ is a shape function that includes dependences on f_i/f_m and u/c_m. The directional characteristics can be approximated by considering the total wind input over a time step and recognizing that the wave–wave interactions cannot alter the mean direction since they are conservative. Hence the mean wave direction at the end of a time step can be estimated by the wind speed and direction over a time step and the initial directional spectrum. Details of this procedure are beyond the scope of this paper since we will be using the model here to look only at cases where the wind direction is constant and colinear with the wave direction.

e. Evaluation of constants in the f^{-4}-based model

In section 2b, we showed that the model for wave growth proposed here is consistent with the existence of a fixed momentum flux from the wind to the waves. This in turn was shown to be consistent with the JONSWAP results, which suggest a linear growth of wave energy with fetch. In that section, we were more interested in the functional forms for various terms so we did not treat the accompanying empirical constants in any detail. Before showing our calculated growth rates, we must estimate the constants that were inherent in some of the equations presented in section 2b.

Let us begin by looking at the growth of waves with fetch from the JONSWAP experiment:

$$E_0 = 1.6 \times 10^{-7} \frac{u^2}{g} x. \quad (2.54)$$

If we want to convert this relationship into a friction velocity form, we must obtain an estimate of the characteristic coefficient of drag for the winds in that experiment. Assuming that the coefficient of drag was $\sim 1.1 \times 10^{-3}$ or so, we can convert Eq. (2.54) into the form

$$E_0 = 1.45 \times 10^{-4} \frac{u_*^2}{g} x. \quad (2.55)$$

If we convert this again into a duration-limited form, we see that

$$\frac{\partial E_0}{\partial t} = 1.45 \times 10^{-4} B_1 c_{g_m} \frac{u_*^2}{g} \quad (2.56)$$

where B_1 is a dimensionless constant that relates the average group velocity of the waves along the X-axis to the group velocity of the spectral peak. Thus, M_1 in Eqs. (2.13) and (2.15) is 1.45×10^{-4}. Substituting the deep-water group velocity formulation into Eq. (2.56) after some algebraic manipulation of the relationship between E_0 and f_m yields

$$\frac{\partial E_0}{\partial t} = 0.725 \times 10^{-4} B_1 (2\pi)^{-1} Q_1{}^{-3/10} E_0{}^{3/10} u_*^2 \quad (2.57)$$

where Q_1 is a factor such that

$$E_0 = Q_1 T_m{}^{10/3} \quad (2.58)$$

where T_m is the peak period (i.e., $T_m = 1/f_m$) and is given by

$$Q_1 = \frac{\lambda_4 \alpha'_* u_*^{2/3} g^{4/3}}{\frac{10}{3}(2\pi)^{10/3}} \quad (2.59)$$

where λ_4 is a multiplier that relates the total energy in a spectrum to an integral of the equilibrium range formula from f_m to infinity, as seen in Eq. (2.6). Intuitively λ_4 can be seen to relate to "peakedness" of the spectrum and the percentage of the total energy in the wave spectrum at frequencies less than f_m. The value of λ_4 typically ranges from 1.3 to 1.7 in measured spectra. Taking the dependence of c_{g_m} on T_m onto the right-hand side of Eq. (2.57) and integrating yields

$$E_0 = [Q_2 Q_1^{-3/10} u_*^2 t]^{10/7} \quad (2.60)$$

where Q_2 is given by

$$Q_2 = 0.5075 \times 10^{-4} \frac{B_1}{2\pi}. \quad (2.61)$$

If we factor out g and u_* from Q, we obtain our final form for the duration rate of wave growth as

$$E_0 = Q_3 \frac{u_*^{18/7}}{g^{4/7}} t^{10/7} \quad (2.62)$$

where Q_3 is given by

$$Q_3 = \frac{1.23 \times 10^{-6} B_1{}^{10/7}}{\lambda_4{}^{3/7} \alpha'^{3/7}_*} \quad (2.63)$$

which for values of $B_1 = 0.78$, $\lambda_4 = 1.60$ and $\alpha'_* = 0.051$ yields as a dimensionless time growth relationship

$$\hat{E}_0 = 2.52 \times 10^{-6} \hat{t}^{10/7} \quad (2.64)$$

where $\hat{t} = g(t/u_*)$. The choice of B_1 is motivated by the relationship between the average phase speed in a spectrum relative to the phase speed of the spectral peak. These values for λ_4 and α'_* are based on comparisons of spectral shapes and total energies in observed spectra.

It should be noted here that we are not attempting simply to fit a number of coefficients independently. Instead, we have attempted to demonstrate how all of these coefficients relate to each other and directly affect wave growth rates. Also, we will show subsequently that the selection of reasonable values for these coefficients produce wave growth rates not inconsistent with observational data.

If we had followed through with an analogous treatment of energy growth based on Eq. (2.52), i.e., Toba's (1973) and Kitaigorodskii's (1983) form for an f^{-4} spectrum, we would have obtained an energy growth equation such that $\hat{E}_0 \sim \hat{t}^{3/2}$, similar to that derived by Toba (1973). Over a wide range of physical scales, this growth form will have significant differences from the $\hat{E}_0 \sim \hat{t}^{10/7}$ form derived here.

3. Discussion

In this paper, we began by noting that the observed fetch-growth behavior in wind waves is consistent with the hypothesis that a constant portion of wind-to-water momentum flux is retained by the wave field. Using this hypothesis as a point of departure and including some scaling relationships inherent in the wave–wave interaction integral, we showed that an f^{-4} equilibrium range, in a somewhat modified form from that originally proposed by Toba (1973) and Kitaigorodskii (1983), is compatible with the balance of energy fluxes through a wave spectrum. Interestingly, an additional consequence of these assumptions is that the idealized fetch- and duration-growth rates are almost identical to those presented in Hasselmann et al. (1976) and Resio (1981), both of which were derived for spectra with an f^{-5} equilibrium range. Figure 5 presents a comparison of fetch-limited growth rates for several contemporary wave models, relative to the original JONSWAP results (assuming a mean value for C_D of 1.1×10^{-3} in the original data). The results of TOHOKU, the model of Toba (1973), are taken from the intercomparison of Allender et al. (1985). WAM is the third generation wave model described in Hasselmann et al. (1985). These data are being reviewed at present and some variation in the location of the line may result in the future; however, the characteristic linear relationship between dimensionless energy and dimensionless fetch should remain.

Figure 6 presents a comparison of the duration-growth characteristics in the same models. Most of the

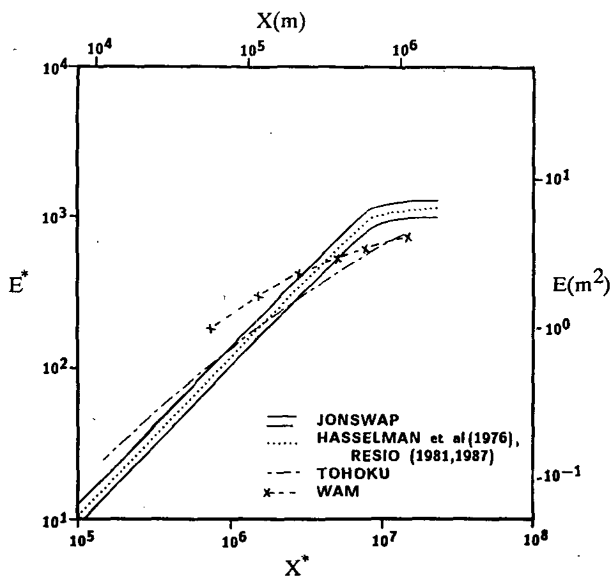

FIG. 5. Nondimensional fetch-limited growth curves for the total energy. TOHOKU is the model of Toba (1973) as reported in Allender et al. (1985). WAM is the third generation model whose growth curves are first described in Hasselmann et al. (1985).

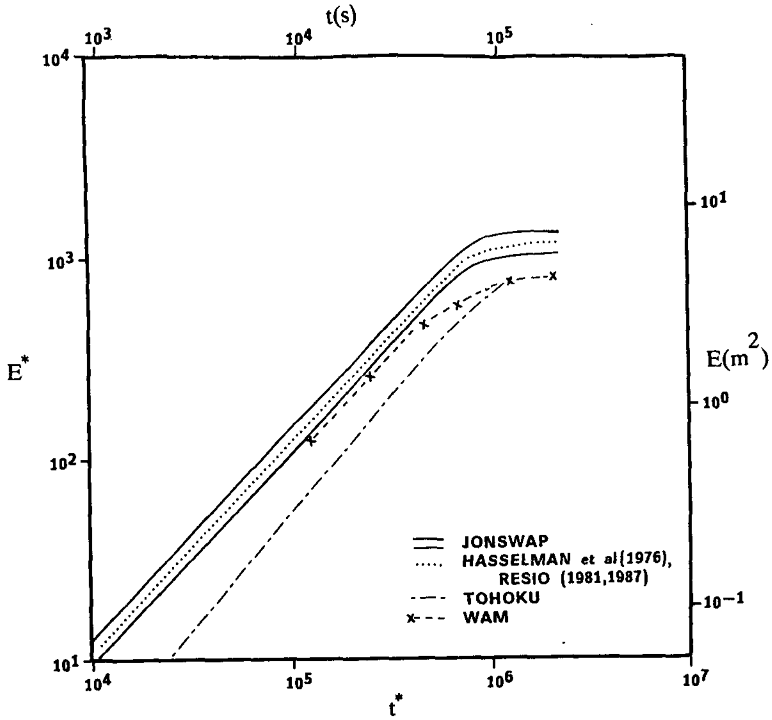

FIG. 6. Nondimensional duration-limited growth curves for the total energy.

models seem to be in reasonable agreement with the $E \sim \hat{t}^{10/7}$ relationship derived in this paper.

The results derived in this paper have some implications for the relative importance of various source terms acting on a wave spectrum. In an historical perspective, our results might be viewed as renewed support for the hypothesis of a self-similar wave generation process. In the 1960s, wave growth was envisioned as the sum of two source terms transferring energy directly from the atmosphere to an entire spectrum of uncoupled wave components (Phillips 1957; Miles 1957). Following JONSWAP (Hasselmann et al. 1973, 1976) the concept that wave–wave interactions represented a dominant source term began to find wide acceptance. Toba (1978) presented evidence that wave growth appeared to be strongly governed by a stochastic process, which produced self-similar spectra. Resio (1981) showed that the direct transfer models did not produce a growth pattern equivalent to that in models dominated by wave–wave interactions. Resio and Vincent (1982) showed that duration-fetch-growth relationships in a model with large source functions independent of f_m could not be transformed into a form equivalent to models dominated by source functions related to f_m. Kitaigorodskii (1983) presented an analysis of energy fluxes in deep-water gravity waves and concluded that the observed equilibrium range characteristics seemed consistent with the concept that the primary process governing the spectral shape was a balance between wind input and fluxes due to wave–wave interactions.

The above discussion suggests that external source functions, which do not scale in a self-similar fashion, may not produce appropriate balances among momentum fluxes within the front-face, midrange, and high-frequency regions of the spectrum, for all fetches and wind speeds. The results of this paper indicate that, until a better understanding of source terms exists, a two-term source balance (considering wind inputs and nonlinear energy fluxes) may be used to provide a theoretical framework which is consistent with observation of both equilibrium range characteristics and wave growth along a fetch. A separate paper by the authors is in preparation which calculates energy fluxes from the complete Boltzmann integral and will attempt to focus on the relative values and roles of various source terms in different regions of the spectrum.

4. Conclusions

The f^{-5} equilibrium range and associated variable α, which originated with Phillips (1958), has dominated experimental and theoretical concepts almost to the present. Our aim was to consider revised concepts of surface gravity wave dynamics brought on by an experimentally validated f^{-4} equilibrium range (Donelan et al. 1985), following the hypothesis of a Kolmogorov regime functioning within the equilibrium range (Kitaigorodskii 1983).

We have shown that a momentum transfer law that is consistent with the fetch relations of JONSWAP and theoretical constraints on energy fluxes due to wave-wave interactions implies a spectral form for one-dimensional midrange energy that is consistent with the field observations of Donelan et al. (1985). Given appropriate parameter relations it has been possible to show that this new spectral form and related fetch- and duration-limited wave growth rates are essentially equivalent to those based on the f^{-5} equilibrium range, as presented by Hasselmann et al. (1976) and Resio (1981).

Acknowledgments. The wave program at BIO is funded by the Office of Energy Research and Development. We are grateful for the support which allowed us to meet several times for extended periods and thus complete this work.

REFERENCES

Allender, J. H., and Co-Authors, 1985: Sea wave modeling project (SWAMP): An intercomparison study of wind-wave prediction models. Part 1—Principal results and conclusions. *Proc. IUCRM Symp. on Wave Dynamics and Radio Probing of the Ocean Surface,* Miami, Plenum.

Barnett, T. P., 1968: On the generation, dissipation and prediction of ocean wind waves. *J. Geophys. Res.,* **73,** 513–530.

Burling, R. W., 1959: The spectrum of waves at short fetches. *Dtsch. Hydrogr. Z.,* **12**(2), 45–117.

Dobson, F. W., and J. A. Elliott, 1978: Wave-pressure correlation measurements over growing sea waves with a wave follower and fixed-height pressure-sensors. *Turbulent Fluxes Through the Sea Surface, Wave Dynamics and Prediction.* A. Favre and K. Hasselmann, Eds., Plenum.

Donelan, M. A., J. Hamilton, and W. H. Hui, 1985: Directional spectra of wind-generated waves. *Phil. Trans. Roy. Soc. London,* **A315,** 509–562.

Forristall, G. Z., 1981: Measurements of a saturated range in ocean wave spectra. *J. Geophys. Res.,* **86**(C9), 8075–8084.

Garrett, J. R., 1969: Some new observations on the equilibrium region of the wind-wave spectrum. *J. Mar. Res.,* **27,** 273–277.

Hasselmann, K., 1962: On the nonlinear energy transfer in a gravity wave spectrum. Part 1. *J. Fluid Mech.,* **12,** 481–500.

——, 1963a: On the nonlinear energy transfer in a gravity wave spectrum. Part 2. *J. Fluid Mech.,* **15,** 273–281.

——, 1963b: On the nonlinear energy transfer in a gravity wave spectrum. Part 3. *J. Fluid Mech.,* **15,** 385–398.

——, and Co-Authors, 1973: Measurements of wind-wave growth and swell decay during the Joint North Sea Wave Project (JONSWAP). *Dtsch. Hydrogr. Z.,* **A8**(Suppl.), No. 12, 95 pp.

——, D. B. Ross, P. Muller and W. Sell, 1976: A parametric wave prediction model. *J. Phys. Oceanogr.,* **6,** 200–228.

Hasselmann, S., and K. Hasselmann, 1985: Computations and parameterizations of the nonlinear energy transfer in a gravity-wave spectrum. Part I: A new method for efficient computations of the exact nonlinear transfer integral. *J. Phys. Oceanogr.,* **15,** 1369–1377.

——, ——, J. H. Allender and T. P. Barnett, 1985: Computations and parameterizations of the nonlinear energy transfer for application in wave models. *J. Phys. Oceanogr.,* **15,** 1378–1391.

Kahma, K. K., 1981: A study of the growth of the wave spectrum with fetch. *J. Phys. Oceanogr.,* **11,** 1503–1515.

Kawai, S., K. Okuda and Y. Toba, 1977: Field data support of three-seconds power law and $gu_*\sigma^{-4}$ spectral form for growing wind waves. *J. Oceanogr. Soc. Japan,* **33,** 137–150.

Kinsman, B., 1960: Surface waves at short fetches and low wind speed—a field study. Tech. Rep. No. 19. Johns Hopkins University, 581 pp.

Kitaigorodskii, S. A., 1962: Applications of the theory of similarity to the analysis of wind-generated wave motion as a stochastic process. *Bull. Acad. Sci.,* USSR Geophys. Ser. 1, 105–117.

——, 1983: On the theory of the equilibrium range in the spectrum of wind-generated gravity waves. *J. Phys. Oceanogr.,* **13,** 816–827.

——, V. P. Krasitskii and M. M. Zaslavskii, 1975: On Phillips' theory of equilibrium range in the spectra of wind-generated gravity waves. *J. Phys. Oceanogr.,* **5,** 410–420.

Komen, G. J., S. Hasselmann and K. Hasselmann, 1984: On the existence of a fully developed wind-sea spectrum. *J. Phys. Oceanogr.,* **14,** 1271–1285.

Lazanoff, S. M., and N. M. Stevenson, 1975: An evaluation of a hemispheric operational wave model. U.S. Navy Fleet Numerical Weather Central, Tech. Note 75-3, 103 pp.

Longuet-Higgins, M. S., 1969: On wave breaking and the equilibrium spectrum of wind-generated waves. *Proc. Roy. Soc. London,* **A310,** 151–159.

Miles, J. W., 1957: On the generation of surface waves by shear flows. *J. Fluid Mech.,* **3,** 185–204.

Mitsuyasu, H., 1968: On the growth of wind-generated waves (I). *Rep. Res. Inst. Appl. Mech. Kyushu University,* **16,** 459–482.

——, F. Tasai, T. Suhara, S. Mizuno, M. Ohkuso, T. Honda and K. Rikiishi, 1975: Observations of the directional spectrum of ocean waves using a cloverleaf buoy. *J. Phys. Oceanogr.,* **5,** 750–760.

——, ——, ——, ——, ——, —— and ——, 1980: Observations of the power spectrum of ocean waves using a cloverleaf buoy. *J. Phys. Oceanogr.,* **10,** 286–296.

Phillips, O. M., 1957: On the generation of waves by turbulent wind. *J. Fluid Mech.,* **2,** 417–445.

——, 1958: The equilibrium range in the spectrum of wind-generated waves. *J. Fluid Mech.,* **4,** 426–434.

——, 1985: Spectral and statistical properties of the equilibrium range in wind-generated gravity waves. *J. Fluid Mech.,* **156,** 505–531.

Ramamonjiarisoa, A., 1973: Sur l'evolution des spectres d'energie des vagues de vent a fetchs courts. *Mem. Soc. R. Sci. Liege Vol. Hors Ser. 6,* **6,** 47–66.

Resio, D. T., 1981: The estimation of a wind wave spectrum in a discrete spectral model. *J. Phys. Oceanogr.,* **11,** 510–525.

——, 1987: Shallow water waves, Part I: Theory. *J. Waterway, Port, Coastal Ocean Eng.,* **113,** 266–283.

——, and C. L. Vincent, 1982: A comparison of various numerical wave prediction techniques. *J. Soc. Pet. Eng.,* **22,** 764–774.

Snyder, R., and C. S. Cox, 1966: A field study of the wind generation of ocean waves. *J. Mar. Res.,* **24,** 141–177.

——, F. W. Dobson, J. A. Elliott and R. B. Long, 1981: Array measurements of atmospheric pressure fluctuations above surface gravity waves. *J. Fluid Mech.,* **102,** 1–59.

Toba, Y., 1973: Local balance in the air–sea boundary processes, III. On the spectrum of wind waves. *J. Oceanogr. Soc. Japan,* **29,** 209–220.

——, 1978: Stochastic form of the growth of wind waves in a single-parameter representation with physical implications. *J. Phys. Oceanogr.,* **8,** 494–507.

Tracy, B. A., and D. T. Resio, 1982: Theory and calculations of the nonlinear energy transfer between sea waves in deep water. WES Rep. No. 11, U.S. Army Engineer Waterways Experiment Station, Vicksburg, MS.

Webb, D. J., 1978: Nonlinear transfers between sea waves. *Deep-Sea Res.,* **25,** 279–298.

Zakharov, V. E., and N. N. Filonenko, 1967: Energy spectrum for stochastic oscillations on the surface of a liquid. *Soviet Phys. Dokl.,* **11,** 881–883.