Geomorphology 114 (2010) 276–283

Contents lists available at ScienceDirect

Geomorphology

journal homepage: www.elsevier.com/locate/geomorph

Sand size versus beachface slope — An explanation based on the Constructal Law

A. Heitor Reis [a,*], Cristina Gama [b]

[a] Department of Physics and Geophysics Centre of Evora, University of Évora, R. Romão Ramalho 59, 7000-671 Evora, Portugal
[b] Department of Geosciences and Geophysics Centre of Evora, University of Évora, R. Romão Ramalho 59, 7000-671 Evora, Portugal

ARTICLE INFO

Article history:
Received 19 February 2009
Received in revised 21 July 2009
Accepted 24 July 2009
Available online 3 August 2009

Keywords:
Beachface dynamics
Sediment size
Beach slope
Constructal Law

ABSTRACT

The relationship between beachface slope and sand grain size has been established based on multiple observations of beach characteristics in many parts of the world. We show that this observational result may be understood in the light of the Constructal Law (Bejan, 1997). A model of wave run-up and run-down along the beachface (swash) was developed to account for superficial flows together with flows through the porous sand bed of average porosity 0.35, the permeability of which may be related to grain diameter and sphericity (0.9 for sand grains) through the Kozeny–Carmán equation. Then, by using the Constructal Law, we minimized the time for completing a swash cycle, under fixed wave height and sand grain diameter. As the result, a relationship involving sand grain size, beachface slope and open ocean wave height has been obtained, and then discussed and validated against experimental data. In addition, this relationship has also been used to illuminate beachface dynamic processes, namely the reshaping of sandy beachfaces in response to changes in wave height. Though the model used in this work may be improved further, the results appear to show, as with other natural systems, that beachface morphing in time may be understood based on a unifying principle — the Constructal Law.

© 2009 Elsevier B.V. All rights reserved.

1. Introduction

The beachface is the sub-aerial beach sector, below the berm, that presents the steepest slope. This sector is exposed to wave swash that is responsible for sediment transport. Beachface gradient in relation to sediment transport and beach profile evolution has been studied from different perspectives: (i) by considering the beachface sediment characteristics, sediment grain size and sorting (Bagnold, 1940; Bascom, 1951; Wiegel, 1964; Turner, 1995; Wilson et al., 2008); (ii) or by accounting for swash infiltration/exfiltration effects on the equilibrium beachface profile in the cross-shore sediment transport (Grant, 1948; Kemp, 1975; Quick, 1991; Turner, 1995; Hughes et al., 1997; Masselink and Hughes, 1998; Turner and Masselink, 1998; Butt and Russell, 1999; Hughes and Turner, 1999; Puleo et al., 2000; Butt et al., 2001; Masselink and Li, 2001; Baldock and Hughes, 2006; Masselink and Puleo, 2006); and (iii) beach groundwater flow in the swash zone (Hegge and Masselink, 1991; Turner, 1993; Kang and Nielsen, 1996; Turner, 1998; Nielsen, 1999; Li et al., 2002; Horn, 2006).

The relationship between beach grain size and beach states ranging from those with steep reflective slopes to those with shallow, dissipative slopes has been examined by Wright and Short (1984) in terms of the dimensionless sediment fall velocity parameter, $\Omega = H_b / W_s T$, where H_b = breaker height; W_s = sediment fall velocity and T = wave period (Gourlay, 1968). The sediment fall velocity parameter has been judged as a rather crude estimator of beach morphodynamic states (e.g. Anthony, 1998; Levoy et al., 2000; Masselink and Pattiaratchi, 2001; Jackson et al., 2005), and may, therefore, be considered as inadequate in reliably characterising beachface types within the reflective–dissipative beach continuum. Anthony (1998) has suggested that beach parameters based on slope, such as the Iribarren number (see Section 2), are better indicators of beach morphodynamic type than sediment size.

The proportionality between beachface gradient and sediment size was described by several authors (e.g. Bascom, 1951; McLean and Kirk, 1969; Dubois, 1982; Sunamura, 1984; Komar, 1998) who related this aspect to swash infiltration and hydraulic conductivity (permeability). If swash infiltration is significant, the water that infiltrates will not participate in the backswash (Kemp, 1975), and this controls beachface slope in relatively coarse sand beds (Bagnold, 1940; Quick, 1991). Swash infiltration depends on the permeability of the sediments bed which increases with grain size and sorting (Bascom, 1951; Shepard, 1963; Pryor, 1973; Selley, 1988; Masselink and Li, 2001). In coarse sand beaches (>1 mm) where swash infiltration is very important the beachface slope tends to increase with permeability (Quick, 1991; Komar, 1998; Masselink and Li, 2001).

With respect to importance of infiltration and exfiltration in the swash zone sediment transport and beach profile evolution, Butt et al. (2001) suggested that in coarse sand beds onshore sediment transport due to the effects of swash infiltration/exfiltration becomes important. These authors pointed out that a critical grain size might

* Corresponding author. Tel.: +351 967324948; fax: +351 266745394.
E-mail address: ahr@uevora.pt (A.H. Reis).

0169-555X/$ – see front matter © 2009 Elsevier B.V. All rights reserved.
doi:10.1016/j.geomorph.2009.07.008

exist below which infiltration/exfiltration promotes offshore transport and above which it promotes onshore transport. Additionally, Masselink and Li (2001) concluded that infiltration augments onshore transport and berm formation. A number of studies have indicated that infiltration in the swash zone is only important for profile evolution on coarse-grained beaches (e.g. Bagnold, 1940; Packwood, 1983; Quick, 1991). On the other hand, simulations carried out by Masselink and Li (2001) showed that swash infiltration on coarse beaches produced steeper gradients than on fine-grained beaches.

In the present paper we focus on the relationships between beachface slope, sand grain size and wave forcing. With the purpose of developing a comprehensive framework of these relationships, and drawing from the body of works cited above, we build a model of the swash process that takes into account uprush and downrush together with infiltration/exfiltration. In this model, the morphology of the sand bed is accounted for through sand grain size and sphericity, and bed porosity.

We will use the Constructal Law (Bejan, 1997) as the principle that governs flow processes in relation to beachface morphodynamics. This law has proven to be an important principle for flow architectures that evolve in time in systems out of equilibrium (see Bejan, 1997, ch.13) in that: "Flow systems morph in time in order to provide easier and easier flow access to the currents that flow through it, under the system constraints". Said another way, the beachface slope will change in time such as to maximize the global currents (swash flows) that flow over and inside it". The Constructal Law has proven to be successful in describing evolving flow architectures in many natural and engineered systems (see reviews by Bejan, 2000; Bejan and Lorente, 2004; Reis, 2006a). With respect to natural systems, it has been applied successfully to biological flow architectures (see for example Reis et al., 2004; Bejan and Marden, 2006; Miguel, 2006), and inanimate flow systems (Bejan and Reis, 2005; Reis, 2006b; Reis and Bejan, 2006).

2. Swash flow modelling

In the following, swash flows at the low tide were modelled. The scheme in Fig. 1 represents a beachface of slope $\beta = \tan\theta$ bathed by swash flows. The uprush flow Q_u has height H (wave set-up) above the average sea level, and spreads over the beachface up to a maximum height $H + H'$ (wave run-up) above the sea level. The flow

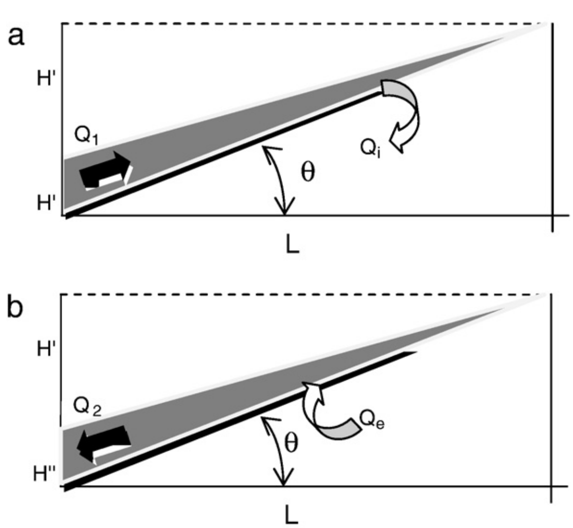

Fig. 1. Modelling of the swash zone: (a) Wave run-up; (b) Wave run-down at beachface; Q_1 — wave run-up flow; Q_2 — wave run-down flow; Q_i — infiltration flow; Q_e — exfiltration flow; H — wave set-up; H' — water height above mean sea level at the rim; H'' — water height above mean sea level at the rim (run-down); θ — angle between beachface and mean sea level ($\beta = tg\theta$).

Q_u splits into the overflow Q_1 and the infiltration flow Q_i that penetrates the porous sand bed:

$$Q_u = Q_1 + Q_i. \tag{1}$$

Under gravity, both flows return to the sea. The total downrush Q_d is composed of the descending overflow Q_2 of average height H'' (see Fig. 1) and the exfiltration flow Q_e from the beachface:

$$Q_d = Q_e + Q_2. \tag{2}$$

By considering the illustration in Fig. 1(a), the overflow Q_1 is estimated as:

$$Q_1 = H(H + H')/(2\beta t_1) \tag{3}$$

where β is beachface slope, and t_1 is uprush time. Because swash zone is usually saturated at the low tide (i.e., the water table is above mean sea level) the infiltration flow Q_i may be calculated with the help of Darcy's Law as:

$$Q_i = K(H + H')g/(\nu\beta) \tag{4}$$

where K is sand bed permeability, and ν kinematic viscosity. By the Kozeny–Carmán equation, the permeability of the sand bed may be related to its porosity (ϕ), to the average sand grain sphericity (S), and to the diameter of the related spherical particle (d) in the following way:

$$K = d^2 S^2 \phi^3 / \left(150\left(1 - \phi^2\right)\right). \tag{5}$$

The descending overflow Q_2 is assumed to follow Manning's law for turbulent flow driven by gravity in open channels. In this case, the width of the "channel" matches the alongshore distance washed by waves, which is much larger than its height (h). Therefore the hydraulic radius, i.e. the ratio of the flow cross-sectional area to the wetted perimeter, is practically given by h. Hence, with n denoting Manning's coefficient, the descending overflow reads:

$$Q_2 = h^{\frac{5}{2}} \beta^{\frac{1}{2}} / n. \tag{6}$$

Analogously to the case of infiltration flow, by considering Fig. 1(b) and Darcy's law, the exfiltration flow Q_e reads:

$$Q_e = K(H + H')g\beta/\nu. \tag{7}$$

Furthermore, mass conservation implies:

$$Q_u t_1 = Q_d t_2 \tag{8}$$

where t_1 and t_2 represent uprush and downrush times, respectively.

Finally, we use Hunt's formula (see Hughes, 2004, p. 1086) which relates maximum regular run-up height, $H + H'$, to open ocean wave height H_0 in the following way:

$$(H + H')/H_0 = \xi_0 \tag{9}$$

where ξ_0 is the Iribarren number $\xi_0 = \beta/\sqrt{H_0/L_0}$ with $L_0 = (g/2\pi)T^2$, where g is the acceleration due to gravity and T is wave period. The general form of Eq. (9) holds either for surging waves ($\xi_0 \sim 3$ in this case) or for the case where waves break onto the beachface slope — with $0.1 < \xi_0 < 2.3$ (see Mase, 1989). For irregular run-up Mase (1989) presented the following relation:

$$(H + H')/H_0 = 2.32\xi_0^{0.77}. \tag{10}$$

On the other hand, for moderately steep beaches H is approximately given by (see Guza and Thornton, 1981):

$$H \sim 0.17 H_0 \tag{11a}$$

while for dissipative beaches with spilling waves ($\xi_\downarrow 0<0.3$) the following relationship holds (see Stockdon et al., 2006):

$$H = 0.35 \xi_0 H_0. \tag{11b}$$

The relationships (11a) and (11b) were derived as the best fits of experimental data. However, it is commonly recognized that every relation of this kind carries significant uncertainties due to the many factors that may influence wave breaking.

Eqs. (9)–(11) allow run-up height to be referred to the nearshore wave "climate" that is characteristic of each beach. The model developed here is appropriate for describing waves with regular run-up (see Fig. 1). Therefore, in the following we will use Eq. (9).

3. Beachface morphing under fixed constraints

The model presented here implicitly assumes that wave height, beachface slope and mean sand grain diameter are interrelated. However, Eqs. (1)–(11) are merely constitutive and say nothing about beachface slope evolution under the existing constraints. Among these constraints, which may vary from time to time, are the wave "climate" — that may be represented by the characteristic offshore wave height H_0 — and characteristic sand grain diameter, d. Then, the only free parameter is beachface slope.

The Constructal Law offers the theoretical basis for determining beachface slope evolution so as to adjust in time to the actual existing constraints, i.e. the beachface slope will morph in time so as to provide globally easier and easier access to both the uprush and downrush flows. The equilibrium slope will be the one that simultaneously maximizes both flows. Equivalent to flow maximization is minimization of total time for a fixed amount of water to complete a swash cycle. In terms of non-equilibrium physics, the Constructal Law in the preceding formulation corresponds to the statement that in the presence of constraints the time for transition between two equilibrium states is the shortest. For a more detailed analysis on the physical grounds of the Constructal Law the reader is directed to Reis (2006a). We will use this last formulation of the Constructal Law for determining the equilibrium slope of a beachface of fixed sand grain size, washed by waves of fixed height. The result will provide the slope to which the actual beachfaces would evolve if they were to reach the respective equilibrium flow architecture.

Then, according to the Constructal Law, for a fixed amount of water V resulting from a breaking wave of fixed offshore height H_0, the time needed to perform a swash cycle on a sand bed of grain diameter d must be the shortest one that matches the existing constraints. The mathematical formulation reads:

$$d(t_1 + t_2)_{H_{0,d}} = 0. \tag{12}$$

The amount of water involved in the swash cycle is also fixed, and following Eq. (8), is given by $V = Q_u t_1 = Q_d t_2$, which allows the expression of t_1 and t_2 as:

$$t_1 = V / Q_u; t_2 = V / Q_d. \tag{13}$$

Therefore, by using Eqs. (3)–(10) and (13), one obtains:

$$t_1 = (V\beta - A) / \left(ad^2\right) \tag{14}$$

where

$$a = \xi_0 H_0 S^2 \phi^3 g / \left(150 \nu \left(1 - \phi^2\right)\right) \tag{15a}$$

and

$$A = 0.085 \xi_0 H_0^2. \tag{15b}$$

Analogously, one obtains t_2 in the following form:

$$t_2 = V / \left(B\beta^{\frac{1}{2}} + ad^2\beta\right) \tag{16}$$

with

$$B = h^{\frac{5}{2}} / n. \tag{17}$$

We assumed that $h = H/2$, and $H'' = H'$ as the acceptable approximations. This later approximation means assuming that the average height of the downrush is of the same order of that of uprush. With respect to the first one, if $\bar{v}$ denotes the average downrush speed, we have: $\bar{v}ht_2 = H(H + H')/2\beta$ and $\bar{v}t_2 = (H + H')/\sin\theta$ (see Fig. 1), which leads to $h = H\cos\theta/2 \sim H/2$.

Eqs. (14) and (16) enable to explicitly develop Eq. (12) in terms of the free variable β, and the fixed constraints d and H_0. The result is the 4th-order algebraic equation with d as the unknown,

$$\left(ab - b^2\beta^2\right)d^4 - \left(2bB\beta^{\frac{3}{2}} - a\left(\frac{B}{2}\right)\beta^{-\frac{1}{2}}\right)d^2 - B^2\beta = 0 \tag{18}$$

the unique real and positive solution of which is:

$$d = \left\{B\left(2\beta^{\frac{3}{2}} - \left(\frac{1}{2}\right)\beta^{-\frac{1}{2}}\right) + B\left[\left(2\beta^{\frac{3}{2}} - \left(\frac{1}{2}\right)\beta^{-\frac{1}{2}}\right)^2 + 4\beta\left(1 - \beta^2\right)\right]^{\frac{1}{2}} / 2a\left(1 - \beta^2\right)\right\}^{\frac{1}{2}}. \tag{19}$$

Eq. (19) relates the slope to which the beachface would evolve in time to the existing constraints, here represented by d and H_0.

The curves in Fig. 3 show sand grain diameter against beachface slope (Eq. (19)) for various characteristic wave regimes. In this determination, we used $S = 0.9$ for the sand grain sphericity (which corresponds to the mean sphericity of objects from cube to icosahedron), $\O = 0.35$ for the porosity of the sand bed (Soulsby, 1991), and $\nu = 10^{-6}$ m^2/s as the kinematic viscosity of water. Manning's coefficient n, is about 0.02 for a sand bed of grain diameter close to 1 mm. As n varies with grain diameter, we used the formula $n = 0.015 + 0.2(d - 0.0003)$, which represents the variation, within the range of interest, of Manning's coefficient. The validation of these results against experimental data will be the subject of the next section.

However, we should note that the exact result of Eq. (6) may be presented in a simplified but fairly accurate form. In reality, the term that contains d in the denominator of the 2nd member of Eq. (16) is much smaller than the other term by several orders of magnitude. This aspect allows us to neglect it not only in this equation but also in the first derivative of t_2 with respect to β. Therefore, a result approximate (with relative error smaller than 1/1000) to that of Eq. (19) is:

$$\beta = (a/2)^{\frac{2}{3}} B^{-\frac{4}{3}} d^{\frac{4}{3}}. \tag{20}$$

Eq. (20) is much simpler than Eq. (19) and may be used for practical purposes. However, we note that some restrictions emerge from the fact that Eq. (20) was derived under the assumptions of fixed bed porosity, sorting and grain sphericity.

4. Data analysis and discussion in the light of the model

4.1. Data collection

Data of characteristic sand grain sizes together with local beachface slopes have been collected on the southwestern coast of Portugal on a seasonal basis over two and a half years (see Fig. 2).

Topographic beach profiles were obtained by using classical methods (e.g. theodolite) for describing shore-normal transects between the dune/sea-cliff base and the surf zone. In each field survey, a beachface slope together with a mean sand grain size has been assigned to each beach. Sampling of superficial beach sediments and beach profiles has been carried out during the monthly spring tides and at the lower tide level. For all the beaches considered in this study, the field data have been collected under a broad range of wave conditions during a 2.5-year period (May 1998 to July 2001).

Beachface slopes were estimated by using a graphical method that adjusts a straight line to the beachface segment of each profile, at the mid-tide level reference ("reference point") as described by Bascom (1951). The slope of the line obtained in this way was assumed to define the beachface slope. In each of the field surveys only superficial beach sediments were collected.

Beach sediments were studied from the perspective of textural analysis by using classical sieving methods (dry-sieved at 1/2 ϕ intervals). The textural parameters used for describing the grain size distribution (mean grain size, sorting and skewness) were obtained through the logarithmic graphical method (Folk and Ward, 1957) applying the software GRADISTAT® package (Blott and Pye, 2001), and by considering the logarithmic Folk and Ward (1957) graphical measures (Gama, 2005). In this study, only the mean grain size was considered.

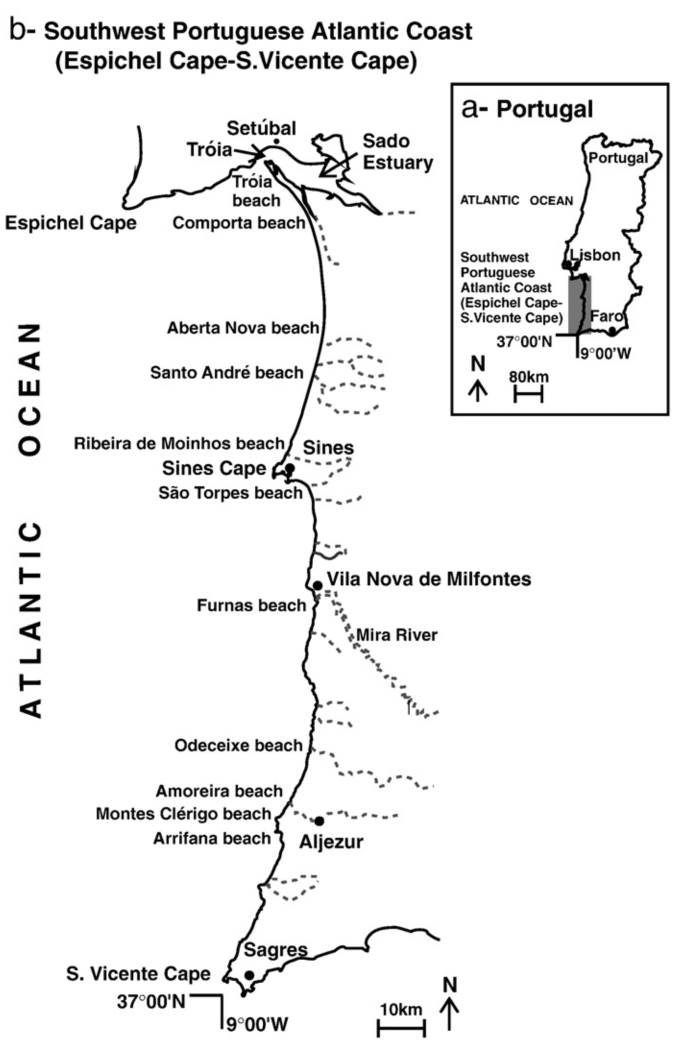

Fig. 2. Chart of the southwestern coast of Portugal showing location of the beaches under study.

We covered two different sectors the southwestern Portuguese coast: the Tróia–Sines sandy embayed coast (northern sector of 60 km length, that includes Tróia, Comporta, Aberta Nova, Santo André and Ribeira de Moinhos beaches), and the Southwest Alentejo–Vicentina rocky coast (sector of 95 km length that comprises São Torpes, Furnas, Odeceixe, Amoreira, Monte Clérigo and Arrifana beaches).

The offshore wave climate characteristic of this coast was studied by Costa et al. (2001). These authors used the offshore wave climate recorded by the Sines directional wave rider (37° 55′ 16″ N, 8° 55′ 44″ W, placed at 98 m water depth) during an 11-year period. The deepwater data indicated a mean annual significant wave height (H_s) of 1.7 m and peak period (T_p) of 10.8 s. The wave directions associated with T_p are distributed by a dominant NW sector (77.3%), followed by a W sector (20%), SW sector (2.4%) and S sector (0.2%). Tidal range for the southern Portuguese coast varies from 3.9 m on spring tides to 0.2 m on neap tides, with a semidiurnal meso-tidal regime.

4.1.1. Tróia–Sines sandy embayed coast

In this sector of the coast, 102 pairs of data (beachface slope and mean sand grain size) have been collected during the above-mentioned period of two and a half years. Gama (2005), based on the morphodynamic classification proposed by Wright and Short (1984) and Wright et al. (1985), classified four of these five beaches of the Tróia–Sines sandy embayed coast as "intermediate", between the "transverse bar and rip" and "rhythmic bar and beach" stages. Ribeira de Moinhos beach is the only exception and was found to be better described as "reflective". The same author also recognized plunging waves to be dominant at Aberta Nova, Santo André and Ribeira de Moinhos beaches. At Comporta beach, whenever a submarine sand bar was present it was possible to observe a combination of spilling and plunging waves. At Tróia beach, waves are of small amplitude and of spilling type during the low tide.

The obtained mean grain size variations revealed longshore progressive coarsening southwards (from fine sand on Tróia beach to very coarse sand on Ribeira de Moinhos beach). The beachface gradient ranged from 0.06 to 0.18. The seasonal analysis of the longitudinal textural parameters indicated that together with a general increase in the mean grain size in the southward direction, sorting also increased from moderately to very well sorted sediments.

4.1.2. Southwest Alentejo–Vicentina rocky coast

In total, 87 data of beachface slope and mean sand grain size have been collected in this coastline. The five beaches on the Sines–São Vicente rocky coast, which are distributed between the Sines and São Vicente capes, are of the sandy pocket type (Gama, 2005). Following the classification by Wright and Short (1984) all these beaches are intermediate and have a low tide terrace with a ridge and runnel system. The beachface gradient varies from 0.03 to 0.14. The sediment mean grain size ranges from medium to fine sand. The sands analysed ranged from well to moderately sorted. In these five beaches, the surf zone is wide and in the low tide the nearshore waves are reformed waves — mainly of spilling type — that result from waves that broke at the seaward limit of the ridge and runnel system.

4.2. Analysis

4.2.1. Data clustering

Data of the mean grain size *versus* beachface slope from various beaches along the Tróia–Sines sandy embayed coast are shown in Fig. 3.

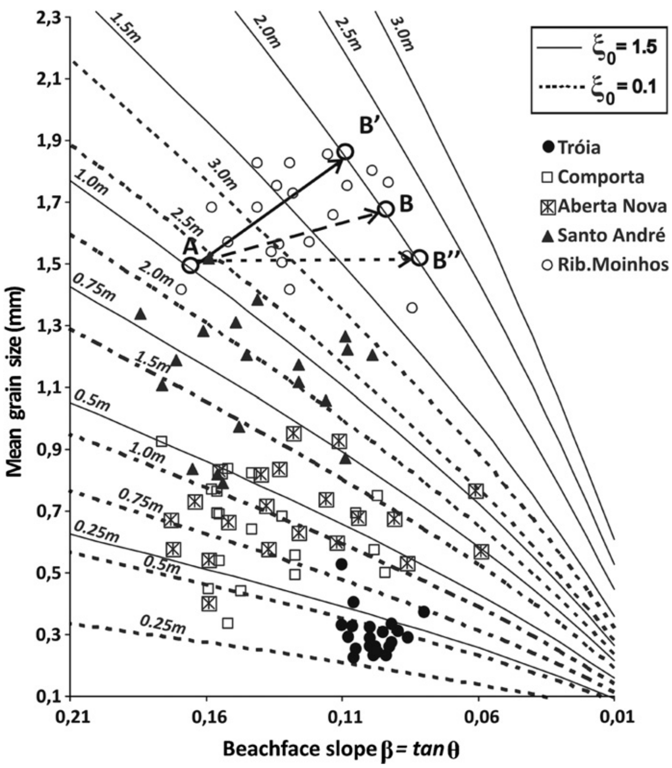

Fig. 3. Curves of equilibrium grain size versus beachface slope for each offshore wave height for Iribarren numbers 0.1 and 1.5. The marks in the figure represent data collected on the beaches from Tróia to Sines Cape (see legend). The curves AB, AB′ and AB″ represent processes involving changes in beachface slope and sediment grain size as responses to wave height variations in the range 1–2 m.

Despite the general grain size variation tendency, these data reveal that grain size is not entirely proportional to beachface gradient as described by Wiegel (1964), Bascom (1951) and Dingler and Reiss (2002). The Tróia beach defines a cluster with no significant grain size and beach slope variations. On Santo André and Ribeira de Moinhos beaches, beachface slope falls in the range 0.1–0.19, and do not show evident proportionality between grain size and beachface slope. The particular distribution patterns of the Santo André and Ribeira de Moinhos beaches suggest that other factors besides grain size must exist to explain the patterns shown in Fig. 3. The fact that the same range of high beachface slopes was found for the Tróia–Sines sandy embayed coast, whose sediment grain size ranges from medium to coarse and very coarse sand, seems to indicate joint influences of wave energy and sand bed permeability. Santo André and Ribeira de Moinhos beaches besides having relatively coarse and very coarse sand (>1 mm) also contain the most sorted sediments.

Curves describing beachface slope in relation to grain size are represented in Fig. 3 for various wave heights and for Iribarren numbers 0.1 and 1.5. Curves representing Iribarren numbers in the range 0.1–1.5 might also be drawn and would appear in between the curves corresponding to the same wave height. In this way, the diagram shown in Fig. 3 may accommodate data from beaches with a relatively wide range of plunging and spilling wave regimes. For the same wave height — represented by each curve — the equilibrium beachface slope increases with sand grain size.

For all the beaches considered in Fig. 3 we can observe clusters of data. The energy of waves that is characteristic of each beach increases along the vertical axis. Tróia beach is washed by small amplitude waves that show little variation around the mean wave amplitude. On the other hand, Ribeira de Moinhos beach is washed by high amplitude waves of moderate deviation around the mean. Each cluster shows different shapes that can be distinguished by both the vertical and the horizontal data ranges of variation.

According to the model presented above, each beach will adjust its beachface slope so as to minimize the time for completing a swash cycle. The equilibrium beachface slope is given by Eq. (20) and is a function of the sand grain size, d, and the wave height (through a, see Eq. (15a)). Usually, the grain size varies in some range, but we can take d as the grain size that is dominant in a particular beachface. If the grain size is allowed to vary in some range, i.e. if the water flows may differentially transport sediment of different sizes across-shore, this represents an extra degree of freedom in addition to beachface slope. Therefore, the extended interpretation of Eq. (20) reads: "The beachface slope and the dominant sand grain size will adjust together so as to minimize the time needed to complete a swash cycle".

In this light we can say that the horizontal range of variation of each cluster (see Fig. 3) corresponds to beachface slope adjustment (through cross-shore sediment transport) while the vertical range corresponds to the variation of sand grain size through *differential* cross-shore sediment transport. The actual changes in the beachface must result from a composite effect of both processes, i.e. beachface slope adjustment and changes in sediment grain size. Therefore, each cluster in Fig. 3 may be viewed as the domain in the (d, β) space that includes the points that represent possible adjustments of beach slope and sand grain size to wave forcing. In this way, in Fig. 3, the curve AB represents the average adjustment of Ribeira de Moinhos beach to plunging waves of height in the range 1–2 m. However, this might represent some particular behaviour we note that some other adjustment may follow the curve AB′, providing that coarser sediment is available to be transported to the beachface, which in this case will present a steeper slope (see Fig. 3). If the sediment available is that of point A, then the adjustment to equilibrium conditions corresponding to wave height of 2 m will follow the curve AB″, and the beachface will end with a less steep slope (point B″). These are examples of the many curves of beachface evolution that can be accommodated in the equilibrium domain of the (d, β) space.

With respect to analysis of the clusters, we may conclude that Ribeira de Moinhos beach may react to wave forcing with a broad range of combinations of beachface slope and grain diameter, which implies a capacity for sediment transport. Despite the fact that the data relating Ribeira de Moinhos beach cover a broad range of wave heights (1–2.5 m), the beachface slope varies in a small range (0.09–0.15), this also being the case of Santo André beach. According to our analysis, the reason for this behaviour must be due to the role played by swash infiltration/exfiltration in beaches with coarse and very coarse sands, such as Ribeira de Moinhos and Santo André. In fact Eq. (20) shows that for the same wave height, beachface slope is proportional to sand grain diameter raised to the power 4/3. Because both infiltration through Eq. (4) and exfiltration through Eq. (7) are proportional to permeability, which in turn increases with grain diameter (see Eq. (5)), we see how swash infiltration/exfiltration is related to permeability and beachface slope. In this context, we can understand the conclusions of Bagnold (1940), Quick (1991) and Packwood (1983) that infiltration in the swash zone is very important for profile evolution on coarse-grained beaches. The same explanation justifies the conclusions of Quick (1991), Komar (1998) and Masselink and Li (2001) that in beaches with very coarse sands (>1 mm) swash infiltration is very important, the beachface slope tending to increase with permeability. This last result is also anticipated by our model, through Eqs. (5) and (20), from which for constant wave height one finds $\beta \propto K^{\frac{2}{3}}$.

With the exception of Tróia beach, the other beaches in Fig. 3 adjust by changing beachface slope preferentially to changing dominant sediment size, though we note that the last aspect is not absent and must play an important role. All these clusters fall in the wave amplitude range characteristic of each of these beaches. Tróia beach shows concentration of data in a small domain of the (d, β) space. This aspect is compatible with the small wave forcing of this beach that requires only small adjustments of beachface morphology either through slope change or sand grain size change.

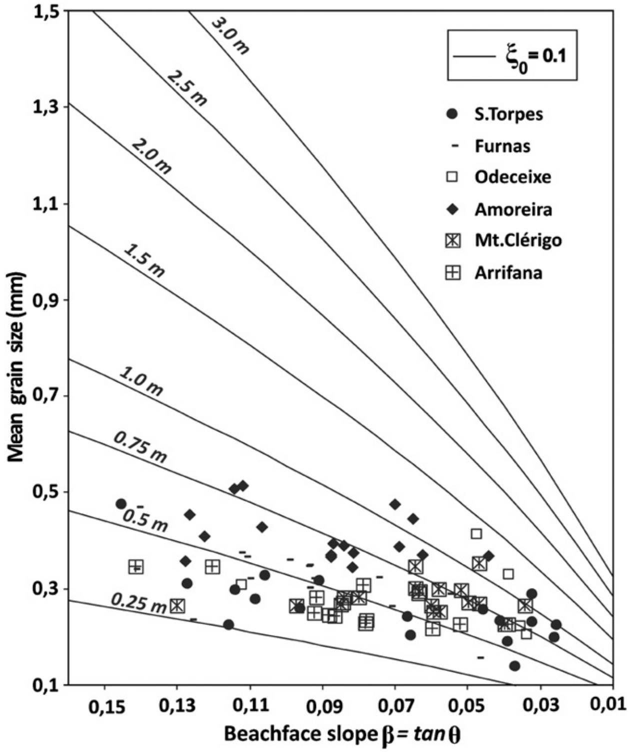

Fig. 4. Curves of equilibrium grain size versus beachface slope for each offshore wave height for Iribarren number equal to 0.1 corresponding to spilling waves. The marks in the figure represent data collected in the pocket beaches from Sines Cape to St. Vincent Cape (see legend).

Fig. 4 shows data from the pocket beaches located south from Sines Cape. These beaches, which occur in small bays along from the main rocky coastline and are locally protected from the offshore waves by natural headlands, exhibit minor variation in grain size (medium sand). In this way, and according to the present framework, the adjustment to wave forcing must occur mainly by changing beachface slope, as observed in Fig. 4. We can also observe that every beach decreases its beachface slope with increasing wave height, i.e. becomes more dissipative. Data from these beaches correspond to wave heights ranging from 0.25 to 1.0 m. In fact, these beaches are somehow protected from the offshore wave climate by wave diffraction that occurs in natural headlands, and by the wave refraction induced by the ridge and runnel system developed at the top of the submerged beach.

4.2.2. Cross-shore sediment transport

The present framework, namely Eq. (20), provides a relationship between wave height, sand grain size and beachface slope that might illuminate the dynamics of the beach profile as we will see next.

Butt et al. (2001) pointed out that critical grain size values might exist below which infiltration/exfiltration promotes offshore transport and above which infiltration/exfiltration promotes onshore transport. Additionally, some authors (e.g. Masselink and Li (2001), Horn (2006, p. 647), Masselink and Puleo (2006, p. 672) concluded that infiltration enhances onshore transport and berm formation. These results can be easily accommodated in the framework developed here. For example, if we consider 0.5 mm as the equilibrium grain size with respect to a definite wave height (see Figs. 3 and 4), for that wave height we may conclude that sand beds made of grains of smaller size will have lower equilibrium beachface slopes. Therefore, for constant wave height and for sand grain sizes smaller than 0.5 mm, the respective equilibrium beachface slope is lower than that corresponding to sand grains of 0.5 mm. Then, that reduction in beachface slope indicates that equilibrium is reached through offshore sediment transport. On the other hand, and again for the same wave height and for sand beds composed of grain sizes larger than 0.5 mm the respective equilibrium beachface slope is higher than that corresponding to sand grains of 0.5 mm, which indicates onshore sediment transport, mainly composed of grains of size bigger than 0.5 mm. The sand grain size of 0.5 mm was considered here as the reference for the equilibrium grain size for the case analysed above, and to our view, it does not represent any universal value. A similar analysis might be performed with reference to equilibrium grain sizes corresponding to some particular beach and wave height.

The analysis also allows explaining why in case that the sand bed grain size spans a broad range, the beachface slope will change to an extent smaller than in the case where it is composed of only one textural class. As shown, the reasons ground in the extra degree of freedom that is provided by differential sand grain transport, which allows beachface to reach equilibrium in combination with beachface slope variation.

4.2.3. Beachface profile engineering

The usefulness of the above results for beachface engineering becomes clear. As an illustration, let us consider point E in Fig. 5 which represents the equilibrium slope (0.16) of a beachface with an average sediment grain size of 1.3 mm, washed by waves of 1 m height. If the wave regime changes to waves of 2 m height, the beachface will evolve in time so as to adjust to the new constraint. This might correspond to the EF process in Fig. 5, during which the same sand bed would evolve in order to smooth the beachface to the new value of 0.06. However, the most common process is of the type EF′ with changes in the sand bed, so as the dominant sand grain size moves to the new value of 1.5 mm. However, if the bedrock underneath does not allow the mobile sand bed to reach such a slope value, all sand will be removed to the sea, and therefore, the equilibrium slope will never be reached. However, by rendering the sea bottom shallower, the sand

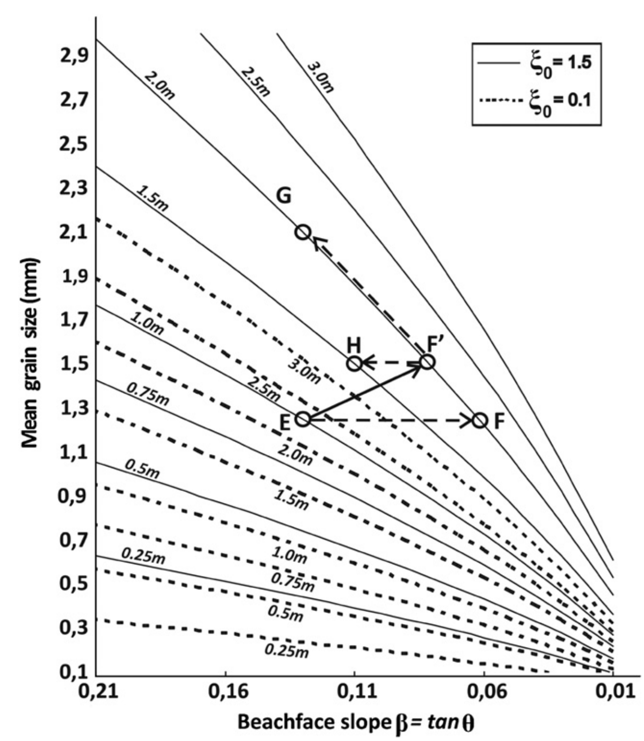

Fig. 5. Beachface processes: E–F′ — Evolution of beachface slope and dominant sand grain size as a response to changes in wave height in the range 1–2 m; E–F — Evolution of beachface slope at constant sand grain size; F′–G — Beachface slope restoration as the result of supply of coarser sediment. F′–H — Wave height reduction as the result of formation of an immersed sand bar.

transported offshore would lead to the development of a submarine sand bar that might induce earlier wave modification, and therefore, smaller uprush height. Then, a new beachface might form with a slope in between E and F (process F′H).

The initial beachface slope may be restored if coarser sediment, of size 2.1 mm, is carried onto the shore (process FG in Fig. 5). This new beach sediment will be in equilibrium with breaking waves of 2 m height. If it is not possible to change the sediment size, the beachface will adjust to waves of 2 m height by changing its slope only (process EF). As seen before, in this case the equilibrium slope will be lower than that of process EF′ and of the order of 0.06.

This example illustrates the usefulness of Eqs. (19) and (20) in the analysis of beachface dynamics. In a certain sense, Fig. 5, is a diagram of the "equilibrium" beachface shapes that will develop as the response to changes in sand size and wave height. Also, it opens the possibility of guiding beachface engineering, namely beachface reconstruction by feeding it with sand with the appropriate grain size.

5. Conclusions

The Constructal Law provides a useful theoretical basis for understanding beachface adjustment as a response to wave swash forcing in the presence of constraints like sand grain size, grain sphericity, and wave height. The Constructal Law, conceived as a principle regulating the dynamics of beachface flows, was applied to a model developed in this paper to account for the swash cycle, and has identified a relationship between offshore wave height, beachface slope and sediment grain size. This relationship showed that beachface slope varies with wave height raised to the power 3/4, and sand grain size raised to the power 4/3. Curves of "equilibrium" beachface slopes against grain size, for each wave height, provided a diagram suitable for data analysis and interpretation.

The diagram, which comprises curves in the range of important wave heights, turns out to be a useful framework for interpretation of field data collected from beaches of the southwestern coast of mainland Portugal. Additionally, it throws light on some findings about beachface slope in relation to sediment grain size and cross-shore sediment transport, published previously in the literature. It has also been shown how beach profile engineering may be guided by the diagram comprising the curves of "equilibrium" beachface slope against grain size, for each wave height.

However, it is important to note that the parameter H describing wave set-up is related to open ocean wave height ($(H)_0$) by the relationships given in Eqs. (11a) and (11b) that carry significant uncertainties. If these relationships are discarded, wave set-up (H) will appear explicitly, and therefore the diagrams of Figs. 3–5 will show H curves.

The model is amenable to improvement, namely by allowing variation of porosity with grain diameter. In this case, however, solution of the model equation would only be possible with the help of numerical analysis. It shows significant potential for understanding beachface adjustment, and adds credence to the Constructal Law as a regulating principle of structured flow systems.

Acknowledgements

This work has been supported by FCT (Portugal) under the contract PTDC/EME-MFE/71960/2006. The authors also thank the (unknown) reviewers of this paper for their useful comments and suggestions.

References

Anthony, E.J., 1998. Sediment-wave parametric characterization of beaches. Journal of Coastal Research 14, 347–352.
Bagnold, R.A., 1940. Beach formation by waves: some model experiments in a wave tank. Journal of the Institution of Civil Engineers 15, 27–52.
Baldock, T.E., Hughes, M.G., 2006. Field observations of instantaneous water slopes and horizontal pressure gradients in the swash-zone Continental Shelf Research 26, 574–588.
Bascom, W.N., 1951. The relationship between sand-size and beach face slope. Transactions of the American Geophysical Union 32, 866–874.
Bejan, A., 1997. Advanced Engineering Thermodynamics, Chapter 13, second ed. Wiley, New York.
Bejan, A., 2000. Shape and Structure, from Engineering to Nature. Cambridge University Press, Cambridge, UK.
Bejan, A., Lorente, S., 2004. The Constructal Law and the thermodynamics of flow systems with configuration. International Journal of Heat and Mass Transfer 47, 3203–3214.
Bejan, A., Marden, J.H., 2006. Constructing animal locomotion from new thermodynamics theory. American Scientist 4, 342–349.
Bejan, A., Reis, A.H., 2005. Thermodynamic optimization of global circulation and climate. International Journal of Energy Research 29, 303–316.
Blott, S.J., Pye, K., 2001. Gradistat: a grain size distribution and statistics package for the analysis of unconsolidated sediments. Technical communication. Earth Surface Processes and Landforms 26, 1237–1248.
Butt, T., Russell, P., 1999. Suspended sediment transport mechanisms in high-energy swash. Marine Geology 161 , 361–375.
Butt, T., Russell, P., Turner, I., 2001. The influence of swash infiltration–exfiltration on beach face sediment transport: onshore or offshore? Coastal Engineering 42, 35–52.
Costa, M., Silva, R., Vitorino, J., 2001. Contribuição para o estudo do clima de agitação marítima na costa portuguesa (in Portuguese). 2as Jornadas Portuguesas de Engenharia Costeira e Portuária. Associação Internacional de Navegação. Sines. CD-ROM, 20 pp.
Dingler, J.R., Reiss, T.E., 2002. Changes to Monterey Bay beaches from the end of the 1982–83 El Niño through the 1997–98 El Niño. Marine Geology 181, 249–263.
Dubois, R.N., 1982. Relation among wave conditions, sediment texture, and rising sea level: an opinion. Shore Beach 50, 30–32.
Folk, R.L., Ward, W.C., 1957. Brazos River Bar: a study in the significance of grain size parameters. Journal of Sedimentary Petrology 27, 2–26.
Gama, C., 2005. Dynamics of Sediment Systems in the Western Coast of Portugal South Cape Espichel. PhD thesis (in Portuguese), University of Évora, 359 p.
Gourlay, M.R., 1968. Beach and Dune Erosion Tests. Delft Hydraulics Laboratory, Delft, Netherlands. Report M935/M936.
Grant, U.S., 1948. Influence of the water table on beach aggradation and degradation. Journal of Marine Research VII, 655–660.
Guza, R.T., Thornton, E.B., 1981. Wave set-up on a natural beach. Journal of Geophysical Research 86, 4133–4137.
Hegge, B.J., Masselink, G., 1991. Groundwater-table responses to wave run-up: an experimental study from Western Australia. Journal of Coastal Research 7, 623–634.
Horn, D.P., 2006. Measurements and modeling of beach groundwater flow in the swash-zone: a review. Continental Shelf Research 26, 622–652.
Hughes, S.A., 2004. Estimation of wave run-up on smooth, impermeable slopes using the wave momentum flux parameter. Coastal Engineering 51, 1085–1104.
Hughes, M., Turner, I., 1999. The beachface. In: Short, A.D. (Ed.), Handbook of Beach and Shoreface Morphodynamics. Wiley, Chichester, pp. 119–144.
Hughes, M., Masselink, G., Brander, R.W., 1997. Flow velocity and sediment transport in the swash zone of a steep beach. Marine Geology 138, 91–103.
Jackson, D.W.T., Cooper, J.A.G., del Rio, L., 2005. Geological control of beach morphodynamic state. Marine Geology 216, 297–314.
Kang, H-Y., Nielsen, P., 1996. Watertable dynamics in coastal areas. Proceedings of the 25th International Conference Coastal Engineering. American Society of Civil Engineers, New York, pp. 4601–4612.
Kemp, P.H., 1975. Wave asymmetry in the nearshore zone and breaker area. In: Hails, J., Carr, A. (Eds.), Nearshore Sediment Dynamics and Sedimentation. Wiley-Interscience, London, pp. 47–67.
Komar, P., 1998. Beach Processes and Sedimentation, 2nd ed. Prentice-Hall, New Jersey, p. 544.
Levoy, F., Anthony, E.J., Monfort, O., Larsonneur, C., 2000. The morphodynamics of megatidal beaches in Normandy, France Marine Geology 171, 39–59.
Li, L., Barry, D.A., Pattiaratchi, C.B., Masselink, G., 2002. BeachWin: modelling groundwater effects on swash sediment transport and beach profile changes. Environmental Modelling and Software 17, 313–320.
Mase, H., 1989. Random wave run-up height on gentle slope. Journal of Waterway, Port, Coastal, and Ocean Engineering 5, 649–661.
Masselink, G., Hughes, M.G., 1998. Field investigation of sediment transport in the swash zone. Continental Shelf Research 18, 1179–1199.
Masselink, G., Li, L., 2001. The role of swash infiltration in determining the beachface gradient: a numerical study. Marine Geology 176, 139–156.
Masselink, G., Pattiaratchi, C.B., 2001. Seasonal changes in beach morphology along the sheltered coastline of Perth, Western Australia. Marine Geology 172, 243–263.
Masselink, G., Puleo, J.A., 2006. Swash-zone morphodynamics. Continental Shelf Research 26, 661–680.
McLean, R.F., Kirk, 1969. Relationship between grain size, size-sorting and foreshore slope on mixed sand-shingle beaches. New Zealand Journal of Geology and Geophysics 12, 138–155.
Miguel, A., 2006. Constructal pattern formation in stony corals, bacterial colonies and plant roots under different hydrodynamics conditions. Journal of Theoretical Biology 242, 954–961.
Nielsen, P., 1999. Groundwater dynamics and salinity in coastal barriers. Journal of Coastal Research 15, 732–740.
Packwood, A.R., 1983. The influence of beach porosity on wave uprush and backwash. Coastal Engineering 7, 29–40.

Pryor, W.A., 1973. Permeability–porosity patterns and variations in some Holocene sand bodies. Bulletin of the American Association of Petroleum Geologists 57, 162–189.
Puleo, J.A., Beach, R.A., Holman, R.A., Allen, J.S., 2000. Swash zone sediment suspension and transport and the importance of bore-generated turbulence. Journal of Geophysical Research 105, 17021–17044.
Quick, M.C., 1991. Onshore–offshore sediment transport on beaches. Coastal Engineering 15, 313–332.
Reis, A.H., 2006a. Constructal theory: from engineering to physics, or how flow systems develop shape and structure. Applied Mechanics Reviews 59, 269–282.
Reis, A.H., 2006b. Constructal view of scaling laws of river basins. Geomorphology 78, 201–206.
Reis, A.H., Bejan, A., 2006. Constructal theory of global circulation and climate. International Journal of Heat and Mass Transfer 49, 1857–1875.
Reis, A.H., Miguel, A.F., Aydin, M., 2004. Constructal theory of flow architectures of the lungs. Medical Physics 31, 1135–1140.
Selley, R.C., 1988. Applied Sedimentology. Academic Press, London, p. 446.
Shepard, F.P., 1963. Submarine Geology, 2nd ed. Harper and Row, New York.
Soulsby, R.L., 1991. Dynamics of Marine Sands. Thomas Telford Publ., London, UK, p. 249.
Stockdon, R.A., Holman, P.A., Howd, Sallenger Jr., A.H., 2006. Empirical parameterisation of setup, swash, and run-up. Coastal Engineering 53, 573–588.
Sunamura, T., 1984. Quantitative predictions of beachface slopes. Geological Society of America Bulletin 95, 242–245.
Turner, I.L., 1993. Water table outcropping on macro-tidal beaches: a simulation model. Marine Geology 115, 227–238.
Turner, I.L., 1995. Modelling the time-varying extend of groundwater seepage on tidal beaches. Earth Surface Processes and Landforms 20, 833–843.
Turner, I.L., 1998. Monitoring groundwater dynamics in the littoral zone at seasonal, storm, tide and swash frequencies. Coastal Engineering 35, 1–16.
Turner, I.L., Masselink, G., 1998. Swash infiltration–exfiltration and sediment transport. Journal of Geophysical Research 103, 30813–30825.
Wiegel, R.L., 1964. Oceanographical Engineering. InPrentice-Hall, Englewood Cliffs, N.J., p. 532.
Wilson, A.M., Huettel, M., Klein, S., 2008. Grain size and depositional environment as predictors of permeability in coastal marine sands. Estuarine, Coastal and Shelf Science 80, 193–199.
Wright, L.D., Short, A.D., 1984. Morphodynamic variability of surf zones and beaches: a synthesis. Marine Geology 56, 93–118.
Wright, L.D., Short, A.D., Green, M., 1985. Short-term changes in the morphodynamics state of beaches and surf zones, an empirical predicative model. Marine Geology 62, 339–364.