Mar. Freshwater Res., 1995, **46**, 419–25

Observational Evidence of the Interaction of Ocean Wind-sea with Swell

J. S. Reid

CSIRO Division of Oceanography, GPO Box 1538, Hobart, Tasmania 7001, Australia.

Abstract. More than 10 000 spectra of ocean wave data were acquired from a series of buoys moored in the Southern Ocean off the west coast of Tasmania for a period of seven years. Spectra were grouped according to the wind speed and whether the wind direction was onshore or offshore and the mean spectrum found for each group. The frequencies of the low frequency cut-offs and of the spectral peaks of the resulting mean spectra were found to be independent of the wind speed in contrast to self-similar standard spectra such as JONSWAP. This property is attributed to the presence of a swell background which controls the evolution of wind-seas in the open ocean. During offshore winds, the spectral variance between 0·04 Hz and 0·16 Hz was found to be negatively correlated with wind speed indicating that swell is in turn shaped either by the wind-sea or by the wind itself.

Relationships between various spectral and time domain parameters and the wind speed were investigated empirically. The slope/acceleration measures such as m_4, the spectral fourth moment, were found to be highly correlated with wind speed.

Introduction

The shape of the frequency spectrum of wind-generated ocean surface waves has been discussed since the 1950s. In the absence of swell the spectrum exhibits a single peak with a sharp cut-off on the low-frequency side and a less dramatic decay on the high-frequency side of the peak.

Bretschneider (1959) proposed a spectrum of the form

$$\phi(\omega) = \alpha_p g^2 \omega^{-5} e^{-0{\cdot}675(2\pi/T\omega)^4} \tag{1}$$

where ϕ is the variance spectral density, g is the acceleration due to gravity, ω is the angular frequency, T is a mean period and α_p is a constant. The high -frequency side of Bretschneider's spectrum follows a fifth-power law. Phillips (1958) had used dimensional arguments for a fifth-power law decay with frequency.

Kitaigorodskii (1961) applied the idea of similarity to wave spectra. He suggested that for sufficiently long fetch and sufficiently prolonged wind action the growth of waves can terminate to form a 'fully developed sea' whose average statistical properties depend only on the wind velocity. Using similarity arguments he showed that for a fully developed sea the spectrum must be a function of dimensionless frequency alone, that the high-frequency tail must follow a fifth-power law, that wave height must follow a U^2 law and that

$$\omega_{max} U/g = k \tag{2}$$

where ω_{max} is the angular frequency of the spectral peak, U is the wind speed or some similar velocity parameter and k is a constant. Thus, for a fully developed sea, wave spectra all have the same shape and are scaled by U. Pierson and Moskowitz (1964) proposed a standard spectrum with these properties for a fully developed sea and evaluated the constants from experimental data. Their observations were based on a working definition of a fully developed sea by Moskowitz (1964) whose criteria included the absence of swell from the wave spectrum. Later Hasselmann *et al.* (1973) proposed a standard spectrum for fetch-limited conditions known as the 'JONSWAP' spectrum. A further feature in the form of an 'overshoot effect' or 'peak enhancement' had been noted by Barnett and Wilkerson (1967) in spectra obtained from airborne measurements in fetch-limited conditions. The JONSWAP spectrum included a factor to describe this.

More recently Donelan *et al.* (1985), using an array of wave staffs on Lake Ontario, demonstrated a fourth-power frequency dependence, viz

$$\phi(\omega) = \alpha g \omega^{-4} \omega_p^{-1} e^{-(\omega_p/\omega)^4} \gamma^{\Gamma} \tag{3}$$

where the peak enhancement exponent is given by

$$\Gamma = e^{-(\omega-\omega_p)^2 / 2\sigma^2 \omega_p} . \tag{4}$$

The formula is adapted from the JONSWAP formula with the original JONSWAP parameters, i.e. ω_p is the frequency of the spectral peak, α is the equilibrium range (rear face) parameter, γ is the peak enhancement factor σ and s is the peak width parameter. These were found experimentally to be dependent on the 'inverse wave age', U_p/c_p, where U_p is the component of the wind in the direction of travel of the waves and c_p is the phase velocity of the waves near the spectral peak.

1323-1650/95/020419$05.00

All of the above spectra are single peaked and were derived from and apply to wind-seas generated in the absence of swell. Swell comprises wave energy propagated into the study area from distant sources. Swell in the range 0·05 to 0·08 Hz was observed to travel across the entire Pacific Ocean from New Zealand to Alaska by Snodgrass *et al.* (1966) with little diminution of energy. Because swell propagates on an oceanic scale and because at any time an ocean will contain a number of wave sources, swell is ubiquitous in the open ocean. It follows that the 'fully developed sea' criteria used by Moskowitz to select wave spectra are rarely satisfied in the open ocean because of the presence of swell and that the various standard spectra may not be typical of the wave conditions prevailing over most of the world's oceans for most of the time.

It might be argued that in order to predict or hindcast a composite spectrum for a given wind speed to first order, the wind-sea spectrum appropriate in the absence of swell might be found and superimposed on the swell spectrum to give a spectrum applicable to the open ocean. This process would ignore the role played by non-linear processes in shaping surface wave spectra. The results presented below suggest that swell does interact with and significantly modify the wind-sea and *vice versa*.

Here, more than 10 000 spectra derived from wave data collected from the Southern Ocean near the west coast of Tasmania over a period of several years are examined.

Two classes of spectra were selected according to whether the wind direction was unambiguously onshore or offshore. The onshore and offshore classes were further divided into groups according to the measured wind speed and the mean spectrum found for each group. The frequencies of the low-frequency cut-offs and of the spectral peaks of the resulting mean spectra were found to be independent of the wind speed in contrast to the various self-similar standard spectra discussed above. This property is attributed to the presence of an average background swell, which conditions the evolution of wind seas in the open ocean.

With the breakdown of similarity, various spectral scaling laws such as Eqn 2 above no longer apply. Relationships between various spectral and time domain parameters and the wind speed were investigated empirically. The quantity m_4, the spectral fourth moment, was found to be highly correlated with wind speed.

Data

Wave Spectra

A 'Waverider' buoy was moored near Cape Sorell (Fig. 1) on the west coast of Tasmania (near 42°7′S,145°3′E). The buoy was moored near the 100 m depth contour on the continental shelf 17 km from the shore and 25 km from the shelf edge. Sea surface height data were transmitted ashore and archived on a routine basis. 'Bursts' of data comprising 2048 heights sampled at a rate of 2·56 samples s^{-1} were saved every 3 h.

The spectral analysis was carried out in the usual way, viz each burst of data was multiplied by a 10% raised cosine filter and the resulting series transformed using a 2048-point fast Fourier transform. The squared modulus of the transform then yielded a 1025-point long periodogram, P_n, of the data from zero to the Nyquist frequency, 1·28 Hz, with a frequency resolution, *df*, of 1/800 Hz. The variance-density spectral estimate of sea surface displacement $S(f)$ is found by averaging P_n over a range of frequencies, viz

$$S(f) = \sum_{n=n_1}^{n_2} P_n / (n_2 - n_1) df. \tag{5}$$

It is often more informative with ocean wave spectra to plot the variance density of the vertical component of sea surface acceleration, $S_a(f)$, rather than of sea surface displacement. Thus

$$S_a(f) = \sum_{n=n_1}^{n_2} (2\pi f)^4 P_n / (n_2 - n_1) df \tag{6}$$

where $f = (n-1)df$. The total variance associated with a given frequency band is found by summing P_n across the band.

In practice owing to the transfer function of the buoy telemetry system, the variance density falls off very rapidly with frequencies above 0·5 Hz. This is well below the Nyquist frequency implying that the spectra were free from aliasing effects.

Winds

At Granville Harbour, 30 km to the north of the wave measurement site, wind speed and direction were routinely logged. The wind measurement site is 1 km inland and 40 m above mean sea level. Wind measurements from 10 m altitude at this site in the form of hourly records of 10-min vector averaged speed U and direction ϕ, were saved for comparison with the wave measurements for the period 18 November 1985 to 24 September 1992.

The coast and continental shelf are roughly parallel with azimuth 343°. In order to avoid coastal effects and to ensure that the wind was unambiguously either onshore or offshore the spectra chosen for study were confined to those for which the wind vector azimuth ϕ, lay within 45° of the normal to the coast in one of the ranges

$$28° < \phi < 118° \text{ (onshore set)}$$
$$208° < \phi < 298° \text{ (offshore set).}$$

There were 1896 spectra in the onshore set and 2742 in the offshore set.

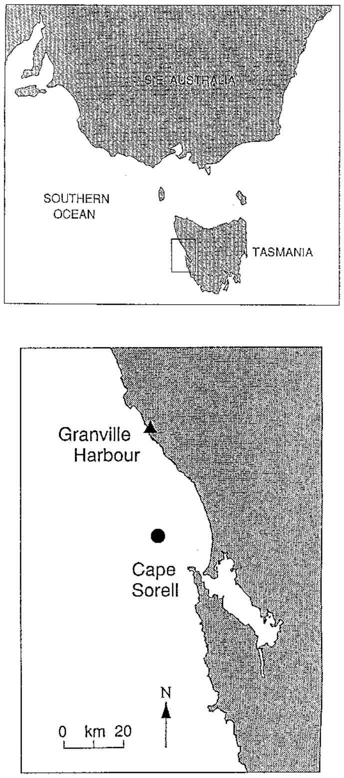

Fig. 1. Map showing the locations of (●) the Waverider buoy and (▲) the wind tower.

Relation between Wind and Variance Density

The relation between variance and wind speed was found by computing the correlation coefficient, r, for various frequency bands in the range 0·04 to 0·5 Hz. The results for the two classes are shown graphically as the solid curves in Fig. 2. The maximum value of the correlation coefficient was $r = 0{\cdot}85$ for the frequency range 0·25 to 0·30 Hz for the onshore data set. The correlation was negative at frequencies below 0·16 Hz with a minumum of –0·235 for the frequency range 0·10 to 0·12 Hz for the offshore data set. Because of the large number of points in the sample, this negative correlation coefficient is significantly different from zero to better than one part in 10^{20}.

The persistence of the wind might also be expected to have some effect on the correlation coefficient. A wind-sea takes a finite time to become 'developed'. For this reason, the data were further restricted to 'persistent' winds, i.e. those for which (*i*) valid wind data were available for the given time and for 3 h and 6 h previously, (*ii*) the wind was consistently either onshore or offshore for the three 3-hourly periods according to the above definitions and (*iii*) the wind speed differed by no more than 2 m s^{-1} from that measured 3 h and 6 h previously.

The results are shown as the dashed lines in Fig. 2. There were 492 cases in the upper, onshore sample and 723 cases in the lower, offshore sample.

The onshore and offshore sets were divided into eight groups according to the absolute magnitude of the wind speed. The ranges and the number of spectra in each group are shown in Table 1. The periodograms for each group were averaged to produce a mean periodogram from which mean spectral estimates were derived by Eqn 5 and Eqn 6 above. The resulting mean displacement and acceleration spectra are shown in Figs 3 and 4. The spectra of Figs 3 and 4 were averaged over 21 neighbouring values of P_n giving a frequency resolution of 0·02625 Hz. This is about half the width of the narrower peaks in Fig. 4(*b*).

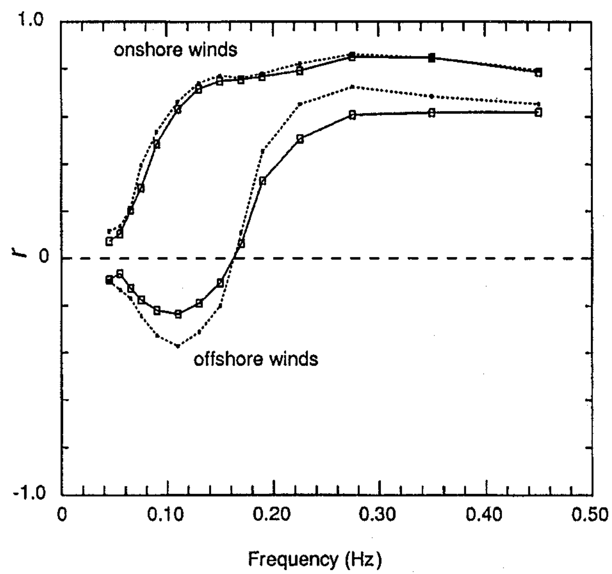

Fig. 2. Sample correlation coefficient, r, of sea surface displacement variance density v. wind speed as a function of frequency. Upper curves, onshore winds; lower curves, offshore winds. The dotted curves show the values for 'persistent' winds (see text).

Table 1. Numbers of spectra averaged in each wind group

Group	Range (m s^{-1})	Number onshore	Number offshore
1	$0 \leqslant U < 2$	138	278
2	$2 \leqslant U < 4$	351	764
3	$4 \leqslant U < 6$	369	747
4	$6 \leqslant U < 8$	344	433
5	$8 \leqslant U < 10$	390	277
6	$10 \leqslant U < 12$	194	169
7	$12 \leqslant U < 14$	75	51
8	$14 \leqslant U < 16$	35	23

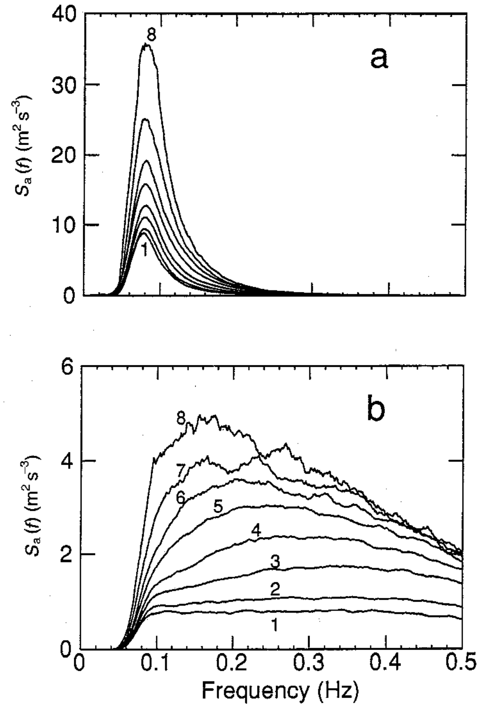

Fig. 3. (*a*) Mean sea-surface displacement variance-density spectra and (*b*) mean sea-surface acceleration variance-density spectra for eight onshore wind speeds. The spectra are labelled according to the wind-speed groups shown in Table 1 and are smoothed with a 21-point running mean.

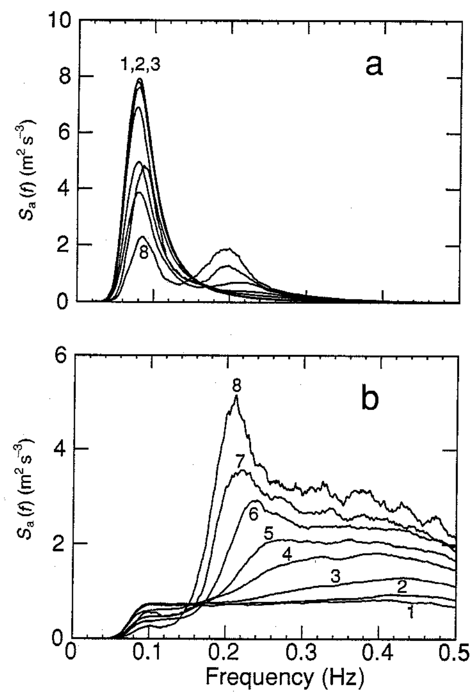

Fig. 4. (*a*) Mean sea-surface displacement variance-density spectra and (*b*) mean sea-surface acceleration variance-density spectra for eight offshore wind speeds. The spectra are labelled according to the wind-speed groups shown in Table 1 and are smoothed with a 21-point running mean.

Relationship between Wind Speed and Wave Statistics

The correlations holding between wind speed and various wave sample statistics are shown in Table 2. Correlation coefficients were computed for four cases: onshore instantaneous winds, onshore persistent winds, offshore instantaneous winds and offshore persistent winds. The terms 'onshore' and 'offshore' are defined in the same way as above, i.e. wind direction within 45° of the normal to the coast, and the term 'persistent' is as defined in the previous section. By 'instantaneous' is meant the 10-min vector averaged mean wind at the time of the wave sample.

Table 2. Correlation coefficients calculated for wind speed *v*. various sample statistics for 'instantaneous' and 'persistent' winds

Statistic	Onshore		Offshore	
	inst.	pers.	inst.	pers.
H_s	0·579	0·610	−0·170	−0·229
T_z	−0·157	−0·043	−0·567	−0·664
H_s/T_z^2	0·819	0·809	0·567	0·686
$\hat{m}_4$	0·883	0·900	0·603	0·692
Number	1094	713	2744	1020

In Table 2, the significant wave height, H_s, is defined as four times the standard deviation of the sample of sea surface heights and T_z is the mean zero crossing period of the sample. The quantity H_s/T_z^2 is a mean vertical acceleration or mean steepness for the sample (since wavelength is proportional to the square of the period). The quantity m_4 is the fourth moment of the displacement spectrum defined by

$$m_4 = \int_0^\infty S(f) f^4 df. \tag{7}$$

From (6)

$$m_4 = (2\pi)^{-4} \int_0^\infty S_a(f) df; \tag{8}$$

that is, m_4 is proportional to the area enclosed by the acceleration spectrum.

In practice, wave buoys do not respond to displacements and accelerations much above 0·5 Hz but accelerations at frequencies much greater than this contribute to the integral in (8). Consequently the statistic $\hat{m}_4$, defined by

$$\hat{m}_4 = (2\pi)^{-4} \int_p^q S_a(f) df \tag{9}$$

where p = 0·04 Hz and q = 0·5 Hz, was used as an approximation to m_4.

Discussion

Spectra

The correlation between variance density and wind speed shown by the solid curves in Fig. 2 is certainly remarkably good at frequencies greater than about 0·25 Hz, particularly in the onshore case. At lower frequencies the curves are quite different, the variance density being negatively correlated with wind speeed for the offshore case at frequencies below 0·2 Hz.

The persistence of the wind appears to make little difference in the onshore case although it does have a small but significant effect on the offshore correlations. Evidently, in the open ocean the waves are generally close to equilibrium with the wind in terms of the 3 h sampling time of this study.

The smoothed average displacement and acceleration variance density spectra for the onshore and offshore cases for the eight wind-speed ranges are shown in Figs 3 and 4. All of the mean displacement spectra in Figs 3*a* and 4*a* are dominated by a single peak at 0·08 Hz. Only the mean spectra for high offshore winds show a separate wind-sea peak. Apart from this, the only major difference between displacement spectra for the different wind-speed groups lies in the size of this peak; for onshore winds the peak increases in size with increasing wind speed, whereas for offshore winds it generally decreases with increasing wind speed. More detail can be seen in the acceleration spectra of Figs 3*b* and 4*b* where the density spectra have been multiplied by the function $(2\pi f)^4$. The differences between the two sets of mean acceleration spectra are more striking; in the onshore case there are no sharp peaks, whereas in the offshore case the spectra are sharply peaked for the high wind-speed groups. This is the 'overshoot effect' of Barnett and Wilkerson (1967), which is characteristic of short fetch spectra.

The onshore and offshore acceleration spectra for winds of < 2 m s^{-1} (Group 1) are almost identical and are flat in both cases with a low-frequency cut-off near 0·07 Hz. This indicates that the high-frequency tails of the corresponding displacement spectra have a fourth-power decay with frequency. These spectra describe an average 'swell background' for this site since negligible wind-sea would be generated in such light winds. Effectively this is the shape of the average spectrum at the site in the absence of local wind.

This swell background appears to control the shape of spectra at higher wind speeds in the onshore case, in that all the mean displacement spectra have a similar shape and peak frequency to the Group 1 spectrum. Even in the offshore case the shapes of the displacement spectra are dominated by this peak, even though the spectra of the higher wind-speed groups exhibit a second or 'wind-sea' peak. The offshore acceleration spectra on the other hand are dominated by this 'enhanced' wind-sea peak similar to that observed at higher wind speeds in swell-free conditions, for example by Donelan *et al.* (1985).

Thus, an onshore wind or wind-sea enhances the pre-existing swell background without changing its frequency, whereas an offshore wind or wind-sea diminishes the swell background and superimposes a wind-sea on it.

Other Observations

Donelan *et al.* (1985) have shown a strong dependence of peak enhancement on inverse wave age with little peak enhancement for U/c_p values of < 2. The absence of peak enhancement in the mean spectra shown in Fig. 3 is consistent with this result; the peak frequencies of around 0.08 Hz correspond to a phase velocity, c_p , of 17 m s^{-1} and inverse wave ages of less than unity for all of the onshore spectra. On the other hand, the groups that show strong peak enhancement in the offshore wind case of Fig. 4*b* have peaks at lower frequencies and consequently larger inverse wave ages. Groups 6, 7 and 8 have spectral peak-phase velocities of 6·5, 7·1 and 7·4 m s^{-1} giving inverse wave ages of 1·5, 1·4 and 1·3 respectively. Although larger, these values are still not sufficient to account for the peak enhancements of 20–50% above background observed in Fig. 4. In the experiments of Donelan *et al.* (1985), inverse wave age values of from 2 to 4 were required to give peak enhancements this large. This suggests that the opposing swell may have played some role in generating peak enhancement.

Donelan (1987), using a wind wave tank, noted that 'swell' in the form of longer paddle-generated waves inhibited the growth of wind waves. That the wind waves also enhanced the swell is evident from his fig. 1. His fig. 6 shows how the presence of swell leads to a broader peak in the wind-sea part of the spectrum as observed here in the onshore case.

The interaction of swell with wind waves has also been discussed by Phillips and Banner (1974), Wright (1976) and more recently by Chu *et al.* (1992). These workers all used tanks and dealt primarily with the suppression of wind waves by the swell; the effect on the swell itself has largely been ignored since an early paper by Phillips (1963) who showed, on theoretical grounds, that swell should be attenuated by breaking wind waves travelling in the same direction, rather than augmented as observed here.

The effect of non-linear coupling due to resonant interactions in bimodal spectra has been modelled by Masson (1993). She found that when such a spectrum is integrated over time under the influence of non-linear interactions, the spectral distribution gradually changes into a unimodal shape where the local sea peak has disappeared and the swell peak has broadened. These results are in agreement with the laboratory work of Donelan (1987) and with the observations presented here.

In the open sea, an effect of swell on peak enhancement has been reported by Kahma and Calkoen (1992). Reduced peak enhancement was found in their Bothnian Sea spectra when swell arrived in an area after a period of exceptionally steady offshore wind; they quoted a similar effect that had been noted by Donelan in Lake Ontario. In these cases, the swell acted to remove the peak enhancement in offshore wind-sea spectra rather than to enhance or support it as suggested here, but without a knowledge of the precise circumstances a direct comparison with the present observations is not warranted. The main thrust of Kahma and Calkoen's paper was a detailed exploration of the discrepancies in the dimensionless fetch laws as derived from data from various sites. The differences were ascribed by them to differences in the stability of atmospheric stratification prevailing during the different experiments. They found that the presence or absence of swell did not appear to influence the dimensionless energy in the JONSWAP spectra which they examined. They also noted that peak enhancements at the same dimensionless fetch varied considerably between Bothnian Sea spectra and those from Lake Ontario.

A wide range of peak enhancements from $\gamma \approx 1{\cdot}5$ to $\gamma \approx 6{\cdot}0$ was also observed during the JONSWAP experiment.

These peak enhancements appeared to be quite uncorrelated with dimensionless fetch (Hasselmann *et al.* 1973, fig. 2·8). Although swell was present for about one-third of the time no attempt was made to discriminate between peak enhancements occurring in the presence or absence of swell.

In summary, it appears that the shape of the wind-sea spectrum may be influenced by swell, that large and unexplained variations in the degree of peak enhancement have been observed and that swell can act to reduce peak enhancement in some circumstances.

Statistics

The correlation coefficients listed in Table 2 show that commonly used time-domain statistics H_s and T_z are not well correlated with wind speed. In fact H_s shows a slight negative correlation with offshore winds as a consequence of the erosion of the swell peak mentioned earlier.

On the other hand, those quantities that are related to the mean vertical acceleration generally show a much better correlation with the wind, particularly in the onshore case. The value of 0·9 for the correlation coefficient relating $\hat{m}_4$ to persistent onshore winds is remarkably high considering the distance between the wave buoys and the wind tower (30 km). The regression relationship between measured mean-square acceleration and onshore wind speed was found to be

$$\hat{m}_4 = a_0 + a_1 U \qquad (10)$$

where

$$a_0 = 1{\cdot}34 \times 10^{-4}\ \mathrm{m^2\,s^{-4}}$$
$$a_1 = 0{\cdot}36 \times 10^{-5}\ \mathrm{m\,s^{-3}}.$$

A corresponding relationship between wind speed and mean-square slope was noted by Cox and Munk (1954); see Appendix.

The lower values of the correlation between the slope/acceleration statistics and the wind in the offshore cases may be due to differences in wind speed between the wind tower and the open ocean. Measured offshore winds are less likely to be representative of the open sea owing to topographic effects. The west coast of Tasmania is mountainous with elevations of 1000 m being common within 40 km of the sea. Nevertheless, even in this case the correlation is still as good as would be expected between two sets of wind measurements from localities 30 km apart in a coastal environment.

Conclusions

Mean wave spectra observed in an open ocean regime where swell was almost always present differed from self-similar wave spectra observed previously by other workers in the absence of swell. Mean displacement spectra were dominated by a single peak at a frequency which was independent of wind speed and equal to the peak frequency of the background swell observed in light winds. Increasing wind speeds caused the peak to be enhanced or diminished according to whether the wind was onshore or offshore but did not alter its frequency. These observations suggest that in the open ocean the development of wind-sea is conditioned by the pre-existing swell and that energy input from the wind is ultimately transferred to frequencies at which energy was already present in the spectrum.

Fetch-limited wind-seas generated by offshore winds developed in a similar way to those in swell-free conditions and showed strong peak enhancement at higher wind speeds. During offshore winds, wave energy at the low-frequency end of the spectrum was found to be negatively correlated with the wind speed in accordance with the nautical maxim: 'an opposing wind flattens a swell'.

The significant wave height, H_s, and mean zero crossing period, T_z, did not correlate well with wind speed. However 'surface slope' parameters such as H_s/T_z^2 were much better correlated with the wind, and the sample acceleration variance, $\hat{m}_4$, showed an exceptionally high correlation with onshore winds. This is consistent with the relationship between wind speed and the mean square slope of the sea surface found by Cox and Munk (1954).

Acknowledgments

This work was supported financially by grants from Steedman Ltd of Perth, Western Australia, and from the Fisheries Industry Research Trust Account. I thank the

Tasmanian Hydro-Electric Commission for provision of the Granville Harbour wind data and the Forestry Commission for housing with the shore base facility. Thanks also to Trevor McDougal and Stephen Walker for helpful advice during the preparation of the manuscript.

References

Barnett, T. P, and Wilkerson, J. C. (1967). On the generation of ocean wind waves as inferred from airborne radar measurements of fetch limited spectra. *Journal of Marine Research* **25**, 292–321.

Bretschneider, C. L.(1959). Wave variability and wave spectra for wind-generated gravity waves. Beach Erosion Board Technical Memorandum No.118.

Chu, J. S., Long, S. R., and Phillips, O. M. (1992) Measurements of the interaction of wave groups with shorter wind generated waves. *Journal of Fluid Mechanics* **245**, 191–210.

Cox, C., and Munk, W. (1954). Measurement of the roughness of the sea surface from photographs of the sun's glitter. *Journal of the Optical Society of America* **44**, 838–50.

Donelan, M. A. (1987). The effect of swell on the growth of wind waves. *Johns Hopkins APL Technical Digest* **8**, 18–23.

Donelan, M. A., Hamilton, J., and Hui, W.H. (1985). Directional spectra of wind generated waves. *Philosophical Transactions of the Royal Society, London* **A315**, 509–62.

Hasselmann, K., Barnett, T. P., Bouws, E., Carlson, H., Cartwright, D. E., Enke, K., Ewing, J.A., Gienapp, H., Hasselmann, D. E., Kruseman, P., Meerburg, A., Muller, P., Olbers, D. J., Richter, K., Sell, W., and Walden, H. (1973). Measurements of wind-wave growth and swell decay during the Joint North Sea Wave Project (JONSWAP). *Deutsche Hydrographische Zeitschrift* **A8**, 1–95.

Kahma, K. K., and Calkoen C. J. (1992). Reconciling discrepancies in the observed growth of wind generated waves. *Journal of Physical Oceanography* **22**,1389–405.

Kitaigorodskii, S. A. (1961). Applications of the theory of similarity to the analysis of wind generated wave motion as a stochastic process. *Izvestia of the Academy of Science of the USSR, Geophysics Series* **1**, 105–17.

Masson, D. (1993). On the nonlinear coupling between swell and wind waves. *Journal of Physical Oceanography* **23**,1249–58.

Moskowitz, L. (1964). Estimates of the power spectrums for fully developed seas for wind speeds of 20 to 40 knots. *Journal of Geophysical Research* **69**, 5161–79.

Phillips, O. M. (1958). The equilibrium range in the spectrum of wind-generated waves. *Journal of Fluid Mechanics* **4**, 426–34.

Phillips, O. M. (1963). On the attenuation of long gravity waves by short breaking waves. *Journal of Fluid Mechanics* **16**, 321–2.

Phillips, O. M., and Banner, M.L. (1974). Wave breaking in the presence of wind drift and swell. *Journal of Fluid Mechanics* **66**, 625–40.

Pierson, W. J., and Moskowitz, L. (1964). A proposed spectral form for fully developed wind seas based on the similarity theory of S. A. Kitaigorodskii. *Journal of Geophysical Research* **69**, 5181–90.

Snodgrass, F. E., Groves, G.W., Hasselmann, K.F., Miller, G.R., Munk, W.H., and Powers, W.H. (1966). Propagation of swell across the Pacific. *Philosophical Transactions of the Royal Society* **A259**, 431–97.

Wright, J. W. (1976). The wind drift and wave breaking. *Journal of Physical Oceanography* **6**, 402–5.

Appendix

Cox and Munk (1954) used photogrammetric measurements of sun glitter to estimate the distribution of slopes, viz

$$\sigma_x^2 + \sigma_y^2 = 0.003 + 5.12 \times 10^{-3} U \tag{11}$$

where $\sigma_x{}^2$ and $\sigma_y{}^2$ are the mean-square slopes in the upwind and crosswind directions and U is the wind speed (at 19·5 m altitude) in m s^{-1}. The mean-square slope is given in terms of the two-dimensional wave-number spectrum $S(k_x,k_y)$ as follows

$$\sigma_x^2 + \sigma_y^2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S(k_x,k_y)(k_x^2 + k_y^2)dk_x dk_y \tag{12}$$

$$\sigma_x^2 + \sigma_y^2 = \int_0^{\infty}\int_0^{2\pi} S(k,\theta)d\theta k^3 dk \tag{13}$$

where $k_x = k\cos\theta$ and $k_y = k\sin\theta$. It follows from the dispersion relation, $k = \omega^2 / g$, that

$$\int_0^{2\pi} S(k,\theta)d\theta k dk = S(\omega)2\omega d\omega. \tag{14}$$

Hence,

$$\sigma_x^2 + \sigma_y^2 = 2\int_0^{\infty} S(\omega)\omega^4 d\omega / g^2. \tag{15}$$

Hence,

$$\sigma_x^2 + \sigma_y^2 = 2(2\pi)^5 m_4 / g^2. \tag{16}$$

Substituting Eqn 10 in Eqn 16 yields $7{\cdot}26 \times 10^{-4}$ m^{-1}s for the regression coefficient, a value considerably smaller than that of $5{\cdot}12 \times 10^{-3}$m^{-1}s in Eqn 11 above. This discrepancy can be attributed to the approximation made by limiting the domain of integration of the integral in Eqn 9. The mean-square slopes found by Cox and Munk (1954) were larger than those derived here because their optical technique was sensitive to slopes covering a wider range of spatial frequencies than is taken into account by the integration.

Manuscript received 7 February 1994; revised 9 May and 2 August 1994; accepted 6 September 1994