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On the Propagation of Gravity Waves in Randomly Inhomogeneous Nonsteady-State Currents

M. A. RAYEVSKIY

Scientific-Research Radiophysics Institute

Closed equations are obtained and analyzed for the average field and the average intensity of gravity waves, propagating along a current whose velocity is a random function of the coordinates and time. The scattering of gravity waves by large-scale turbulence near the surface and random internal waves is examined. It is shown, in particular, that the average energy of the gravity waves increases monotonically during scattering.

The effect of currents on the propagation of surface waves has been considered in great detail (see, for example, [1]) for those cases when the current velocity is some determinate (usually smooth) function of the coordinates and time. However, it is clearly of interest to investigate surface waves propagating along a current with a random velocity profile. Let us note that such currents are very typical of the ocean and they can include currents caused by a random field of internal waves, by turbulence near the surface, by a random configuration of vortices, etc.

In this paper we obtain closed equations for the average field and intensity of gravity waves, propagating along a current, whose velocity is a random function of time and the horizontal coordinates. These equations are used to examine the variation of the average field, the average distribution and the energy of the gravity waves in those cases when the current is caused by a large-scale turbulence near the surface and by random internal waves. Estimates for the characteristic time of these effects under typical ocean conditions show that the scattering processes can have a significant effect on the propagation of gravity waves.

BASIC EQUATIONS

Let us consider gravity waves, propagating along a given randomly inhomogeneous nonsteady-state current. We write the equations for the gravity waves in canonical form with the Hamiltonian function [2]:

$$H=\frac{1}{2}\int d\mathbf{r}_\perp\int_{-\infty}^{\eta}(\nabla\Phi+\mathbf{u})^2dz+\frac{g}{2}\int\eta^2d\mathbf{r}_\perp, \qquad (1)$$

where the generalized variables are the vertical displacement η of the free surface and the velocity potential Φ. Let us restrict our consideration to the case when the velocity $\mathbf{u}(\mathbf{r}_\perp, t)$ of the given current is a random function of the horizontal coordinates and the time and does not depend on the vertical coordinate z. Assuming the amplitudes of the surface waves are quite small, we expand the Hamiltonian function (1) in powers of Φ and η. Retaining quadratic terms in this expansion, we obtain:

$$H=\frac{1}{2}\int d\mathbf{r}_\perp\int_{-\infty}^{0}(\nabla\Phi)^2\,dz+\frac{g}{2}\int\eta^2\,d\mathbf{r}_\perp+\int(\mathbf{u}\nabla_\perp\Phi_{z=0})\,\eta\,d\mathbf{r}_\perp, \qquad (2)$$

where the first two terms describe gravity waves in stationary water while the third term describes the interaction of the waves with a current and it is assumed that the interaction energy is small compared with the self-energy of the surface waves. Let us convert to the normal variables a_k and a_k^*, in which

$$\eta=\frac{1}{2\pi}\int\left(\frac{k}{4g}\right)^{1/4}(a_\mathbf{k}+a_{-\mathbf{k}}^*)e^{i\mathbf{kr}}\,d\mathbf{k},$$
$$\Phi=-\frac{i}{2\pi}\int\left(\frac{g}{4k}\right)^{1/4}(a_\mathbf{k}-a_{-\mathbf{k}}^*)e^{kz+i\mathbf{kr}}\,d\mathbf{k}. \qquad (3)$$

In these variables the Hamiltonian function has the following form:

$$H=\int\omega_\mathbf{k}a_\mathbf{k}a_\mathbf{k}^*\,d\mathbf{k}+ \qquad (4)$$
$$\frac{1}{2}\sum_n\int[U^n_{\mathbf{k}_1\mathbf{k}\mathbf{k}_2}u_{\mathbf{k}n}a_{\mathbf{k}_2}a_{\mathbf{k}_1}^*\delta(\mathbf{k}-\mathbf{k}_1+\mathbf{k}_2)+\text{const}\,]d\mathbf{k}\,d\mathbf{k}_1\,d\mathbf{k}_2,$$

where $\omega_k=\sqrt{gk}$ is the frequency of the gravity wave, u_{kn} are the Fourier-transformed components of the current velocity, and the matrix coefficient of the interaction has the form

$$U^n_{\mathbf{k}\mathbf{k}_1\mathbf{k}_2}=\left(\frac{k}{k_2}\right)^{1/4}k_{2n}+\left(\frac{k_2}{k}\right)^{1/4}k_n. \qquad (5)$$

Below we shall limit our consideration to those cases when the characteristic time scales of the current variation are large compared with the periods of the gravity waves; therefore, the nonresonance terms, proportional to $a_{k_1}a_{k_2}$ and $a_{k_1}^*a_{k_2}^*$, can be discarded in the Hamiltonian function. By varying Eq. (4) and converting to the slow complex amplitude $a_k' = a_k \exp(-i\omega_k t)$ (we omit the prime below), we obtain for it the stochastic equation

$$\frac{da_{\mathbf{k}}}{\partial t} = -i\sum_n \int U^n_{\mathbf{k}\mathbf{k}_1\mathbf{k}_2} u_{\mathbf{k}_1 n} a_{\mathbf{k}_2} e^{i(\omega_{\mathbf{k}_2}-\omega_{\mathbf{k}})t}\,\delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\,d\mathbf{k}_1\,d\mathbf{k}_2. \tag{6}$$

We will assume the velocity field $\mathbf{u}(\mathbf{r}_\perp, t)$ is statistically homogeneous and stationary, i.e.,

$$\langle u_{\mathbf{k}n}(t)\,u_{\mathbf{k}_1 n_1}(t_1)\rangle = B_{\mathbf{k}}^{nn_1}(t-t_1)\,\delta(\mathbf{k}+\mathbf{k}_1). \tag{7}$$

Then in the Burr approximation the equation for $\langle a_k\rangle$ (the symbol $\langle\ldots\rangle$ denotes an averaging over an ensemble of realizations of the random field $u(r_\perp, t)$) is the following:

$$\frac{\partial\langle a_{\mathbf{k}}\rangle}{\partial t} = -\sum_{m,n}\int U^m_{\mathbf{k}\mathbf{k}_1\mathbf{k}-\mathbf{k}_1} U^{n*}_{\mathbf{k}\mathbf{k}_1\mathbf{k}-\mathbf{k}_1}\int_0^t B^{mn}_{\mathbf{k}_1}(\tau)\,\langle a_{\mathbf{k}}(t-\tau)\rangle\, e^{i(\omega_{\mathbf{k}-\mathbf{k}_1}-\omega_{\mathbf{k}})\tau}\,d\mathbf{k}_1\,d\tau \tag{8}$$

(it is assumed that the initial conditions are specified at time $t = 0$). Equation (8) has the solution $\langle a_k(t)\rangle = a_k(0)\exp(-\gamma_k t)$, in this case the damping of the average field of the gravity wave is determined by the decrement $\mathrm{Re}\,\gamma_k$, the expression for which in the case $\mathrm{Re}\,\gamma_k \ll \omega_k$ is

$$\mathrm{Re}\,\gamma_{\mathbf{k}} = \pi\sum_{mn}\int U^m_{\mathbf{k}\mathbf{k}_1\mathbf{k}-\mathbf{k}_1} U^{n*}_{\mathbf{k}\mathbf{k}_1\mathbf{k}-\mathbf{k}_1} B_{mn}(\mathbf{k}_1\Omega)\,\delta(\omega_{\mathbf{k}}-\omega_{\mathbf{k}-\mathbf{k}_1}-\Omega)\,d\mathbf{k}_1\,d\Omega, \tag{9}$$

where

$$B_{mn}(\mathbf{k},\Omega) = \frac{1}{2\pi}\int B_{\mathbf{k}}^{mn}(\tau)\,e^{-i\Omega\tau}\,d\tau.$$

Let us now consider the equation for the quantity $n_k = \langle a_k a_k^*\rangle$, in terms of which it is easy to express any quadratic quantities, integrated over the volume of the wave packet, such as the average energy $\langle E\rangle = \int \omega_k a_k a_k^*\,d\mathbf{k}$, by using the formulas of the linear problem. In this same approximation it is convenient to write this equation as

$$\frac{\partial n_{\mathbf{k}}}{\partial t} = \int (W_{\mathbf{k}\mathbf{k}_1\mathbf{k}_2} + W_{\mathbf{k}_2\mathbf{k}_1\mathbf{k}})(n_{\mathbf{k}_2} - n_{\mathbf{k}})\,d\mathbf{k}_1\,d\mathbf{k}_2, \tag{10}$$

where $W_{\mathbf{k}\mathbf{k}_1\mathbf{k}_2} = \pi\sum_{mn} U^m_{\mathbf{k}\mathbf{k}_1\mathbf{k}_2} U^{n*}_{\mathbf{k}\mathbf{k}_1\mathbf{k}_2}\int B_{mn}(\mathbf{k}_1,\Omega)\,\delta(\omega_{\mathbf{k}}-\omega_{\mathbf{k}_2}-\Omega)\,\delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)$. Equation (8) is valid if the damping of the average field is small in the correlation scale l_{cor} of the current (this, of course, imposes limitations on the intensity of the fluctuations of the velocity $\mathbf{u}(\mathbf{r}_\perp, t)$). In addition to the obvious condition of a small variation of the intensity in the l_{cor} scale it is also necessary that the effects of the multiple passage of a wave through one and the same inhomogeneity of the velocity field $\mathbf{u}(\mathbf{r}_\perp, t)$ be negligibly small in order for Eq. (10) to be valid.

The latter condition is satisfied in the obvious case when the characteristic scale of current variation L is much greater than the length of the gravity wave $\lambda = 2\pi/k$, i.e., $kL \gg 1$, since in this case backscattering is small. It can be shown that Eq. (10) is also valid in the case $kL \leqslant 1$ if the characteristic time scale of the intensity variation is much greater than the correlation τ_{cor} of the current velocity field. Let us note that Eqs. (8) and (10) describe arbitrary (quite slow), rather than small, variations of the average field and intensity.

In the case $kL \gg 1$ we can convert from the integral equation (10) by the standard method to the diffusion equation

$$\frac{\partial n_{\mathbf{k}}}{\partial t} = \frac{\partial}{\partial k_i} D_{ij}\frac{\partial n_{\mathbf{k}}}{\partial k_j} \tag{11}$$

with the diffusion tensor

$$D_{ij} = \int k_{1i}k_{1j} W_{\mathbf{k}\mathbf{k}_1\mathbf{k}_2}\,d\mathbf{k}_1\,d\mathbf{k}_2. \tag{12}$$

SCATTERING OF GRAVITY WAVES BY TURBULENCE NEAR THE SURFACE

Let us consider the variation of the gravity wave parameters when they are scattered by turbulence near the surface, assuming that the characteristic dimensions of the vortices are large compared with the scales of the gravity waves. It is obvious that the interaction in this case is determined by the character of the turbulence near the surface only (in a layer of the order of the length of the gravity wave), where, as is known (see, for example, [1]), the characteristic vertical velocities of the turbulent fluctuations are small compared with the horizontal velocities. In addition, in this case we can ignore the dependence of the horizontal components of the velocity on the vertical coordinate, i.e., it should be expected that the two-dimensional current model considered above correctly describes the large-scale turbulence. Assuming the velocity field $\mathbf{u}(\mathbf{r}_\perp, t)$ is isotropic in the horizontal plane, we write $B_{ij}(\varkappa, \Omega)$ as:

$$B_{ij}(\varkappa,\Omega) = B^t(\varkappa,\Omega)\,\delta_{ij} + [B^l(\varkappa,\Omega) - B^t(\varkappa,\Omega)]\frac{\varkappa_i\varkappa_j}{\varkappa^2}, \tag{13}$$

where B^t and B^l are, respectively, the transverse and longitudinal spectral correlation functions. Let us note that $B^l=0$ does not follow from the incompressibility condition since, although $u_z \ll u_x$, u_y near the free surface, in the general case $\partial u_z/\partial z$ is of the order of $\partial u_x/\partial x$ and $\partial u_y/\partial y$ and therefore $|\partial u_x/\partial x+\partial u_y/\partial y \neq 0$.

The expression for the damping decrement of the average field in the case $k_1 \ll k$ is as follows:

$$\mathrm{Re}\,\gamma_k=4\pi\int\left\{B^t(k_1,\Omega)\left[k^2-\frac{(\mathbf{k}\mathbf{k}_1)^2}{k_1^2}\right]+B^l(k_1,\Omega)\frac{(\mathbf{k}\mathbf{k}_1)^2}{k_1^2}\right\}\delta(\omega_k-\omega_{k-k_1}-\Omega)\,d\mathbf{k}_1\,d\Omega. \tag{14}$$

It follows from the frequency synchronism condition $\omega_k=\omega_{k-k_1}+\Omega$ that only those $\mathbf{k}_1$, for which $(\mathbf{k}\mathbf{k}_1)/kk_1 \approx k_1/2k+2(\Omega/\omega_k)k/k_1$, contribute to $\mathrm{Re}\,\gamma_k$. The characteristic velocities of the turbulent fluctuations are usually small compared with the phase velocities of the gravity waves; therefore $(\mathbf{k}\mathbf{k}_1)/kk_1 \ll 1$ and the damping decrement is given by the expression

$$\mathrm{Re}\,\gamma_k=8\pi\frac{k^3}{\omega_k}\int B^t(k_1\Omega)\,dk_1\,d\Omega. \tag{15}$$

Thus, $\mathrm{Re}\,\gamma_k$ is determined primarily by the transverse correlation function B^t and increases proportionally to $k^{5/2}$ with an increase in k.

Let us now consider some consequences of the diffusion equation (11). Taking into consideration that $(\mathbf{k}\mathbf{k}_1)/kk_1 \ll 1$, we can write the diffusion tensor as

$$D_{ij}=4\pi k^2\int k_{1i}k_{1j}B^t(k_1,\Omega)\,\delta(\omega_k-\omega_{k-k_1}-\Omega)\,d\mathbf{k}_1\,d\Omega, \tag{16}$$

i.e., the variation of the intensity of the gravity waves is also determined primarily by the transverse correlation function. Let us estimate the characteristic time for isotropization of a plane wave. In a coordinate system with the x axis oriented along the direction of initial wave propagation, it can be estimated as $\tau_\theta=k^2D_{yy}^{-1}$. It is easy to obtain the expression

$$\tau_\theta^{-1}=\frac{8\pi k}{\omega_k}\int k_1^2B^t(k_1,\Omega)\,dk_1\,d\Omega. \tag{17}$$

for τ_θ.

It is interesting to consider the variation of the average energy of the gravity waves in the scattering process. From Eq. (11) we can obtain the equation

$$\frac{\partial\langle E\rangle}{\partial t}=\int n_k\frac{\partial}{\partial k_i}\left(D_{ij}\frac{\partial\omega}{\partial k_j}\right)d\mathbf{k}, \tag{18}$$

for $\langle E\rangle$. The components of the diffusion tensor can be written as:

$$D_{xx}=\frac{8\pi k^3}{\omega_k}\int\cos^2(\theta+\alpha)\,k_1^2B^t(k_1,\Omega)\,dk_1\,d\Omega,$$

$$D_{xy}=D_{yx}=\frac{4\pi k^3}{\omega_k}\int\sin 2(\theta+\alpha)\,k_1^2B^t(k_1,\Omega)\,dk_1\,d\Omega, \tag{19}$$

$$D_{yy}=\frac{8\pi k^3}{\omega_k}\int\sin^2(\theta+\alpha)\,k_1^2B^t(k_1,\Omega)\,dk_1\,d\Omega,$$

where $\alpha=\arccos[(2\Omega/\omega_k)k/k_1]$, θ is the angle between the vector $\mathbf{k}$ and the x axis, and it is assumed that $k_1/k \ll (\Omega/\omega_k)k/k_1 \ll 1$. By using Eq. (19), after some manipulations we can reduce Eq. (18) to:

$$\frac{\partial\langle E\rangle}{\partial t}=64\pi\int_{-\pi}^{\pi}d\theta\int_0^\infty\frac{k^4}{\omega_k^2}(3+\cos 2\theta)\,n(k,\theta)\int\Omega^2B^t(k_1,\Omega)\,dk_1\,d\Omega \tag{20}$$

(the natural symmetry of the angular distribution of the waves relative to the x axis, i.e., $n(k,\theta)=n(k,-\theta)$), is assumed here). It is easy to see that for any intensity distribution $n(k,\theta)$ the right side of Eq. (20) is positive, i.e., the average energy of the gravity waves increases monotonically during scattering by large-scale turbulence. Turbulence near the surface often has an extended spectrum. In this case interaction with small vortices leads to absorption of the wave energy; therefore, the character of the energy variation in the general case is determined by the competition of the effects of interaction with large and small vortices and cannot be determined without extremely detailed information about the spatial and temporal structure of the turbulence near the surface. The results obtained are clearly directly applicable in the case of degenerate surface turbulence when small vortices dissipate quickly and only large energy-containing vortices "survive."

As far as we know, there are no reliable experimental data for either the form of the function B^t, or for the energy spectrum $\varepsilon(\varkappa)=\pi\varkappa\int(B^t+B^l)\,d\Omega$; this prevents a direct computation of the integrals in Eqs. (15), (17) and (20). Assuming that the energy-containing vortices with scales L and velocities v_L make the major contribution to scattering, we obtain the following estimates for the characteristic times of the effects being considered: $\mathrm{Re}\,\gamma_k\sim v_L^2Lk^3\omega_k^{-1}$, $\tau_\theta^{-1}\sim 10v_L^2L^{-1}k\omega_k^{-1}$, and $\tau_{\langle E\rangle}^{-1}\sim\langle E\rangle^{-1}\partial\langle E\rangle/\partial t\sim 10^3\,v_L^4L^{-1}k^3\omega_k^3$. Hence, for example, for decimeter gravity waves for typical values of the surface turbulence parameters $L\sim 10^2$ cm and $v_L\sim 1$ cm/sec it follows that $\mathrm{Re}\,\gamma_k\sim 10$ sec^{-1} and $\tau_\theta\sim\tau_{\langle E\rangle}\sim 10^2$ sec. Let us note that even more intense surface turbulence with $v_L\sim 10$ cm/sec^{-1} is observed in the ocean. Thus, interaction with a surface turbulence can significantly affect the dynamics of gravity waves.

SCATTERING OF GRAVITY WAVES BY RANDOM INTERNAL WAVES

Let us now consider the interaction of gravity waves with the current created by random internal waves. It is known (see, for example, [3]) that near the surface the vertical velocities of such a current are small compared with the horizontal velocities. In addition, the length of the gravity waves is usually small compared with the scales of the internal waves, and we can use Eqs. (9) - (11) with the matrix coefficient (5) to describe the scattering process.

For a plane internal wave the vector of the horizontal velocity is parallel to the wave vector; therefore, the correlation tensor of the current velocities is

$$B_{ij}(\varkappa, \Omega)=\frac{\varepsilon(\varkappa, \Omega)}{\pi\varkappa^3}\varkappa_i\varkappa_j, \tag{21}$$

where $\varepsilon(\varkappa, \Omega)$ is the one-dimensional energy spectrum of the internal waves near the surface. For waves of sufficiently small amplitude

$$\varepsilon(\varkappa, \Omega)=\sum_n \varepsilon_n(\varkappa)\delta(\Omega-\Omega_\varkappa^n) \tag{22}$$

(here $\varepsilon_n(\varkappa)$ is the spectral density of the energy of the mode with index n, $\Omega_\varkappa^n$ is its natural frequency). In this case the damping decrement is defined by the expression

$$\operatorname{Re}\gamma_{\mathbf{k}}=4\sum_n\int\frac{(\mathbf{k}\mathbf{k}_1)^2}{k_1^3}\varepsilon_n(k_1)\delta(\omega_{\mathbf{k}}-\omega_{\mathbf{k}-\mathbf{k}_1}-\Omega)\,d\mathbf{k}_1\,d\Omega. \tag{23}$$

Taking into consideration that the phase velocities of the internal waves are usually small compared with the phase velocities of surface waves, it is easy to obtain

$$\operatorname{Re}\gamma_{\mathbf{k}}=\frac{8k^3}{\omega_{\mathbf{k}}}\sum_n\int\frac{\varepsilon_n(k_1)}{k_1}\left(\frac{k_1}{2k}+2\frac{\Omega_{\mathbf{k}_1}^n}{\omega_{\mathbf{k}}}\frac{k}{k_1}\right)^2 dk_1. \tag{24}$$

The dispersion relations $\Omega_\varkappa^n$ and the energy spectra $\varepsilon_n(\varkappa)$, observable in the ocean, are extremely diverse; therefore, it seems more interesting to us to arrive at general estimates for them without computing the obtained integral expressions in any particular case. We assume that internal waves, whose horizontal scales do not exceed the ocean depth and the variation scale of the Väisälä frequency $N(z)$ along the z axis, take part in the scattering. Then $\Omega_\varkappa^n$ is of the order of the characteristic value of $N(z)$ (denoted by N^* below) and $\operatorname{Re}\gamma_{\mathbf{k}}$ is of the order of the largest of the terms $v_L^2\varkappa_L k\omega_{\mathbf{k}}^{-1}$, $v_L^2N^*\varkappa_L^{-1}k^3\omega_{\mathbf{k}}^{-2}$, and $v_L^2\varkappa_L^{-3}(N^*)^2k^5\omega_{\mathbf{k}}^{-3}$, where $\varkappa_L$ are the wave numbers, making the primary contribution to Eq. (24), and v_L are the corresponding amplitudes of the internal waves. It is obvious that $\operatorname{Re}\gamma_{\mathbf{k}}$ is determined by the scattering by internal waves with the maximum horizontal scales if $\varepsilon_n(\varkappa)$ decreases faster than $\varkappa^{-2}$ as $\varkappa$ increases.

Let us now estimate the characteristic time for isotropization of a plane wave. Taking into consideration that the diffusion tensor in this case is

$$D_{ij}(k)=4\sum_n\int k_{1i}k_{1j}\frac{\varepsilon_n(k_1)}{k_1^3}(\mathbf{k}\mathbf{k}_1)^2\delta(\omega_{\mathbf{k}}-\omega_{\mathbf{k}-\mathbf{k}_1}-\Omega_{\mathbf{k}_1}^n)\,d\mathbf{k}_1, \tag{25}$$

analogously to Eq. (17), we can obtain the expression

$$\tau_\vartheta^{-1}=\frac{8k}{\omega_{\mathbf{k}}}\sum_n\int k_1\varepsilon_n(k_1)\left(\frac{k_1}{2k}+2\frac{\Omega_{\mathbf{k}_1}^n}{\omega_{\mathbf{k}}}\frac{k}{k_1}\right)^2 dk_1 \tag{26}$$

for τ_ϑ^{-1}. Thus, τ_ϑ^{-1} is of the order of the largest of the terms

$$v_L^2\varkappa_L^3/k\omega_{\mathbf{k}},\ v_L^2\varkappa_LN^*k/\omega_{\mathbf{k}}^2,\ v_L^2(N^*)^2k^3/\varkappa_L\omega_{\mathbf{k}}^3.$$

The expression

$$\frac{\partial\langle E\rangle}{\partial t}=64\sum_n\int_{-\pi}^{\pi}d\vartheta\int_0^\infty\frac{k^6}{\omega_{\mathbf{k}}^4}(4+\cos 2\vartheta)n(k,\vartheta)\,dk\int\frac{(\Omega_{\mathbf{k}_1}^n)^4}{k_1^3}\varepsilon_n(k_1)\,dk_1 \tag{27}$$

can be obtained for the average energy of the gravity waves (it is also assumed here that the condition $k/k_1\ll(\Omega_{\mathbf{k}_1}^n/\omega_{\mathbf{k}})k/k_1\ll 1$ is satisfied and the angular distribution with respect to the direction of initial wave propagation is $n(k,\vartheta)=n(k,-\vartheta)$). Thus, the average energy of the gravity waves increases monotonically in the process of scattering by internal waves. For the characteristic variation time of the energy it is easy to obtain $\tau_{\langle E\rangle}^{-1}\sim 10^2v_L^2(N^*)^4k^5/\varkappa_L^3\omega_{\mathbf{k}}^5$ from Eq. (25). Let us estimate the damping time of the average field, the isotropization time and the energy variation time for meter-length gravity waves, assuming that scattering occurs at internal waves with wave numbers $\varkappa_L\sim 10^{-3}$ cm^{-1} and amplitudes $v_L\sim 10$ cm/sec. Considering the case of stratification with a well-defined thermocline, where $N^*\sim 10^{-2}$ sec^{-1}, we obtain $\operatorname{Re}\gamma_{\mathbf{k}}\sim 10^{-1}$ sec^{-1}, $\tau_\vartheta\approx 10^3$ sec and $\tau_{\langle E\rangle}\sim 10^5$ sec. Estimates for other situations also show that the damping of the average field is the fastest effect while the increase of $\langle E\rangle$ is the slowest. Let us note that the energy variation of meter waves due to other factors also occurs quite slowly. Thus, for example, their damping time due to viscosity is of the order of 10^5 sec, the "acceleration" time by a moderate wind is of the order of 10^4 - 10^5 sec, etc.; this indicates the importance of the considered effect when studying the energy balance of gravity waves.

Since large-scale turbulence and the internal waves change slowly with time, a sufficient condition for the applicability of the obtained expressions is the smallness of the correlation scale l_{cor} of the velocity field $\mathbf{u}(\mathbf{r}_{\perp}t)$ compared with the spatial scales of the effects being considered $v_{gr}/\mathrm{Re}\,\gamma_k$, $v_{gr}\tau_\varphi$, and $v_{gr}\tau_{(E)}$, where v_{gr} is the group velocity of the gravity wave. In the turbulence case l_{cor} is of the order of the vortex size ($l_{cor} \sim L$). If we also assume $l_{cor} \sim L$ for the internal waves, then this condition is satisfied for the parameter values considered in this paper.

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