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1	Laboratory experiments on the effects of a variable current field on the
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ABSTRACT

Laboratory experiments were performed to investigate the effects of a co-15 flowing current field on the spectral shape of water waves. Our results indicate 16 that refraction is the main factor in modulating wave height and overall wave 17 energy. Although the structure of the current field varies considerably, some 18 current-induced patterns in the wave spectrum are observed. In high frequen-19 cies, the energy cascading generated by nonlinear interactions is suppressed 20 and the development of a spectral tail is disturbed, as a consequence of the 21 detuning of the four-wave resonance conditions. Furthermore, the presence 22 of currents slows the downshifting of the spectral peak. The suppression of 23 the high frequency energy under the influence of currents is more prominent 24 as the spectral steepness increases. The energy suppression is also more ac-25 centuated and longstanding along the fetch when the directional spreading of 26 waves is sufficiently broad. This result indicates that the current-induced de-27 tuning of resonant conditions is more effective when exact resonances are the 28 primary mechanism of nonlinear interactions than when quasi-resonances pre-29 vail (directionally narrow cases). Additionally, the directional analysis shows 30 that the highly variable currents broaden the directional spreading of waves. 31 The broadening is suggested to be related to random refraction and scattering 32 of wave rays. The random disturbance of wavenumbers alters the nonlinear 33 interaction conditions and weakens the energy exchanges among wave com-34 ponents, which is expressed in the suppression of the high frequency energy. 35

36 1. Introduction

Wave-current interactions have been the subject of theoretical (e.g. Longuet-Higgins and Stewart 37 1961) and experimental studies (e.g. Lai et al. 1989) for decades. It is well known that currents 38 can affect significantly ocean wave generation and propagation. In deep water, the refraction of 39 waves on mesoscale current features can be significant (Rapizo et al. 2014) and be potentially 40 responsible for energy convergence and enhanced probability of freak wave occurrence (White 41 and Fornberg 1998). Typical conditions of fetch-limited wave growth can also be considerably 42 affected by background currents due to changes in the relative wind and shift of the wind stress 43 away from the mean wind direction (Haus 2007). 44

Based on the radiative transfer equation (RTE), current-induced effects on waves are limited 45 to linear refraction and energy exchanges originated from the work done by the radiation stress 46 against the current strain (e.g. Phillips 1977), which is also linear in terms of energy. However, it 47 has been shown that nonlinearities can play an important role in modulating the wave parameters, 48 especially when waves steepen due to negative current gradients (e.g Babanin et al. 2011; Moreira 49 and Peregrine 2012). In widely used wave models based on the RTE, dissipation and nonlinear 50 four-wave interaction terms do not directly take into account the mean flow, since little is known 51 about the influence of currents on these processes. 52

A few studies have tested the role of the dissipation and nonlinear interaction terms in spectral models under influence of currents with interesting results. Ardhuin et al. (2012), for example, pointed out that none of the parameterizations for wave dissipation proportional to wave steepness to the fourth power are satisfactory when the dissipation is induced by adverse currents. Tamura et al. (2008) showed that the use of the discrete interaction approximation (DIA) method represents poorly the spectral transformations on mesoscale eddies. Therefore, an explicit inclusion of current on these two terms (dissipation and nonlinear interactions) must bring a considerable improvement
 on modeling waves at certain conditions.

Relatively little work has been done on observed wave spectral changes due to the interaction 61 with a variable current field. Numerical simulations of uni-dimensional transformations have been 62 attempted (Trulsen et al. 1990), however field or laboratory results are scarce (Chawla and Kirby 63 2002), especially concerning the directional properties. By propagating over an area of strong 64 current gradients, the wave components are expected to experience differed modification patterns, 65 and the distribution of energy in the spectrum would be affected. It is reasonable to think that 66 energy convergence and divergence would narrow and broaden, respectively, the directional dis-67 tribution of the wave spectrum in opposing and co-flowing current jets (e.g. Kenyon 1971). By 68 perturbing the original spectral form, nonlinear interactions would act in the way to stabilize the 69 spectral shape (e.g. Young and Van Vleder 1993; Tamura et al. 2008). This process would become 70 more complex in highly variable current fields and observations are needed for a more thorough 71 understanding. Nonetheless, they are practically nonexistent. 72

This study aims to provide observations of the effects of a variable co-flowing current on the 73 spectral shape of water waves. Laboratory experiments were performed with background currents 74 highly variable in space and time. The experiments were designed to investigate how a random 75 current field affects the wave spectral geometry for different initial spectra, varying directional and 76 non-directional parameters. Although the current is unsteady, consistent wave spectral patterns 77 are observed under current influence. These transformations are shown and the possible causes 78 discussed. In the following section, a brief theoretical background will be given. Section 3 will 79 depict the experimental methods, including a description of the wave facility and the characteristics 80 of the current field as well as initial wave conditions. Results and discussions are presented in 81 section 4, separated by the main groups of experiments, which are represented by regular wave 82

trains, unidirectional and directional irregular waves with varying steepness and irregular waves
 with varying directional spreading. Concluding remarks are presented in the last section.

2. Relevant effects of currents on waves

The relevant current-induced effects on gravity waves are briefly reviewed to support the further 86 discussions of this study. The *Doppler-shifted* dispersion relation, i.e. $\omega - \mathbf{k} \cdot \mathbf{U} = \sigma$, affects con-87 siderably the kinematics of a propagating wave, where **k** is the wavenumber vector, $\boldsymbol{\omega}$ the absolute 88 frequency and σ the frequency in a frame of reference moving with the current U. Variations in 89 wavenumber and, consequently, wave refraction become dependent on the spatial gradients of the 90 current field by $\frac{d\mathbf{k}}{dt} = -\nabla[\mathbf{k} \cdot \mathbf{U}(x, y)]$, where $\frac{d\mathbf{k}}{dt}$ is the Lagrangian derivative or the rate of change 91 of wavenumber following a wave packet. If the current field is steady, the absolute frequency ω is 92 constant. For an unsteady current, changes in ω are dependent on local accelerations of the mean 93 flow $\partial \mathbf{U}/\partial t$. 94

⁹⁵ Under the influence of a variable current, wave energy is no longer conserved, and instead, wave ⁹⁶ action is conserved (e.g. Phillips 1977). The evolution of spectral wave action density $N(f, \theta) =$ ⁹⁷ $E(f, \theta)/\sigma$, where $E(f, \theta)$ is spectral wave energy density, is usually expressed by the radiative ⁹⁸ transfer equation (action balance)

$$\frac{\partial N}{\partial t} + \nabla_x \cdot (\mathbf{C}_{\mathbf{x}} N) + \nabla_k \cdot (\mathbf{C}_{\mathbf{k}} N) = F,$$
(1)

⁹⁹ where ∇_x and ∇_k are divergence operators in geographical and wavenumber space, respectively, ¹⁰⁰ $\mathbf{C_x} = \mathbf{c_g} + \mathbf{U}$ is the advective velocity, in which $\mathbf{c_g}$ is the intrinsic wave group velocity, $\mathbf{C_k} = \frac{d\mathbf{k}}{dt}$ ¹⁰¹ and *F* represents different forcing terms. The underlying assumption of this equation is that the ¹⁰² current field varies in space and time at a much larger scale than the wavelength and wave period. In spectral wave models, for example, the forcings F in equation (1) are subgrouped by energy input from wind, dissipation (wave breaking, bottom friction among others) and nonlinear wavewave interactions. The latter redistributes energy in the spectral space. Little is known about the influence of a background current field on this important process.

¹⁰⁷ Nonlinear interactions are fundamental for spectral evolution and are employed by the models ¹⁰⁸ in terms of the Hasselmann's kinetic equation (Hasselmann 1962). These resonant interactions ¹⁰⁹ were first pointed out by Phillips (1960) with development of deterministic theories coming on a ¹¹⁰ subsequent number of papers (Benney 1962; Longuet-Higgins 1962; Longuet-Higgins and Phillips ¹¹¹ 1962; Zakharov 1968). Third order resonant interactions occur only when the wavenumbers meet ¹¹² the quadrilateral conditions

$$\begin{cases} \mathbf{k_1} + \mathbf{k_2} &= \mathbf{k_3} + \mathbf{k_4} \\ \omega_1 + \omega_2 &= \omega_3 + \omega_4 + \Delta \omega, \end{cases}$$
(2)

where the detuning term $\Delta \omega$ is zero for exact resonance. The resonance detuning term plays a pivotal role in the instability of Stokes wave, which was first experimentally and theoretically discovered by Benjamin and Feir (1962). The role of the resonance detuning has been highlighted in the past decade because of its relevance in the generation of freak waves in random directional seas (e.g. Janssen 2003; Onorato et al. 2004).

A deterministic spectral evolution equation considering both the exact four-wave resonant and quasi-resonant interactions, was first derived by Zakharov (Zakharov 1968)

$$i\frac{\partial b_0}{\partial t} = \omega_0 b_0 + \int T_{0123} b_1^* b_2 b_3 \delta_{0+1-2-3} \,\mathrm{d}\mathbf{k}_{123},\tag{3}$$

where $b_i(\mathbf{k})$ are canonical complex variables obtained by a transformation using the Fourier coefficients of the surface elevation and velocity potential, in an integral-power series (Krasitskii 1994). The indices i = 0, 1, 2, 3 are a compact notation of wavenumbers k_i , so that $T_{0123} = T(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ is the interaction coefficient, $\delta_{0+1-2-3} = \delta(\mathbf{k} + \mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3)$ is the Dirac delta function and asterisk represents complex conjugation. Details of the derivation of (3) and each of its terms can be found in Krasitskii (1994). Apparently, the exact resonance condition in terms of the wave frequency does not appear in (3) and, therefore, it contains exact as well as quasi-resonance interactions when $\Delta\omega/\omega_0 \sim O(\varepsilon^2)$, where $\varepsilon = ak$ is the representative steepness.

Equation (3) describes the evolution of a wave component through wave interaction in a fourwave combination. In the ocean, where different wavenumbers may interact in different resonant sets, a stochastic approach is more suitable. An evolution equation of the wave energy density can be readily derived from the Zakharov's equation under the quasi-Gaussian closure hypothesis (e.g Hasselmann 1962; Yuen and Ferguson 1982; Krasitskii 1994). At a relatively long kinetic time scale ($O(\varepsilon^{-4})$), Hasselmann's equation can be retrieved. The Hasselmann equation (Hasselmann 1962) describes the evolution of the wave action spectral density

$$\frac{\partial N_0}{\partial t} = 4\pi \int T_{0123}^2 f_{0123} \delta_{0+1-2-3} \delta(\omega_0 + \omega_1 - \omega_2 - \omega_3) \,\mathrm{d}\mathbf{k}_{123},\tag{4}$$

where $f_{0123} = N_2 N_3 (N_0 + N_1) - N_0 N_1 (N_2 + N_3)$. Third generation wave forecast models do only 135 consider exact resonance conditions, which are essential to the spectral evolution by redistributing 136 and downshifting the input energy. However, quasi-resonances, i.e. $\Delta \omega \neq 0$, are also important for 137 the evolution of statistical properties of a wave system (Annenkov and Shrira 2006) particularly 138 in conditions of fast $O(\varepsilon^{-2})$ time scale evolution (Gramstad and Babanin 2016). The deviation 139 from the Gaussian statistics at this dynamical time-scale is considered to be responsible for the 140 enhanced occurrence of freak waves in the ocean, as demonstrated theoretically, numerically and 141 experimentally by numerous authors (e.g. Janssen 2003; Onorato et al. 2004). 142

The roles and interplay of exact and quasi-resonant interactions are discussed in Waseda et al. (2009b) by analysing the evolution of random waves in a tank. The directional distribution of ¹⁴⁵ wave energy is tightly connected to the nature of nonlinear interactions, if driven by resonant or ¹⁴⁶ quasi-resonant conditions. It would then determine the primary force which controls the nonlinear ¹⁴⁷ interactions and the spectral evolution. It is noteworthy that in the case of a unidirectional wave ¹⁴⁸ field resonant quartets do not occur and the spectral evolution is exclusively due to quasi-resonant ¹⁴⁹ interactions (Waseda et al. 2009a). Quasi-resonances also play a key role in the development of ¹⁵⁰ a spectral tail, which at dynamical time scale can be observed in the wave tank used in this study ¹⁵¹ (Tanaka 2001; Waseda et al. 2009a).

A number of studies have investigated the influence of a background current on the nonlinear interactions. Qingpu (1996) derived a modified Zakharov equation which includes the effects of a shear current in the interaction coefficients. Others analyzed modulation instability of a Stokes wave (e.g. Toffoli et al. 2013). The amplification of wave instability by adverse current gradients have been shown theoretically (Onorato et al. 2011) and experimentally (e.g. Toffoli et al. 2015; Babanin et al. 2011).

The exact resonance case under influence of a random current field was recently studied by 158 Waseda et al. (2015) through a series of experiments performed in the same facility of this study. 159 By generating conditions suitable for triad interactions, where one wave component is repeated 160 in the conditions (2), they found that the energy transfer to a third originally non-existent wave 161 is suppressed by the background current field. The authors associated this effect to a modified 162 wavenumber-dependent *Doppler* velocity due to vertical current shear (Stewart and Joy 1974), 163 random linear refraction due to horizontal current shear and temporal variations of the current 164 field. Through the numerical integration of the discretized Zakharov's equation, they have shown 165 that a constant resonance detuning results in recurrence, whereas a randomly varying detuning 166 suppresses the growth of the originally non-existent wave. In other words, the short-term fluctua-167 tion of the current speed was more important than the magnitude of the mean current speed. The 168

argument, however, is somewhat qualitative as the relevant time scale of the current variation was
 not identified in their study.

The possible corollary of the aforementioned experimental result is that the background random current field will suppress the nonlinear energy transfer. To prove this hypothesis, they additionally presented a few tests for irregular directional waves, keeping the same wave-maker signal for different current conditions. The spectral tail, which is developed due to nonlinear energy cascading, is suppressed and steepens as the current speed increases.

Thus, the dynamics of nonlinear wave-current interactions is rich and many aspects are not even 176 known. This experiment was intended to isolate and study a selection of those. The experimen-177 tal cases of the present study did not present dissipation by breaking and the waves propagate 178 in deep water. Therefore, it is considered that changes in the wave spectrum are caused by con-179 servation of action due to current spatial gradients and nonlinear wave-wave interactions, which 180 will be the two basis of our discussions. By generating a variety of initial wave conditions in a 181 co-flowing current field with similar mean velocity, we have an interesting opportunity to confirm 182 the assumptions from Waseda et al. (2015) of current-induced detuning of four-wave interaction 183 conditions. We further extend their analysis by investigating the impact of the random current field 184 on different directionally distributed wave fields, from unidirectional to extremely broad spread-185 ing. Therefore, these tests provide an insight on the significance of each mechanism, i.e. exact 186 or quasi-resonances, on the interaction with the currents and, particularly, how the detuning force 187 discussed in this section impacts each case. 188

189 3. Methods

¹⁹⁰ a. Facility and Measurements

The Ocean Engineering Basin of the Institute of Industrial Science, University of Tokyo (Ki-191 noshita Laboratory and Rheem Laboratory) has dimensions of 50 m x 10 m with 5 m depth. The 192 multi-directional wave-maker is composed of 32 independent plungers, which are able to generate 193 regular and irregular directional waves for a specified spectral shape. A current field can be gener-194 ated in the wave tank both following and opposing the direction of wave propagation. The water 195 flow is created by the circulation of the entire volume of the tank, through a water inlet and outlet 196 located at both extremes underneath the water. A turbine moves and directs the water inside a pipe 197 of 3 m in diameter, positioned on the side of the tank. The inlet and outlet have width of 10 m but 198 different heights of 3 and 3.5 m, respectively. The generated current field is not uniform and the 199 opposing current is considerably more unstable, with larger variation in time and space than the 200 co-flowing current, mainly due to the differences of inlet/outlet dimensions (Waseda et al. 2015). 201 The instability increases for higher current speeds. A vertical shear is also observed (Toffoli et al. 202 2013). Fig. 1 shows the schematic of the tank and location of wave gauges and current meters 203 used in this experiment. 204

In order to investigate the spatial evolution of the one-dimensional spectrum, 8 wave-wire gauges forming a linear array along the tank were positioned at 2.5 m from the left-side wall. The first 5 gauges are separated by 3 m whereas the 3 last gauges are 2 m apart. Besides analyzing the evolution of the one-dimensional spectrum, a geometrical configuration of 6 wave gauges was positioned at 32 m from the wave-maker to measure the directional spectrum. The 6 gauges form a pentagon with a central gauge and the directional spectrum is obtained by using the wavelet directional method (WDM), which assumes a nonstationary wave field to determine the direction of random waves (Donelan et al. 1996). The method is based on the idea that the sea surface is represented by the superposition of wave groups propagating in different directions along the time. In order to compare the ability of different methods to evaluate the directional spreading of the waves in the tank, Waseda et al. (2009a) compared the WDM against the maximum likelihood (MLM) and the maximum entropy (MEM) methods. Among the three methods, the WDM was the only one capable of distinguishing the different directional spreadings generated and, therefore, is the chosen method to be applied in the present study.

²¹⁹ The wave-maker generates random waves with directional spreading as defined by Mitsuyasu ²²⁰ et al. (1975)

$$G(\theta) = G_n cos^n(\theta). \tag{5}$$

Here we analyze the directional distribution of the generated waves based on the integral of the normalized directional distribution, as proposed by Babanin and Soloviev (1998)

$$A(f)^{-1} = \int_{-\pi}^{\pi} K(f,\theta) \,\mathrm{d}\theta. \tag{6}$$

where $K(f, \theta)$ is the directional distribution normalized with respect to the maximum value in the dominant direction θ_{max} , so that $K(f, \theta_{max}) = 1$ and $G(f, \theta) = AK(f, \theta)$. The parameter *A* represents the inverse normalized directional spectral width. It is related to the parameter *n*, of *cosⁿ* spreading forms, by

$$A = \frac{\Gamma(n/2+1)}{\pi^{1/2}\Gamma(n/2+1/2)}$$
(7)

²²⁷ where Γ is the gamma-function. The convenience of using the parameter *A* to represent the direc-²²⁸ tional spreading is that the integration of the normalized directional distribution avoids uncertain-²²⁹ ties in the directional width resultant from irregularities of a measured spectrum. Moreover, Ba-²³⁰ banin and Soloviev (1998) provide its relation to other existing and widely used spreading forms. ²³¹ Hereinafter parameter *A* refers to the value of the inverse normalized directional distribution at the peak frequency, i.e. $A(f_p)$. Therefore, higher values of *A* correspond to narrower directional distribution of wave energy at the peak frequency.

234 b. Characteristics of the Current Field

Prior to the wave experiments, a current field was generated in the tank with magnitude set to 235 7.5 cm s⁻¹ towards the end of the tank (beach), which was the chosen current to be generated for 236 all the wave cases analyzed in this study. In order to confirm the non-homogeneity of the current 237 observed in previous experiments (e.g. Toffoli et al. 2013; Waseda et al. 2015), an electro-magnetic 238 current meter was positioned at 30 m from the wave-maker in 5 different locations, 2 m apart (red 239 triangles in Fig. 1). Although the 5 measurements were not performed simultaneously, the results 240 provide a sense of the cross-section profile of the current field. Speed and direction for the 5 241 locations are shown in Fig. 2. R2 and L2 are positions at the most extreme right and left sides, 242 respectively, relative to the direction of wave propagation. Mean value (μ) and standard deviation 243 (std) at each point are shown in table 1. 244

The most noticeable feature observed from Fig. 2 is the significantly higher standard deviation 245 (std, in table 1) of the current speed at the right-hand side of the tank (position R2). The variability 246 of the current reaches values of 40% of the mean speed at position R1. Conversely, the left side 247 (points L1 and L2) shows mean speed of more than 2 cm s⁻¹ slower than at R2 and a comparatively 248 steady flow. The directional variation follows the same patterns as the current magnitude, with 249 overall deviation to positive values (0° is longitudinal). Therefore, the intensity and direction of 250 horizontal shear is modified along the time. A similar behaviour of the currents in the tank was 251 observed by Waseda et al. (2015) for opposing currents. Simultaneous measurements performed 252 by Toffoli et al. (2013) also show that the spatial and temporal variability of the flow is substantial. 253

A Lagrangian approach to estimate the spatial distribution of the current field in the tank was applied by Takahashi (2011) using Particle Tacking Velocimetry (PTV) methods. The floating drifters converged on the left side, indicating slower velocities on this side, as also shown in our analysis. Although the analyzed domain covers only a rectangular section of the tank, the most relevant result is that the current distribution is highly variable in both the cross- and along-channel directions.

A more unsteady flow is associated with stronger shear. The fluctuations of velocity and direc-260 tion on the right-hand side (positions R1 and R2) can be used to estimate the main eddy scales 261 in the tank. We can observe different fluctuations with specific time scales. Considering fluctua-262 tions greater than 10% with respect to the mean velocity, we identify a few longer fluctuations at 263 position R1 (highest std) of approximately 30 seconds with fluctuating velocity of $\sim 2 \text{ cm s}^{-1}$. If 264 we consider the advective velocity as $u_{ad} = \overline{u} + u'$, where \overline{u} and u' are the mean and fluctuating 265 components, respectively, the length scale of these larger eddies would be of ~ 3 m. However, the 266 most common and defined fluctuations observed are of 7-10 s with fluctuating velocities in the 267 range 1-1.5 cm s⁻¹, which would be associated to length scales of 0.8-1.1 m. Other well defined 268 fluctuations would be related to ~ 0.4 m length scales. Therefore, the scales of the main eddies 269 where the flow is more turbulent range from tens of centimeters to few meters, with a progressive 270 weakening towards the left side of the channel. The current on the left side will thus produce less 271 intense wave refraction, while the direction of wave propagation on the right-hand side is expected 272 to be deviated more sharply. 273

For all the experimental cases the generated current follows the direction of wave propagation (positive sign). The electro-magnetic current meter was positioned in the middle of the tank at 30 m from the wave-maker (position 'M' in Fig. 1). Thus, it was not possible to measure the transversal current gradient for all experiments. Based on our observations and on the results in Takahashi (2011), Toffoli et al. (2013) and Waseda et al. (2015), we consider the current field as randomly variable, which hinders a precise modeling of wave propagation.

280 c. Experimental Set-up

The analysis of 4 groups of experiments are presented, in each of which the same initial wave 281 conditions are generated in the absence and presence of a co-flowing current field. In the first 282 group, we test the propagation of an initial single monochromatic wave train characterized by 283 period (T) and steepness (ak, where a is wave amplitude and k wavenumber). The second set of 284 experiments was performed for initial unidirectional random waves based on the Joint North Sea 285 Wave Atmosphere Project spectral formulation (JONSWAP, Hasselmann et al. 1973) with varying 286 steepness. The third and fourth groups are represented by directional random waves, firstly varying 287 the steepness for a same directional spreading and, finally, varying the directional distribution for 288 a similar 1d spectral form. For the random cases, the wave steepness is defined as $\varepsilon = ak_p$, where 289 $a = \sqrt{2}\sqrt{m_0}$ is the mean amplitude (in which m_0 is the total variance or integrated spectral energy) 290 and k_p is wavelength at the spectral peak. Table 2 shows the list of experiments performed. All 291 the irregular wave cases were recorded for 15 minutes, whereas the regular waves, for 10 minutes. 292 The experiments were planned in order to generate a broad range of initial wave conditions. The 293 two main wave characteristics we judge to be fundamental in this study are the wave steepness 294 and the directional distribution. Therefore, the the experiments aim to vary one of these two 295 parameters within the respective groups. As a consequence, nonlinear wave-wave interactions are 296 expected to increase between the cases of each group. Waseda et al. (2015) briefly investigated 297 the effect of adverse currents on a same wave field and their impact on quasi-resonant interaction. 298 They observed the steepen of the high frequency spectral tail (f in the range 1.56–3.03 Hz) as 299 the current speed increases. Here we add different types of initial conditions for random waves 300

with varying wave steepness and directional spreading on co-flowing currents. By doing so, the nonlinear interactions potentially intensify within the groups and a similar background current field is expected to affect each case differently. It should be noted that no breaking was observed, eliminating any dissipation concerns.

Table 3 compares the aimed to the measure values of the main wave parameters and mean current speed (\overline{U}). The wave measurement corresponds to the mean value among the gauges in the pentagon. The relation \overline{U}/c_g , where c_g is the wave group velocity, is also shown. Although there are differences between aimed and measured values, the main goal of the experiments were achieved, i.e. to generate cases with considerable differences in wave steepness (first three groups) and keep a similar steepness with varying directional spreading (last group).

4. Results and Discussion

Our interest is on the transformation of the wave spectrum by comparing the same generated 312 wave field (signal sent to wave-maker) in the absence and presence of currents. For the monochro-313 matic cases, our analysis is also focused on the wave parameters, such as wave height and period, 314 which provides a sense of the current distribution in the tank. For irregular waves, we turn our 315 attentions mainly to the high frequency part of the spectrum and directional distribution of energy. 316 The wave-maker frequency upper limit is 2.5 Hz, however it has been shown that, despite the 317 short time scale of tank experiments, the nonlinear energy transfer to higher frequencies occurs in 318 such a fast rate (Tanaka 2001) that a tail beyond this limit is developed by dynamical cascading 319 (Waseda et al. 2009a). Therefore, the spectral energy observed above 2.5 Hz is purely originated 320 from four-wave interactions [equation (3)] and hence the wave components must meet the quadri-321 lateral condition (2). This process gives us the opportunity to investigate how the random current 322 field would perturb the nonlinear interactions and the development of the spectral tail, according 323

to the discussion in the end of section 2. Transformations of the directional spectrum of irregular waves caused by the current field are shown and the results discussed.

326 a. Group 1: Regular Waves

³²⁷ The first group of experiments are represented by single wave trains with T = 0.9s and varying ³²⁸ steepness. The measured wave parameters of significant wave height (H_s), spectral peak period ³²⁹ (T_p) and steepness (ak, where a is wave amplitude) are shown in Fig. 3. H_s exhibits little variation ³³⁰ along the tank when currents are absent. T_p shows no change when in the presence of currents ³³¹ and no variation along the tank. According to the linear theory, current-induced changes in the ³³² absolute frequency ω are related to the temporal variability of the current $\partial U/\partial t$ (see section 2), ³³³ which indicates that the current at these points is not effectively unsteady.

The two wave trains exhibit different patterns when in the presence of currents. Both have their 334 wave energy decreased at most gauges of the linear array (eight initial points). However, at the 335 pentagon gauges (last five points), the first case (ak=0.08) shows an increase while the second 336 (ak=0.05), decrease of energy. These patterns are probably being controlled by refraction-induced 337 convergence or divergence of wave energy, and it varies among the cases due to temporal variations 338 in the spatial distribution of the current field (see Fig. 2 and the discussion in the appendix). 339 Variations in steepness follow variations in H_s . The time series of surface elevation indicate that 340 the energy varies considerably in all wave sensors along the time, with no clear pattern. 341

³⁴² An alternating pattern of refractions can be seen by means of the directional spectrum (Fig. 4). In ³⁴³ order to show variations along the time, the measured time series were divided in three segments. ³⁴⁴ Fig. 4 shows the example of case 1. By obtaining the directional spectrum of each segment, it ³⁴⁵ is possible to see that the current-induced refraction is not constant along the recorded time, but ³⁴⁶ exhibits random variations. The bottom panels show the directional shift of the spectral peak ($\Delta\theta$). This unpredictable behavior creates zones of convergence or divergence of energy in the tank as well as consecutive disturbances in wavenumbers. The spectra are apparently broadened and the final spectrum is highly perturbed.

If we invoke "frozen turbulence", assuming that the mean current speed observed at the two 350 sides of the tank (R2 and L2, see table 1) generates a homogeneous cross-channel gradient along 351 the tank, we can apply the geometrical optics approximation to infer refractions undertaken by 352 the wave rays. Refractions of a regular wave of T = 0.9 s (L = 1.26 m) would be of 3.5° at 353 the point where the directional spectrum is obtained. However, the random fluctuations in current 354 speed and direction generate a highly variable current field. By dividing the time series of surface 355 elevation and obtaining the directional spectrum of a regular wave (Fig. 4), we observe that the 356 wave direction randomly varies along the time, with $\Delta\theta$ from 2.6° to 15.2°. This supports the 357 assumption of a variable and unsteady current field, which is associated with sharp gradients. 358

The mean current speed values (table 1) suggests a non-homogeneous cross-section gradient, 359 which would consequently create zones of convergence and divergence of wave energy. The results 360 of Takahashi (2011), applying a PTV method, showed that the current velocity in the along-tank 361 direction is also considerably variable. Furthermore, the right-hand side of the tank exhibits a 362 significantly higher standard deviation of current speed and direction, which would contribute to 363 the random character of the refraction process. Current-induced focusing and defocusing of wave 364 energy are likely to be responsible for the changes in wave height observed in Fig. 3 (see the 365 appendix for a discussion on wave refraction and energy focusing in the tank). 366

From the analysis of single wave trains we can conclude that the current field is highly variable in its spacial distribution. Although temporal variations exist, they were not effective in changing the wave absolute period. However, wavenumber and direction are randomly modified. Our results indicate that energy focusing/defocusing originated from the alternating refraction patterns is the ³⁷¹ cause of wave height variations. The task of representing a good approximation of the current field
 ³⁷² for modeling purposes is thus complicated with many uncertainties involved.

373 b. Irregular Waves

From the first group of experiments it was possible to have a more thorough understanding of the current field and its effects on single wave trains. Groups 2, 3 and 4 are represented by irregular wave fields based on the JONSWAP spectral formulation, which assumes an equilibrium tail proportional to f^{-5} :

$$E(f) = \alpha g^2 (2\pi)^{-4} f^{-5} \exp\left(\frac{-5}{4} (f/f_p)^{-4}\right) \gamma^{\exp\left[(f-f_p)^2/(2\sigma^2 f_p^2)\right]},\tag{8}$$

where $\sigma = 0.08$ and f_p is the peak frequency (which is set to ~1.25Hz for all cases), α controls 378 the energy level and γ affects the spectral peakedness. The steepness can be manipulated by 379 varying both α and γ . The latter, however, has direct implications on the frequency bandwidth and, 380 consequently, on the instability of nonlinear groups (Ribal et al. 2013) by strengthening the quasi-381 resonant interactions. The peakedness γ assumes values of 1, 5.5 and 17 for the unidirectional 382 waves (group 2) whereas α is the chosen parameter which controls the steepness of directional 383 waves (group 3), assuming values of 0.0041, 0.0080 and 0.0164. Thus, our attentions are on 384 steepness variations via changing one of both, γ or α , while the other is retained. In the last group 385 of experiments (group 4) we investigate different directional spectral width (A=1.5, 2.5 and 3.5) 386 for a similar directionally integrated spectrum. 387

388 Spectral downshifting

The distribution of wave parameters along the tank follow similar patterns to the regular waves, except for peak period T_p . In the presence of currents, the integrated spectral energy, represented ³⁹¹ by parameter H_s , is always decreased at the linear array (left-hand side of the tank), whereas the ³⁹² pentagon of gauges show at times increasing and at times decreasing of energy (not shown here). ³⁹³ This support the assumption of a randomly variable current field with different time scales of ³⁹⁴ temporal variation.

³⁹⁵ A progressive downshifting of the spectral peak along the fetch is observed in the absence of ³⁹⁶ currents, which, since energetic breaking is absent, is associated to nonlinear interactions (Waseda ³⁹⁷ et al. 2009b). In the presence of currents, the downshifting can also be observed, however it is ³⁹⁸ slower or less intense. To quantify the downshifting, we calculate the ratio between the average ³⁹⁹ peak frequency between 10–17 m (ω_i , first 3 gauges) and at 32 m (ω_f , pentagon of gauges). The ⁴⁰⁰ relations of the ratio ω_i/ω_f with steepness ε and the directional parameter *A* are shown in Fig. 5 ⁴⁰¹ for all the irregular wave cases (groups 2–4 in table 2).

When currents are absent, the downshifting ratio is within 5%, which agrees with the experi-402 mental results of Waseda et al. (2009b) for a similar fetch range and peak wavelength. It seems 403 that there is no clear dependence on the initial steepness or directional width. In the presence of 404 currents, the rate of spectral downshifting decreases for all cases, except one. This case is repre-405 sented by the first case of the varying γ group, having the lowest steepness and broadest frequency 406 bandwidth within the group. For this case, the spectra showed an almost "flat" or slightly bi-407 modal shape (see Fig. 7 upper left panel), the identification of the spectral peak was inaccurate 408 and no progressive downshifting was seen along the fetch. Because in the absence of breaking 409 the downshifting is connected to both exact and near resonant interactions, we conjecture that the 410 presence of a background current field perturbs the conditions of four-wave interactions [equation 411 (2)] as suggested by Waseda et al. (2015). Consequently, the rate of peak downshifting is slowed. 412 The process of current-induced detuning and its consequences on the high frequency part of the 413 spectrum will be discussed in details in the following sections. 414

415 1) GROUP 2 AND 3: VARYING STEEPNESS CASES

The comparison of the spectra in the absence of currents for groups 2 and 3 is shown in Fig. 6 416 (top and bottom panels, respectively). As expected, the steepness ($\varepsilon = ak_p$) increases considerably 417 as γ and α assume greater values. The initial JONSWAP form imposes an energy decay at a 418 rate proportional to f^{-5} , up to around 2.5 Hz. However, the spectra within each group show 419 different characteristics in the high frequency region, above the wave-maker upper limit of 2.5 420 Hz (vertical dashed line). This limit is clearly marked for some cases and is associated with a 421 sudden steepen of the spectral tail. Remarkably, the sudden drop of the tail is more prominent 422 for the lowest ε cases and it is almost imperceptible for the highest ε spectra. A spectral tail is 423 developed beyond 2.5 Hz by wave-wave interactions (Waseda et al. 2009a). From Fig. 6, we see 424 that the development is stronger for the steeper spectra and the tail approximates the f^{-5} decay. 425 Therefore, the tail developed beyond the generated frequencies will be used as an indicator of the 426 strength of nonlinear interactions and energy cascading. 427

By adding a background current field the wave spectrum undertakes considerable changes (Fig. 428 7). The energy decreases around the peak probably due to divergence effects on the left-hand side 429 of the tank (see Fig. 3). These changes are somehow random and not clearly patterned. However, 430 the high frequency tail (i.e. for f > 2.5 Hz) exhibits an interesting pattern when in the presence of 431 currents. The tail is suppressed and steepened. The tail differences between corresponding spectra 432 in absence and presence of currents are greater for the steeper cases. The larger is the spectral 433 steepness ε the more intense the nonlinear interactions and energy cascading. As a consequence, 434 the current-induced perturbations of the tail is more evident. 435

⁴³⁶ Our results corroborate the arguments of Waseda et al. (2015) of current-induced detuning of ⁴³⁷ the conditions of four-wave interactions. The detuning would be a result of the *Doppler* shift and

20

⁴³⁸ random refraction. The cumulative effect of a wavenumber-dependent *Doppler* velocity caused ⁴³⁹ by current vertical shear (Toffoli et al. 2013) would be significant for the detuning term $\Delta \omega$ in (2), ⁴⁴⁰ since $\Delta_{kU} = \mathbf{k}_1 \cdot \mathbf{U}_1 + \mathbf{k}_2 \cdot \mathbf{U}_2 - \mathbf{k}_3 \cdot \mathbf{U}_3 - \mathbf{k}_4 \cdot \mathbf{U}_4 \neq 0$. Additionally, random refraction will perturb ⁴⁴¹ the resonance condition of wavenumber

$$\mathbf{k}_{1} + \mathbf{k}_{2} - \mathbf{k}_{3} - \mathbf{k}_{4}|_{t=t_{1}} = \int_{t_{0}}^{t_{1}} \nabla \Delta_{kU} \,\mathrm{d}t \tag{9}$$

The authors support their hypothesis by numerically simulating the Zakharov equation (3) for the exact resonance case with the addition of a detuning term to represent $\Delta \omega$ in (2). As the randomly varying detuning increases, the growth of the originally non-existing wave slows and eventually saturates. They found the results in agreement with the experimental simulation of a same quartet. Therefore, the experimental and numerical results of Waseda et al. (2015) support the hypothesis that the random current field annuls the resonant conditions, with the consequent suppression of the spectral tail.

2) GROUP 4: VARYING DIRECTIONAL SPREADING CASES

The last group of experiments is represented by initial spectra with similar directionally inte-450 grated form, represented by $f_p = 0.8$ Hz, $\gamma = 5.0$ and $\alpha = 0.194$, but varying directional spreading 451 (A = 1.3, 2.1 and 4.0). The cases of A=1.3, 2.1 and 4.0 correspond to n=10, 32 and 100 of the 452 Mitsuyasu directional spreading function $G(\theta) = G_n \cos^n \theta$, respectively. We chose a reasonably 453 high value for steepness ($\varepsilon = 0.12$) in order to activate the nonlinear interactions but not produce 454 energetic breaker. In directionally confined wave fields, the evolution of the spectrum is primarily 455 controlled by quasi-resonant interactions. When the spectrum is considerably broad, exact reso-456 nances are the principal mechanism of energy exchanges. Therefore, this group of experiments 457 aims to investigate the effect of the current field on the interplay between quasi-resonant and res-458 onant interactions. 459

The measured spectra in the absence of currents are shown in the left panel of Fig. 8. As intended, the 1d spectral geometry are remarkably similar and little difference can be noticed. Differently from the previous groups, the wave-maker limit is not distinguishable and all the spectra develop a high frequency tail which approximates an f^{-5} decay above 2.5 Hz. The values of H_s obtained were 0.064, 0.070 and 0.070 cm for the cases A = 1.3, 2.1 and 4.0, respectively. A small difference in peak frequency was observed, with measured values of 1.20, 1.15 and 1.17 Hz, respectively.

Comparison with the corresponding spectra in the presence of co-flowing currents are shown 467 in the right-hand side panels of Fig. 8. In the absence of currents, the equilibrium tail in the 468 frequency range $1.35f_p - 2f_p$ (indicated in the plots) shows a higher stage of development as the 469 directional spreading broadens. The values of the exponent v of the tail decay f^{-v} obtained for 470 the equilibrium range were of 3.9, 4.62 and 5.05 for A = 1.3, 2.1 and 4.0, respectively. This result 471 agrees with Waseda et al. (2009b), where a larger number of initial directional distributions were 472 tested for a same JONSWAP spectral shape. It is worth noting that a decay rate to the power of -4473 is representative of resonant interactions. As the energy is distributed in a sufficiently broad range 474 of directions, exact resonances control the evolution of the spectral form, and are associated with 475 downshifting of the spectral peak as well as the maintenance of the equilibrium tail (Waseda et al. 476 2009b). This explains the slower decay in the equilibrium range for the broadest case. However, 477 beyond 2.5 Hz the energy decay is highly similar for the three cases (A = 1.3, 2.1 and 4.0), with 478 v = 5.40, 6.00 and 5.44, respectively, which are similar to the highest steepness cases of group 2 479 and 3. 480

In order to have a better understanding of the relative significance of the mechanism at work (resonances or quasi-resonances) in the three cases, we calculate the kurtosis as a function of the Benjamin-Feir Index (BFI) and the directional spreading, as proposed by Mori et al. (2011). The ⁴⁸⁴ kurtosis μ_4 is related to the wave grouping and quasi-resonant interactions in an unidirectional ⁴⁸⁵ wave field based on the ratio of steepness to frequency bandwidth. It was extended by Mori et al. ⁴⁸⁶ (2011) to include the directionality of the wave field using a directional or effective Benjamin-Feir ⁴⁸⁷ Index (BFI_{2D}):

$$\mu_4 = 3 + \frac{\pi}{\sqrt{3}} \operatorname{BFI}_{2D}^2, \tag{10}$$

488 where

$$BFI_{2D}^{2} = \frac{BFI^{2}}{1+cR},$$
(11)

in which BFI = $(\sqrt{2}\sqrt{m_0}k_p)/\delta_{\omega}$ is the the BFI for unidirectional waves, δ_{ω} is the frequency 489 bandwidth, $R = \frac{1}{2} \delta_{\theta}^2 / \delta_{\omega}^2$ is an effective bandwidth and c is an empirical coefficient found to be 490 c = 7.1. For obtaining a representative δ_{ω} , all the wave gauges in the pentagon are considered. 491 The kurtosis, parameterized by the BFI_{2D} (Mori et al. 2011), has shown to monotonically increase 492 as the directional spreading narrows, with values of 3.02, 3.06 and 3.2 for A=1.3, 2.1 and 4.0, 493 respectively. Therefore, the role of the quasi-resonances become more important as the directional 494 spreading narrows. Furthermore, Waseda et al. (2009a) analyzed the relation of kurtosis and the 495 spreading parameter A and suggested that a value of A=4 is the transition from the predominance 496 of exact resonances to quasi-resonances in the nonlinear energy transfer, which reinforce the sug-497 gestion that in the last case (A=4.0) quasi-resonances are the primary mechanism at work. 498

⁴⁹⁹ When the currents are present, the high frequency energy is suppressed. This effect, which was ⁵⁰⁰ evident for the high ε cases of groups 2 and 3, is more prominent as the directional spreading ⁵⁰¹ broadens. This suggests that the detuning of nonlinear interaction conditions caused by the ran-⁵⁰² dom currents is more efficient when exact resonances are dominant. As the spectrum narrows in ⁵⁰³ direction, quasi-resonances are in turn the primary mechanism of interactions. The tail is also ⁵⁰⁴ suppressed, however not as intensely, which indicates that the detuning force is less effective. The impact of currents on the different spectra is better visualized by looking to the evolution of individual cases along the fetch. Fig. 9 shows the spatial evolution of the wave spectrum for the directionally broadest (A = 1.3) and narrowest (A = 4.0) cases ('a' and 'b', respectively). The 1d spectra at the 8 wave gauges of the linear array are plotted. Spectra with and without currents are compared (panels 'a₁' and 'b₁'). To quantify the suppression of the high frequency energy, we calculate the integral of the normalized difference in the high frequency spectral energy in the absence (*E*) and presence (E_c) of currents, i.e.

$$\Delta \overline{E}_{hf} = \int_{2}^{7} \frac{E(f) - E_c(f)}{E(f)} \,\mathrm{d}f \tag{12}$$

Here, the frequency limits applied were 2–7 Hz, hence emphasizing the energy generated exclu-512 sively by nonlinear interactions. The random detuning force tends to weaken for both cases as 513 the waves propagate along the fetch and to eventually reach a saturation stage. The evolution of 514 $\Delta \overline{E}_{hf}$ (panels 'a₂' and 'b₂') decreases along the fetch, however in different manners. In direction-515 ally broad initial waves (plots 'a'), the current-induced suppression of the high frequency energy 516 is evident. The detuning force (represented by $\Delta \overline{E}_{hf}$) persists and reaches its maximum at ~ 14 517 m, decreasing as waves approach the last three wave gauges (25-29m). For the narrowest initial 518 wave field (A = 4.0), the difference between the spectra in the absence and presence of currents 519 is practically only noticed at the first 3 gauges. $\Delta \overline{E}_{hf}$ rapidly decreases to null or even negative 520 values. Panels ' a_3 ' and ' b_3 ' of Fig. 9 shows the evolution of the decay exponent v of the high 521 frequency tail $f^{-\nu}$. The tail of the spectra, both in the presence and absence of currents, tends 522 to steepen between the 11m and 25m gauges with very similar patterns regardless the presence 523 of background currents. These results reinforces the suggestion that the current-induced detuning 524 of nonlinear interactions is more effective for spectra in which exact-resonances prevail. When 525

quasi-resonances are dominant, the suppression of high frequency energy is less prominent and short-lived along the fetch.

From the observations of irregular waves it is observed that as the steepness increases, the spectral tail is more developed and, consequently, more affected and suppressed by the random current. Moreover, it seems that the resonance detuning is more effective as the distribution broadens (lower values of *A*). To expresses the effectiveness of the detuning force, we introduce a straightforward parameter which weights the steepness ε with the directional spreading parameter *A*

$$\varepsilon_A = \frac{\varepsilon}{A} \tag{13}$$

Fig. 10 shows the evolution of the decay exponent v of the tail f^{-v} as a function of steepness (panel 'a') and parameter A (panel 'b'), for all irregular wave cases (i.e. cases 3–11). Panels 'd' and 'e' show $\Delta \overline{E}_{hf}$ as a function of ε and A, respectively. The results show the mean value among gauges 2–5, at which the effects of currents were seen to be more evident. The tail decay exponent v was obtained from the frequency interval 2–4 Hz.

Results of Fig. 10a demonstrate that the development of the tail is tightly related to the increase of steepness and it reaches a saturation stage close to f^{-5} . The relation of v with ε seems to not distinguish between unidirectional (group 2) and directional (groups 3 and 4) wave fields. In panel 'd', ($\varepsilon vs \Delta \overline{E}_{hf}$) we can see the impact of the current on this process. For steeper waves, parameter $\Delta \overline{E}_{hf}$, which quantifies the suppression of the tail energy, reaches larger values.

⁵⁴³ By analyzing the relations of v and $\Delta \overline{E}_{hf}$ with the directional parameter A (middle column plots ⁵⁴⁴ of Fig. 10) an interesting characteristic is seen. There seems to be no direct dependence of the ⁵⁴⁵ high frequency tail exponent v on parameter A (panel 'b'), however the energy difference at high ⁵⁴⁶ frequencies $\Delta \overline{E}_{hf}$ is always small for directionally confined wave fields (panel 'e'). For low values ⁵⁴⁷ of A (broad waves), $\Delta \overline{E}_{hf}$ is highly variable and it depends on the spectral steepness. This evidence

suggests that when quasi-resonant interactions are predominant, the detuning force caused by cur-548 rents is always limited ($\Delta \overline{E}_{hf} < 0.3$), including high steepness waves. This explains two features 549 observed: firstly, there are two distinct evolution curves for the dependence of $\Delta \overline{E}_{hf}$ on the steep-550 ness ε (panel 'd'): one for narrow wave fields (including the unidirectional group, represented by 551 circles and one case of group 4, plotted by a triangle) and another curve for directionally broad 552 spectra (all the other markers). Secondly, we can see an isolated point of high steepness ($\varepsilon > 0.12$) 553 but at which $\Delta \overline{E}_{hf}$ was relatively low. This case corresponds to the narrowest case of group 4, 554 where the initial waves are steep with directionally confined energy distribution (A = 4.0), thus 555 the spectral evolution is controlled by quasi-resonant interactions. Despite the high steepness and 556 a tail decay close to the power of -5, the current does not suppress the energy cascading process as 557 intensely as for those broader cases, where exact resonances play a fundamental role. 558

⁵⁵⁹ Another interesting characteristic between directionally broad and narrow fields is noticed when ⁵⁶⁰ we look at the standard deviation of $\Delta \overline{E}_{hf}$ (vertical bars) among the gauges considered in Fig. 10. ⁵⁶¹ As shown in Fig. 9, the evolution of $\Delta \overline{E}_{hf}$ rapidly decreases along the fetch for the case A = 4.0. ⁵⁶² From panel 'e' it is observed that the high standard deviation (vertical bars) are predominantly ⁵⁶³ related to directionally confined energy, which is a consequence of the rapid decrease of $\Delta \overline{E}_{hf}$ ⁵⁶⁴ along the initial sensors of the fetch.

⁵⁶⁵ By weighting the steepness with parameter *A* [eq. (13)], the balance of quasi-resonances and ⁵⁶⁶ exact resonances are included and the effect of the current-induced detuning is thus better repre-⁵⁶⁷ sented (Fig. 10f). However, some of the narrowest cases (unidirectional wave fields, plotted by ⁵⁶⁸ circles) are 'overweighted' by their high *A* values and, within the unidirectional cases (group 2), ⁵⁶⁹ the dependence is lost. To properly represent the weight of the directional parameter *A*, a correc-⁵⁷⁰ tion is introduced into parameter ε_A , based on the argument that directionally narrow wave fields with *A* above a certain threshold must be weighted equally, i.e.

$$\varepsilon_{A} = \frac{\varepsilon}{A_{n}}, \quad \text{where} \begin{cases} A_{n} = A, & \text{if } A < A_{thr} \\ A_{n} = A_{thr}, & \text{if } A \ge A_{thr} \end{cases}$$
(14)

The use of parameter A_n considers any value of A above a certain threshold A_{thr} as having the 572 same weight on parameter ε_A and, therefore, the same impact on the current-induced detuning of 573 resonance conditions. Following our arguments that detuning of the quadrilateral conditions are 574 more effective on cases controlled by exact resonances, the threshold A_{thr} would then determine 575 the transition between the predominance of resonant and quasi-resonant interactions. Waseda et al. 576 (2009b), based on the analysis of spectral downshifting and kurtosis, suggested that a value of A 577 around 4 is the transition from the predominance of exact resonance to quasi-resonance in the 578 nonlinear energy transfer. To verify this hypothesis, we test different threshold values in the new 579 parameter ε_A . 580

Fig. 11 shows the relation of $\Delta \overline{E}_{hf}$ with the new parameter ε_A from (14) with $A_{thr} = 2, 4, 6$ and 581 8. The range would thus include the limit proposed by Waseda et al. (2009b). We show only these 582 four threshold values, however all values in the range 1.3-11.3 (minimum and maximum A from 583 our cases) were tested. It is clear that a low threshold around 2 does not unify the distinct curves 584 correspondent to narrow and broad distributions. Values around 8 or above fall in the limitations 585 discussed above, i.e. the threshold is too high and overweights the unidirectional cases (circles). 586 It is observed that the best fit which unifies all the A values in a single relation, would lie in the 587 range 4–7. No visible differences are seen for A_{thr} values adopted in this range and they all show 588 a high correlation between ε_A and $\Delta \overline{E}_{hf}$ (> 0.9). A value close to 4, as suggested by Waseda et al. 589 (2009b), is the minimum value for which parameter $\Delta \overline{E}_{hf}$ highly correlates to ε_A . 590

⁵⁹¹ c. Current-induced Broadening of Directional Spreading

There is a lack of observations regarding the transformation of the wave directional spectrum under the influence of currents, with the notable exception of Toffoli et al. (2011). In this section the results of the observed current-induced changes in the directional spectrum and, especially, in the directional spreading are shown.

By analyzing the spreading parameter A, it is observed that practically all initial spectra are 596 considerably broadened in the presence of currents, which is more evident in the narrower cases. 597 Cases with an initial broad spreading (i.e. A < 2) showed little changes, although perturbations 598 of the directional spectrum can be seen. Toffoli et al. (2011) also observed broadening of the 599 directional spreading of waves propagating over an oblique current field. Two examples are shown 600 in Fig. 12. The spectra of the narrowest and broadest cases (top and bottom panels, respectively) 601 of the varying spreading group (group 4), are plotted in the absence and presence of currents (left 602 and right-hand columns, respectively). 603

The systematic broadening observed is suggested to be related to the high spatial and tempo-604 ral variability of the current field in the tank, which would randomly refract the wave rays with 605 a consequent scattering of wave energy. This process is comparable to wave propagation over 606 intermediate to shallow water depths. Scattering of waves by irregular bottom topography was 607 firstly investigated by Long (1973). Additionally, Ardhuin et al. (2003) implemented a numerical 608 Eulerian-Lagrangian model to account for wave ray refraction over small scale bottom features. 609 From data analysis along the continental shelf, they observed that the distribution of wave energy 610 was broadened in direction, particularly for directionally narrow swells at the inner shelf. The 611 broadening was associated with the scatter of wave rays due to small scale bottom topography and 612 agreed with predictions from their Bragg scattering model. We believe a similar process occurs 613

when waves propagate over a variable current field, which is observed from our results. Waves are scattered in random directions due to the highly variable current field. The wavenumbers also change in time due to the unsteadiness of the current field. This results in variations of direction of propagation of each component with a consequent broadening pattern of the final spectrum.

The difference of parameter A in the absence and presence of currents are shown in Fig. 13 618 against the initial A (i.e. A in the absence of currents), for all irregular wave experiments. ΔA 619 represents the difference $A_{NOcurr} - A_{curr}$. Panel 'b' shows changes in peak direction ($\Delta \theta_p$). The 620 predominant direction in which the energy propagates shows no relation with the initial directional 621 spreading. The spatial distribution of the currents is modified along the time and it produces 622 different values of $\Delta \theta_p$. It was observed that the refraction of the main energy peak is considerably 623 less intense than it is for regular waves. However, the directional spreading exhibits an evident 624 broadening pattern. Since the spectral shape of a narrow initial spectra (i.e. A>3) is subject to 625 more drastic changes, the broadening is more evident for narrower cases and, consequently, ΔA 626 highly correlates to the initial value of A. 627

To complement our analysis, we use some of the results of the experiments performed and 628 presented in Toffoli et al. (2015) and Waseda et al. (2015). 5 different opposing current 629 fields are generated with increasing mean speed, from which the obtained mean values are 630 U = -4.13, -5.99, -10.48 and -13.39. The initial wave field, in the absence of currents, is 631 represented by: $T_p = 0.871$, $\varepsilon = 0.095$ and A = 3.3. The directional distribution is considerably 632 narrow and the broadening is expected to be evident. The purpose of analyzing only additional 633 opposing current cases is to complement our results, where co-flowing currents were investigated. 634 Therefore, we can verify if the broadening occurs regardless the mean current direction. 635

Waseda et al. (2015) shows that the currents are more unsteady as the mean velocity increases. The spatial distribution of the current field is expected to be less homogeneous. According to

our assumptions of wave rays scattering, as the standard deviation of the currents increase, the 638 directional broadening of wave spectrum would be expected to be more intense. This is exactly 639 what we see from the additional results (Fig. 14). The directional spectra are shown in the top 640 panels (including for U = 0). In the bottom panels, the measured directional spreading parameter 641 A is plotted against the mean (left-hand side) and standard deviation (right-hand side) of the time 642 series of current speed. The broadening of the directional distribution in the presence of currents 643 is evident. Since the variability of the current increases for higher mean speed fields (Waseda et al. 644 2015), parameter A is inversely proportional to the mean and standard deviation of currents. We 645 can also see that the spectral energy in high frequencies is progressively suppressed as the current 646 standard deviation (and mean speed) increase, which was also observed by Waseda et al. (2015) 647 through the analysis of the 1d spectra. 648

Therefore, the broadening and suppression of high frequency energy occurs for co-flowing as 649 well as opposing currents. The main factor is how variable and unstable the background current 650 field is and not a direct consequence of the *Doppler* effect. The random refraction and scattering 651 of wave components result in the final broadening of the directional spectrum. If the wavenumbers 652 are randomly refracted and the energetic part of spectrum is perturbed in time, the energy transfer 653 to high frequencies and the maintenance of a high frequency spectral tail is suppressed. A remain-654 ing question is whether this perturbation prevails in space and time, i.e. whether wave interactions 655 would act in a way to restore the equilibrium spectral shape under the broadening force at longer 656 space-time scales and more tests and observations are needed to investigate this balance. 657

658 Conclusions

⁶⁵⁹ This study investigates the effects of a highly variable current field on the spectral shape of prop-⁶⁶⁰ agating waves. Despite the spatio-temporal variability of the currents in the tank, some patterns ⁶⁶¹ in the wave spectrum are observed, which are mainly concerned with the suppression of energy ⁶⁶² cascading to higher frequencies and directional broadening.

The main current-induced effects on single wave trains are related to random directional changes, which consequently modulates wave height. The spatial structure of the current field is variable and, consequently, the refraction patterns are modified for each experiment. The presence of currents has no influence on the wave absolute frequency, which suggests that a stationary or slow varying current approximation holds.

Having the previous background results for monochromatic waves, our analysis of irregular waves was focused on the spectral geometry only. In the absence of currents, a progressive downshifting of the spectral peak along the fetch is observed. Unlike the regular wave cases, the peak frequency is changed under the influence of currents and the downshifting of the peak, represented by the ratio between the peak frequency in advanced and early stages, is reduced. This result indicates that the nonlinear interactions responsible for downshifting the spectral peak is perturbed when currents are present.

The wave energy is transferred to frequencies beyond the generated wave frequencies via wave-675 wave energy exchanges. This process offers a valuable opportunity to study current-induced ef-676 fects on the nonlinear interactions. The investigation of unidirectional and directional irregular 677 waves shows that the interaction between wave components is more intense as the spectral steep-678 ness increases. In the presence of currents, the energy transfer to higher frequencies beyond the 679 wave-maker upper limit is suppressed, which can be explained by the detuning of the four-wave 680 resonant conditions, proposed by Waseda et al. (2015). To quantify this process, we calculate the 681 integrated normalized energy difference in high frequencies between current and no current spectra 682 $\Delta \overline{E}_{hf}$. The high frequency energy suppression is more intense for steeper waves and, interestingly, 683 broader directional spreading. Therefore, parameter $\Delta \overline{E}_{hf}$ shows to be a function of the initial 684

spectral steepness ε , but it is limited when the spectrum is sufficiently narrow. By introducing a new parameter ε_A , which relates the steepness to the inverse normalized directional distribution *A*, we found that the relation of $\Delta \overline{E}_{hf}$ better correlates with ε_A .

If the steepness is high enough and the directional spectrum sufficiently broad, exact resonances 688 are the main mechanism of wave energy exchanges and maintenance of a high frequency tail. The 689 results suggest that, for these cases, the random current field is more effective in detuning the 690 four-wave interaction conditions than for waves with directionally confined energy, where quasi-691 resonant interactions are predominant. The detuning force seems to not prevail along the fetch 692 and the tail tends to an equilibrium as the waves propagate, which occurs more rapidly for the 693 directionally narrow waves. Therefore, parameter ε_A is an attempt to include the physics observed 694 from our findings that the the detuning of resonance conditions caused by the background current 695 is more effective when the spectral evolution is controlled by exact resonances (broad directional 696 distribution) over quasi-resonances (directionally confined energy distribution). Furthermore, it 697 was observed that a threshold for A must be included in \mathcal{E}_A , which would represent the switch 698 between exact and quasi-resonances. Thus, any value of A above the applied threshold assumes 699 the same value. For an observed threshold in the range 4–7, $\Delta \overline{E}_{hf}$ highly correlates with ε_A in an 700 universal dependence. 701

The impact of the random current field extends to the directional distribution of wave energy. Random refraction scatters the wave energy and a consequent broadening of the directional spreading occurs. All the experimental cases showed broadening of the observed wave spectrum, except for one of the broadest case. As the directional spectrum narrows, the current-induced broadening is more evident. Additional results with opposing current fields also show that the broadening is intensified as the standard deviation of the current increases. Therefore, the broadening effect is not related to the direction of the current relative to the wave propagation, but instead to the spatiotemporal variability of the current field. It is suggested that the mechanism behind this process is
similar to wave ray scattering over small scale variable bottom features (Long 1973), which is
related to broadening of the directional spread of waves approaching coastal waters (Ardhuin et al.
2003). The disturbance of wavenumbers caused by wave refraction thus weakens the nonlinear
interactions by detuning the resonant conditions and, consequently, the energy inflow into shorter
waves is suppressed.

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721

APPENDIX

722

Focusing/defocusing of wave energy induced by the currents in the tank

The quantification of the observed spectral modifications based on the arguments of focus-723 ing/defocusing of wave energy is a complicated task. Since the currents in the tank are highly 724 variable in space and time, the induced refraction pattern becomes extremely hard to be repro-725 duced. However, one possibility is to consider the statistics shown in table 1. The mean current 726 speed at the 5 positions suggests that the gradient is variable along the cross-section. The ray tra-727 jectories can be numerically simulated through the geometrical optics approximation (e.g. Kenyon 728 1971) and considering the values of table 1 constant along the tank. This approach can provide 729 an average refraction pattern. We can estimate the degree of convergence using a straightforward 730

method proposed in Rapizo et al. (2014), based on ray counting. This method is applied for an 731 incoming number of 300 rays, which provides a reasonable estimation of the degree of conver-732 gence eventually induced by the mean current profile. Fig. A1 shows the ray tracing simulation 733 (top panel). The bottom panel shows the focusing/defocusing of rays in relation to the original 734 incoming number of rays per grid cell, which was made by dividing the grid in $1m \times 1m$ cells. 735 Although fig. R1 provides a sense of the intensity of energy focusing/defocusing in the tank, it 736 is a rough estimation and can be misleading. Firstly, because the current field is not homogeneous 737 in the along-channel direction (Takahashi 2011; Toffoli et al. 2013). Second, the variability of the 738 current on the right-hand side can reach 40% of the mean value, while the current on the left side 739 is comparatively steady. Despite the lower mean value of 8 cm s⁻¹ at position L2, the values at 740 positions R2 and R1, for example, reach lower values than 8 cm s⁻¹ (see Fig. 2). This would make 741 the gradient vectors completely inverse their direction and change the refraction patterns along the 742 time. As a consequence a region where the energy converges, can experience a divergence of 743 wave rays at other times. The reason of the variation of wave energy over the sensors in the 744 pentagon for different experiments in the presence of currents is probably associated with the 745 variability of focusing/defocusing patterns. To exemplify this process, Fig. A2 shows the same 746 ray diagram of Fig. A1, but considering a different and possible distribution of the current field 747 due to its variability, based on measured values. The rays now strongly diverge from the center. 748 This estimation again considers a velocity field constant along the tank. It is important to stress 749 that figures A1 and A2 are potential snapshots only and not meant to be predictive. Therefore, the 750 spatio-temporal variability of the current field is significant and thus treated in the present study as 751 random. 752

⁷⁵³ Finally, we can have an estimation of focusing/defocusing by analyzing the time series of surface
 ⁷⁵⁴ elevation of a regular wave. Fig. A3 shows an example of a time series recorded in the absence

⁷⁵⁵ (black line) and presence (blue line) of currents. In the presence of currents, the amplitude varies ⁷⁵⁶ considerably and can be reduced in 35% and increased in 30% comparing with conditions of ⁷⁵⁷ U = 0. The modulation of the wave on currents is rather random and, based on the aforementioned ⁷⁵⁸ argument, likely to be associated with refraction induced by the current field.

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nosition	$\mu(U)$	$\operatorname{std}(U)$	$\mu(\theta_U)$	$\operatorname{std}(\theta_U)$
position	[cm s ⁻¹]	$[\mathrm{cm}\ \mathrm{s}^{-1}]$	[deg]	[deg]
R2	10.31	0.86	6.76	4.78
R1	9.30	1.00	9.42	4.89
М	7.94	0.74	5.52	4.22
L1	7.73	0.41	4.01	3.72
L2	7.97	0.25	3.71	1.90

TABLE 1. Statistics of current speed (*U*) and direction (θ_U) of the plots of Fig. 2. μ is mean value and std, standard deviation. Positions are shown in Fig. 1 by the red triangles.

group	case	type	$f_p(\text{Hz})$	k_p (rad m ⁻¹)	kh	α	γ	ε	Α
1	1	Mono	1.11	4.96	24.79	-	-	0.05	-
1	2	Mono	1.11	4.96	24.79	-	-	0.08	-
	3	JON-1d	1.25	6.28	31.44	0.0063	1	0.05	-
2	4	JON-1d	1.25	6.28	31.44	0.0063	5.5	0.07	-
_	5	JON-1d	1.25	6.28	31.44	0.0063	17	0.10	-
	6	JON-2d	1.25	6.28	31.44	0.0041	3.3	0.05	2
3	7	JON-2d	1.25	6.28	31.44	0.0080	3.3	0.07	2
	8	JON-2d	1.25	6.28	31.44	0.0164	3.3	0.10	2
	9	JON-2d	1.25	6.28	31.44	0.0194	5.0	0.12	1.5
4	10	JON-2d	1.25	6.28	31.44	0.0194	5.0	0.12	2.5
	11	JON-2d	1.25	6.28	31.44	0.0194	5.0	0.12	3.5

TABLE 2. Relation of experiments performed. The terms 'Mono', 'JON-1d' and 'JON-2d' refer to regular monochromatic, unidirectional and directional waves based on the JONSWAP spectral formulation, respectively

group	case	$T_p(s)$	$T_p^*(s)$	ε	$oldsymbol{arepsilon}^*$	Α	A^*	$\overline{U} \ (\mathrm{cm} \ \mathrm{s}^{-1})$	$\overline{U}^* ({ m cm}~{ m s}^{-1})$	\overline{U}^*/c_g
1	1	0.9	0.89	0.05	0.051	-	21.9	+7.5	+6.68	0.097
1	2	0.9	0.89	0.08	0.077	-	33.8	+7.5	+7.17	0.103
	3	0.8	0.88	0.05	0.056	-	7.9	+7.5	+7.28	0.106
2	4	0.8	0.85	0.07	0.073	-	11.4	+7.5	+6.90	0.104
	5	0.8	0.89	0.1	0.090	-	11.3	+7.5	+7.08	0.102
	6	0.8	0.85	0.05	0.044	2.0	2.26	+7.5	+7.54	0.121
3	7	0.8	0.85	0.07	0.070	2.0	3.08	+7.5	+7.57	0.122
	8	0.8	0.87	0.1	0.106	2.0	2.32	+7.5	+7.33	0.108
	9	0.8	0.84	0.12	0.121	1.5	1.3	+7.5	+7.17	0.111
4	10	0.8	0.91	0.12	0.110	2.5	2.1	+7.5	+7.30	0.103
	11	0.8	0.92	0.12	0.119	3.5	4.0	+7.5	+7.26	0.098

TABLE 3. Aimed (P) and measured (P^*) parameters of the experiments performed.

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FIG. 1. Schematic of the Ocean Engineering Tank of the Institute of Industrial Sciences, University of Tokyo.
Wave gauge locations are represented by blue circles and current meters by red triangles.



FIG. 2. Time series of current speed (thick black line) and direction (dashed gray line) at the 5 locations in a cross section at 30 m from the wave-maker, where R1 and L1 are the most right and left points, respectively (from the wave-maker point of view). Positions are shown in Fig. 1 by the red triangles.



FIG. 3. Measured wave parameters of significant wave height (H_s), peak period (T_p) and steepness (ak) (from left to right, respectively) in the presence (filled markers) and absence (open markers) of currents for the two monochromatic cases, 1 (\bigcirc) and 2 (\square). The first 8 gauges are from the linear array on the left side of the tank, while the last 5 compose the pentagon array in the middle of the tank (at 32m from the wave-maker).



FIG. 4. Directional spectra obtained for the monochromatic case 1 in the presence (bottom) and absence (top) of currents. The elevation time series were divided in three segments. Plots from left to right show each segment and full time series spectra. Peak direction θ_p , the difference of θ_p in the absence and presence of currents ($\Delta \theta_p$) and *A* obtained for the full series are shown.



FIG. 5. Relation between frequency downshift and (a) steepness ε and (b) parameter *A* for irregular wave cases: varying γ (\bigcirc , group 2), varying α (\square , group 3) and varying directional parameter *A* (\triangle , group 4). Open symbols show cases in the absence of currents and filled symbols, when currents are present. Downshift is quantified as ω_i/ω_f , where ω_i is average peak frequency between 10–17 m and ω_f at 32 m.



FIG. 6. Frequency spectrum for unidirectional (top) and directional random waves (bottom) in the absence of currents. Measurements are from gauge 4, at 20 m from the wave-maker. Theoretical f^{-5} decay is shown (dashed black line). The spectra were estimated using average periodograms with 50% overlap, Hanning window and frequency resolution of 0.0244. Vertical dashed line draws the wave-maker upper limit.



FIG. 7. Frequency spectrum at gauge 4 for all unidirectional, varying gamma, (top) and directional, varying alpha, (bottom) cases. Solid black line shows spectrum with no current and dashed gray line shows spectrum in the presence of currents. Theoretical f^{-5} decay is shown. Vertical dashed line draws the wave-maker upper limit. Spectral estimation method applied is the same as described in Fig. 6



FIG. 8. Frequency spectrum for directional random waves with varying directional spreading (A=1.5,2.5 and 3) in the absence (left-hand panel) and the comparison of absence and presence of currents (right-hand panels). Measurements are from gauge 3, at 17 m from the wave-maker. Theoretical f^{-5} decay is shown for all plots and line representing decay at the frequency range $(1.35f_p - 2f_p)$ is shown in the right panels. Vertical dashed line draws the wave-maker upper limit. The spectra were estimated using average periodograms with 50% overlap, Hanning window and frequency resolution of 0.0244.



FIG. 9. Spectral evolution along the fetch (a_1, b_1) for the directionally broadest (a) and narrowest (b) cases. Panels a_2/b_2 and a_3/b_3 show evolution of normalized high frequency energy difference $\Delta \overline{E}_{hf}$ and exponent of tail decay v, respectively.



FIG. 10. Relations of steepness ε , parameter A and parameter ε_A defined in (13) with decay exponent of the high frequency spectral tail v and normalized energy difference in the presence and absence of currents $\Delta \overline{E}_{hf}$, as defined in (12): a) ε vs v; b) A vs v; c) ε_A vs v; d) ε vs $\Delta \overline{E}_{hf}$; e) A vs $\Delta \overline{E}_{hf}$; and f) ε_A vs $\Delta \overline{E}_{hf}$. Results correspond to mean values of gauges 2–5 (standard deviation bars shown). Different symbols show irregular wave groups: 2 (varying γ , \bigcirc), 3 (varying α , \Box) and 4 (varying A, \triangle).



FIG. 11. Relation between parameters ε_A , as defined in (14), and the integrated energy difference in the absence and presence of current ($\Delta \overline{E}_{hf}$). Each plot shows a different threshold value A_{thr} applied in (14), as indicated: 2, 4, 6 and 8.



FIG. 12. Directional spectra for the broadest (Case9, top panels) and narrowest (Case 11, bottom panels) directional spreading cases of group 3. Left spectra are in the absence of currents and right, with a background current field. Parameters A and θ_p are shown for each plot.



FIG. 13. Variation of parameter *A* and peak direction θ_p in the absence and presence of currents as a function of *A* in the absence of currents (A_{NOcurr}): a) $\Delta A = A_{NOcurr} - A_{curr}$ and; b) $\Delta \theta_p = \theta_{pNOcurr} - \theta_{pcurr}$. Different symbols show irregular wave groups: 2 (varying γ , \bigcirc), 3 (varying α , \Box) and 4 (varying *A*, \triangle).



FIG. 14. Directional spectra for the same initial wave signal and varying opposing current field (top) and directional spreading parameter A as a function of the mean current speed and standard deviation (bottom, left and right-hand sides, respectively).



Fig. A1. Simulation of wave ray trajectories using the mean current speed values of table 1. Top panel: current vectors and ray paths (red lines); bottom panel: relative changes in the number of rays and position of sensors, where circles represent the wave gauges in the linear array and the 'x' marker shows the location where the directional spectrum is obtained.



⁹⁹⁶ Fig. A2. same ray diagram and estimated focusing/defocusing of rays as in Fig. A1, but considering different ⁹⁹⁷ values for the current speed at positions R2–L2.



Fig. A3. Comparison of time series of surface elevation in the presence (blue line) and absence (black line) of currents for the regular wave case 1 (gauge 11).