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Technical note

Doppler properties of radars in circular orbits

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Abstract. Expressions are presented for Doppler frequency shift, Doppler bandwidth, zero Doppler offset angle in spacecraft yaw and, in the SAR mode, the rate of Doppler frequency modulation, azimuth time-bandwidth product, resolution, available integration time and the location of principal azimuth ambiguities. The equations are simply expressed with virtually no approximations based on angles referenced to the satellite. Earth rotation is included, as is the geosynchronous case. (These results differ from expressions to be found elsewhere in the literature, most of which have been derived using flat-Earth approximations.)

1. Introduction

The purpose of this article is to present compact expressions for the Doppler frequency, frequency linear modulation rate, Doppler bandwidth and the like for radars on a circular (or near-circular) orbit. The work applies both to SARs (Synthetic Aperture Radars) and to scatterometers and, in particular, those on ERS-1 and RADARSAT. An effort is made to offer some insight into the relationships. In particular, the role of Earth rotation and the correct use of spacecraft or footprint velocity are highlighted.

The results are expressed in a spacecraft-centred co-ordinate system, in which the principal variables are location of the spacecraft and relative azimuth and elevation angles from the radar to the scatterer. The notation is defined in the Appendix.

Some background. The work of this paper derives from the author's analysis of SAR systems found on SEASAT, RADARSAT, ERS-1, Magellan, Pioneer Venus and other orbital radars. Such systems are based on a short but honourable history of airborne systems, as described quantitatively by Brown (1967), Harger (1970) and Tomiyasu (1978). Although acknowledged to be a first-order approximation, the simple unforgettable rule about SAR systems is that they are (theoretically) capable of achieving an azimuth resolution that is one-half of the size of the antenna aperture that is used to illuminate the terrain; this is independent of radar wavelength, range or system velocity.

Of course, high-resolution performance with these properties is ideal for applications requiring satellite sensors. Thus there have evolved several systems (and a great deal of literature) centring on satellite SAR. The resolution rule of one-half the aperture has been uniformly adopted in that context. Unfortunately, the original work on SAR was strictly modelled using a flat-Earth approximation which is not appropriate for a satellite radar configuration. The resolution achievable from a satellite is actually significantly better than one-half aperture (Raney 1984), one of the points encompassed in this paper. Indeed, this fact and the circumstances surrounding it seem not to be considered in contemporary literature (see, for example, Tomiyasu 1978, Ulaby *et al.* 1982, Barber 1985).

More motivation. The resolution issue is just one symptom of a much more serious set of issues. Nowhere in the literature does there seem to be a treatment of the Doppler space for a satellite radar that is both sufficiently complete to include the principal properties needed for correct system design and analysis, while at the same time sufficiently simple so that greater insight into the system operation is possible. This article is one modest attempt in that direction.

The issue is focused by consideration of a very simple question, namely 'what is the velocity of a radar on a satellite in circular orbit?' It happens that there are three possible answers to this question, only one of which is correct for a given context. Each of the three may be correct, according to the circumstances. The problem is that the literature abounds with incorrect answers or inappropriate answers to the velocity question. The velocity issue lies at the heart of the azimuth response of an SAR, as it is only a glorified range-Doppler radar. Correct velocity formulation for the orbital case is essential for a proper understanding and design of SARs and other Doppler-dependent radars intended for satellites.

The three possible answers to the velocity question are: (i) the spacecraft velocity V_{sc}, (ii) the footprint velocity V_g, the rate at which the Earth's surface is covered by the sensor, and (iii) the square root of the product of V_{sc} and V_g. For the flat-Earth model, these three velocities have the same value, if not the same meaning. For the satellite case, their specific roles need to be thought through afresh. This is the main theme of this paper.

Examples. In an otherwise excellent text, *Microwave Remote Sensing*, Volume II (Ulaby *et al.* 1982, p. 470) the authors derive the Doppler structure for a flat Earth, stating that for other cases "... the situation is similar, but more complicated" and let the matter go. Unfortunately, the example used in the accompanying text is a satellite SAR.

In a recent paper on ERS–1 SAR design (Sawyer *et al.* 1984) both spacecraft and footprint velocity are introduced, yet employed in a misleading manner. The Doppler bandwidth (of the processed signal in azimuth) is derived by scaling inverse resolution by the footprint velocity, which gives a correct numerical result, but for an incorrect reason! Even more seductive, this approach suggests that the radar Doppler spectrum is dependent on footprint velocity, which is not the case.

In a recent extensive review article on this subject, Barber (1985) seems not to recognize the role of these various velocities. As a result, azimuth time scaling of the point-spread function (p. 1014) is in error, as is the derived resolution. Likewise, scaling in the azimuth parameters of interpulse spacing, aperture time and number of samples in the Doppler replica (pp. 1044–1046) are not correct. Although that article is helpful at several levels, it does not maintain the standards set in the signal-theory sections when it comes to interpretation of that work for the azimuth dimension of an orbital SAR.

Digital processing has been with us for several years. A key parameter in design of the azimuth matched filter is the rate of Doppler frequency linear modulation, the so-called FM rate. This term is derived (Bennett *et al.* 1980) by analysis of the relative velocity between the SAR and an observed scatterer on the Earth. The FM rate is proportional to $V_{sc}V_g$, although one frequently finds V_{sc}^2 in the literature (Harger 1965, Li and Johnson 1983), even in the context of orbital systems. Of course, these may have been intended as approximations, but if the 'approximation' is only good to 15 per cent or worse, and if there is available a simple expression correct to better than 1 per cent, then the more accurate expression should be employed.

Many more examples could be cited. The literature is voluminous, confusing and rarely correct on the matter.

The bottom line. Unfortunately, with so many choices available, it is possible for satellite radar analysis and design to go astray. For satellite systems, and for parameters that may be urged against sensitive margins, variations of 5 or 10 per cent can be very expensive in cost or in performance. For example, if one (incorrectly) uses either V_g or $\sqrt{(V_g V_{sc})}$ for Doppler scaling, then an error of 7–17 per cent can occur in the selection of radar pulse repetition frequency for low Earth-orbit SARs, which in turn implies up to 10 dB worse azimuth ambiguity levels than might be intended, using numbers typical of ERS-1 or RADARSAT baseline design (Raney 1984).

In terms of azimuth resolution, a difference of 17 per cent (RADARSAT) in predicted azimuth resolution can mean an increase of 17 per cent in allowed antenna length, implying a decrease of 1·5 dB in required transmitter power, or a savings of more than 100 W. This, in turn, implies an implementation decision involving tens of millions of dollars.

The intent of these two examples is to show that clarification of the fundamental description of SAR and allied radar systems can have an impact that is significant, even if the magnitude is seemingly small of the corrections on the parameters involved.

With the above as an explanation of the problem motivation, we turn to the matter at hand.

1.1. *Vector solution*

For an arbitrary (vector) geometry, it is well known that the relative Doppler frequency between a radar and a scattering object being observed is

$$f_{\mathrm{Dop}} = -\frac{2}{\lambda}\frac{\dot{\mathbf{r}}\cdot\mathbf{r}}{R} \tag{1}$$

where $\mathbf{r}$ is the range vector, $\dot{\mathbf{r}}$ is the first time derivative of $\mathbf{r}$ and $R=|\mathbf{r}|$ is the range distance. For the problem at hand, there are two sources of motion: spacecraft orbital velocity $\dot{\mathbf{H}}$ and Earth rotation $\dot{\mathbf{R}}_e$. We have range $\mathbf{r}=\mathbf{H}-\mathbf{R}_e$ and the range rate $\dot{\mathbf{r}}=\dot{\mathbf{H}}-\dot{\mathbf{R}}_e$ which express the range and range rate in terms of the spacecraft vector $\mathbf{H}$ and the scatterer location vector $\mathbf{R}_e$.

For a circular orbit, $\dot{\mathbf{H}}\cdot\mathbf{H}=0$ (since these vectors are orthogonal), and $\dot{\mathbf{R}}_e\cdot\mathbf{R}_e=0$ for Earth rotation. Thus,

$$f_{\mathrm{Dop}} = \frac{2}{\lambda}\left(\frac{\dot{\mathbf{H}}\cdot\mathbf{R}_e}{R}+\frac{\dot{\mathbf{R}}_e\cdot\mathbf{H}}{R}\right) \tag{2}$$

This may be simplified further by using the dot product to advantage. Consequently, the Doppler frequency is

$$f_{\mathrm{Dop}} = \frac{2}{\lambda}\left(\frac{V_{sc}R_e}{R}\cos\mathscr{E}_2+\frac{\omega_e H R_e \cos l}{R}\cos\mathscr{E}_1\right) \tag{3}$$

where $\mathscr{E}_2$ is the angle between the spacecraft velocity vector and the scatterer location vector and $\mathscr{E}_1$ is the angle between the spacecraft location vector and the scatterer motion vector produced by Earth rotation. The Earth's rotation is at uniform rate ω_e, which at latitude l leads to a tangential scatterer velocity magnitude of $\omega_e R_e \cos l$.

The results so far, although perhaps obvious, are instructive. We see that the effects of Earth rotation and spacecraft velocity are separated and additive, independent of the specific co-ordinate system. It is also clear that the relative position of scatterer to spacecraft is of importance rather than the absolute scatterer position.

1.2. *Geometry*

To find solutions, it is necessary to employ specific co-ordinate systems.

1.2.1. *Spacecraft*

The spacecraft is confined to an orbital plane at an inclination ψ. (For Sun-synchronous remote-sensing Earth satellites, ψ is near 100°.) The position of the spacecraft in this plane is variously described in remote-sensing references. A very convenient variable is argument of latitude β, which is a standard spacecraft parameter. (Note that this is not the projected latitude onto the Earth's surface, a 'latitude' that is sometimes used for spacecraft position, but can be misleading.) The orbital spacecraft vector is $\mathbf{H}$, with magnitude H measured from the Earth's centre. The angular rate of the spacecraft is ω, and the magnitude of the spacecraft velocity along the orbital path is V_{sc}.

1.2.2. *Earth*

The Earth is taken to be an oblate spheroid (corrected by higher-order terms) and represented by a local radius vector $\mathbf{R}_e$ from the Earth's centre. (The error resulting from this locally spherical model compared with the perfect case is less than 50 m in elevation (Curlander 1982), which is of no consequence in the work presented here.)

1.2.3. *Radar*

The radar looks down from the spacecraft to a scatterer, as in figure 1. The range elevation plane is the one in which a radar is customarily described, in which the elevation angle γ and the range R are the principal definitive variables. Trigonometric identities of use are listed in the figure.

The angle between the range elevation plane and the spacecraft orbital plane is defined as azimuth a in this work, measured from the forward spacecraft direction as in figure 2. Thus, $a = 90°$ corresponds to the side-looking case. An indicator variable ε is helpful to describe radars looking to the right (+1) or left (−1) of the orbital velocity vector.

Note that for people used to working with spacecraft, the variable a is more correctly known as yaw. Furthermore, a is not azimuth as used in describing radar antenna patterns. Antenna azimuth ξ is customarily defined with respect to the boresight directed in the elevation direction γ. This issue is readdressed as required below.

2. Results

It may be shown that solution of the Doppler equation in the desired angles leads to

$$\frac{R_e}{R}\cos\mathscr{E}_2 = \sin\gamma\cos a$$
$$\frac{R_e\cos l}{R}\cos\mathscr{E}_1 = -\sin\gamma(\varepsilon\cos\beta\sin\psi\sin a + \cos\psi\cos a) \qquad (4)$$

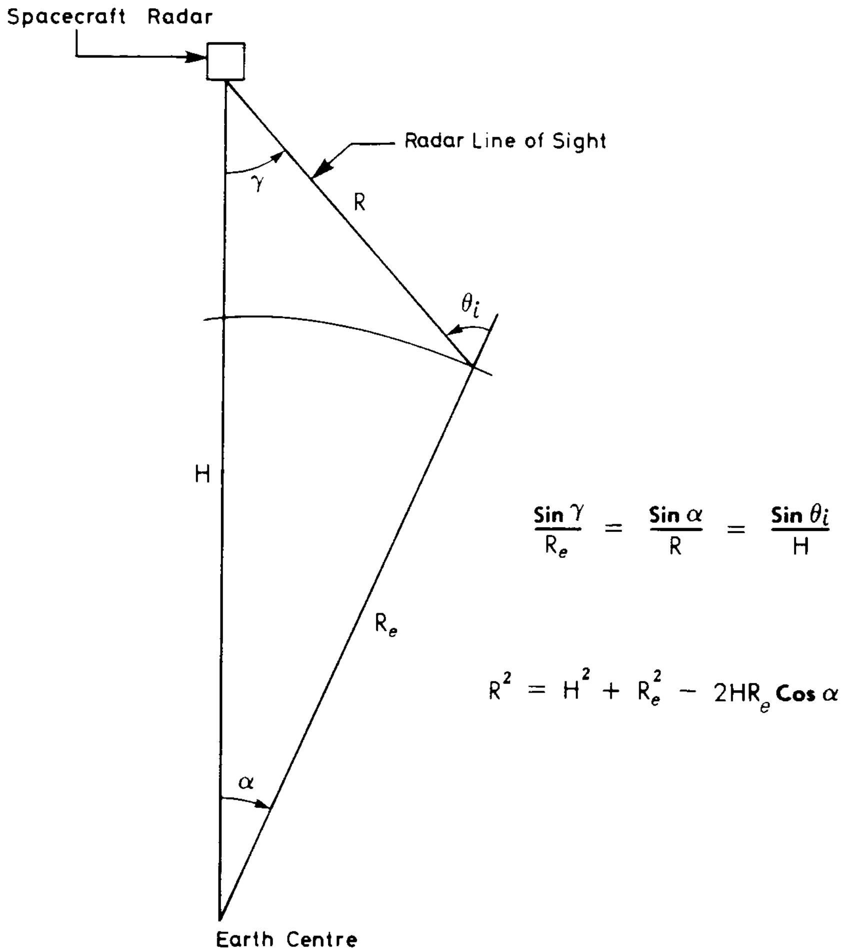

Figure 1. Geometry in the range–elevation plane.

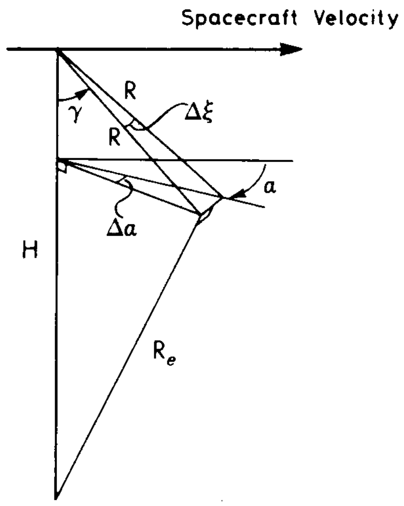

Figure 2. Geometry for Doppler bandwidth of azimuth beamwidth at a given angle a relative to the orbital plane.

Thus, the total Doppler shift is

$$f_{Dop} = (2V_{sc}/\lambda)\sin\gamma\cos a\,[1-(\omega_e/\omega)(\varepsilon\cos\beta\sin\psi\tan a + \cos\psi)] \tag{5}$$

The first term represents Doppler arising from the spacecraft motion, and the remaining two terms come from Earth rotation.

This equation yields several results of specific interest. These are outlined here.

2.1. *Yaw steering*

If it is desired to steer the spacecraft (or antenna) to stabilize the offset angle to zero Doppler, what angle a_0 is required? We need to solve $f_{Dop}(a_0)=0$, leading to

$$\tan a_0 = \varepsilon\left(\frac{\omega/\omega_e - \cos\psi}{\cos\beta\sin\psi}\right) \tag{6}$$

which, for a given radar and orbit, is simply a function of spacecraft argument of latitude. Clearly, the yaw angle a_0 is dependent on which side of the orbit the radar looks (ε), and on the relative spacecraft-to-Earth rotation rate, ω/ω_e. As $\omega/\omega_e \to \infty$, $a_0 \to 90°$, as it should be. Finally, for $\psi \to 0°$ or 180°, $a_0 \to 90°$ since at these inclinations no component of Earth rotation causes a Doppler shift.

2.2 *FM rate (SAR mode)*

Let the radar be side looking, so that $a = 90°$ relative to the spacecraft orbital plane, the customary geometry for an SAR. Then the azimuth FM rate is found by taking the time derivative of the Doppler shift. In the SAR mode, for a constant range contour, $d\gamma/dt = 0$. It may be shown that $da/dt = V_g/R\sin\gamma$ and $d\beta/dt = \omega$ by definition. Then we find by direct calculation that the FM rate is

$$\frac{df_{Dop}}{dt} = -\frac{2V_{sc}V_g}{\lambda R}\left\{1 - \frac{\omega_e}{\omega}[\cos\psi - \varepsilon\sin\psi\sin\beta\cot(90° + \alpha)]\right\} \tag{7}$$

This is an interesting result. The leading term is the known approximate FM rate for an orbital SAR. The parameter V_g is the effective rate of transport of the antenna footprint over the Earth's surface. (Recall that the interplay between V_g and V_{sc} is of major interest in this work, as they are so prone to misuse.)

The terms modulating ω_e arise from Earth rotation. The first of those, $-\cos\psi$, is the component of Earth rotation that is parallel to the spacecraft velocity vector; for inclination angles larger than 90°, $-\cos\psi$ is positive, so that this term effectively increases the FM rate, since the Earth rotation and orbital velocity are in opposing directions. This amounts to an increase of FM rate by about 1 per cent for $\psi = 98{\cdot}5°$, and typical altitudes. For the shuttle radars, the inclination is much less, such as $\psi = 57°$ for SIR-B for example. This leads to a decrease of the FM rate by about 3 per cent.

The final term is dependent on spacecraft position, as it represents the component of Earth rotation orthogonal to the slant range vector. For Earth remote-sensing satellites in near polar orbit, generally $90° + \alpha \leqslant \psi$, so that an upper bound on the third term is $\sin\beta\cos\psi$. Since $\varepsilon = \pm 1$, this term can be nearly as forceful as the $\cos\psi$ term, with the sign dependent on which side the radar is looking, and whether operating in the Northern or Southern Hemisphere. The largest values of this term occur near the poles, and it goes to zero at the equator. Thus, the total Earth rotation correction required ranges from near zero to approximately 2 per cent for low Earth-orbit radars.

There is an interesting special case. For a side-looking radar looking directly on one of the poles, "the still point of the turning world" (Eliot 1942), the effects of Earth rotation should nullify. Thus, for $\beta=90°$ or $270°$,

$$\cos\psi=[\varepsilon\sin\beta]\sin\psi\cot(90°+\alpha) \tag{8}$$

which yields a solution

$$\alpha_0=\psi-90° \tag{9}$$

at the North Pole for a right-looking SAR, or at the South Pole for a left-looking SAR, as expected.

In the event the satellite is at geosynchronous altitude, $\omega_e/\omega=1$, and the results are dominated by Earth rotation acting in concert with orbit inclination.

2.3. *Doppler bandwidth*

Doppler bandwidth is a fundamental design parameter for all pulsed radars, especially orbital ones. Therefore, we are interested in the Doppler bandwidth determined by antenna beamwidth corresponding to a small angular subtense $\Delta\xi$, where ξ is the azimuth angle relative to the antenna boresight at elevation angle γ, as in figure 2. In order to determine the sensitivity to antenna azimuth beamwidth for a given range, we need to hold γ constant, let a vary and calculate the azimuthal dependence between Δa and $\Delta\xi$. For arbitrary azimuth a and elevation γ we find $\Delta\xi=\Delta a\sin\gamma$. Then, taking the derivative d/da of the Doppler frequency expression to determine change of Doppler frequency with angle, we have directly that the Doppler bandwidth Δf_{Dop} for antenna beamwidth $\Delta\xi$ is given by

$$\Delta f_{\text{Dop}}=\frac{2V_{\text{sc}}}{\lambda}\Delta\xi\sin a\left[1+\frac{\omega_e}{\omega}(\varepsilon\cos\beta\sin\psi\cot a-\cos\psi)\right] \tag{10}$$

It is interesting to note that the Doppler bandwidth is not a function of incidence angle, even for the case of arbitrary azimuth a considered here.

2.4. *Doppler bandwidth (SAR mode)*

For the SAR (side-looking) case, $a\simeq 90°$ and

$$\Delta f_{\text{Dop}}=\frac{2V_{\text{sc}}}{\lambda}\Delta\xi\left(1-\frac{\omega_e}{\omega}\cos\psi\right) \tag{11}$$

The dominant approximation in this result is

$$\Delta\xi\simeq\sin\Delta\xi \tag{12}$$

which for the antenna patterns anticipated in space radars is accurate to one part in 10^4. It is satisfying that there is no dependence on latitude for the side-looking case.

The Doppler bandwidth of a radar signal arises from the corresponding definition of antenna beamwidth, $\Delta\xi$. If $\Delta\xi$ is the -3 dB one-way (power) beamwidth, the Δf_{Dop} corresponds to the width in frequency of the (instantaneous) Doppler spectrum to the -6 dB level. The Doppler angular response is frequently approximated by

$$\Delta f_{\text{Dop}}\simeq(2V_{\text{sc}}/\lambda)\Delta\xi \tag{13}$$

neglecting Earth-rotation effects. For remote-sensing satellite configurations, the $(\omega_e/\omega)\cos\psi$ term increases the Doppler bandwidth by about 1 per cent over that found from the spacecraft velocity alone, and for SIR–B it decreases the Doppler bandwidth by about 3 per cent. For comparison between two radars (e.g. RADARSAT and SIR–B) of sensitive parameters such as $PRF/\Delta f_{Dop}$ for which a difference of a few percent can be significant, these Earth-rotation effects should be included.

2.5. *Location of azimuth ambiguities*

The radar pulse repetition frequency (PRF) samples the ensemble of reflecting terrain illuminated by the antenna. From sampling theory, it is known that ambiguities arise from signals whose Doppler frequencies are integer multiples of the PRF. It is of interest to know the angle ξ_0 giving rise to ambiguous azimuth information.

For the principal SAR azimuth ambiguities we have (as a special case of the Doppler/angle relationship derived above)

$$\mathrm{PRF} = \pm\frac{2V_{sc}}{\lambda}\left(1-\frac{\omega_e}{\omega}\cos\psi\right)\xi_0 \tag{14}$$

The relative azimuth displacement $\Delta\chi$ of an ambiguous image element from its proper position is $\Delta\chi \simeq R\xi_0$; therefore

$$\Delta\chi = \pm\frac{R\lambda}{2V_{sc}}(\mathrm{PRF})\left(1-\frac{\omega_e}{\omega}\cos\psi\right)^{-1} \tag{15}$$

Again we find that for accurate work it is necessary to account for Earth rotation, which in this case shrinks the spatial scaling by about 1 per cent compared with that resulting from spacecraft velocity alone for Sun-synchronous sensors and expands the spatial scaling by about 3 per cent or more, for the shuttle imaging radars. Note that the correct velocity scaling parameter is that of the spacecraft.

2.6. *Available integration time (SAR mode)*

Whether used coherently for achieving ultimate resolution or used partially coherently to reduce the inherent variance in imagery of distributed scenes, the integration time employed by the azimuth processor is a fundamental SAR parameter. The integration time ΔT is defined by the time over which the azimuth signal integration occurs. Thus, for known Doppler bandwidth and azimuth linear FM rate we have

$$\Delta T = \Delta f_{Dop}\left(-\frac{df_{Dop}}{dt}\right)^{-1} \tag{16}$$

Each of these two quantities have been evaluated above. We find

$$\Delta T = \frac{R\Delta\xi}{V_g}\,\frac{1-(\omega_e/\omega)\cos\psi}{1-(\omega_e/\omega)(\cos\psi-\varepsilon\sin\psi\sin\beta\cot(90^\circ+\alpha))} \tag{17}$$

For low Earth orbit the correction terms resulting from Earth rotation cause only about a 1 per cent change, which for most applications of ΔT is negligible, so that the approximation is useful

$$\Delta T \simeq R\Delta\xi/V_g$$

This simple result is noteworthy in that it shows integration time to be inversely proportional to footprint velocity V_g, not spacecraft velocity as is usually invoked in this expression. Footprint velocity is appropriate because an SAR operating in a circular orbit slowly rotates as it progresses, thus increasing the amount of time that a given scatterer is illuminated, with the increase in time proportional to V_{sc}/V_g. This has a beneficial and habitually overlooked effect on resolution, as shown below.

2.7. *Time bandwidth product (SAR mode)*

The key parameter describing the azimuth processing complexity of an SAR is the time–bandwidth product, TBP (Brown 1967). Using expressions for ΔT and Δf_{Dop} developed above, we have

$$\mathrm{TBP} \simeq \frac{2R}{\lambda}\left(\frac{V_{sc}}{V_g}\right)\Delta\xi^2 \tag{18}$$

which compares with the expression $(2R/\lambda)\Delta\xi^2$ normally found for linear flight over a flat Earth. Again we seen an enlargement of the TBP over the normally invoked expression (Barber 1985) due to the circular orbital geometry.

2.8. *Resolution (SAR mode)*

The resolution achieved by fully coherent processing over an angular azimuth beamwidth $\Delta\xi$ is given by

$$r_a = R\Delta\xi/\mathrm{TBP} \tag{19}$$

where $R\Delta\xi$ is the width of the processed azimuth antenna angle at range R. Using the TBP evaluated above, we find

$$r_a \simeq \frac{\lambda}{2}\frac{V_g}{V_{sc}}\frac{1}{\Delta\xi} \tag{20}$$

where the Earth-rotation correction terms have been neglected. (For calculations of resolution achieved for high orbits, such as $\omega_e/\omega = 1$ as in the geosynchronous case, these terms are important and cannot be omitted. For low Earth orbits, the approximation is good to 1 per cent or so.)

The azimuth resolution is very interesting in that it compares favourably with the commonly employed expression $\lambda/2\Delta\xi$. Note that the principal effect of the orbital geometry on resolution is to narrow the impulse response width by the ratio V_g/V_{sc}. Although this effect is normally overlooked, it leads to approximately a 15 per cent improvement or more, a number that is quite significant compared with impulse response broadening budgets typically employed in system design. As remarked above, this can have a significant impact on system performance or cost for systems in which the antenna length is constrained above by azimuth resolution.

3. Conclusions

Doppler properties of radars in circular orbit have been explored. The general expression is derived, from which follow results of increasing specificity.

Parameters of interest in azimuth performance are obtained in the SAR case. Several of these explicitly include the effect of orbit geometry and Earth rotation, correct expressions for which seem unavailable elsewhere in the literature. It is shown that Earth rotation and orbital geometry should be considered for comparison between radars, or for conceptual design of azimuth Doppler frequency budgets. For the calculation of integration time or azimuth resolution, non-rotating Earth approximations usually suffice. Appropriate allocations of spacecraft or footprint velocity, subtle parameters subject to capricious appearance in the available literature, has been addressed.

Appendix

Nomenclature

a = angle of the azimuth plane from the $\mathbf{V}_{sc}\mathbf{H}$ plane (Note: this is not the same as the relative azimuth angle within an antenna pattern)
β = argument of latitude: the angle measured between the ascending node and the spacecraft position in the orbit plane from the centre of the Earth
ψ = orbit inclination measured from Earth spin vector to orbit rate vector
γ = elevation angle: included angle at spacecraft between range vector and Earth radius vector from spacecraft
α = Earth centre angle, between spacecraft radius vector and scatterer location vector
V_{sc} = velocity along orbit of spacecraft
ω = spacecraft orbital rotation rate: $V_{sc} = \omega H$
V_g = SAR footprint effective velocity on $R_e(V_g = \omega R_e \cos\alpha)$
ω_e = Earth rotation rate
$\mathbf{r}$ = radar to scatterer range vector
R = magnitude of $\mathbf{r}$
$\mathbf{H}$ = spacecraft radius vector, Earth centre to spacecraft
$\mathbf{R}_e$ = (local) Earth radius vector to scatterer
f_{Dop} = Doppler frequency shift (Hertz)
Δf_{Dop} = Doppler bandwidth corresponding to antenna azimuth increment
ξ = antenna azimuth angle, measured in the slant range plane relative to the antenna boresight (Note: this is not the same as azimuth a)
$\Delta\xi$ = increment of antenna azimuth angles, such as the 3 dB one-way beamwidth
ε = indicator variable. $\varepsilon = 1$ if radar looks to right side of spacecraft velocity vector (e.g. SEASAT and ERS-1), and $\varepsilon = -1$ if radar looks to the left (RADARSAT)
λ = radar wavelength
ΔT = available integration time corresponding to $\Delta\xi$
r_a = azimuth resolution
TBP = (SAR) azimuth time-bandwidth product

References

Barber, B. C., 1985, Theory of digital imaging from orbital synthetic aperture radar. *Int. J. remote Sensing*, **6**, 1009.

Bennett, J. R., Cumming, I. G., and Deane, R. A., 1980, The digital processing of Seasat synthetic aperture radar data. *Proceedings of the I.E.E.E. International Radar Conference*, p. 168.

Brown, W. M., 1967, Synthetic aperture radar. *I.E.E.E. Trans. Aerospace Electron. Syst.*, **3**, 217. (Reprinted in Kovaly, J. J., 1976, *Synthetic Aperture Radar*, Artech House.)

Curlander, J. C., 1982, Location of spaceborne SAR imagery. *I.E.E.E. Trans. Geosci. remote Sensing*, **20**, 359.

Eliot, T. S., 1943, Burnt Norton, from the *Four Quartets*.

Harger, R. O., 1965, An optimum design of ambiguity function, antenna pattern, and signal for side-looking radars. *I.E.E.E. Trans. milit. Electron.*, **9**, 264.

Harger, R. O., 1970, *Synthetic Aperture Radar Systems* (New York: Academic Press).

Li, F. K., and Johnson, W. T. K., 1983, Ambiguities in spaceborne synthetic aperture radar systems. *I.E.E.E. Trans Aerospace Electron. Syst*, **19**, 389.

Raney, R. K., 1984, Conceptual design of satellite SAR. *Proceedings of IGARSS 84 Symposium*, Strasbourg, 27–30 August, 827.

Sawyer, F. G., Cox, R. P. and Joyce, H., 1984, ERS synthetic aperture radar design. *Proceedings of IGARSS 84 Symposium*, Strasbourg, 27–30 August, p. 827.

Tomiyasu, K., 1978, Tutorial review of synthetic aperture radar (SAR) with applications to imaging of the ocean surface. *Proc. Inst. elect. electron. Engrs*, **66**, 563.

Ulaby, F. T., Moore, R. K., and Fung, A. K., 1982, *Microwave Remote Sensing*, Vol. II (Reading, Mass.: Addison-Wesley).