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A Theory of Microbaroms*

Eric S. Posmentier

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Summary

A theory analogous to the Longuet-Higgins theory on the generation of microseisms explains the generation of microbaroms by standing water waves associated with marine storms. The spectral characteristics and the amplitude order-of-magnitude of microbaroms that are predicted by this theory agree well with observations. The theory is based on the oscillations of the centre of gravity of the air above the ocean surface on which the standing waves appear (or of the water below, to explain microseisms). These oscillations are of twice the ocean wave frequency and thereby explain the observed frequency-doubling common to both microbaroms and microseisms. The theory is expanded by statistical methods to predict the microbarom-generating effect of more realistic ocean waves, whose phases vary randomly over the ocean surface. In addition, the effect of the widespread source on microbarom coherence and resolvability at the receiving array is discussed.

1. Introduction

Microbaroms were first reported by Benioff & Gutenberg (1939). They observed pressure fluctuations with periods of $\frac{1}{2}$ to 5 s at their Seismological Laboratory near Pasadena, California, on records from an electromagnetic microbarograph. Further studies (Gutenberg & Benioff 1941) revealed that the pressure variations with 5 s periods were very similar in waveform to microseisms. The horizontal velocities of the waves were reported to be approximately the same as the speed of sound in air, and the directions were mainly from the Pacific Ocean. It was also observed that the activity level of the microbaroms had an annual periodicity, with its maximum during the winter months, and that the occurrence of high microbarom amplitudes correlated with the appearance of low-pressure areas off the coast of Southern California. Although no direct correlation between microbaroms and microseisms was found, the fact that both had the same type of activity pattern and a similar relationship to low-pressure areas led to the conclusion that a similar type of source was indicated.

Baird & Banwell (1940) have described the microbaroms and microseisms recorded by them at Christchurch, New Zealand. They found that the periods of both oscillations fall in the 4–10 s range. In particular, they observed that the amplitudes of microbaroms and microseisms wax and wane together, as do their periods. The microbaroms, however, have a tendency to have an amplitude maximum in the middle night hours, unlike microseisms. The factor of the state of the sea at and near the nearby extended steep coastline was also considered, and the amplitudes of the ocean waves in this area were noted to correlate with the microbarom and microseism amplitudes, although the ocean wave periods did not have any noticeable relationship with the periods of the latter two waves. In addition, the investigators attempted to

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locate the source of the microbaroms and microseisms by considering the wave direction (from microseism data) and distance of source (by taking the time lag between microbarom and microseism wave envelopes). The results indicated that if there was a common source, that it was in the region of the coastal cliffs, and was active corresponding with times of high ocean wave activity.

Microbaroms were observed in Fribourg, Switzerland, by Saxer (1945). Sound pressures sometimes greater than 1 dyne/cm^2 were reported. Dessauer, Graffunder & Schaffhauser (1951) also studied microbaroms in Fribourg, and found that their horizontal phase velocities reached approximately 400 m/s. They attributed this supersonic velocity to two possible explanations. The first was in terms of microbaroms with non-horizontal phase velocity vectors, although no explicit discussion of the entire ray paths was given. Secondly, the suggestion was made that two wave trains travelling at acoustic velocities, and with the same periods, but in different directions, would lead to anomalous measurements of phase velocities, possibly the same as those actually made. In addition to measurement of velocity, the direction of wave approach was determined to be from the NW.

Daniels (1952, 1953) was simultaneously studying infrasound at the U.S. Army Signal Research and Development Laboratory in New Jersey. The earlier of the two works attributes microbaroms to the 'piston effect' of ocean waves, radiating in patches 40m square, and with random phase relations. This theory predicts pressure effects considerably larger than those observed, and the disparity is offered as an explanation of the heating in the 30–50 km and 90–120 km sound channels. However, these layers are recognized to be sound channels in the absence of wind. If wind structure is also considered, the sound velocity maxima, hence the sound channels, may be located differently. In the second paper, Daniels modified his original 'piston effect' suggestion to depend upon the assumed existence of transient groups of standing ocean waves. In a still later contribution (1962), Daniels recognizes that such waves would function only as an acoustical multiplet source which could produce only relatively weak effects. He suggests here that, according to Nanda's theory on the origin of microseisms (1960) which is based on an oscillatory wind drag caused by the periodically varying roughness of the sea surface, the oscillatory wind drag would also cause pressure fluctuations in the atmosphere which would be propagated as a compressional wave.

Further results of the work in Fribourg were reported by Saxer (1954). Records of microbarom activity in Fribourg, of microseism activity in Strasbourg (200 km north of Fribourg), and of ocean wave heights in the North Atlantic, were maintained for over a year. Correlation among the three were clearly established. Saxer, in the same paper, also explains both the diurnal and annual variations in microbarom amplitudes by the wind-currents and temperature changes in the layer of air 50 km up. It is suggested that reflection of microbaroms takes place in this 50 km layer. These conclusions further support the proposal by Dessauer *et al.* (1951), mentioned previously, that the microbarom path may not be horizontal.

Cook (1962a), in reviewing the causal relationship suggested by Saxer's earlier results (1954), dispels the possibility that the vertical ground motion due to microseisms can cause the pressure variations with which they are related. He also shows that, conversely, the pressure due to travelling microbaroms cannot cause elastic deformation of the earth's surface of sufficient amplitude to explain microseisms. Cook also examined the possibility that the vertical motion of ocean waves is sufficient to generate microbaroms. Theory shows that an infinite train of sinusoidal surface waves similar in period and amplitude to ocean waves radiate no net sound power, and that the pressure effect decreases exponentially with height. Cook's conclusion is that the observed relationship between ocean waves and microbaroms is not a causal one.

Shortly later, Cook (1962b) reconsidered the pressure effect of ocean waves.

Although the radiation of sound by an infinite train of sinusoidal 'ocean' waves is still discounted, it is found that a *semi*-infinite wave train has a non-zero energy radiation, and that the pressure field produced by such a source can account for observed microbaroms. This type of radiation is discussed in terms of the effect of the discontinuity, such as at a beach, and is described approximately as a field due to a line source at the line of discontinuity.

Donn & Posmentier (1967) recently reported their observations of a microbarom–microseism storm associated with a marine meteorological storm. They deduce from their data that both the microbaroms and the microseisms were generated by ocean waves over the wide-spread storm area, and that their nearly identical spectral characteristics are a function of the ocean waves, although microbarom amplitudes are affected by the variable atmospheric conditions along the path of propagation.

In a theoretical discussion of the related problem of microseism generation by ocean waves, Longuet-Higgins (1950) has shown, on the basis of an earlier work by Miche (1944), how standing water waves can produce pressure fluctuations beneath the waves, unattenuated with depth and of twice the water wave frequency. This result, verified experimentally, was further shown to be appropriate to overlapping portions of the spectra of realistic ocean wave models. Several observational studies of microseisms have supported the validity of this theory and imply that it explains at least one type of microseism generation (for example, Dinger & Fisher (1955) and Latham & Sutton (1966)).

The purpose of this paper is to suggest a theoretical explanation for the generation of microbaroms. The literature which we have cited, especially Donn & Posmentier (1967) and Longuet-Higgins (1950), suggests the constraints which must be satisfied by the theory:

1. The generating mechanism must be based on a realistic ocean wave model;

2. The ocean waves must generate both microseisms and microbaroms of the same period. Furthermore, since the type of microseisms usually associated with microbaroms are the type discussed by Longuet-Higgins, microbaroms and microseisms must have half the period of the ocean waves;

3. The theoretically expected microbarom amplitudes must be of the same, or possibly one higher, order of magnitude as observed microbaroms (i.e. 1–5 μb).

2. The acoustic field of simple ocean waves

2.1 *A preliminary model*

Consider the problem of the air pressure variations above standing water waves, and an approach similar to that used by Longuet-Higgins (1950). We take the model of incompressible air of density ρ_0 bounded below by a surface perturbed by two sinusoidal wave trains identical in period T, wavelength λ_0 and amplitude a, but travelling in opposite directions. The resulting standing wave pattern is illustrated in Fig. 1, along with a pendulum whose motion is analogous to the motion of the centre of gravity of a vertical column of the air. It may be seen that, although the water–air interface (and the pendulum) oscillate with period T, the *height* of the centre of gravity of the air (and of the pendulum), and hence the vertical force on either, oscillates with period $T/2$. This line of reasoning, as in the analogous case of microseisms, leads to the conclusion that standing waves in water will produce vertically propagating acoustic waves in the air above, with period $T/2$. These waves will not be subject to the exponential decay of the pressure fluctuations above a single sinusoidal wave train, which was described by Cook (1962a).

The surface height, η, in Fig. 1, is given by

$$\eta = 2a \cos k_0 x \cos \omega t, \tag{1}$$

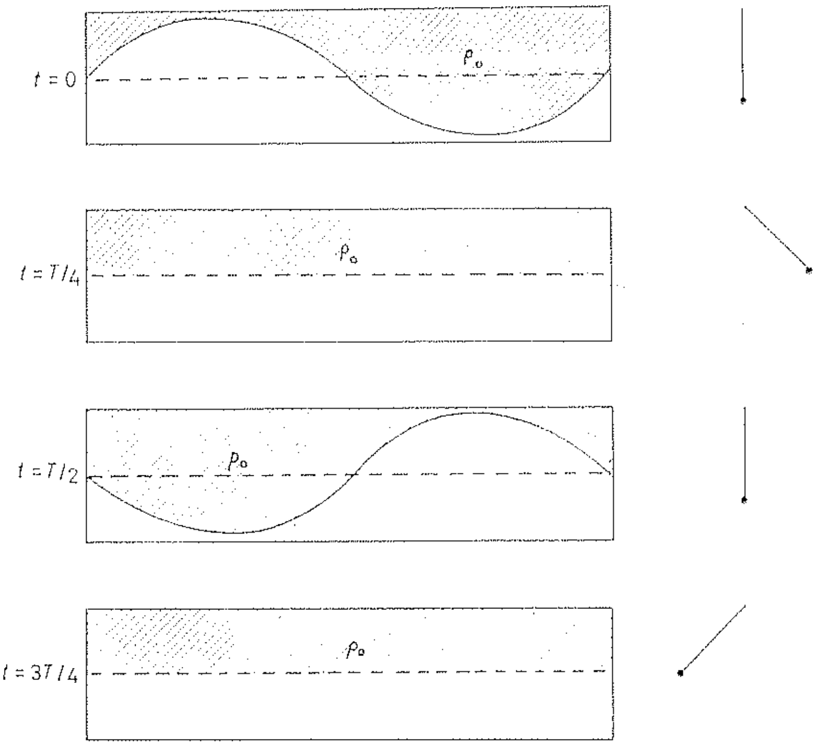

FIG. 1. Comparison of the air above a standing wave, with a pendulum, both of period T. The height of the centre of gravity of either oscillates with period $T/2$.

where $k_0 = 2\pi/\lambda_0$ and $\omega = 2\pi/T$. The height of centre of the gravity of the air in the cross-hatched area is therefore

$$\xi = \frac{1}{\rho_0 \lambda_0 H} \int_0^{\lambda_0} dx \int_\eta^H \rho_0 z \, dz \tag{2}$$

Using (1), expression (2) becomes, upon integration,

$$\xi = \frac{H}{2} - \frac{a^2}{H} \cos^2 \omega t \tag{3}$$

The average perturbation pressure over the surface, required to maintain the periodic motion of the centre of gravity of the air, must be, by Newton's second law

$$\bar{p} = (\rho_0 H) \frac{d^2}{dt^2} \xi \tag{4}$$

Using equations (3) and (4), and performing the indicated differentiation,

$$\bar{p} = -2\rho_0 a^2 \omega^2 \cos 2\omega t. \tag{5}$$

This result verifies the frequency-doubling principle demonstrated on a more intuitive basis, above. We now examine its applicability by taking a quantitative example. If two ocean waves of amplitude 100 cm, period 9 s meet to produce standing waves, the air pressure perturbations produced at any point above the ocean surface would have an amplitude of 13 dynes/cm^2, or 13 μb, and a period of 4·5 s. The amplitude of 13 μb is a full order of magnitude larger than typical storm microbaroms. Since the physical model is idealized, its calculated generating efficiency must be considerably above observations. The periods of the ocean waves and microbaroms of this example are both typically observed values. The correspondence of the theory presented above with actual data will be given further consideration in a separate section of this paper.

2.2 *Compressible air model*

The question immediately arises as to what is the effect, if any, of the compressibility of air. We shall show below that the effect, on the analysis above, is negligible. First, consider the well-known density structure of an unperturbed, isothermal atmosphere in a gravity field (for example, Haltiner & Martin 1957).

$$\rho = \frac{P_0}{R'\theta} \exp[-gz/R'\theta] \tag{6}$$

where

ρ = unperturbed density at height z;
P_0 = total pressure at sea level;
R' = specific gas constant;
θ = absolute temperature;
g = acceleration due to gravity.

The air density structure is further affected by the directly radiated acoustic field of the vibrating sea surface. To calculate this additional effect on density, we first find a velocity potential Φ which satisfies the sound wave equation and is consistent with the boundary condition imposed by equation (1). The result is

$$\Phi = \frac{2ia\omega}{k_z} \exp[i\omega t - k_z z] \cos k_0 x, \tag{7}$$

where $k_z^2 = k_0^2 - \omega^2/c^2$, and c is the velocity of sound in air. From equation (7), we immediately obtain an expression for perturbation pressure, p.

$$p = \frac{2\rho a\omega^2}{k_z} \exp[i\omega t - k_z z] \cos k_0 x. \tag{8}$$

The perturbed density, ρ', of the air when compressed adiabatically according to equation (8) may be given by

$$\rho' = \rho[(P+p)/P]^{1/\nu}, \tag{9}$$

where P is the ambient pressure at the height in question, and is given by

$$P = R'\theta\rho. \tag{10}$$

The constant ν is the ratio of the isobaric to the isochoric specific heats of a fluid ($\nu \cong 1 \cdot 4$ for air). If we assume that $p \ll P$, we may approximate equation (9) by

$$\rho' = \rho\left(1 + \frac{p}{\nu P}\right). \tag{11}$$

Combining (6), (8), (10) and (11), we find that

$$\rho' = \frac{P_0}{R'\theta} \exp(-gz/R'\theta)\left(1 + \frac{2a\omega^2}{\nu R'\theta k_z} . \exp(i\omega t - k_z z) \cos k_0 x\right). \tag{12}$$

We now define

$$\zeta = \int_\eta^\infty z\rho'(x, z, t)\, dz \tag{13}$$

and

$$\delta = \int_0^\lambda \zeta(x, t)\, dx. \tag{14}$$

The average pressure $\bar{p}$ at the sea surface, which will propagate vertically as a plane wave without exponential decay, may be calculated by Newton's second law

$$\bar{p} = \frac{1}{\lambda} \frac{d^2}{dt^2} \delta. \tag{15}$$

We proceed to integrate equation (13), using equation (12), and expand exponentials in a Taylor's series, retaining only those terms up to the second order. Upon integrating these results in equation (14), and applying equation (15), we find, finally

$$\bar{p} = -2\rho_0 a^2 \omega^2 \cos 2\omega t \tag{16}$$

which is identical with equation (5). We may therefore conclude that the compressibility of air has no significant effect on the pressure fluctuations associated with oscillations of the centre of gravity of the air above standing ocean waves.

3. The acoustic field of a storm-generated wave system

The surface of the ocean is an extremely complicated vibrating surface, in areas of both sea and swell. The ocean wave spectrum is a function of both time and position; phases of groups of waves in adjacent areas are randomly related. It is therefore doubtful that any analysis of the ocean's acoustic field (such as the one above or that of Cook (1962a, 1962b) which is based upon an exact functional specification of the entire sea surface can be dependable as a final analysis. Rather, one must examine the random combination of the pressure effects of finite areas of ocean surface whose dimensions are sufficiently small to allow spatial phase-coherency of waves.

In the ocean wave models considered above, we assumed the grossly simplified situation in which standing waves over an entire plane are exactly in phase. We shall proceed to find the acoustic field due to small areas affected by equation (1) the standing wave motion discussed above, and by equation (2) single progressive wave trains. These results will then be used to study the net acoustic fields of both equations (1), an ocean whose surface is moving in randomly related patches of standing waves, and equation (2), an ocean whose surface is moving in randomly related patches of progressive wave trains. We shall refer to the first as centre-of-gravity coupling, and the latter as off-resonant coupling.

For plane acoustic waves, the amplitude of the perturbation pressure is $\rho c w$, where ρ, c, and w are air density, acoustic velocity, and vertical particle velocity, respectively. For hemispherical waves radiating outward from a piston of small area S and maximum velocity w, the pressure amplitude will be, at large distances, $\rho c w K S/2\pi r$, where K is the acoustic wave number, and r is distance from the source. (See, for example, Morse 1948.) The ratio of the latter pressure to the former is $KS/2\pi r$. If we multiply expression (16) by this ratio, we find that the pressure due to a patch of standing waves has an amplitude

$$p = \frac{\rho a^2 \omega^2 K S}{\pi r}. \tag{17}$$

We note that the term a^2 may be rewritten $a_1 a_2$, where a_1 and a_2 are the amplitudes of two waves of same period and opposite directions. This has been demonstrated by Longuet-Higgins (1950).

If we assume that the patches are more or less coherent over a distance proportional to n wavelengths, λ_0, then the contribution per unit area of generating surface to the square of the net pressure, will be from (17)

$$d(p^2) = \left(\frac{\rho a^2 \omega^2 K n \lambda_0}{\pi r}\right)^2. \tag{18}$$

Implicit in the algebra leading to (17) and (18) are the following assumptions:

(1) Perturbations of pressure, density, etc. are all small compared to their respective ambient values;

(2) Distance from source is large compared with one acoustic wavelength;

(3) The size of the 'patches' is small compared with one acoustic wavelength;

(4) Signals of one frequency whose phases are randomly related add incoherently, i.e. in power or in the square of the amplitudes.

Under these same assumptions, we shall proceed to determine expressions analogous to (17) and (18), for the part of the acoustic field due to off-resonant coupling. We shall use as a starting point the expression obtained by Cook (1962b) for the pressure due to a semi-infinite sinusoidal wave train, with wavelength λ_0, terminating discontinuously at the line $x = 0$,

$$|p| = \frac{\rho c a}{(\beta-1)T}\sqrt{\frac{\lambda}{x}}, \tag{19}$$

where β is the ratio of acoustic to ocean wave phase velocities, c/c_0, and λ is the acoustic wavelength. Based on this result we may find, by superposition, the pressure due to two superposed ocean wave trains—each has the same period and travels from $x = -\infty$ to $x = 0$, but they travel at velocities $c_0 - \delta/2$ and $c_0 + \delta/2$.

$$|p| = \frac{\delta}{c_0}\,\frac{\rho c a}{\beta T}\sqrt{\frac{\lambda}{x}}\,. \tag{20}$$

The ocean wave model responsible for the pressure given by (20) does not have the weakness of an assumed discontinuity at $x = 0$; neither does it have waves of constant height, but beats with a spatial periodicity of

$$B = \frac{T(c_0-\delta/2)(c_0+\delta/2)}{\delta}. \tag{21}$$

We note that this more realistic pressure differs in magnitude from Cook's result by a factor of approximately δ/c_0, which is equivalent to λ_0/B, for $c_0 \ll c$.

If we now subtract from the pressure given by equation (20) the pressure due to a similar model terminating at $x = -B$, including the phases in calculations, we find the pressure due to a semi-infinite strip of width B of ocean waves travelling across the strip, beginning and ending at its edges without discontinuity. We apply assumptions (2) and (3), above ($x \gg \lambda$, and $B \ll \lambda$).

$$p = (c_0-\delta/2)(c_0+\delta/2)\rho a K\sqrt{\frac{\lambda}{x}}\exp\left[i(\omega t - Kx)\right]. \tag{22}$$

We multiply this expression by $B\sqrt{[K/(2\pi r)]}$, the ratio of the pressures due to (1) a uniformly oscillating square of side B at a distance r, to (2) a uniformly oscillating strip of width B at a distance x. The result is an estimate of the pressure at distance r from a patch of mean dimension B, affected by progressive waves.

$$|p| = \frac{2\pi(c_0-\delta/2)^2\,(c_0+\delta/2)^2\rho a}{c\delta r}\,. \tag{23}$$

This expression is analogous to (17), if S is taken to have the same significance as B^2. The expression analogous to (18) must therefore be

$$d(p^2) = \left(\frac{K(c_0-\delta/2)(c_0+\delta/2)\rho a}{r}\right)^2. \tag{24}$$

We are now prepared to consider the relative importance of centre-of-gravity and off-resonant coupling mechanisms. The ratio of equations (23) to (17), if $\delta \ll c_0$, is

$$\frac{\lambda_0}{4\pi}\frac{a}{a_1 a_2}\frac{\lambda_0}{B}. \tag{25}$$

The first factor of this ratio shows that increasing λ_0, while keeping the second factor (amplitudes) and third (incoherence of ocean waves) constant, will increase the relative importance of off-resonant coupling. Since either type of coupling may be described in terms of the incomplete cancellation of pressure effects of adjacent phases of ocean waves, it is thus clear that the larger the generating areas of nearly uniform phase, the larger will be the pressure generated. In the case of off-resonant coupling, the dimension of this area of constant phase is some fraction of a wavelength. Hence, it is reasonable to expect that larger ocean wavelengths will enhance off-resonant coupling, as indicated by the first factor of equation (25).

The second factor gives the obvious effect of ocean wave amplitudes. The third factor gives the ratio of ocean wavelength to the dimension of a coherent patch of ocean waves, and is thus an index of the incoherence of the waves. The incomplete cancellation responsible for off-resonant coupling is not dependent on this ratio, as may be seen by examining equation (24), but the centre-of-gravity coupling will be improved for larger ratios, i.e. for more coherent ocean waves, or larger generating areas of nearly uniform phase, which is the usual case with swell, as opposed to sea.

The results given by equations (18) and (24) provide a means of estimating the magnitude of centre-of-gravity and off-resonant coupling of ocean wave energy into atmospheric acoustic energy. We shall now use these results to examine the acoustic field of large, but finite, areas affected by equations (18) or (24). This procedure relies on assumption (4) above. The integration was performed by the IBM 7094 digital computer at the Columbia University Computer Center.

The sea model, illustrated in Fig. 2, consists of two sets of waves of equal and opposite wave numbers. The dimension of the coherent patches of standing waves is $n\lambda_0$. The product of the wave heights (double amplitudes) is 51840 cm^2 (corresponding to two wave trains with heights of 23·6 and 2·36 ft) at the maximum, and decays as the hyperbolic secants of the two components of distance from the maximum

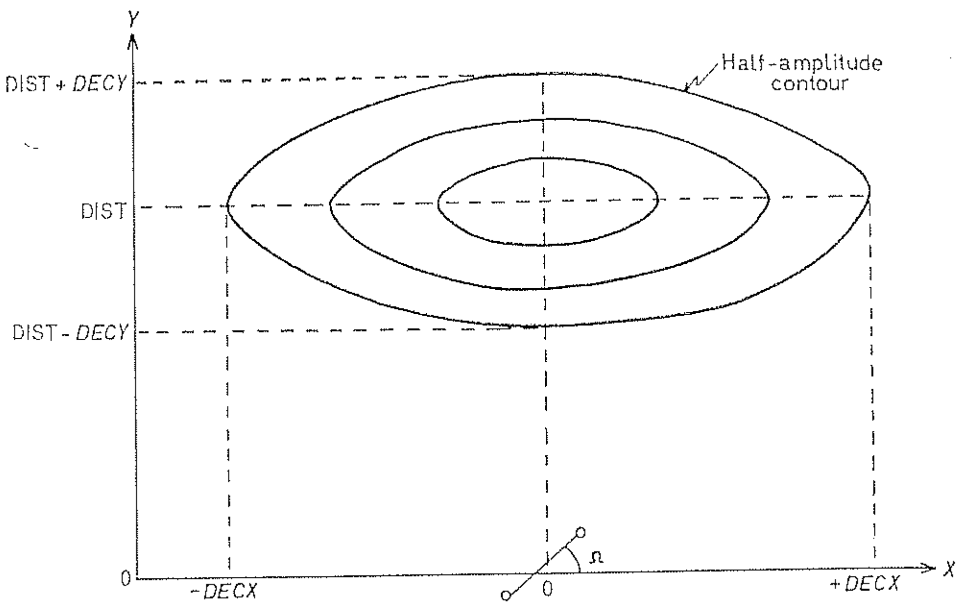

FIG. 2. Configuration of the generating area and receivers. The receivers, denoted by 0, are separated b a distance W.

point, with half the maximum amplitude occurring at points at a distance *DECX* from the centre in the *X*-direction, and *DECY* from the centre in the *Y*-direction. The storm centre is at a distance DIST from the receiver.

With a given wave period, the perturbation pressure at the receiver will be a function of *DECX* and *DECY* (i.e. the size of the source region). In particular, we investigated a model with an ocean wave period of 12·6 s. For $n = 10$, $DECX = DECY =$ 80 km, and DIST = 600 km, the resulting pressure, arrived at by numerical integration of equation (18), is 1·6 μb, which is the same order of magnitude as typical 'strong' microbaroms. As *DECX* is increased (i.e. a larger storm area), the pressure increases in a manner illustrated in Fig. 3. This pressure is generated by the centre-of-gravity coupling, and has a period of 6·3 s, or half the ocean wave period. In this case, the ratio given by equation (25) is about 5, indicating that off-resonant coupling should give rise to an 8 μb signal with a 12·6 s period.

Experimentation with various parameters has led to the conclusion, based on the theory presented in the preceeding sections, that a typical storm wave system can account for acoustic signals of twice the wave frequency and amplitudes at least as great as those actually observed, as well as for acoustic signals of the same ocean wave frequency and amplitudes comparable with the frequency-doubled amplitude. The relative importance of the two types of coupling, however, is quite sensitive to quantitative assumption, and equation (25) can easily vary from 1/10 to 10.

In the course of the computer-oriented numerical computations, we digressed to consider the separate problem of coherence and time lags between two receivers, with the same source regions used above. Barber (1961), discussed the directional resolving characteristics of an array, assuming that he was looking at plane-wave signals. The purpose of this part of our study, in contrast, is to investigate the 'resolvability' of widespread sources. This problem is of interest in the recording of microbaroms, microseisms, and ocean waves, and possibly elsewhere. In this paper, we will only briefly mention our procedure and present a few characteristic results. A more elaborate discussion will be published separately.

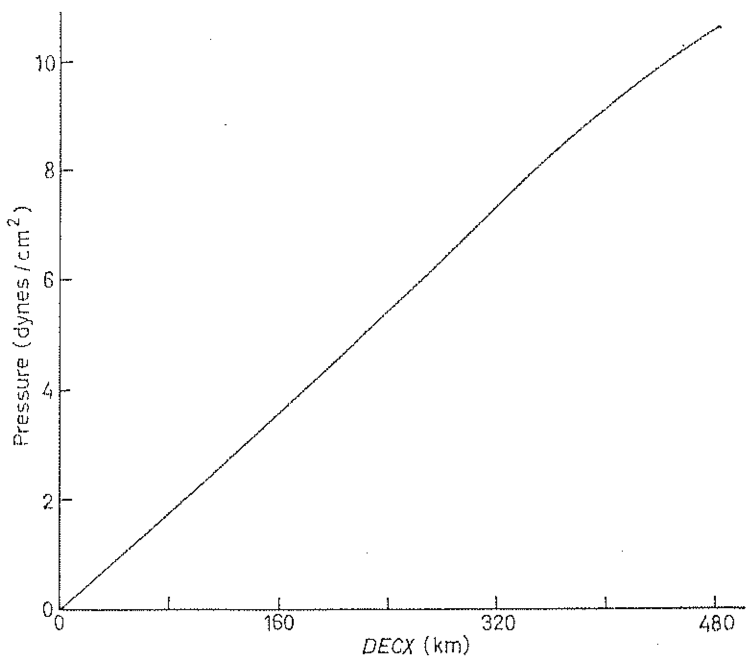

FIG. 3. Signal pressure as a function of storm size.

We assume that the quasi-monochromatic signals from adjacent patches, as given by equation (17), are incoherent. That is, their mean product is zero. For our purposes, we have defined coherence as the maximum value of the usual correlation function between the two stations in Fig. 2. The coherence will be a function of the parameters of the configuration in Fig. 2. The time lag is defined as the lag value at the principal point of maximum correlation. This time lag is the expected value of the actual measured time lag. As may be expected, coherence decreases with increasing station separation or with increasing source size. In addition, it appears that Ω (the angle between the line between receivers, and the wave fronts of waves from the storm centre) affects coherence, with the best coherence for stations in line with the storm centre, and the worst for stations in line with the wave fronts from the storm centre. These results are illustrated in Fig. 4.

From the point of view of the observer, the most desirable value for the expected actual time lag between two stations is the value corresponding to the ideal time delay, which is the time delay between the two stations for a single ray from the centre of the generating area. As the generating area deviates from a point source, however, there occurs a marked discrepancy between the 'expected' and 'ideal' time lags. The resulting 'phase lag error', which we define as $360 \times$ (expected lag $-$ ideal lag) $\div$ wave period, is plotted in Fig. 5, for the same range of $DECX$ and values of Ω and W/cT as in Fig. 4. It is interesting to note that, for $\Omega = 0$, the expected and ideal time lags are at first identical; they are both zero. When the coherence becomes zero, the phase lag error jumps discontinuously to 180°. For $\Omega = \pi/2$, the phase lag errors become negative as $DECX$ begins to increase, and then return to zero and become positive. When $\Omega = \pi$, the phase lag errors decrease monotonically from zero. These phase lag errors arise essentially as a result of the fact that rays from different parts of the generating area give rise to a spectrum of time lags, which do not necessarily 'average out' to the time lag for the ray from the centre of the generating area.

4. Conclusions

A few points of agreement between the theory discussed above, with observations have already been noted. We wish to further examine the consequences of the theory, and their relationship with observations.

1. The theory explains the generation of acoustic waves by two separate mechanisms, operating simultaneously, in the presence of patches of progressive waves and/or standing waves. The existence of the wave motion required for the first (off-resonant) coupling mechanism, i.e. patches of progressive waves with parameters such as those in the example above, is evident. The existence of the wave motion required for the second (centre-of-gravity) coupling mechanism, i.e. patches of standing waves such as those in the example above, perhaps requires some supporting evidence. In particular, we cite some of the results of Dr D. E. Cartwright, of the National Institute of Oceanography, in England, who has made some comparison of ocean wave directional spectra with microseisms (personal communication). Although his conclusions regarding a 2:1 period relationship were negative, we do note that his directional spectra, shown in Fig. 6, illustrate the existence of two sets of strong wave motion, with remarkably good correlation between $E(\omega, \theta)$ and $E(\omega, \theta + \pi)$. We have been assured by Drs Pierson and Neumann, of New York University, that the occurrence of a similar, although less striking spectrum is not an altogether unusual incident. It is clear that the ocean surface in an area specified by such a spectrum will have patches of standing waves.

Our theory shows that the centre-of-gravity coupling mechanism will generate microbaroms if the ocean surface is affected by patches of standing waves, i.e. if the directional ocean wave spectrum contains significant energy in *approximately* opposite wave numbers. This is in contrast with Longuet-Higgins' (1950), who applied the

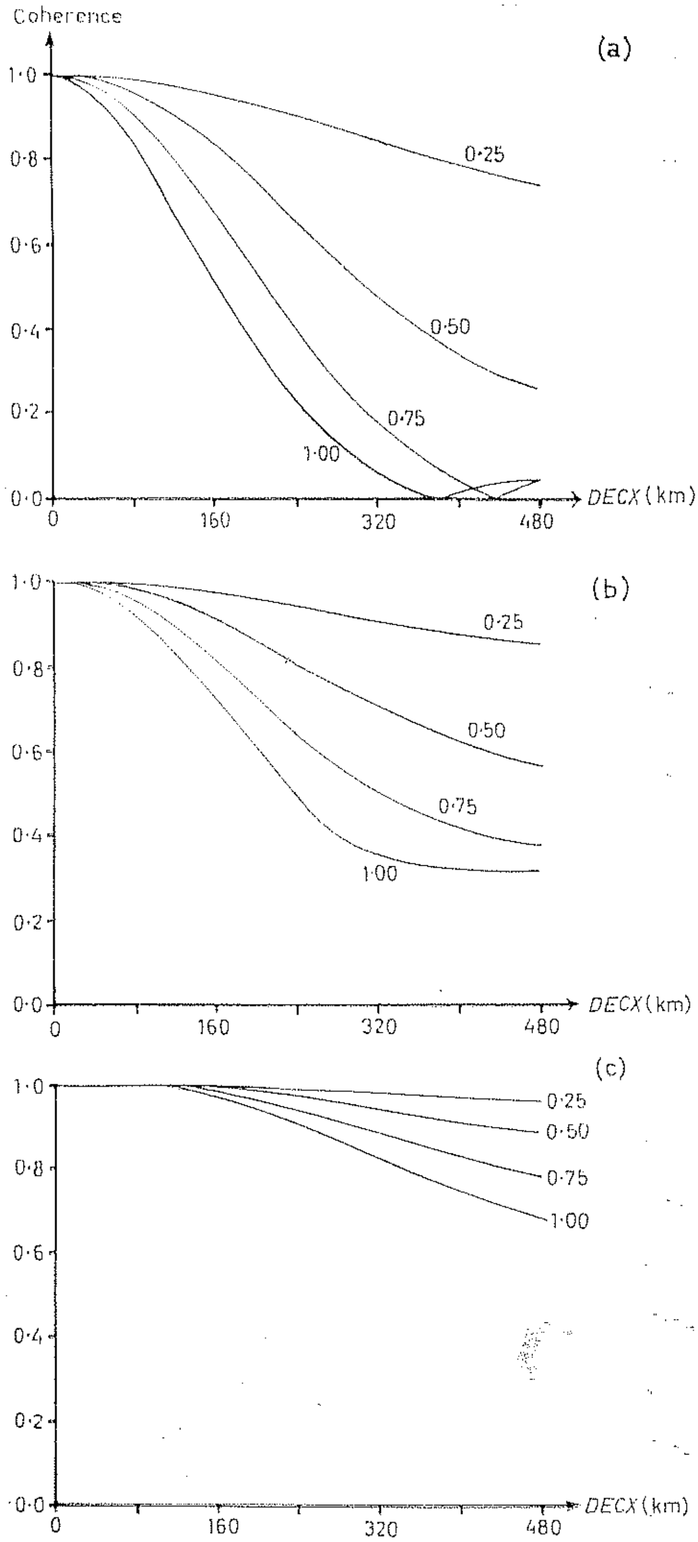

FIG. 4. Coherence vs source width (*DECX*) for various values of $\propto$ (ratio of receiver separation to wavelength) and Ω (angle between line connecting receivers, and wavefront of ray from storm centre).

(a) $\Omega = 0$; (b) $\Omega = \pi/4$; (c) $\Omega = \pi/2$.

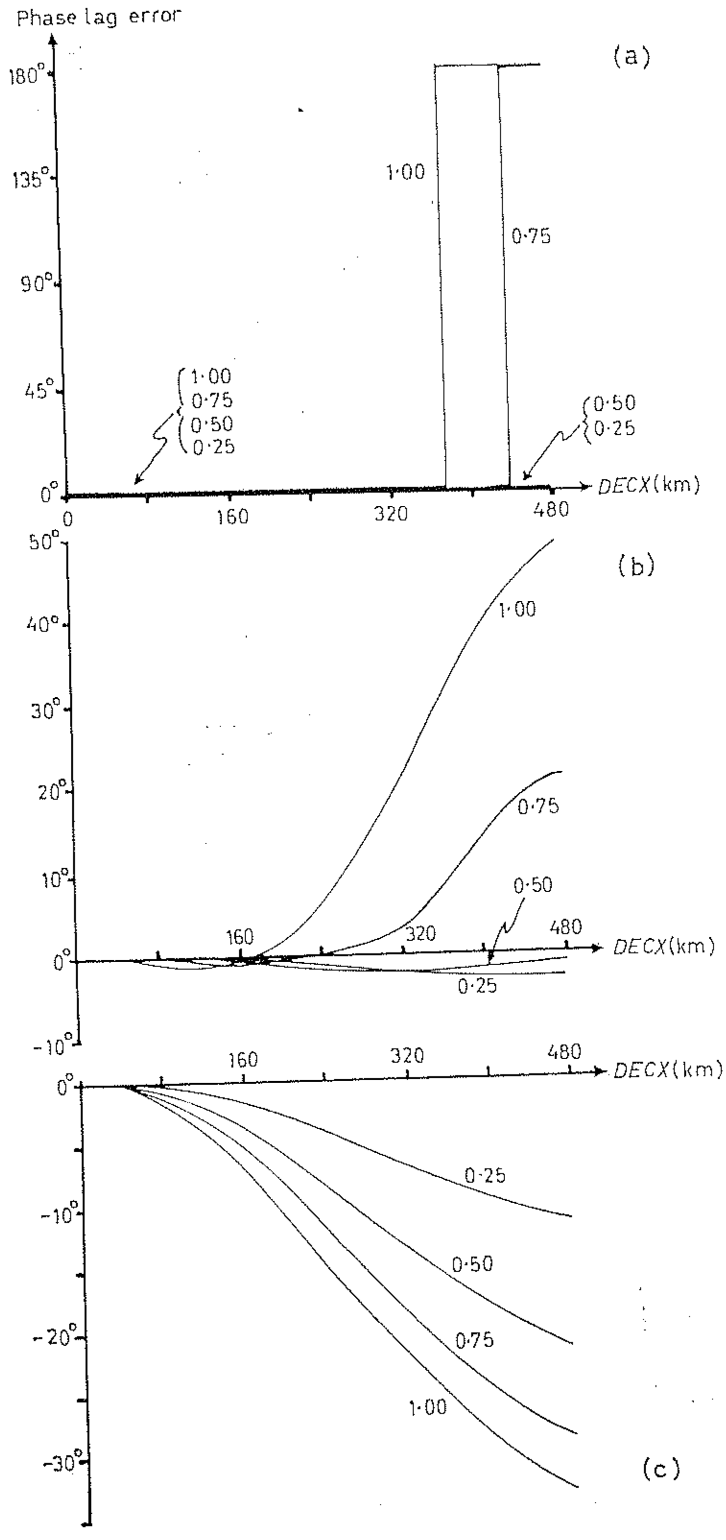

FIG. 5. Phase lag error vs source width (*DECX*) for various values of α (ratio of receiver separation to wavelength) and Ω (angle between line connecting receivers, and wavefront of ray from storm centre).

(a) $\Omega = 0$; (b) $\Omega = \pi/4$; (c) $\Omega = \pi/2$.

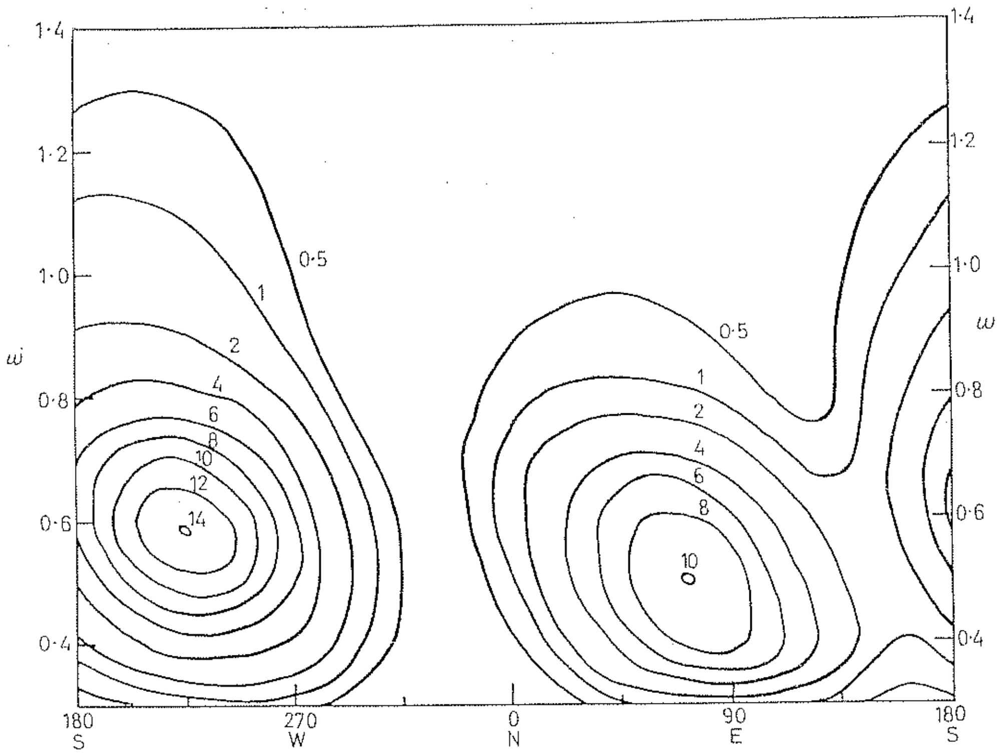

FIG. 6. Directional spectra of ocean waves at 58° 51′ N, 24° 7′ W at 16.15 GMT on 15 November 1960. This position was near the centre of a large double cyclonic depression. (From Dr. D. E. Cartwright, personal communication.) The spectral density, $E(\omega, \theta)$, is in units of ft^2/s.

centre-of-gravity coupling mechanism for microseisms only to the case of an infinite plane of standing waves, or directional wave spectra which have significant energy at *exactly* opposite wave numbers. However, the latter theory was generalized by Hasselman (1963), whose equation (2.13) evaluates the total centre-of-gravity coupling effect for microseisms, for all ocean wave interaction in a completely general ocean wave spectrum. Hasselman then showed that for the case of microseisms his general equation (2.13) can be approximated by (2.15) which evaluates the effect only of ocean waves interacting at exactly opposite wave numbers—the mechanism of Longuet-Higgins.

2. The centre-of-gravity coupling mechanism will generate microbaroms with one-half the ocean wave period. According to the theory of Longuet-Higgins (1950), which has been substantiated by the observation of Dinger & Fisher (1955) and Latham & Sutton (1966), the standing waves which must exist for the centre-of-gravity coupling mechanism to exist, will also generate microseisms with one-half the ocean wave activity. This mechanism, then, can satisfactorily explain the observed spectral characteristics of both microbaroms and microseisms.

3. The amplitudes of microbaroms with one-half the ocean wave period, as predicted by our theory of a centre-of-gravity coupling mechanism, will be at least as large as observed 'strong' microbaroms, when assuming realistic ocean wave heights.

4. A parallel theory of an 'off-resonant' coupling mechanism (equal in period to ocean waves), predicts a second type of microbarom corresponding to the type of microseisms which have been explained by Hasselman (1963) in terms of surf beat. It has not yet been proven experimentally that both types of microbaroms are generated simultaneously. This may be due, at least in part, to the fact that microbaroms equal in period to the ocean waves usually fall outside of the range in which observa-

tions have been made, and we cannot thus confirm this prediction. However, Oliver & Page (1963) have reported concurrent storms of long and ultralong period microseisms, related in period by a factor of two.

5. As a by-product of our numerical analyses, we found that if a pair of nearby stations receive microbaroms from a wide source area, the signal coherence between the two stations will be affected not only by the source width and the station separation, but by the stations' orientation as well. In addition, the expected time lag between the stations will differ from the time lag for a ray from the source's centre by a time lag error which is a function of the same three factors which affect coherence.

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Lamont Geological Observatory,
Columbia University,
Palisades,
New York 10964.

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