Spectral Decomposition of Short Gravity Wave Systems

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ABSTRACT

The component waves of a short gravity wave system generated in a linear wind-wave channel at a fetch of 5 m s^{-1} and air friction velocity of 55 cm s^{-1} were detected and measured with coherent microwave scattering. Phase speeds were determined from the Doppler shifts. Surface displacements deduced from the Doppler spectra were superposed and compared with independently measured slope spectra. It is concluded that at least 80% of the energy of the short gravity wave system is carried by essentially free waves.

1. Wave ensembles

Waves, the most ubiquitous elementary excitations, can be superposed to describe a great many physical systems. Of course, an ensemble of completely free, simple harmonic waves can only oscillate. Changes in the mean properties of the system must be effected by interactions among the component waves. When the interactions are weak the changes take place slowly compared to the frequency of any pertinent wave, and the waves are still essentially free. Curiously, the world's archetypical waves, the wind-generated waves of the ocean, are among the most recent to be successfully treated by weak wave-wave interaction theory (e.g., Hasselmann, 1968). More recently, a number of authors (Lake and Yuen, 1978; Toba, 1978; Rikiishi, 1978) have contended that wind-generated short gravity waves cannot be satisfactorily decomposed into free waves. The purpose of this note is to show that, using microwave scattering, it is experimentally straightforward to decompose the short gravity wave systems which are generated in linear wind-wave channels into their component free waves. Briefly, the dispersion relation for the component waves is deduced from the Doppler shift; the contribution to the surface displacement spectrum is found to be highly concentrated along the dispersion curve, and the surface displacement spectrum, deduced from the scattered microwave power, agrees well with that deduced from the measured slope spectrum.

2. Doppler spectra and slope spectra

It is well established, indeed, it is essentially a tautology, that the electromagnetic field scattered from a rough surface with surface displacement sufficiently small compared to the wavelength of the incident radiation is linearly dependent on the surface displacement (Rice, 1951; Wright, 1966). This fact is sufficient to show that the power spectrum $P(\omega,\theta)$ of the coherently received microwave signal backscattered from such a surface at a depression angle θ depends linearly on the full surface displacement spectrum $\Psi(\mathbf{k},\omega)$. Or, more precisely,

$$P(\omega,\theta) \propto \Psi(\mathbf{k}_B,\omega), \tag{1}$$

where the Bragg wavevector, $\mathbf{k}_B$ is

$$\mathbf{k}_B \equiv (2k_0 \cos\theta, 0). \tag{2}$$

Here k_0 is the microwave number and the x coordinate lies along the intersection of the plane of incidence and the mean surface. If the surface displacement is due to an ensemble of essentially free waves, the spectrum vanishes throughout ω, $\mathbf{k}$ space except in the neighborhood of the surface given by the dispersion relation $\omega(\mathbf{k})$. The Doppler spectrum, as $P(\omega,\theta)$ is called, is then sharply peaked about the frequency of the Bragg wave, the surface wave with wavevector $\mathbf{k}_B$. This property has long been observed in high-frequency scattering from the ocean (Crombie, 1955). A recent study of the growth and equilibrium of short gravity waves in a linear wind wave channel (Plant and Wright, 1977), of which this report is an outgrowth, also depends on this characteristic.

According to (1) a wind wave spectrum may be experimentally decomposed by measuring Doppler spectra in backscattering from wind generated waves as a function of depression angle. We have done this in our wind-wave tank using a fetch of 5 m, an air friction velocity of 55 cm s^{-1}, and a microwave frequency of 1.85 GHz. For these conditions the dominant wavelength, Bragg wavelength and microwavelength were about equal at a depression angle of 60° and the mean squared slope of the dominant wave had attained its maximum value as determined from previous surveys of slope spectra in our tank (Plant and Wright, 1977). Absolute calibration of the backscattering cross section at each angle was accomplished by comparing it with the power backscattered by a monochromatic water

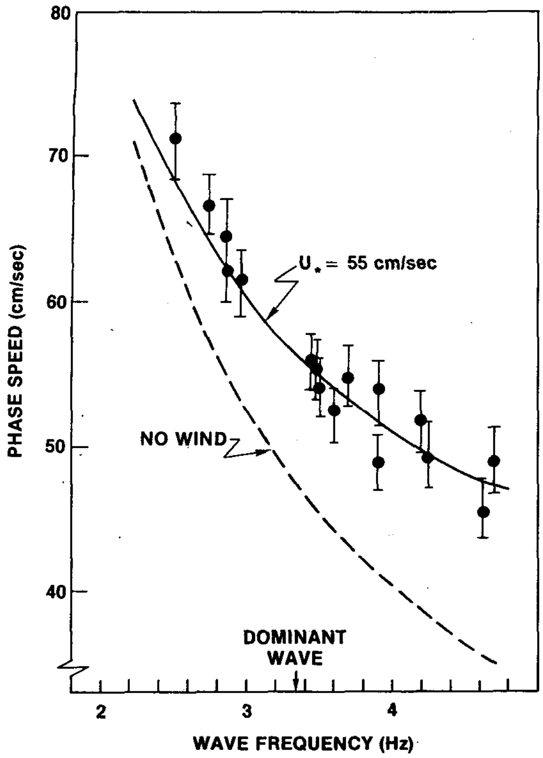

FIG. 1. Phase speeds of wind-generated waves at an air-friction velocity of 55 cm s^{-1} and a fetch of 5 m. Measured values shown as solid circles, solid line is theoretical relation (after Plant and Wright, 1979). Phase speeds for waves with no wind are also shown.

wave, the slope of which was measured using Cox's photometric method (Cox, 1958). The calibration technique, which is described in more detail by Larson and Wright (1974), involves varying the wavelength of the monochromatic wave at a fixed depression angle to measure the modulus of the Fourier transform of the microwave antenna illumination pattern at the surface from which the wavenumber resolution of the measurement, as well as the effective illuminated area, may be obtained.

Wind-wave slope spectra were measured with precisely the same implementation of Cox's method as was used for the monochromatic waves. A source of graded illumination was placed beneath the wind-wave channel which has a transparent bottom for the purpose. Light which entered a telescope focused on the air-water interface originated from a position along the graded illumination which depended on the interfacial slope and was detected with a photomultiplier tube. The gradation in illumination was oriented along the channel axis so that upwind-downwind slopes were measured. The illumination source was designed so that the photomultiplier output deviated from proportionality to surface slope by less than 5% for slopes less than 40°, which is much in excess of the slopes of the dominant waves. The slope measurement was calibrated simply and frequently by measuring the photomultiplier output as the telescope was moved along the channel axis.

3. Experimental results

The narrow Bragg peaks in the Doppler spectra yielded the frequency of the Bragg wave with a precision of about 0.1 Hz. We computed phase speeds (Fig. 1) of the Bragg waves by multiplying this frequency by the Bragg wavelength determined from the antenna calibration measurements. Phase speeds calculated by perturbation of the coupled shear flow at the air-water interface using a variational technique (Plant and Wright, 1977, 1979) are given by the solid line in Fig. 1. The agreement between this line and the data points shows that measured speeds are readily interpretable as the phase speeds of essentially free waves. Deviation of measured phase speeds from those calculated for potential waves (the dashed line in Fig. 1) has been interpreted as evidence for unusually strong interactions in wind wave systems (Lake and Yuen, 1978; Rikiishi, 1978; Toba, 1978). Close examination of this deviation on a free wave picture (Plant and Wright, 1978) reveals that the major cause is advection by the wind drift which is greater for shorter waves. Inertial pressure (the pressure which leads, at sufficiently high winds, to Kelvin-Helmholtz instability) and finite amplitude of the waves play smaller roles.

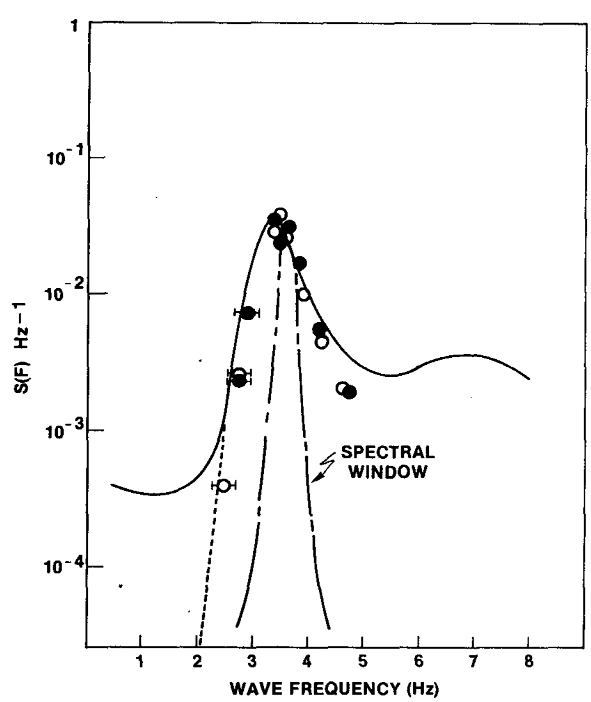

FIG. 2. Comparison of slope spectra measured optically (solid line) and calculated from Doppler spectra (circles).

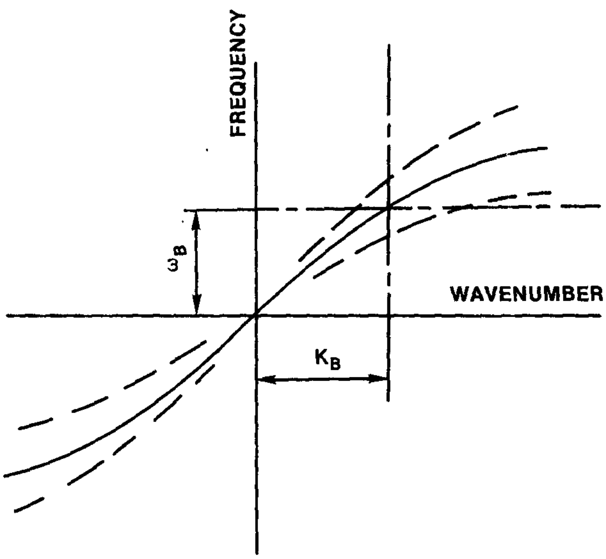

FIG. 3. Schematic diagram showing the dispersion relation for free waves (solid line) and for second-order sidebands (dashed lines) in the case of long-crested waves.

First-order Bragg peaks in the Doppler spectra were integrated over frequency to obtain surface displacement spectra $\Phi(k_B,0)$. The upwind-downwind slope spectrum was then calculated using the relation

$$S(f) = (k_B{}^2/c_g)\Phi(k_B,0) \int_{-\pi}^{\pi} f(\varphi)d\varphi. \quad (3)$$

Group speeds c_g were calculated from the measured Bragg wave frequencies which were also used to convert wavenumber to frequency in the spectral transformation required by (3). A spreading function, $f(\varphi) = \cos^2\varphi$, was obtained by measuring Doppler spectra as a function of angle with respect to the channel axis. The open points (Fig. 2) were individually, absolutely calibrated as described in Section 2; the solid points come from another series of measurements which were relatively, but not absolutely, calibrated and were fit, as a group, to the set of open points. The spectral window imposed by the microwave antenna illumination pattern, measured as described in Section 2, is also shown (Fig. 2). The width of this window is proportional to $\sin\theta$; the example shown was measured at a depression angle of 59°. On the steep, forward face of the spectrum, the window was sufficiently wide that the spectrum varied significantly over the window. The effects of this were corrected using the measured slope and surface displacement spectra. These corrections were less than 40% in the integral over the Bragg peak and less than 0.15 Hz in frequency.

Six slope spectra which were measured by Cox's method in sequence with some of the microwave data shown in Fig. 2 were averaged to obtain the solid line (Fig. 2). Slope spectra measured in wind-wave channels show a low frequency plateau which is not present, for example, in surface displacement spectra measured with capacitance probes. The dashed line (Fig. 2) fits a slope spectrum calculated from measured capacitance probe spectra to the average slope spectrum.

Comparison of the surface wave spectra derived from the microwave data with that measured with the slope and capacitance probes (Fig. 2) makes it clear that the bulk of the contribution to the surface displacement is accounted for by the free waves into which the short gravity wave system is decomposable. The contribution to mean squared height for frequencies between 2.2 and 5 Hz is 0.22 cm^2 for the Doppler-derived surface spectra compared to 0.26 cm^2 for the probe-derived spectra. This difference of 20% is of the order of the estimated experimental error in the comparison.

4. Discussion

Our measurements of phase speeds (Fig. 1) are in reasonable agreement with those of others who have measured the phase of the cross-correlation function between two point probes with a fixed separation, though our interpretation of them is not. It has been found (e.g., Yuen and Lake, 1978; Toba, 1978) that the phase speeds measured with the two probe method decrease with frequency in the neighborhood of the dominant wave, much as do ours, though the decrease is less rapid than ours at frequencies greater than that of the dominant wave. This is not unexpected, for any two-probe method utilizing a fixed separation measures a wave property which is, relatively unselectively, averaged over all wave-vectors. For example, the Doppler spectra corresponding to the highest frequency microwave measurements shown in Fig. 2 exhibited sidebands comparable in magnitude to that of the first order Bragg peak. Such sidebands occur in the water wave spectrum as well, but the wavenumber resolution of probe techniques is usually inadequate to detect them. The higher frequency sideband is characteristically larger (Plant and Wright, 1977), and, at second order, due, roughly, to the product of the spectral amplitude of the dominant wave and that of a wave of wavelength intermediate between that of the dominant wave and that of a Bragg wave. The situation is illustrated in Fig. 3 for the simple case of very long crested waves. The first order surface displacement spectrum is confined to the dispersion curve, the solid curve in Fig. 3. The sidebands occur along the dashed curves which are displaced from the dispersion relation by an amount calculable from the dispersion relation (Wright, 1978). If the spectrum were sectioned along the line $k = k_B$ (Fig. 3) one would obtain the Doppler spectrum discussed earlier. A section along a line ω = constant would yield a qualitatively similar spectrum, but in this case the lower, longer wavelength, sideband would be larger. A measurement with point probes at a frequency where the longer wavelength sideband dominated would associate a wavelength with that frequency which was greater than the wavelength of the corresponding free wave. Thus an apparent phase speed, larger than that of the free wave, would be obtained as is in fact observed.

Contributions from sidebands were deliberately excluded in determining surface wave spectra from the microwave Doppler spectra but, as they are unresolved, they cannot be excluded from the directly measured slope spectra. It is important to recognize that determinations of the surface displacement spectra by these two methods differ significantly only for wave components nearly two orders of magnitude smaller in energy density than that of the dominant wave.

5. Conclusion

At least 80% of the energy in short gravity waves 10–25 cm in length, generated in linear wind-wave channels, is carried by free waves. The adequacy of Hasselmann's (1968) model, which assumes random phases among component waves, to account for energy *transfer* among the components is discussed in a forthcoming report.

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