Comments on "A Parametric Wave Prediction Model"

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1. Introduction

There have been a number of developments in computer-based wave specification and forecasting models in addition to the ones cited in Hasselmann *et al.* (1976) that relate to the efforts described by Pierson *et al.* (1966). For one thing, the model we described then is now being used operationally for the Northern Hemisphere oceans at the Fleet Numerical Weather Central in Monterey, Calif., with fairly satisfactory results. Moreover, a model that includes the effects of wave refraction for forecasting waves in hurricanes has been developed and is being tested at the NOAA Atlantic Oceanographic and Meteorological Laboratory against measurements of hurricane waves. Pertinent references are Cardone (1974), Pierson (1976), Salfi (1974), Lazanoff *et al.* (1973) (for a model for the Mediterranean Sea), Cardone *et al.* (1976) and Lazanoff and Stevenson (1975).

These models are defined in such a way that the winds need not be uniform either in speed or direction over a fetch or with time, and the land boundaries can be highly irregular. They all use the same model of spectral growth based on the Pierson-Moskowitz (PM) spectrum is a way quite different from the way the PM spectrum is used in Hasselmann *et al.* (1976), hereafter referred to as H. The way that the PM spectrum is used in the above models is ever so much simpler than the way it is used in H. Yet, it can easily be shown that comparable results are obtained without the need for a peak enhancement factor, once the errors due to bias, which are introduced in the procedures used in H, are understood and eliminated, and once the high-frequency limitations of the data that were used are considered.

These comments are divided into four main subjects. They are 1) a derivation of γ and α_H as given in H from the model described by Salfi (1974) as specialized to the simple case of a constant wind over a constant fetch, 2) a demonstration that the procedures used to fit the spectrum $E(f)$ as in H bias the value of γ (and hence the spectral peak) to too high a value, 3) a discussion of the use of the wind at 10 m instead of at 19.5 m, and 4) a discussion of the so-called equilibrium range.

2. Derivation of α_H and γ

To keep notation simple, $E(f)$ will represent Eq. (2.1) of H, $S(f)$ will represent the PM spectrum, and $\hat{E}(f)$ will represent a spectrum *estimated* from an ocean-wave time history by either the old Tukey method or the new fast Fourier transform method.

The function $S(\omega)$ given by Pierson and Moskowitz (1964) can be transformed to

$$S(f)=\alpha g^2(2\pi)^{-4}f^{-5}\exp\left[-\frac{5}{4}(f_M/f)^4\right], \qquad (1)$$

where

$$f_M=(2\pi)^{-1}(4\beta/5)^{\frac{1}{4}}gu^{-1}, \qquad (2)$$

and where $\beta=0.74$ and u is measured for a neutral atmosphere at an elevation of 19.5 m. The wind speed is the wind that generated the fully arisen sea. Eq. (1) has been modified in the models cited above for high winds and high frequencies, and this point will be discussed last.

Eq. (H2.1) (i.e., Hasselmann *et al.* Eq. 2.1) has five parameters, α_H, γ, f_m, σ_a and σ_b, and much of the effort in H is related to the study of γ and α_H as a function of the nondimensionalized value of f_m, namely $\nu=uf_m/g$. Note that the α in (H2.1) has been defined to be α_H in these comments because it was determined in a different way from the α in (1).

Eq. (H2.1) can be factored into two functions. The first function looks like (1), except that f_m, which is the frequency of the spectral peak of whatever spectral estimate $\hat{E}(f)$ is being used, has been substituted for f_M, which is uniquely defined by the wind velocity. The second factor is the peak enhancement function defined immediately below Eq. (H2.1).

Given a spectrum $\hat{E}(f)$ as estimated from a wave record, if it has a single peak, the analysis procedure used in H is to calculate γ and α_H in terms of $\hat{E}(f_m)$, which is the peak value of the spectrum, and $\hat{E}(f)$, and then relate γ and α_H to f_m (or ν) and the fetch x.

From the definitions given in H, the value of γ is computed from (3):

$$\gamma=\hat{E}(f_m)(2\pi)^4f_m{}^5\exp\left(\frac{5}{4}\right)(\alpha_H g^2)^{-1} \qquad (3)$$

and the value of α_H is computed from (4) as defined in H (Section 4a, p. 212), apparently with a $(2\pi)^4$ missing:

$$\alpha_H = (0.65 f_m)^{-1} \int_{1.35 f_m}^{2 f_m} (2\pi)^4 f^5 g^{-2} \exp\left[\frac{5}{4}\left(\frac{f_m}{f}\right)^4\right] \hat{S}(f) df. \tag{4}$$

At 1.35 f_m, for the values of σ_b given in H, the effect of the peak enhancement factor is usually negligible and can be neglected in calculating α_H.

In the model being compared with H, the spectrum is specified by integrals of $S(f,\chi)$ over either 13 or 15 frequency bands and over either 12 or 24 direction bands. The frequency bands are 1/180 s^{-1} wide for low frequencies, 2/180 s^{-1} wide for intermediate frequencies and still wider for higher frequencies. The spectral growth is calculated for each time step Δt using $S(f_i; t_0)$ as the initial value and getting $S(f_i, t_0+\Delta t)$ as the next value. The propagation of *each* spectral component at the correct group velocity in the correct direction then accounts for the effect of fetch since zeros are propagated from land. The spectral growth theories of Phillips (1957), Miles (1957, 1959a, 1959b, 1962), and Phillips (1966) are used. When the spectral components propagate out of an area of high wind, they automatically become swell.

Each spectral component will grow in amplitude in the model until it reaches the value given by (1) as defined by the wind speed via f_M, unless the effects of duration or fetch prohibit the growth. The high frequencies grow very rapidly, followed by slightly lower frequencies, and so on to saturation.

For a well-defined fetch and a constant wind over that fetch, the result is quite simple. The spectrum at x is given by Eq. (3),

$$S_x(f) = \begin{cases} G(f,u,x), & 0<f<f_m \\ S(f,u), & f_m<f<\infty \end{cases} \tag{5}$$

where, of course, $S(f, u) = S(f, f_M)$ (see Inoue, 1967).

The values of $G(f,u, x)$ lie between zero and $S(f,u)$ and are less than $S(f_m,u)$ so that $S(f_m,u)$ is the peak of the spectrum. To calculate $S_x(f)$ for $f>f_m$ it is simply necessary to evaluate (1) for $f>f_m$. Those who recall Pierson *et al.* (1955) will recognize this as the equivalent of the co-cumulative spectrum. Of course, $G(f,u,x)$ is also found, but it is not required for this discussion. The function $G(f,u,x)$ is calculated directly from the Phillips-Miles-Phillips theory.

The factor γ in H and the peak enhancement function related to it are simply highly circular ways to correct for the fact that what appears to be the PM spectrum in $E(f)$ [Eq. (H2.1)], really is not because f_m is used instead of f_M when the function is evaluated for $f>f_m$. To demonstrate this, it is only necessary to substitute (5) for $f_m<f<\infty$ as defined by (1) into (3) and (4) as if the spectra being fitted were those predicted from the model being compared with the model given in H. The results are given by (6) and (7), where γ and α_H are functions of f_m and f_M:

$$\gamma = \frac{\alpha}{\alpha_H} \exp\left(\frac{5}{4}(1-(f_M/f_m)^4)\right), \tag{6}$$

$$\alpha_H = \alpha (0.65 f_m)^{-1} \int_{1.35 f_m}^{2 f_m} \times \exp\left\{-\frac{5}{4}\left[\left(\frac{f_M}{f}\right)^4 - \left(\frac{f_m}{f}\right)^4\right]\right\} df. \tag{7}$$

Now let $f/f_M = K$ and $f_m/f_M = K_m$. Then ν is given by

$$\begin{aligned} \nu &= (2\pi)^{-1}\left(\frac{4\beta}{5}\right)^{\frac{1}{4}} K_m \\ &= f_m u/g \quad \text{for} \quad 1<K_m<\infty. \end{aligned} \tag{8}$$

Eqs. (6) and (7) can be put in the form

$$\gamma = \frac{\alpha}{\alpha_H} \exp\left[\frac{5}{4}(1-K_m^{-4})\right], \tag{9}$$

$$\frac{\alpha_H}{\alpha} = (0.65 K_m)^{-1} \int_{1.35 K_m}^{2 K_m} \exp\left[\frac{5}{5}\left(\frac{K_m^4-1}{K^4}\right)\right] dK. \tag{10}$$

A further transformation of variables yields

$$\frac{\alpha_H}{\alpha} = \frac{1.35}{0.65} \int_1^{2/1.35} \exp\left\{\frac{5}{4}\left[\frac{K_m^4-1}{(1.3K_m)^4} y^{-4}\right]\right\} dy. \tag{11}$$

A series expansion about the center of the range of integration then yields (12) for ease of evaluation.

$$\frac{\alpha_H}{\alpha} = \exp[0.1588(1-K_m^{-4})][1+0.0080(1-K_m^{-4}) + 0.00278(1-K_m^{-4})^2]. \tag{12}$$

Clearly $\alpha_H = \alpha$ only if $K_m = 1$, and $\alpha_H > \alpha$ if $K_m > 1$.

The various functions involved are graphed in Fig. 1. The method of calculating α_H in H does not yield the asymptotic value of α in (1) of the PM spectrum. The values of γ range from 1 to 2.95. If the curve for γ in Fig. 1 were plotted on Fig. 2 of H it would fit the data fairly well and explain about 90% of the "average" value.

The average value of γ according to H is 3.3 for the mean JONSWAP spectrum as shown in Fig. 1. This is a pretty good value for ν greater than 0.21, but, of course, the values of γ are biased too high, as will be shown later when the scatter in γ is discussed. It would be interesting, using *all* data sets, to compute the average value of γ over small bands of ν, say, 0.14 to 0.16, 0.16 to 0.18 and so on.

From (H2.2) it is possible to calculate the non-dimensional fetch $\xi = gx/u^2$ for different values of ν,

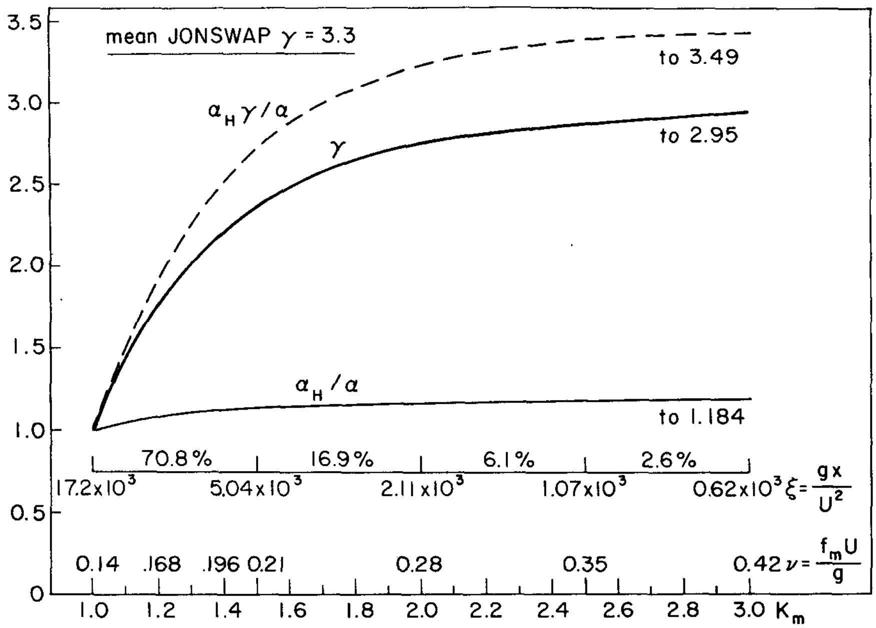

FIG. 1. Graph of γ and α_H as used by Hasselmann *et al.* (1976) as derived from operational numerical wave prediction model described by Salfi (1964).

and the values of ξ are also given in Fig. 1. As ν varies from 0.14 to 0.21, which is 25% of the range shown, ξ varies from 17.2×10^3 to 5.04×10^3 so that a mean value of γ equal to 3.3 only applies to the first 29% of the fetch as measured from the shore. The value of γ decreases quite rapidly as ν decreases from 0.21 to 0.14 so that in terms of f_m, the spectral parameters change rapidly, but in terms of fetch on a linear scale the values of γ change slowly. The final transition to a fully developed PM spectrum occurs in the final stage of development as a function of ν, (H, p. 208) but certainly not as a function of fetch.

From Fig. 1 and statements and equations in H, the following example can be worked out. Consider a nondimensional frequency ν of 0.42; for this value of ν, the nondimensional fetch is 620. If the wind speed is 30 m s^{-1}, then the frequency of the peak of the spectrum for this fetch limited wind-generated sea will be 0.1372 Hz, which corresponds to a wave with a period of 7.29 s. The fetch is 57 km. For these values of ν and ξ, the peak enhancement factor γ is about 2.95 on the average in order to obtain the best fit to $E(f)$ (Eq. H2.1). The spectrum whose peak is enhanced in $E(f)$ is the PM spectrum for a *fully developed* wind generated sea with a peak at 0.1372 Hz, which corresponds to a wind of 10 m s^{-1} and which has been multiplied by 1.18.

The question that arises, however, is that of providing a reason for the use of a spectrum associated with a wind one-third as strong as the actual wind for the underlying spectrum that is to be *enhanced* by the peak enhancement factor in order to explain the observed spectrum. In the models cited above the controlling factor for the calculation of a fetch, or duration, limited spectrum is the actual wind, and the spectrum grows to a limit given by the PM spectrum for the *observed* wind for frequencies greater than f_m and to a value less than the PM spectrum for frequencies less than f_m. If the PM spectrum is evaluated at a frequency of 0.1372 for a wind of 30 m s^{-1}, this value is 3.49 times the value of the PM spectrum at 0.1372 for a wind of 10 m s^{-1}.

The ramifications of this particular example are explored in what follows in greater detail. The main point to be emphasized, however, is that the underlying spectrum, which is enhanced in the procedures given by H, has no connection with the wind that is generating the waves. That the spectrum for a fetch-limited sea for a wind of 30 m s^{-1} has a peak that is about 3.5 times higher than the peak of a fully developed sea at the same frequency that is generated by a 10 m s^{-1} wind is explainable by the model described by Salfi (1974).

It would have been ever so much simpler to evaluate (1) for $f > f_m$ using f_M in the equation in order to define *most* of the fetch limited spectrum. The only remaining area to study is the shape of $G(f,\ u,x)$, if necessary, to see if it is properly predicted by the model. *There would be no need, then, for the peak enhancement function.*

Stated another way, the spectral wave forecasting and hindcasting model being compared with the results of H predicts nearly the same spectral shapes and nearly the same peak spectral values *without* the use of a peak enhancement function and *without* recourse to very intricate appeals to nonlinear wave interactions. Moreover, it is much simpler to apply, and it is a special

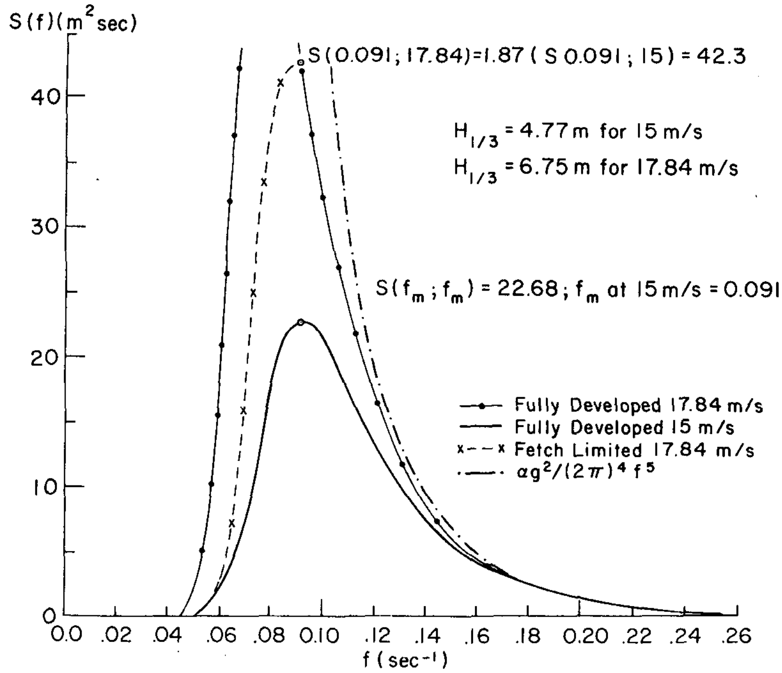

FIG. 2. Example of comparison of the Hasselmann *et al.* (1976) model and the Salfi (1974) model.

case of the general proof of Kitaigorodskii (1961) in which, for $f>f_m$, $S(f)$ no longer depends on x but only on u.

A specific example of these considerations is given in Fig. 2. The partially developed fetch-limited spectrum predicted by the model being compared with the results of H for a wind of 17.84 m s^{-1} is given by (1) evaluated with f_M determined by a wind speed of 17.84 m s^{-1} for $f_m>0.091$. The function $G(f,u,x)$ from (5) is sketched by the dashes and x's. Except that α instead of α_H has been used, the function given by (H2.1), using f_m instead of f_M, is shown by the solid curve before multiplying by the peak enhancement function. This solid curve corresponds to the fully developed PM spectrum for a wind of 15 m s^{-1}. The ratio α_H/α is the amount by which the solid curve has to be multiplied in order to make it agree with the 17.84 m s^{-1} curve for frequencies between 0.122 and 0.182 Hz. In this example, the value of $\alpha_H\gamma/\alpha$ is 1.87 so that $\nu\cong 0.168$. The peak enhancement function inflates a spectrum that is too low (because a different wind speed has been used than the one actually present) to the values of the PM spectrum evaluated with f_M as a parameter for $f>f_m$.

3. Sampling variability and bias

The spectra estimated from an ocean-wave time history as in $\hat{E}(f)$ are statistics in the true sense of the word (Mood *et al.*, 1974, for example). As shown by Tukey (1949), each point of an estimated spectrum is distributed according to chi-square with the degrees of freedom determined from the length of the record, the value of Δt and the resolution. Missing from Fig. 1 of H are the 90% fiducial confidence intervals on $\hat{E}(f)$ or at least an indication of the degrees of freedom of the spectral estimates. A spectrum predicted by any prediction model can only be verified against spectral estimates, and the exact value of a particular estimate used for verification cannot be predicted because of sampling variability.

The arguments in H do not justify throwing out the cases of multiple peaks. They are a real part of the problem even for theoretically unimodal spectral forms. The problem becomes doubly complicated if one attempts to separate double peaks caused by a real physical effect from double peaks caused by sampling variability.

Typically, most estimates of $\hat{E}(f)$ have from 20 to 40 degrees of freedom. For 20 degrees of freedom, the 90% fiducial confidence interval values are about 1.5 and 0.6 times the spectral estimate. For a particular point of an estimated spectrum, the estimate can easily fluctuate by ± 20 to 30% or more of the value actually obtained.

Efforts to parameterize ocean-wave spectra by referring everything to the frequency of the spectral peak f_m as done originally by Darbyshire (1952), and as is done in H are fraught with grave difficulties. Spectra with two or more peaks are unjustifiably excluded from the sample although it is perfectly possible for $\hat{E}(f)$ in a pure, simple fetch-limited seaway to have more than one peak. Were the spectra in the extensive data set of H analyzed at twice the resolution (and consequently half the degrees of freedom per estimate) many of the single-peaked spectra would be replaced by multiple-peaked spectra. An analysis technique should be general enough to handle all possible estimates of the spectra being studied and not just a subset of them.

Multiple-peaked spectral estimates serve to emphasize how poorly defined, from a sampling point of view, the location of the peak in the spectrum actually is. This effect is partially masked in the data sets analyzed by H because equal resolution over ν and not f would be desirable if spectra truly scale according to $\nu = uf/g$ (Pierson, 1964). When spectra for all wind speeds are estimated with the same resolution, the resolution for ν for a 20 m s^{-1} wind is only half as good as for a 10 m s^{-1} wind. Thus spectral estimates for high waves and high winds tend to have more single peaks than spectra for low winds for the same ν.

That the location of f_m, for a given wind speed and fetch is itself a random variable can be seen from the plots in Fig. 1 of H. With confidence intervals of say 0.6 and 1.5, or perhaps even 0.8 and 1.3, it is a matter of mere chance that the first of the two highest points in the west Atlantic example is the larger of the two.

For the AVA example, the points on each side of the peak are about 0.8 times the peak. Since 1.3×0.8 is larger than 0.8×1, the peak could just as easily have been one spectral band width to either side of the peak that happened to have occurred by chance.

The simplest way to show that the selection of the peak as a dominant scaling parameter for a spectral model biases the data is to show what happens near a flat peak that is four, three or two spectral bands wide and to assume that the spectral estimates are independent as with the fast Fourier transform method. For low values of chi-square the expected value of chi-square does not equal the 50% value. The difference is small compared to the range of variability so that there is almost a 50% chance that a given estimate will exceed the expected value, and, consequently, there is almost a 50% chance that it will be less than the expected value. What happens at four, three or two spectral bands near a fairly flat peak will be worked out as examples. The details become somewhat different near the peak of a spectrum that may not be flat, but similar results can be obtained.

TABLE 1. Sampling variability outcomes for four spectral bands near a flat peak of an estimated spectrum. A plus means that the estimate exceeds the expected value, and a minus means that it is less than the expected value.

Outcome	Spectral band	Probability of single peak	Probability of multiple peak
	1 2 3 4		
1	+ + + +	1/3	2/3
2	+ + + −	2/3	1/3
3	+ + − +		1
4	+ + − −	1/2	1/2
5	+ − + +		1
6	+ − + −		1
7	+ − − +		1
8	+ − − −	1/6	5/6
9	− + + +	2/3	1/3
10	− + + −	1	
11	− + − +		1
12	− + − −	1/2	1/2
13	− − + +	1/2	1/2
14	− − + −	1/2	1/2
15	− − − +	1/6	5/6
16	− − − −	1/3	2/3

P(single peak) = 1/3. P(multiple peak) = 2/3.

TABLE 2. Sampling variability outcome for three spectral bands near the peak of an estimated spectrum.

Outcome	Spectral band	Probability of single peak	Probability of double peak
	1 2 3		
1	+ + +	2/3	1/3
2	+ + −	1	
3	+ − +		1
4	+ − −	1/2	1/2
5	− + +	1	
6	− + −	1	
7	− − +	1/2	1/2
8	− − −	2/3	1/3

P(single peak) = 2/3. P(double peak) = 1/3.

Table 1 illustrates the outcomes if four spectral bands can be resolved near the peak. Of the 16 possible outcomes, only one can yield values for all four estimates below the expected value. Consider outcome number 2. The permutations of 1 (low), 2 (middle), and 3 (high) for the three plus signs yield four possible outcomes with a single peak and two with a double peak so that if this pattern occurs (+++−) there are two chances out of three for a single peak. Similar analyses for outcomes 3 to 15 yield the other tabulated probabilities. The probabilities for outcomes 1 and 16 have been found from the permutations of the numbers (1, 2, 3, 4). For a relatively broad spectral peak of four bands, the probability that the estimated spectrum will have a single peak is 0.33. The single peak will be a biased overestimate of the true shape of the spectrum near the peak 15 times out of 16 if it is used. Approximately seven out of ten spectra cannot be fitted by the technique under discussion, since sampling variability will produce a double peak.

Table 2 illustrates the sampling variability outcomes when only three spectral bands can be resolved near a relatively flat peak. The probability structure for the eight possible outcomes is much simpler. Two out of three spectra will have a single peak. If the spectral estimate has a single peak, there are seven chances out of eight that the peak will be an overestimate of the true spectral value.

For a peak resolved by only two spectral bands, the outcomes are simple and need not be tabulated. A single peak results every time, and three times out of four the fit to the peak overestimates the expected value.

For typical record lengths, as the waves get higher, the spectra tend to look more peaked simply because of the decreased relative resolution near the peak. The probability that chance fluctuations near the peak will produce a single peak increases from 0.33 through 0.67 to 1 as the resolution at the peak decreases from four bands to three bands to two bands.

For spectral estimates of nearly fully developed seas, generated by low winds, rejecting multiple peaks may cause as many as two-thirds of the available data to be rejected unjustifiably. For those that are kept, the estimate at the peak [i.e., $\hat{S}(f_m)$] will exceed the expected value 93% of the time, and hence γ will be an overestimate of the true value of the peak. For intermediate winds, one out of three spectra will be rejected, and of those used 87% will have peaks exceeding the expected value. For higher winds, where only two bands can be resolved, the peak exceeds the expected value 75% of the time. The penalty, of course, is that in the long run the waves will be forecasted to be too high with such a model.

The data (Moskowitz *et al.*, 1963) used by Moskowitz (1964) were reanalyzed by H. More than half of the estimated spectra contained multiple peaks, as would be expected from the above tabulations. Those for the lower wind speeds had the most multiple peaks, which again is explained by the tabulations. The multiple-peaked spectra were discarded, which biased the data. Those that had single peaks yielded a γ of 1.40, which shows that selecting only single-peaked spectra biased the peak to 40% higher than the value calculated theoretically in the preceding section.

Moskowitz (1964) also illustrated the effect of using a higher resolution on the spectral estimates. Even the "averaged" spectra had multiple peaks at a resolution increased by a factor of three. The procedures that use (H2.1) are very sensitive to record length and spectral resolution.

In the work of Moskowitz (1964) for five different wind speeds, 12, 8, 12, 8 and 14 spectra were averaged, respectively. It was shown that a large number of the estimated values for different spectral bands in the data set did indeed behave as if they had been drawn from an appropriate chi-square distribution. Spectra from wave records drawn at random from those available and stratified only as to wind speed were shown not to have come from the same population.

The main reason for trying to obtain a number of spectra for each wind speed was to increase the number of degrees of freedom for each spectral estimate at the given wind speed. Since each separate spectrum had 19.33 degrees of freedom for each frequency band, the averaged spectra had 232, 155, 232, 155 and 271 degrees of freedom, respectively, for each frequency band. The increased number of degrees of freedom made the "averaged" spectra the equivalent of the spectra that would have been obtained from wave records 8 to 14 times longer than the individual records, and accordingly stabilized the frequency of the peak and the value at the peak to a greater extent than estimates from a single spectrum.

Fitting the spectra by the techniques given in H gives more sharply peaked spectra than are actually found "on the average" because of the coupling between $\hat{S}(f_m)$ and f_m as random variables. There may be a difference between the average shape of the spectrum and the shape of the average spectrum, but in most statistical estimation procedures the estimate of a parameter should converge to the true population parameter. A biased estimation procedure does not do this and is, therefore, undesirable.

The values of α_H couple into the calculation of γ to complicate the interpretation of γ even further. If because of sampling variability, α_H is 0.8 times its expected value and if $\alpha_H\gamma/\alpha$, which is independent of α_H, is 1.6 times its expected value, then γ will be twice its expected value. Values of γ equal to 6 and higher would, therefore, not be unusual.

The parameter γ is a random variable (as well as a statistic) since it is estimated from values of $\hat{E}(f)$, which are themselves random variables. The different values of γ are not all drawn from the same population since all spectra are not resolved equally over ν and since the length of the integral that determines α_H is variable. However, the probability density function for γ could probably be derived.

The distribution of α_H in the denominator of the definition of γ is probably close to chi-square with the degrees of freedom given by sums of the degrees of freedom of the spectral bands in the integration given by Eq. (4). The distribution of $\hat{S}(f_m)$ would be something like Eq. (20) of Mood *et al.* (1974, p. 183) for $m=2$, 3 or 4, and the distribution of the quotient could be found from Eq. 28, of Mood *et al.* (1974, p. 187). The values of γ would have many different detailed distributions based on such an analysis. The simplest way, however, is not to use the techniques of H and return to the well-known properties of spectral estimates given by Tukey (1949).

4. The use of winds at 10 m

In the model described by H the winds were referred to an elevation of 10 m instead of to 19.5 m as in Pierson and Moskowitz (1964). The separation of sea from swell at a ν of 0.14 is inconsistent with a wind measured at 10 m since this value of ν corresponds to a fully developed form like Eq. (1) with the wind defined at 19.5 m. Since the PBL model of Cardone (1969) is used, the ratio of u_{10} to $u_{19.5}$ is not strictly a constant.

The ratio u_{10} to $u_{19.5}$ is approximately 0.93 so that $\nu=0.13$ more nearly corresponds to the fully developed situation. Moreover, unless some correction is made, the wave heights would be 86% of the values calculated from a 19.5 m wind if the 10 m wind were substituted into (1), and the spectral areas would be 75% of those for a correct 19.5 m wind.

A quick fix to make the situation consistent would be to change β to 0.56 to compensate for the lower wind speed at 10 m. This would require the replotting of γ and α_H.

An alternate correction would be to use a 19.5 m wind, which would require the recalculation of all of the nondimensionalized plots. Either way, as now presented, the results of H are internally inconsistent.

5. The equilibrium range

Although the value of α in (1) apparently does well in describing the dominant shape of the spectrum over 70% of ocean scale fetches and for most of the frequency range of mid-ocean waves, there is increasing evidence that there is no equilibrium range of the form $\alpha g^2(2\pi)^{-4}f^{-5}$ for high frequencies. For frequencies above 0.2 Hz, or so, the spectrum appears to respond rapidly to the local wind, and these waves appear to grow and shrink in direct response to the wind. This variation is particularly important from 5 to 30 Hz in the capillary range. Further discussion of these matters can be found in Pierson and Stacy (1973), Stacy (1974) and Pierson (1976). The growth of short waves with wind speed is clearly shown in "State of Sea Photographs for the Beaufort Wind Scale 1971" (Meteorological Branch, Department of Transport, Canada). Clearly, the spectrum for waves 3 m long and shorter is growing with wind speed at these wavelengths in the presence of extremely high seas in mid-ocean, so that high spectral values for these wavelengths (and frequencies) are *not* due to a short fetch.

More needs to be learned about the high-frequency behavior of waves. The short fetch data of JONSWAP can provide this information, but it might be better to study the behavior of a given frequency solely as a function of wind speed and not after scaling by means of $\nu=fu/g$. For the hurricane model (Cardone *et al.*, 1976) a correction for high frequencies is added to the PM spectrum of the form

$$S_c(f)=\alpha u_* g^2/u_{*m}3(2\pi)^4 f^4, \tag{13}$$

according to a theory of Kitaigorodskii (1961), where $u_{*m}=12$ cm s^{-1} and the number 3 is a frequency (Hz). The function $S_c(f)$ intersects $S(f)$ at

$$f_i=3u_{*m}/u_* \tag{14}$$

and $S_c(f)$ replaces $S(f)$ for $f>f_i$.

The friction velocity ($u_{*m}=12$ cm s^{-1}) corresponds to a wind of 3.5 m s^{-1} at 19.5 m based on the results of Cardone (1969). No waves at all are forecasted to grow for winds associated with u_* values less than 12 cm s^{-1}. For a u_* of 36, corresponding to a 19.5 m wind of 10.0 m s^{-1}, $f_i=1$, and for winds around 20 m s^{-1} the correction begins to be important.

The Baylor gage[1] has a different frequency response from that of the Tucker shipborne wave recorder, and there are many problems associated with recording the spectral components from about 0.2 to 5 Hz under conditions of high wind and waves.

The modification introduced by Eq. (13) affects the higher frequencies in the spectrum for a given wind speed. For high values of f_m (short fetches) portions of the spectrum as defined by (13) will be included in the interval 1.35 $f_m<f<2f_m$ as used by H to obtain α_H. As f_m decreases, this interval moves toward lower frequencies and less and less of (13) is included. Finally, for f_m low enough, no part of (13) may be included. The method for finding α_H would then yield a decreasing α_H as a function of ν, and yet the spectrum at the higher frequencies, not sampled when f_m is small, might not have changed at all.

From Eqs. (H4.13) and (H4.14), it follows that

$$\alpha_H=3.3\times10^{-2}(f_m u/g)^{\frac{2}{3}}, \tag{15}$$

so that asymptotically

$$E(f)\sim3.3\times10^{-2}(2\pi)^{-4}g^2(f_m u/g)^{\frac{2}{3}}f^{-5}. \tag{16}$$

For a fixed f_m, the JONSWAP spectrum grows as $u^{\frac{2}{3}}$, whereas in (13) it grows as u_*. However, as f_m decreases, according to H, the spectral values at high frequencies decrease, whereas in (13) these values remain high. The JONSWAP data could help clarify this difference if the wave recorders have the same response at high frequencies at all fetches.

The excellent array of wave recorders used in JONSWAP is providing much valuable data for programs in remote sensing. Compared to the Tucker (1956) shipborne wave recorder that cannot sense waves with frequencies much higher than 0.25 Hz, the JONSWAP data provide reliable information on much higher frequencies. However, it does not appear that these effects scale according to $\nu=uf/g$, and so no attempt has been made to incorporate (13) into the comparison of the two models. The high values for α_H for moderate and high values of ν may be in part due to differences in wave recorders, and in part, due to effects such as those just described. There is much data in the literature that shows that these higher frequencies grow with wind speed just as much as shown in H—but very far from land and in the presence of fully developed waves.

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[1] Used to measure the hurricane waves in the report by Cardone *et al.* (1976).

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REFERENCES

Cardone, V. J., 1969: Specification of the wind field distribution in the marine boundary layer for wave forecasting. Rep. TR 69-1, Geophys. Sci. Lab., New York University.

——, 1974: Ocean wave prediction: Two decades of progress and future prospects. *Seakeeping 1953–1973*, T and R Symp. S-3, Soc. Naval Arch. Mar. Eng.

——, W. J. Pierson and E. G. Ward, 1976: Hindcasting the directional spectra of hurricane-generated waves. *J. Petrol. Tech.*, April, 385–394.

Darbyshire, J., 1952: The generation of waves by wind. *Proc. Roy Soc. London*, **A215**, 299–328.

Hasselmann, K., D. B. Ross, P. Müller and W. Sell, 1976: A parametric wave prediction model. *J. Phys. Oceanogr.*, **6**, 200–228.

Inoue, T., 1967: On the growth of the spectrum of a wind generated sea according to a modified Miles-Phillips mechanism and its application to wave forecasting. TR-67-5, Geophys. Sci. Lab., New York University.

Kitaigorodskii, S. A., 1961: Applications of the theory of similarity to the analysis of wind-generated wave motion as a stochastic process. *Izv. Akad. Nauk SSSR, Ser. Geofiz.*, No. 1, 73–80.

Lazanoff, S. M., N. Stevenson and V. J. Cardone, 1973: A Mediterranean Sea wave spectral model. Tech. Note 73-1, Fleet Numerical Weather Central, Monterey, Calif.

——, and ——, 1975: An evaluation of a hemispherical operational wave spectra model. Tech. Note 75-3, Fleet Numerical Weather Central, Monterey, Calif.

Miles, J. W., 1957: On the generation of surface waves by shear flow. Part 1. *J. Fluid Mech.*, **3**, 185–204.

——, 1959a: On the generation of surface waves by shear flow. Part 2. *J. Fluid Mech.*, **6**, 568–582.

——, 1959b: On the generation of surface waves by shear flow. Part 3. *J. Fluid Mech.*, **6**, 583–598.

——, 1962: On the generation of surface waves by shear flow. Part 4. *J. Fluid Mech.*, **13**, 433–448.

Mood, A. M., F. A. Graybill and D. C. Boes, 1974: *Introduction to the Theory of Statistics*. 3rd ed. McGraw-Hill. 564 pp.

Moskowitz, L., 1964: Estimates of the power spectrums for fully developed sea for wind speeds of 20 to 40 knots. *J. Geophys. Res.*, **69**, 5161–5179.

——, W. J. Pierson and E. Mehr, 1963: Wave spectra estimated from wave records obtained by the OWS *Weather Explorer* and the OWS *Weather Reporter* Parts (I) and (II). Tech. Rept., Navy Oceanogr. Office, Contract N62306-1042, New York University, School of Engineering and Science.

Phillips, O. M., 1957: On the generation of waves by turbulent wind. *J. Fluid. Mech.*, **2**, 417–445.

——, 1966: *The Dynamics of the Upper Ocean*. Cambridge University Press, 261 pp.

Pierson, W. J., 1964: The interpretation of wave spectrums in terms of the wind profile instead of the wind measured at a constant height. *J. Geophys. Res.*, **69**, 5191–5203.

——, 1976: The theory and applications of ocean wave measuring systems at and below the sea surface, on the land, from aircraft and from spacecraft. Contractor's Report, NASA Goddard Space Flight Center, NASA CR-2646, 388 pp.

——, G. Neumann and R. W. James, 1955: Practical methods of observing and forecasting ocean waves by means of wave spectra and statistics. H. O. Pub. No. 603, U. S. Navy Hydrographic Office, 284 pp.

——, and L. Moskowitz, 1964: A proposed spectral form for fully developed wind seas based on the similarity theory of S. A. Kitaigorodskii. *J. Geophys. Res.*, **69**, 5181–5190.

——, and R. A. Stacy, 1973: The elevation, slope and curvature spectra of a wind roughened sea surface. NASA Contractor Report CR 2247 (Langley Research Center).

——, L. J. Tick and L. Baer, 1966: Computer based procedures for preparing global wave forecasts and wind field analyses capable of using wave data obtained by a spacecraft. *Proc. Sixth Naval Hydrodynamics Symp.*, 499–532.

Salfi, R. E., 1974: Operational computer based spectral wave specification and forecasting models. The University Institute of Oceanography of the City University of New York prepared for SPOC (NOAA/NESS).

Stacy, R. A., 1974: Spectral analysis of high-frequency gravity waves generated by a hurricane. Ph.D. thesis, New York University, 63 pp.

Tucker, M. J., 1956: A ship-borne wave recorder. *Trans. Inst. Naval Arch.*, **98**, 236–250.

Tukey, J. W., 1949: The sampling theory of power spectrum estimates. *Proc. Symp. Applications to Autocorrelation Analysis to Physical Problems*. NAVEXOS-0-735, Office of Naval Research.

Reply

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1. Introduction

In his comments Pierson has raised a number of interesting questions. To place the discussion in proper perspective, we should perhaps first point out that the purpose of our paper was not to experiment with various parametrical representations of the wave spectrum—as interesting as these are—but to apply recent results on the energy balance of the wave spectrum to the practical task of wave prediction. These results, summarized in Hasselmann *et al.* (1973, hereafter called J), have changed the structure of the wave prediction problem.

On the one hand, the problem has apparently be-

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