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Surf Zone Currents[1]

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Abstract. A large fraction of the water-wave energy incident on beaches is dissipated as the waves break and travel towards the shore through the surf zone. However, the momentum associated with the incident waves is not destroyed and drives other motions within the surf zone. An analysis is given of the unsteady and irregular currents that may occur in the surf zone. The forcing due to a discrete group of waves, and the vorticity of the surf-zone currents are the major topics. In particular, the almost two-dimensional nature of the flow implies that significant eddies are likely to be generated. There is evidence that eddies arise from long-shore currents, and the combination of eddies of opposite sign gives rip currents. It is noted that generation of vorticity by non-uniformities in bores may be a useful way of considering the forcing of surf zone motion. Expressions for the rate of increase in the circulation about material circuits and the vorticity generated at bores are derived.

1. Introduction

Many, if not most, beaches made of sand or fine sediment have relatively gentle slopes, less than 1:20. Almost all wind-generated waves incident on such beaches break and propagate towards the shore line as bores. The region of breakers and bores is the surf zone and the strong turbulence generated by both breakers and bores gives a clear visual indication that much of the incident wave energy is dissipated. However, there is no direct momentum loss at bores or breakers; the momentum is either transferred to currents or to the maintenance of pressure gradients such as cause "set-up" of the mean water level towards the shoreline. The "shoreline" is not a fixed line, but moves back and forth over the swash zone.

The main features of surf zone currents, include:

(i) Long-shore currents, i.e., flow roughly parallel to the shoreline.
(ii) Rip currents, i.e., relatively narrow and fast currents flowing away from the shore.
(iii) Low frequency wave motions, these are gravity waves with typical periods of more than twice the period of the incident waves forcing the surf zone currents. These were first described by Munk (1949) who noted their association with groups of incident waves, and introduced the term "surf beat" to describe them. The name "infra-gravity waves" is also applied to these motions but is not used here since these are essentially gravity waves with periods in the range 20 seconds to 10 minutes

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and do not have sufficiently low frequencies to be strongly affected by Earth's rotation, and hence infra-gravity, like inertial waves, for example.

The major step in the theoretical study of surf zone currents came from Longuet-Higgins and Stewart's work on the interaction of waves and currents which they summarized in their 1964 paper. They introduced the name "radiation stress" for the average momentum flux associated with a wave train and showed that gradients of radiation stress characterize the momentum transfer between waves and currents. Longuet-Higgins (1970a,b) further clarified the way in which the flux of the long shore component of momentum flux towards the shore acts to drive long-shore currents in the surf zone, the region where the waves are dissipated. These concepts based on results for very slowly varying wave trains are now well established in textbooks such as Dean and Dalrymple (1984).

There are difficulties associated with these models of long shore currents. Right from Longuet-Higgins's (1970a) initial paper, high bed friction coefficients were needed to obtain even rough agreement with the velocity magnitudes in measurements, and relatively large horizontal mixing is needed to obtain fair agreement on long-shore current profiles.

A similar difficulty occurs with low frequency wave generation where most of the studies have been on modulated wave trains composed of two wave trains with nearby frequencies. There is rough agreement on amplitude between theoretical results such as Schäffer (1993) and the experiments of Kostense (1984) but not enough agreement on the distribution of wave amplitude across the shoaling region to give confidence that the forcing mechanisms are understood.

Rip currents appear to fall into two categories. Some are clearly associated with bed topography. For example, there is often a sand bar where waves first break, and at a gap in such a sand bar, where waves are less likely to break, there is an outward flow of the water which has been driven over the bar by the breakers. Other rip currents on more uniform beaches may occur sporadically, or regularly, in both space and time. One problem in the modelling of rip currents is that their width is often less than the length of the incident waves so that the applicability of some wave-averaging approaches is doubtful.

In all the wave-averaging approaches there has been a simplification of the shoreline conditions by not taking into account the motion of the instantaneous shoreline up and down the beach. The problem of averaging over the shoreline motion has been recently addressed by Brocchini and Peregrine (1996). The terms that arise from averaging over the swash zone and also averaging over the waves are derived and discussed in that paper. Difficulties arise in providing a good model of the surf zone waves, so further developments need to be made. The discussion of vorticity in this paper may also help to clarify important aspects of the swash but this is left for future study.

The aim of this paper is to discuss aspects of the non-uniform forcing of surf zone currents and to highlight features that may be of value for interpretation of measurements. The next section gives an overview of recent studies at Bristol of the onshore–offshore motions generated by a single group of incident waves. The remainder of the paper concerns three-dimensional flows, but those that are almost two-dimensional because of the gentle slope of beaches. Hence, there is, or should be, a strong tendency to form eddies. Eddies and vorticity are the theme for the rest of the paper which includes discussion of circulation and vorticity generation at bores.

2. OnShore–OffShore Motion and Wave Groups

There are still considerable difficulties in modelling the hydrodynamics of wave breaking, the transition to bores, and the bores themselves. However, in this context we are not concerned with the detailed hydrodynamics of an individual wave crest but in the transfer of momentum. Thus the simplest model of a bore is adequate. It is then consistent to use the non-linear shallow water (NLSW) equations: i.e., the mass conservation equation

$$\frac{\partial \zeta}{\partial t} + \nabla \cdot [(h + \zeta)\mathbf{u}] = 0 \tag{1}$$

and the momentum equation

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + g\nabla\zeta = 0, \tag{2}$$

where $\zeta(x, y, t)$ is the height of the water surface and $h(x, y)$ is the depth of the bed from some horizontal datum. The velocity $\mathbf{u}(x, y, t)$ is the mean horizontal velocity. These equations are obtained by assuming that both the surface and bed slopes are so gentle that vertical accelerations of the water are negligible compared with gravity. This is not the case at bores, or close to wave crests. Bores are represented by discontinuities in the solution across which both mass and momentum are conserved. At wave crests before wave breaking neglect of vertical water accelerations is inappropriate. The next approximation, for shallow water is the Boussinesq equations, e.g., see Peregrine (1967) or (1972). For two-dimensional motion it is also now possible to use full irrotational flow computations, as demonstrated by Barnes and Peregrine (1995), for the transition of a wave group from deep water to breaking.

Viscous and bed friction terms are omitted from (2), usually a Chézy bed friction term $-C|\mathbf{u}|\mathbf{u}/(h + \zeta)$ is added to the right-hand side. None the less, the equations are not inviscid since the energy dissipation at bores is included in each discontinuity. Watson *et al.* (1994) discusses the effect of bed friction. It is not negligible but for the common range of beach slopes 1:20 to 1:50 it is not of great significance.

Our studies on wave groups have used both numerical solutions and experiments, with Hydraulics Research Ltd. Wallingford. The aim is to identify the origin of low frequency waves generated by the shorter incident waves more clearly. As it turned out, the numerical modelling, with NLSW equations, or a combination of NLSW and Boussinesq equations, gave satisfactory comparisons between experiment and computation except for some discrepancies close to wave crests through the breaking process, as might be expected (Barnes *et al.*, 1994). Figure 1 shows a comparison between experiment and computation for the waves reflected from beaches which have the shoreline on a 1:20 slope and on a 1:100 slope. Incidentally these illustrate the importance of bed friction on the gentler slope.

Conventional analysis of the results of this wave-group study has proved more difficult than expected. Whereas over a long period Fourier analysis of a signal can divide the short period incident waves from the low frequency waves, for a wave group of, say, four or five waves the division is not so clear cut. Use of more complex methods with wavelets did not appear to give much improvement. Here, the aspects of the flow which appear to be relevant from visual inspection of the records are described and accentuated to show more clearly what still needs to be done for successful modelling.

In the experiments and most computations the wave groups were generated in shallow water. The conventional model of an incident wave using a Stokes approximation gives an accompanying "set-down"

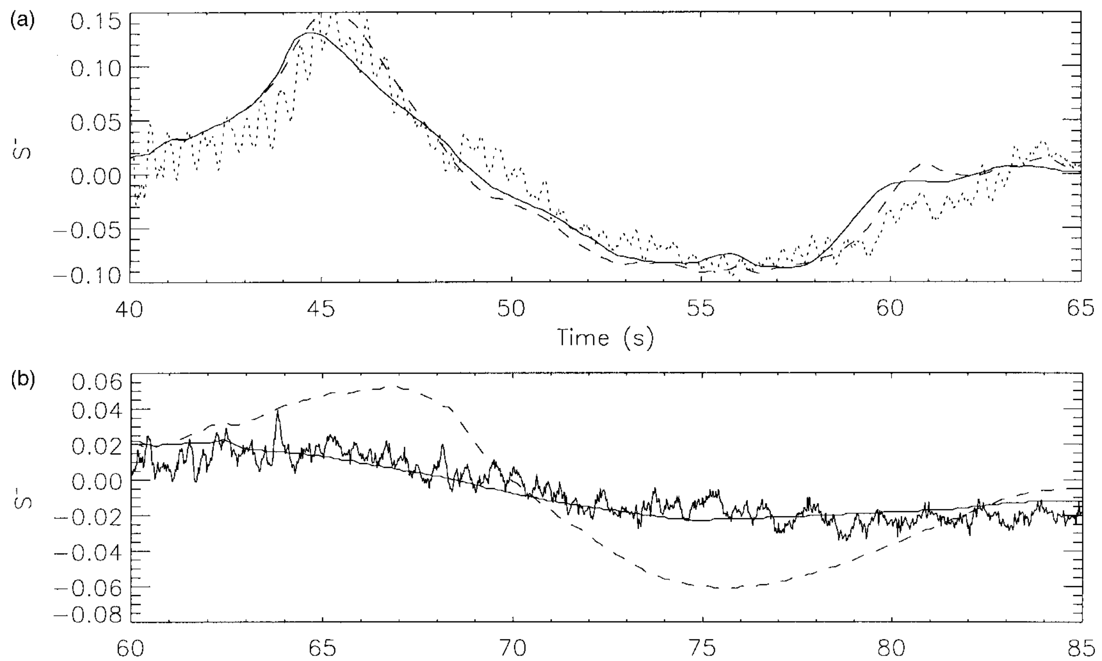

Figure 1. The reflected low frequency wave generated by a group of waves incident on a beach. Solid line irregular line: experimental measurement. Dashed line: inviscid solution of the NLSW equations. Smooth solid line: NLSW solution including a friction term. (a) Beach slope 1:20. (b) Beach slope 1:100. [Courtesy of T. Barnes.]

long wave as described by Longuet-Higgins and Stewart (1964). However, this theory is not applicable in shallower water, e.g., where individual wave crests can be treated like solitary waves. Initially sinusoidal wave groups were used, but the strong generation of higher harmonic free waves gave unrealistic wavefields, e.g., see Galvin (1972) and Bryant (1973). Instead wave groups were formed from a set of solitary waves superposed on a longer set-down wave. Now, the set down wave in shallow water is not defined by the shorter waves as in deep or intermediate water depths. In fact it can have different forms depending on the bed topography over which the waves have travelled. Further, it is not directly linked to the short waves as they propagate, steepen, break, and approach the shore. Except on very gentle slopes incident long waves come into a beach and are almost perfectly reflected: e.g., see Guza and Thornton's (1985) analysis of field measurements.

Various different set-down long waves were used as initial conditions, but it is clear in many of the records that the long set-down wave travels at the linear long-wave velocity. The individual shorter waves travel, like solitary waves, at a higher velocity, so they climb out of the depression associated with the set-down. The end result in water of constant depth is a group of solitary waves, with no depression below the mean water surface, followed by the depression plus any short oscillatory waves that develop. Although the solitary waves correspond to the short incident waves, any attempt at analysing for long waves gives a long-wave signal from the set of solitary waves.

Further, both solitary waves and bores travel at speeds which increase with their amplitude, thus in a typical wave group, with the highest wave at the centre, there is a change in relative positions of the crests once they are in shallow water and have broken. For the waves preceding the highest wave each succeeding wave travels faster than the one in front. Thus these wave crests converge. Depending on how gentle the beach is this convergence can occur either before breaking, or when the waves have become bores, or in the swash zone.

On beaches of near-zero slope where an appreciable length of solitary-wave type propagation is expected, it is possible for the wave crests to converge into a single more substantial crest. This corresponds directly to well-known solutions of the Korteweg–de Vries equation on water of constant depth. Any sufficiently large initial crest of water eventually splits up into a set of solitary waves plus some shorter oscillatory waves. Here we have the same scenario but starting at an earlier time when the same set of solitary waves is converging to form that "initial" large wave. Of course in our case the beach is unlikely to be of precisely constant depth, the waves may not converge perfectly, and breaking may intervene before the waves reach their closest proximity. Figure 2 illustrates this behaviour with a computation of the Boussinesq equations on constant depth. A spatially periodic domain is used and both the emergence of the waves from the set down and their convergence may be seen.

As already noted, wave breaking does not diminish the tendency for crest convergence since bores can also converge. This phenomenon is already known from the analogous gas dynamic situation with shock waves, e.g., see Whitham (1974). This convergence of bores is more effective than the convergence of solitary waves since once one bore catches another they combine and continue as one. There has been little study of this effect for water-wave application. Peregrine (1974) analyses the result of such a combination of bores, drawing attention to a small reflected wave. However, this type of crest convergence is readily observed in any reasonably wide surf zone. Observations of a surf zone on a natural, plane, 1:60, beach at Putsborough, Devon were made with a time-lapse move from the top of the cliff bounding a beach. Simple counting of crests over a 28 minute period at different distances from the shoreline gave the results of Table 1, which clearly indicate the degree of crest convergence. Figure 3 is a photograph taken from beside the

Table 1. Number of waves at different offshore distances, Putsborough Beach, Devon, 25 June 1980.

Distance from shoreline in metres	Number of wave crests in 1560 seconds
30	230
20	133
10	68
0	18

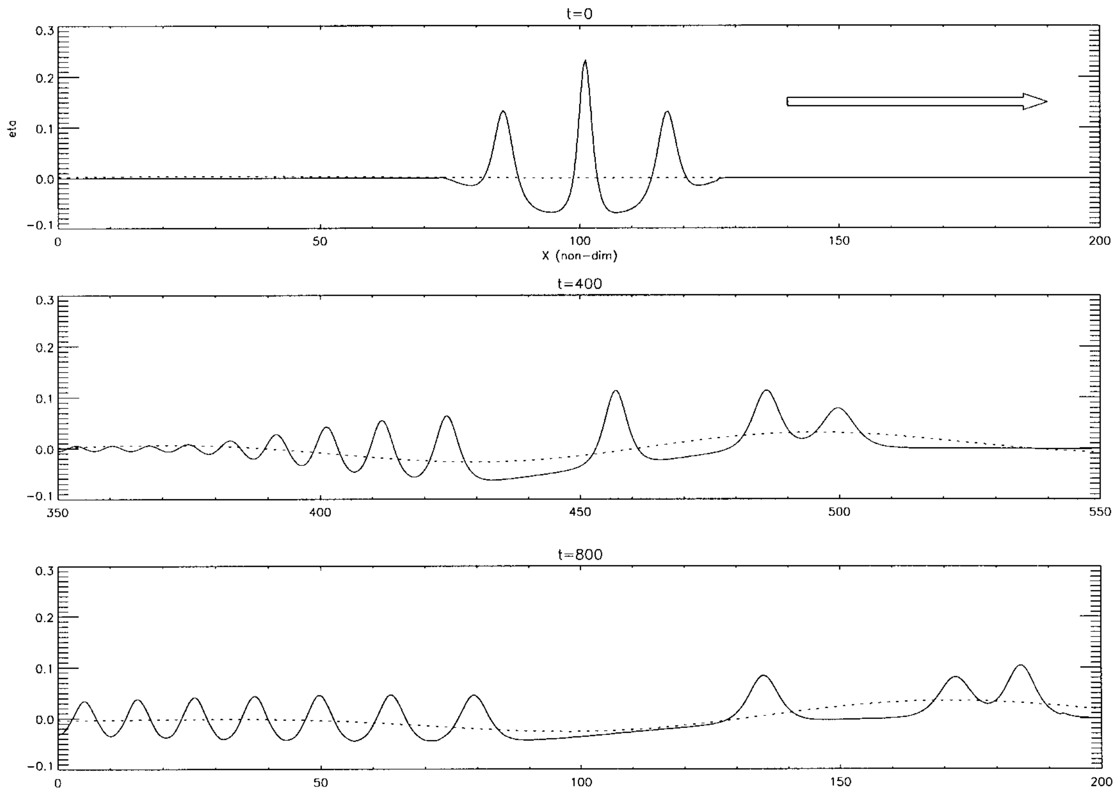

Figure 2. Evolution of a wave group computed using Boussinesq equations (solid), filtered long wave (dotted). The initial group has a set down that gives it zero excess mass. The wave crests leave the set-down behind. Dispersive effects at the tail of the depression generate the shorter waves seen on the left of the figure. [Courtesy of T. Barnes.]

movie camera after the time-lapse sequence. The beach had been surveyed and a measured rule of 100 m marked upon it for reference at low tide.

Even if crests have not merged as they approach the shoreline there is a strong tendency for the swash resulting from successive waves to combine in the swash zone to give just one single swash from a wave group, e.g., Guza and Thornton (1985) found very little energy at the incident wave frequency in their measurements of run-up, shoreline movement, from swell. Figure 4 shows a computational example corresponding to one of the experiments with wave groups. Watson *et al.* (1994) discusses the relationship between wave groups, their constituent waves and their combination in the run-up, but further study is needed to provide predictive tools.

3. Three-Dimensional Surf Zone Currents

The surf zone currents are driven by the incident waves: thus they correspond to forced motions within the surf zone. However, this forcing is rarely steady, or uniform, so the free motions of the surf zone are also relevant to understanding of the whole current field. For example, an isolated three-dimensional group of waves may arrive in the surf zone and over a short time deposit the waves' momentum in the surf zone. This can then be treated as an initial value problem for the NLSW equations. Ryrie (1983) discusses this problem for the linearized long wave equations on a plane beach, and it is clear that his primary results also hold with only a little modification for the NLSW equations.

The free motions of the surf zone fall into two categories:

(i) Irrotational flows, which are waves that propagate offshore or propagate along the shore as trapped waves.
(ii) Rotational flows with vorticity.

Figure 3. Photograph of waves on Putsborough Beach, Devon, 25 June 1980. [Photograph: D.H. Peregrine.]

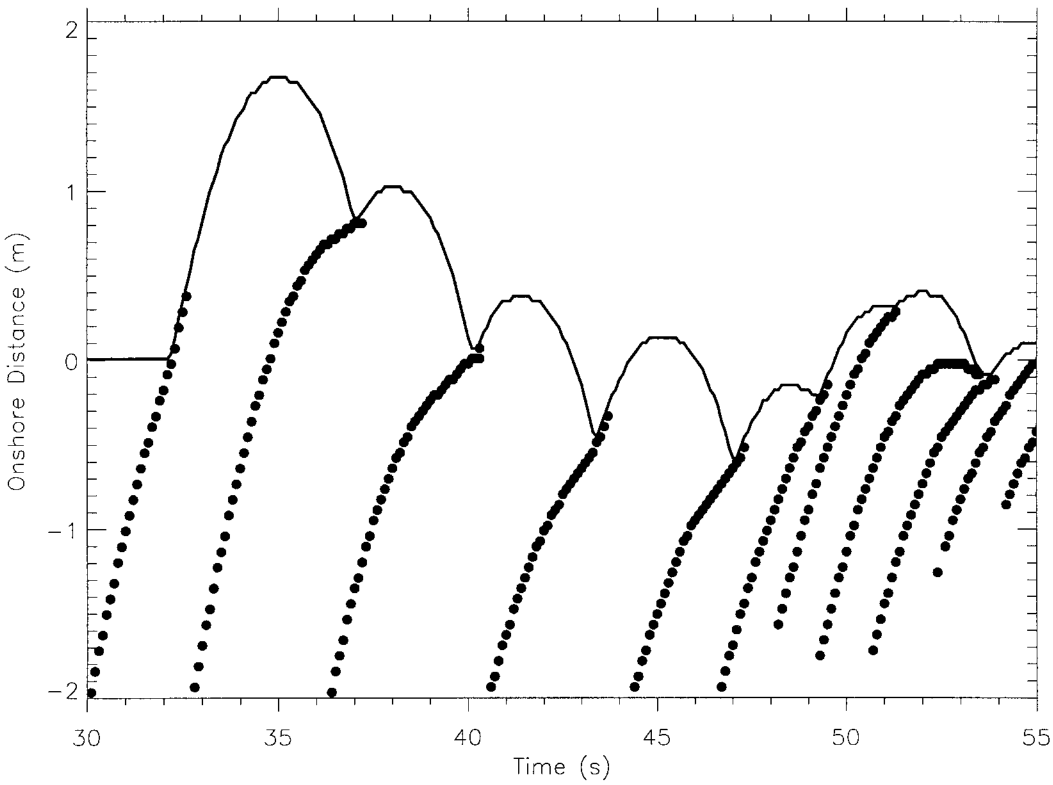

Figure 4. Space–time diagram of the shoreline movement due to a group of bores, from an NLSW computation of an experiment. Note how the first set of bores almost make one longer swash movement. The smaller succeeding bores do combine into a single swash movement. [Courtesy of T. Barnes.]

For the linearized equations these are independent modes of motion, but for the non-linear equations there are interactions. The irrotational waves may steepen and break, thus creating vorticity. Unsteady vortical flows can, in general, excite wave motions. However, these two types of flow form a useful division for the study of surf zone currents.

3.1. Three-Dimensional Wave Motions

The three-dimensional water wave solutions not only include onshore–offshore waves as in the two-dimensional case, but there are three-dimensional wave motions which are localized to the neighbourhood of the coast. On long uniform beaches these are the edge wave modes. For a plane beach, Stokes (1846) discovered the zero mode and Ursell (1952) found the higher modes which become possible for beaches of gentle slope. (The linear long wave equations have an infinite set of such modes for a plane beach, but the long wave approximation fails for higher modes at any finite beach slope.) Linear edge wave solutions are available for a range of bed topographies (LeBlond and Mysak, 1978).

Edge waves have modes which are numbered according to the number of zeros of wave elevation between the shore and the outermost part of the wave which decays exponentially seawards. For the higher modes of edge waves refraction of waves away from deeper water is seen to be responsible for their confinement to shallower regions. For the lowest-order modes, this interpretation is less appropriate but the effect is the same. These modes have been recognized from spatially distributed observations, e.g., Huntley (1976), Huntley *et al.* (1981), and Oltman-Shay and Guza (1987).

For beaches bounded by headlands or by an entrance to a bay standing edge waves corresponding to the modes fitting the beach length are most likely to be observed. On long clear beaches progressive edge waves may occur but standing edge waves are also observed. Note: no edge waves are possible for a straight coast with a vertical cliff bounding an ocean of constant depth, however, there are "whispering-gallery" modes along a curved bay in such a coast. Although it is simpler to study coasts which are uniform in an along shore direction, natural coasts show greater variety of topography. Study of the interaction of long waves with more varied topography is under way at Bristol University. It appears, from the work of Linton and Evans (1993) and Evans and Fernyhough (1995), that regular bays in a straight coast bounding an ocean of constant depth can also support an intriguing variety of edge wave modes. Further work (Santos and Peregrine, in progress) looks at the reflection, transmission, and radiation from edge waves when they meet an abrupt change in the coastal cross-shore profile.

3.2. Vortical Motions

First, consider the case of an isolated patch of vorticity such as may be deposited in the surf zone by a single wave group. We can learn a little of its general behaviour by considering two special beach topographies. These have already been noted in the above discussion: (i) the plane beach, and (ii) a vertical cliff bounding an ocean of constant depth. Typically the surf zone currents are of sufficiently low Froude number, $u/[g(h+\zeta)]^{1/2}$, that surface fluctuations, $\zeta(x, y, t)$, may be ignored to a first approximation, except close to the shoreline. This has a considerable advantage for the two special beach profiles. The vertical cliff topography simply corresponds to the case of ordinary two-dimensional flow bounded by a wall. The plane beach topography corresponds to flow in a wedge of small angle, which may be considered to be a slice from an axisymmetric flow field, without swirl, where the axis of symmetry coincides with the shoreline.

For axisymmetric motion there is a well-known solution corresponding to a patch of vorticity next to the axis of symmetry. It is Hill's spherical vortex (Hill, 1894; see Batchelor, 1967, p. 526). We can think of this as an eddy moving along the beach under the influence of its image on the sloping bed. That is, the part of the full axisymmetric circular vortex lines outside the water may be considered to be the image of the portion of vortex line between the free surface and the bed. Hill's spherical vortex is not a stable flow (Pozrikidis, 1986) but it is just one member of a whole set of steadily propagating vortex patches with uniform potential vorticity (Norbury, 1973). These lead to the tentative conclusion that a patch of vorticity of one sign would propagate parallel to the shoreline, or perhaps, in more complicated topography such as when there is a sand bar, a patch of vorticity may tend to propagate along bed contour lines if depth variations are sufficiently gentle.

There is a two-dimensional equivalent of Hill's spherical vortex. It is a patch of uniform vorticity, known as a Lamb dipole, see Meleshko and van Heijst (1994b) for a historical discussion of this and related flows. Indeed, Jiménez and Orlandi (1993) have studied the behaviour of an initial patch of vorticity by a wall. An initial elongated patch reorganizes itself into a sequence of Lamb dipoles. It should be noted that the widest of these extends twice as far from the wall as the initial distribution of vorticity. Thus one might expect irregular distributions of surf zone vorticity evolving in a similar manner to spread a similar distance outside the surf zone. This may, in part, be a reason for most measured long-shore current profiles extending outside the breaker zone. On the other hand vortices do develop on long-shore currents, as noted below.

When more than one patch of vorticity, or eddy, is considered then the range of behaviour of coaxial vortex rings gives some indication of possible motions, as does the behaviour of line vortices. Thus we can expect like-signed pairs of eddies to circle each other if close enough and proceed to merge, whereas a couple of opposite-signed eddies will tend to propagate, either shoreward and then separate, or travel offshore where they would slow down. Couder and Basdevant (1986) introduces the nomenclature of pairs and couples to distinguish two eddies of like and opposite sense, respectively. That paper also gives fine photographs of the behaviour of a two-dimensional flow with eddies, in a soap film.

The axisymmetric vortex-ring model of surf-zone eddies can be useful for eddy couples that are moving into deeper water. The effect of deepening water is captured by the vortex ring image which can be further adjusted for the local bed slope. Shariff and Leonard (1992) give a review of vortex rings. Chu *et al.* (1995) describe a detailed experimental and numerical investigation of the head on collision of equal axisymmetric vortex rings, including dissipative effects which makes good analogy for an eddy couple propagating directly offshore.

Alternatively the change in velocity as a couple of eddies propagate offshore is described by noting that potential vorticity, $\Omega/(h + \zeta)$, is conserved following a fluid particle, where Ω is the vorticity,

$$\Omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}.$$

Thus as an eddy couple moves offshore into deeper water the vorticity increases, but the area of the vortex patch decreases, with the net result that the circulation around the eddy remains constant. Since the area is reduced this means the magnitude of both the water velocities at a given distance away from the patch and the eddy-couple velocity are also reduced. Despite the influential role of potential vorticity in studies of geophysical fluid dynamics over much of this century, there has been little application of the concept to surf zone problems, except for a paper by Arthur (1962) on the particular example of rip currents.

Of course the surf zone does not receive just a few patches of vorticity. There is a continuous forcing from the dissipation of the incident waves which is irregular in both space and time. Thus it may be more constructive to think of flow in the surf zone as being a strip of two-dimensional turbulence along the coastline. Two-dimensional turbulence differs markedly from three-dimensional turbulence. In two dimensions, enstrophy (square of the vorticity magnitude) is conserved, and there are a number of illustrations of how this leads to local concentrations of vorticity. McWilliams's (1984) numerical computations on a spatially periodic domain show how concentrated eddies develop. Dracos *et al.* (1992) experiments with a turbulent jet emitted into successively shallower water show how larger vortex scales become dominant as the water depth is decreased. Completely different experiments, in stratified flow, such as those of Boubnov *et al.* (1994) also show stable vortex structures. The observational evidence from the ocean is also strong (Robinson, 1983; McWilliams, 1985).

Observation of the velocity field in the surf zone is difficult. The motions are dominated by the incident breaking waves so that in most circumstances the motions on longer time scales are only obtained by low-pass filtering of measurements from current meters. Even in the large-scale U.S. experiments at Duck, NC, with dozens of current meters it is difficult to spot the signature of a passing eddy. There are some velocity measurements which may be interpreted as a sequence of eddies. Oltman-Shay *et al.* (1989) describe large oscillations of a long-shore current. They had a sufficient number of current meters to show that the flow structures that were causing the oscillating measurements travel with a velocity which is within the range of mean velocities measured across the current. This is just the type of velocity that is to be expected for eddies carried by the current. Experiments by Reniers and Battjes (1997) show some of the same features. Initial theoretical studies of the phenomenon were concerned with linear stability (Bowen and Holman, 1989; Dodd *et al.*, 1992), but numerical studies show concentrations of vorticity (Nadaoka and Yagi, 1993; Allen *et al.* 1996).

Figure 5. Rip currents, Rosarita Beach, Baja California, Mexico, October 1956. The wide round head of the currents indicates the existence of an eddy couple [Courtesy D.L. Inman.]

There is one type of occasion when surf zone eddies may become clearly visible. Sometimes the strength of the breakers means that there is substantial suspension of sediment in the surf zone, but not outside the surf zone. This means that when an eddy couple forms and propagates outside the surf zone, the eddy cores transport sediment laden water out into clear water. Inman *et al.* (1971) have a photograph, reproduced in Figure 5, showing such eddy couples, which are usually described as rip currents. Figure 6 shows another example. Smith and Largier (1995) describe the detection of such an event with acoustic-Doppler observations from Scripps pier.

These eddy couples may be a potent source of sediment transport out of the surf zone. The photograph in Figure 7 shows a number of areas of discoloured water stretching out from the surf zone which seem to indicate an episodic character for offshore sediment transport such as can occur from eddy couples propagating offshore. Inman *et al.* (1971) have a similar photograph. The topic of transport of pollutants or sediment by persistent eddies is ripe for further development. Meleshko and Van Heijst (1994a) give an account of the topic. Lingevitch and Bernoff (1994) study particles carried by a two-dimensional vortex couple.

3.3. Generation of Vorticity

Forcing of surf zone motions is usually assigned to gradients of radiation stress. This leads to a view of the surf zone with emphasis on the gradients of surface elevation driving currents. Lighthill (1963) showed how useful it is in other areas of fluid mechanics to study a flow's vorticity; so it is natural to review the vorticity of surf zone currents. Thus it is desirable to have a more transparent analysis of vorticity generation. If the radiation stress forcing terms are carried through to the potential vorticity equation, the resulting expression is a little too complex to be useful. We thus proceed to a direct consideration of vorticity and circulation without averaging over the incident waves.

Using the NLSW equations as our model, note that Kelvin's circulation theorem holds for any material circuit in the smooth parts of the wavefield. However, it does not apply to circuits cutting across bores. In

Figure 6. Photograph of the surf zone looking north at Nazaré, Portugal, 20 September 1996, showing an intermittent rip current. [Photograph: D.H. Peregrine.]

particular, consider the case illustrated in Figure 8 where a wave has a finite crest length leading to a bore of finite length. The material circuit C crosses the bore just once, that is, it goes around one end of the bore. To find the rate of change of circulation we need to break the circuit C at the bore and consider

$$\Gamma(t) = \int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{u} \cdot d\mathbf{r},$$

where $\mathbf{r}_1$ and $\mathbf{r}_2$ are adjacent points on each side of the bore. Material "particles" here are columns of liquid satisfying the NLSW equations, note the particles at $\mathbf{r}_1$ and $\mathbf{r}_2$ are moving at differing velocities $\mathbf{u}_1$ and $\mathbf{u}_2$, thus we need to be careful in evaluating

$$\frac{d\Gamma}{dt} = \lim_{\tau \to 0} \frac{\Gamma(t+\tau) - \Gamma(t)}{\tau}.$$

Let A_1 and A_2 be the adjacent material points on C that are on either side of the bore at time t. Similarly let B_1 and B_2 be such points at time $t + \tau$, see Figure 9. We take the bore to have a velocity such that its component along the material circuit is in the direction from A_2 towards A_1 and from B_2 towards B_1, thus the total water depth h_2 at A_2, is greater than the total water depth, h_1, at A_1. This means the points are in the order A_1, B_2, B_1, A_2 around C, if it is taken with the sense indicated in the diagram. Thus

$$\Gamma(t) = \int_{A_1}^{A_2} \mathbf{u} \cdot d\mathbf{r} = \int_{A_1}^{B_2} + \int_{B_1}^{A_2} [\mathbf{u} \cdot d\mathbf{r}]_t$$

and

$$\Gamma(t+\tau) = \int_{B_1}^{B_2} \mathbf{u} \cdot d\mathbf{r} = \int_{B_1}^{A_2} + \int_{A_1}^{B_2} [\mathbf{u} \cdot d\mathbf{r}]_{t+\tau}.$$

Figure 7. Grande Beach on the east coast of Santa Catarina Island, Brazil, 20 January 1996. A number of plumes of sediment are visible indicated an episodic nature for offshore sediment transport. [Photograph: D.H. Peregrine.]

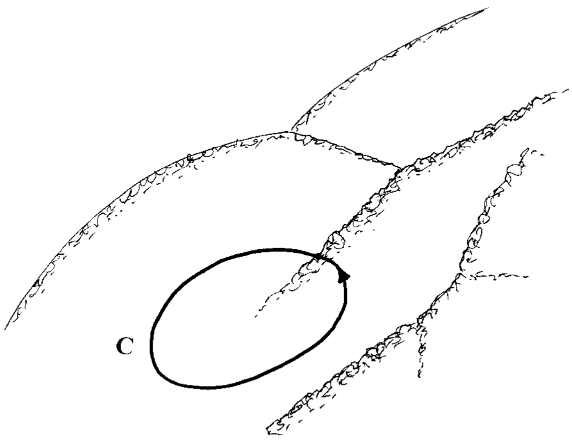

Figure 8. A sketch of a bore of finite length with a material circuit cutting it.

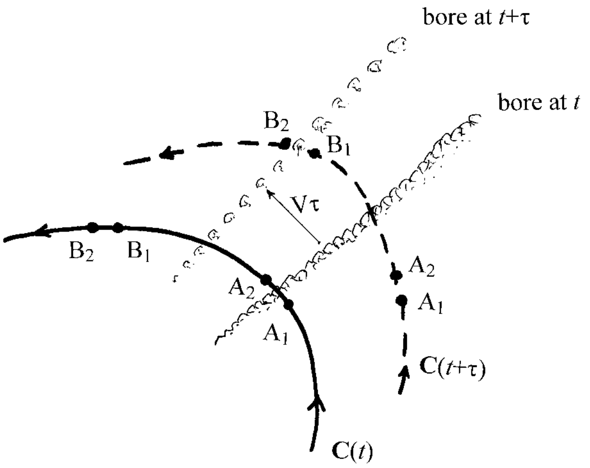

Figure 9. Sketch of the motion of a material circuit and a bore that it crosses. The broken lines indicate the position at $t + \tau$.

Hence,

$$\Gamma(t+\tau) - \Gamma(t) = \int_{B_1}^{A_2} \{[\mathbf{u}\cdot d\mathbf{r}]_{t+\tau} - [\mathbf{u}\cdot d\mathbf{r}]_t\} + \int_{A_1}^{B_2} [\mathbf{u}\cdot d\mathbf{r}]_{t+\tau} - \int_{A_1}^{B_2} [\mathbf{u}\cdot d\mathbf{r}]_t.$$

The first term in this expansion gives a contribution

$$\tau \int_{B_1}^{A_2} \left(\frac{d\mathbf{u}}{dt}\cdot d\mathbf{r} + \mathbf{u}\cdot d\mathbf{r}\right) = \tau[-g\zeta + \tfrac{1}{2}\mathbf{u}^2]_{B_1}^{A_2}$$

after following the usual steps in deriving Kelvin's circulation theorem. For the other two terms, it is convenient to introduce a local co-ordinate system with the x direction normal to the bore so that the bore has velocity $V\mathbf{i}$ and the y direction is parallel to the bore. Thus the velocities on the two sides of the bore are

$$u_1\mathbf{i} + v_1\mathbf{j} \quad \text{and} \quad u_2\mathbf{i} + v_2\mathbf{j},$$

respectively, and the bore has a forward velocity $W = V - u_1$ relative to the water in front of it.

The two integrals over A_1B_2 are on different sides of the bore and so we represent them as

$$[\mathbf{u}_2\cdot d\mathbf{r}_2]_{t+\tau} - [\mathbf{u}_1\cdot d\mathbf{r}_1]_t$$

since they are both over a distance of $O(\tau W)$. In the time interval τ the bore travels $W\tau$ in the x direction, so we may write

$$d\mathbf{r}_1 = W\tau\mathbf{i} + dy\mathbf{j}, \qquad d\mathbf{r}_2 = dx_2\mathbf{i} + dy\mathbf{j},$$

since the change in the material displacement component dy is of a higher order in small quantities.

Now through the bore the conservation of volume gives both

$$h_1\, dx_1 = h_2\, dx_2 = h_1 W\tau$$

and

$$(V - u_1)h_1 = Wh_1 = (V - u_2)h_2.$$

We also have

$$v_1 = v_2.$$

Combining the above results:

$$\begin{aligned}\frac{d\Gamma}{dt} &= [-gh_2 + \tfrac{1}{2}(u_2^2 + v^2)] - [-gh_1 + \tfrac{1}{2}(u_1^2 + v^2)] + \frac{u_2 h_1 W}{h_2} - u_1 W \\ &= -g(h_2 - h_1) + \tfrac{1}{2}W^2\left(1 - \frac{h_1^2}{h_2^2}\right),\end{aligned}$$

or, using the result $W^2 = gh_2(h_1 + h_2)/2h_1$, we find that

$$\frac{d\Gamma}{dt} = \frac{g(h_2 - h_1)^3}{4h_1h_2} = E_D,$$

say. This result shows that the rate of change of circulation, in the sense chosen, equals the rate of loss of energy by the water passing through the bore, e.g., see equation (19) of Section 187 in Lamb (1932).

The local rate of generation of vorticity can be deduced from this result by considering a small material circuit which after passing through the bore is rectangular with sides Δx and Δy. While the circuit is passing through the bore

$$\frac{d\Gamma}{dt} = E_D(y + \Delta y) - E_D(y) = \Delta E_D$$

since the circuit passes through the bore twice in opposite directions. The time taken for a side Δx to emerge from the bore is

$$\frac{\Delta x}{V - u_2} = \frac{h_2 \Delta x}{h_1 W}.$$

Thus the increase in circulation after passing through the bore is

$$\frac{h_2 \Delta x \Delta y E_D}{h_1 W}.$$

On the other hand, if the corresponding change in vorticity is $\Delta\Omega_D$, Stokes' theorem can be applied to the rectangle and gives from the integral over the rectangle that the circulation is $\Delta\Omega_D \Delta x \Delta y$. Hence by equating these two expressions

$$\Delta\Omega_D = \frac{h_2}{h_1 W}\frac{dE_D}{dy} = \left[\frac{2h_2}{gh_1(h_1 + h_2)}\right]^{1/2}\frac{dE_D}{dy}.$$

However, this not the only cause of a change in vorticity. The pre-existing circulation is conserved, but the *area* of the rectangle is changed. We have a change from this cause with

$$\Delta x_1 \Delta y \Omega = \Delta x_2 \Delta y(\Omega + \Delta\Omega_A)$$

giving

$$\Delta\Omega_A = \left(\frac{h_2}{h_1} - 1\right)\Omega$$

and hence the total change in vorticity is $\Delta\Omega_A + \Delta\Omega_D$. The change in potential vorticity is $\Delta\Omega_D/h_2$.

The derivation of this result neglects a number of terms of higher order in small quantities which are unimportant. Note that we do not expect any term depending simply on the bore's curvature since a circular bore in axisymmetric flow generates no vorticity.

Previous studies include Nof (1986), which considers the special case of flow perpendicular to a straight jump such that the flow on each side is forced to satisfy the geostrophic equation. The results are obscured by unnecessary complications in the mathematics. However, Pratt (1983), working directly from the NLSW equations in a rotating frame of reference, derives the result for the jump in potential vorticity at a bore, but does not discuss the pre-existing vorticity.

The results for the generation of circulation and vorticity are sufficiently simple that they can be used to gain a qualitative overview of the forcing of surf zone currents from a visual inspection of the breaking waves and bores. For example, when there is a gap in a breaker line, the sense of circulation around circuits cutting a breaker on just one side, or the other, indicates that there is a relative outflow current through the gap in the breakers. The existence of such an outflow can also be deduced from consideration of radiation stress gradients that lead to a set-up on both sides of the gap, but consideration of circulation, or vorticity, is more direct.

An example from analogous two-dimensional gas dynamics flows is given in numerical calculations reported by Botta (1995). The transonic flow past a circular cylinder has a finite shock wave on each side of the cylinder as the only dissipative effect in an otherwise inviscid flow. These shock waves generate substantial vorticity which forms large eddies as soon as there is any disturbance to the initially symmetric flow. The derivations of vorticity generated at shock waves take a differential, rather than an integral, approach to the problem, e.g., see Kevlahan (1997).

4. Conclusion

A number of topics relating to unsteady surf zone currents have been discussed. The currents are forced by the dissipation, through breaking, of incident waves. Consideration of the momentum deposited in the surf zone by a single wave group of limited extent helps to clarify the problem. In particular, it is useful to distinguish between the low frequency waves, both onshore and offshore propagating and the edge waves that propagate along the shore, and the vortical motions.

Since the surf zone has predominantly horizontal currents, and irregular forcing, the vortical motions are likely to take the character of two-dimensional turbulence. The most distinctive feature of such turbulence is the way in which the vorticity becomes concentrated into strong eddies. Most studies of two-dimensional turbulence have been for unbounded or periodic domains. In the case of the surf zone the turbulent region is finite. From observation, and other examples, see especially the photographs in Couder and Basdevant (1986), we can expect eddy couples to migrate out of the turbulent region. On most beaches their migration is limited by the deepening water where a useful correspondence with axisymmetric vortex rings is noted.

There are indications from the two-dimensional studies of Jiménez and Orlandi (1993) that the concentration of vorticity could lead to cases where vorticity that is initially generated in a relatively thin layer along the coast may concentrate into a patch of vorticity with a greater offshore dimension. Such behaviour, and the occurrence of vortex couples may explain the large horizontal eddy viscosities needed to model measured mean long-shore velocity profiles.

New results are presented for the generation of circulation and vorticity by bores. These results can be helpful in interpreting surf zone flows on the basis of their vorticity, even though measurement of large-scale vorticity, and even recognition of eddies, in the surf zone is difficult. The value of being able to discuss flows from two different viewpoints, namely, with pressure and velocity or with vorticity, is widely appreciated, especially since Lighthill's (1963) exposition of the vorticity viewpoint.

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