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LARGE-SCALE VORTICITY GENERATION BY BREAKERS IN SHALLOW AND DEEP WATER

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Abstract – Water wave breaking is of considerable importance in the transfer of momentum, and in other transfers, between the atmosphere and oceans. Typically breaking occurs on deep water as events that have finite duration and finite spatial extent. Near shore lines most of the water motions are dominated by breaking waves. Recent work on the generation of vorticity by breaking waves and bores in the surf zone on beaches is considered and typical vortical structures are briefly discussed. Consideration of deep water breaking leads to the proposal that the end result of a breaking event in deep water may be a coherent structure within the resulting current field. Such a structure is topologically equivalent to half a vortex ring. © Elsevier, Paris

Introduction

The role of eddies in surf zone currents, and the generation of their vorticity are discussed in Peregrine (1995, 1998), with the latter paper giving a quantitative measure of vorticity generation by bores. The aim of this paper is to give an indication of the implications of this vorticity generation from three-dimensional waves, i.e. breaking waves of finite crest length, for the structure of surf-zone currents; and to extend the discussion to the effects of deep water breakers. Wave breaking on coastal beaches, due to waves propagating into shallower water, is so commonplace that no introduction is necessary. On deeper water breaking is less common and usually stimulated by the wind.

When wind blows over the ocean surface it is frequently generating water waves. These waves usually sustain a significant density of transient breakers. Sometimes these breakers are large enough to be seen as white caps. More commonly, if not almost all the time, there are smaller breakers, micro-breakers where the effects of capillarity are strong enough to prevent air entrainment but insufficient to prevent development of small scale turbulent patches similar to those created by larger breakers. Banner & Peregrine (1993) review the topic of wave breaking on deep water: the discussion below spans the whole range of breakers.

For ocean wave studies wave breaking is most commonly thought of as the cause of dissipation when wave growth and decay is being predicted, e.g. Komen et al (1994). However, whether one views the fluid dynamics on the scale of a wave, or on an oceanic scale, one of the most significant features of breaking is that it transfers momentum from the relatively well-organised motions of surface waves into a turbulent flow that represents a large part of the wave-induced currents; or, taking one step back up the chain of momentum transfer, these also represent the wind-induced currents. Breaking is also of considerable importance in the transfer of momentum from wind to waves, which is the dominant part of the wind stress (Banner & Melville 1976). It is clear that greater understanding of the dynamics of the breaking process is important for ocean and atmosphere dynamics.

The usual way to model the transfer of momentum from waves to currents is to average over the wave motion, assuming the waves to be sufficiently regular for this to be appropriate. The resulting equations have a momentum transfer term, that is known as radiation stress following its development by Longuet-Higgins &

Stewart (1964), e.g. see Phillips (1977), or Leblond & Mysak (1978). These averaged equations include mean currents and mean pressures: a standard way of studying fluid motion. However, an alternative way of analysing flows is to consider their vorticity. In the somewhat simpler environment of waves breaking in shallow water on a beach Peregrine (1998) has discussed vorticity generation. An outline of that work is given below. The vorticity from breakers of finite crest length, and duration, is discussed for shallow water waves. These results guide a further discussion of deep water breakers which indicates that their legacy is a vortical structure, which has a noticeably longer life time than the breakers themselves. In that case such structures may be among the typical coherent structures of near-surface wind driven currents.

Note: the vorticity that is being discussed here is not the vorticity caused directly by the breaking of the wave and the subsequent organised and turbulent motions on the scale of the wave crest. Rather, we are concerned with vertical vorticity on a larger scale (horizontal currents), at the scale of the crest length or wave length of the wave, such as is important in describing mean currents generated by breakers.

Vorticity generation by bores in shallow water

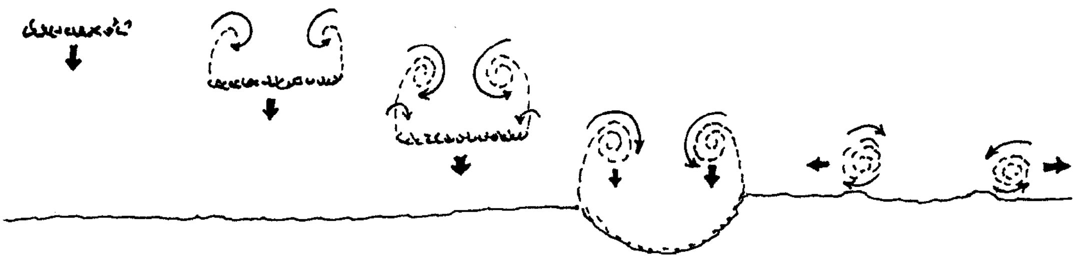
The simplest dynamic model for waves breaking on a beach of gentle slope is for shallow water. The equations for finite amplitude shallow water waves, e.g. see Stoker (1957), permit waves to steepen until the necessary approximation of gentle surface slopes is no longer valid. In practice, if the variation of surface elevation is large enough compared with the depth, waves break shortly after they steepen significantly. Thus, if details of the breaking and the associated turbulent motions are on shorter length and time scales than are of immediate interest, the breaking event can be modelled as the development of a surface and current discontinuity in the shallow water equations. Such discontinuities are bores and are dynamically consistent if mass and momentum are conserved.

For the development of wave generated currents it is these longer scales that are most relevant. Modelling of waves on a beach with the shallow water equations plus bores has developed from early numerical models (Keller, Levine & Whitham, 1960; Hibberd & Peregrine; 1979) to more effective one-dimensional models (Kobayahsi & Wurjanto, 1992; Watson, Peregrine & Toro, 1992). Only now are models with two horizontal dimensions and bores being used in this area (Peregrine & Bokhove, 1998). However, comparisons with experiment described in Barnes, Peregrine & Watson (1994) show that although details of wave breaking are poorly modelled the overall generation of currents is well described.

Kelvin's circulation theorem can be derived for the shallow water equations, as can the conservation, following fluid particles, of potential vorticity: (vorticity)/(total waterdepth). However, both of these properties require that the flows be represented by continuous and differentiable functions. The development of bores introduces a new feature: the discontinuity of velocity and depth that represents a bore gives a rate of change of circulation in any material circuit that cuts through the bore unless it has another section through the bore in the opposite direction at a point where the bore has the same properties. It is clear that velocity, and hence circulation along a line, is changing most rapidly at bores. By considering the effect of a bore on a material circuit over a small time interval Peregrine (1998) derives the rate of change of circulation due to one intersection with the bore. It turns out to have a magnitude equal to the rate of energy dissipation in the bore at that point, divided by the water density. This neat result has the nice feature that to a large extent the rate of dissipation is related to the visible strength of a bore in terms of the intensity of splashing and air entrainment.

However, here we need to discuss the large-scale vorticity generation. Peregrine (1998) derives a formula for the generation of vorticity by considering the change in vertical vorticity, Ω, that occurs in an infinitesimal material circuit as it passes through a bore. This was originally found by Pratt (1983) using the shallow water equations and bore relations. The result, at a bore that has an increase in water depth from h_1 to h_2, is $\Delta\Omega_A + \Delta\Omega_D$. Here $\Delta\Omega_A$ is the change in vorticity due to the vertical stretching of fluid elements:

$$\Delta\Omega_A = \left(\frac{h_2}{h_1} - 1\right)\Omega\,,$$



(a) (b) (c)

FIGURE 1. Shallow water bore of finite length: depicted by an irregular heavy line. Vorticity generated by the breaker denoted by circular arrows. (a) Shortly after the formation of the bore by wave breaking. (b) A little time later. (c) Just after cessation of breaking at the shoreline.

where Ω is the original vorticity of the fluid element. This part represents no change in the potential vorticity. The motion of the bore, over running water in front of it gives the change of vorticity

$$\Delta\Omega_D = \left[\frac{2h_2}{gh_1(h_1+h_2)}\right]^{\frac{1}{2}} \frac{dE_D}{dy}, \quad \text{where} \quad E_D = \frac{g(h_2-h_1)^3}{4h_1h_2}$$

is the dissipation rate at the bore divided by fluid density.

As may be seen a bore with uniform properties, such that E_D is constant, conserves potential vorticity. On the other hand if the bore varies due to non-uniformities in E_D, then new potential vorticity is generated. Non-uniformities in E_D come from variations in the water depths h_1 and h_2, either because of non-uniformities in the bed, or because of variations along the wave crest. The effect of bed non-uniformities is discussed in Peregrine & Bokhove (1998) with numerical examples. Here we discuss along-crest variations. In particular, for any finite length breaking wave crest the most rapid along-crest variation is at the ends where there is usually an abrupt demarcation between the breaking crest and an unbroken water surface.

Shallow water examples of finite breaking wave crests

Figure 1 is a sketch of the evolution of vorticity due to a breaker crest of finite length. In figure 1(a) the wave has recently started breaking and the vertical vorticity is being most strongly generated at the margins of the breaker. Once vorticity is generated it is convected with the fluid particles. However, the motion of the water is influenced by the vorticity. As is well known the 'free' end of a vortex sheet rolls up: for example the starting vortex from an airfoil at incidence, or from the edge of a plate caused to move perpendicular to its plane. Thus once the breaker has moved on, in figure 1(b), vorticity is still being generated, but the trailing vorticity is beginning to roll up. Further on where the breaking has ceased, the whole strip of vorticity can roll up as sketched in figure 1(c). However, at this point there are various possible scenarios.

In shallow water on a beach breaking doesn't usually cease until the wave meets the shoreline, or else meets deeper water as when passing over a bar to an inshore trough in the bed. In either case the vortex arising from each side of the crest begins to move according to its environment. If no other current structures are around and the vortices are not near the shoreline, they influence each other and proceed onward in the wave's direction as a vortex pair. On the other hand if they are near the shore line, they have an effective image in the shoreline which acts to move each vortex parallel to the shore such that their separation increases. [The action of an effective image of a vortex over a sloping bed near the shore line can be modelled with the family of axisymmetric vortices that has Hill's spherical vortex as a limiting member: see Peregrine (1998).]

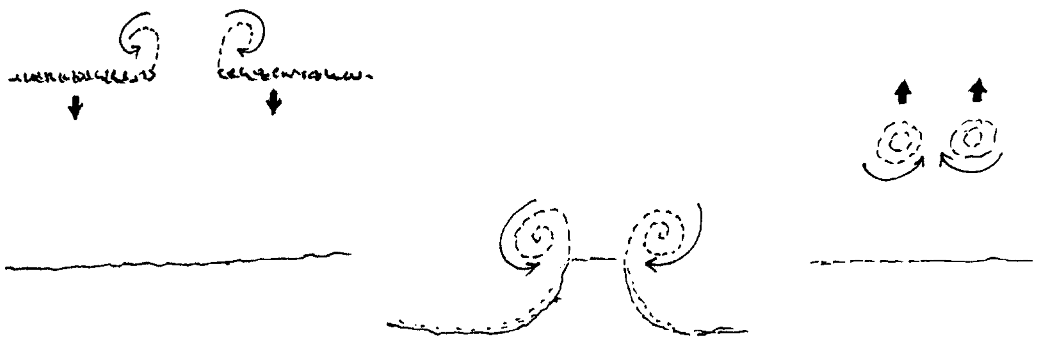


FIGURE 2. A gap in the breaking of a bore and its vorticity. Conventions and stages as in figure 1.

If the breaking wave crest is not very long, so that the strips of vorticity that are generated are much longer than the spacing between them, a more complex rolling up of the vorticity is likely. For example, when a ring vortex is generated by expelling fluid from a sharp edged tube there is a maximum size of vortex generated. Such experiments are described and discussed by Gharib, Rambod & Shariff (1998).

On beaches wave crests are usually so organised that they are rather long and the greatest variation in the breaking comes from a gap in a breaking crest. As such a gap propagates with a wave the ends of the breakers leave strips of vorticity that can roll up as sketched in figure 2. This time if two vortices form and migrate under their mutual influence they move in a direction opposite to the that of wave propagation. This is a relatively common feature on beaches. Just occasionally such a vortex pair becomes visible due to having formed in a region with suspended matter and then moved with that water into a clear region. More commonly they are registered as causing an intermittent rip current. Once again if the spacing between the vortex strips is much shorter than their length the flow is not so simple. The two strips, even if they become convoluted, then form the boundaries of a longer lasting rip current.

Deep water breakers

As yet there is no simplifying model of breakers in deep water like that of a bore in shallow water. However, if one looks at a surface layer of water, in either case, the surging forward of the water in a breaker can be expected to have the same qualitative effect as it transfers momentum from the wave motion to form currents. Thus the sketches in figures 1 and 2 should bear some resemblance to the surface motions due to deep water breakers. Given the usual character of deep water waves with relatively short wave crests it is only figure 1 that has relevance. However, such a picture of the flow is incomplete. Vortex lines can not end within the interior of the fluid. For shallow water they easily reach the bottom. For deep water they must return to the surface. Thus the only sensible interpretation of figure 1 for a deep water breaker is that the vortex lines for the two vortices must be connected beneath the water surface. A sketch is given in figure 3.

There is no difficulty in appreciating the vortex connection between the two vortices since the surface water in a breaker must induce an appropriate shear relative to the undisturbed water beneath it, as in Peregrine & Svendsen's (1978) description of the breaker turbulence as being like a mixing layer between two flows. The turbulence left by the breaker spreads downward to give the vorticity the same, larger, length scale as the trailing vortices. The same shear beneath the surface occurs for breakers in shallow water, but before the shear spreads to the larger scale relevant to the currents it has met the bed where vorticity is absorbed. The current length scale is usually many times the water depth in the surf zone.

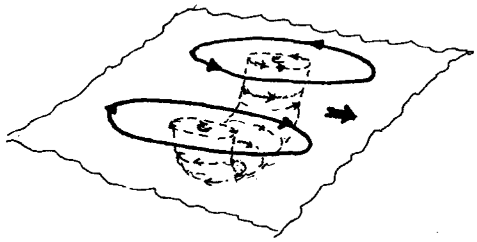


FIGURE 3. A sketch of the current field derived from a deep water breaking wave shortly after the event.

A following paper of the Symposium, Nepf & Wu (1998) gave a description of measurements of the current generated by a three-dimensional breaker which corroborates and quantifies the description given here.

Discussion

On a beach the currents are almost two-dimensional. The general properties of two-dimensional flows, especially the flow of enstrophy to larger scales leads one to expect that vortices/eddies are dominant and long lasting. There are good numerical examples of such behaviour (Özkan-Haller & Kirby, 1998), but identifying eddies from measurements made at a restricted number of points is difficult. Sancho & Svendsen (1998) give evidence of eddies in laboratory experiments. Thus further detailed study of these shallow water flows is continuing.

For deep water waves, the above discussion suggests that the residual flow after a typical finite breaking event is something like half a vortex ring with the diameter in the free surface. It seems likely that such a flow may last significantly longer than both the lifetime of a breaker and a typical wave period. Thus it might be seen as a coherent structure within a wind driven current field. In the longer term such a vortex ring structure is likely to be much less durable than the eddies in two-dimensional flows. Presumably as it becomes more diffuse it suffers three-dimensional distortions in the ambient current field. The vertical shear can stretch it out, and eventually it contributes to that shear. A interesting question is the relationship of such structures to the longitudinal-roll type of flows, known as Langmuir vortices, that are frequently seen in wind-driven seas. See Thorpe (1995) for a review of motions in a wind-driven sea.

While there is no problem in identifying the current with a vortex ring structure generated from a three-dimensional deep water breaking wave in a laboratory, it is very much harder it the field. The waves do not break to order. The vortex structure when at its most coherent and strongest is likely to be of a similar scale to the wavelength so that any current measurement needs to be extricated from the currents due to passing waves as well as other current structures. Even so it is worth pursuing these ideas further to complete the chain of momentum transfer from wind to waves, to breakers, to vortex structures, to the mean wind-driven current.

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