Ann. Rev. Fluid Mech. 1983. 15:149-78

BREAKING WAVES ON BEACHES

D. H. Peregrine

School of Mathematics, University of Bristol, Bristol BS8 1TW, England

INTRODUCTION

Visitors to any coastline exposed to open water can see the dramatic transformation of surface waves that occurs as they advance onto a beach. The waves offshore have a relatively smooth water surface, whereas the waves arriving at the shoreline have rough white fronts of spray and bubbles. The transition between these two types of waves is the subject of this review; the term "wave breaking" is used here to describe the transition from a smooth wave to the quasi-steady state with a white-water front rather than to any particular instant within the transition.

The most prominent stage of wave breaking is the initial overturning motion of the wave crest that creates spray and white water, often by the forward projection of a jet of water. Much wave-breaking research has consisted of experiments to determine, for given offshore wave characteristics and beach slope, when and where waves first break and what type of initial breaking motion results. Galvin (1972) gives a review of such work.

The descriptive terms for breaker type use the initial motion to characterize them and are as follows (Galvin, 1968, 1972):

Spilling White water appears at the wave crest and spills down the front face, sometimes preceded by the projection of a small jet.

Plunging Most of the wave's front face overturns and a prominent jet falls near the base of the wave, causing a large splash.

[1]Copyright covers text matter only.

0066-4189/83/0115-0149$02.00

Collapsing The lower portion of the front face overturns and behaves like a truncated plunging breaker.

Surging No significant disturbance of the smooth wave profile occurs except near the moving shoreline.

There often appears to be a smooth gradation between all these types of waves, which hints at the possibility of a one-parameter family of breaking "events," once allowance is made for the geometric scale of the wave. However, it is the beach that causes wave-breaking, and beach shapes vary widely, so that it is not difficult to find occasions when waves do not fit well into the above set of descriptions. Perhaps the most frequent such exception is the *shore break*, where the whole face of the wave from trough to crest becomes vertical with relatively little or no water in front of it. Very strong turbulent motions result; these waves seem to be among those that surfers call "sand-busters" or "dumpers."

Most experiments on breaking waves are influenced by the example of a single wave train specified in deep water by frequency ω, and amplitude a_0, and incident on a plane beach of slope α. Results of experiments of this type appear to depend on two parameters: the beach slope α and initial wave steepness, e.g. $a_0\omega^2/g$. Summary diagrams for wave height and water depth at breaking may be found in texts and manuals such as Wiegel (1964), Silvester (1974), Horikawa (1978), and Coastal Engineering Research Center (1977). Furthermore, some experimental results depend on a single parameter $a_0\omega^2/g\alpha^2$, discussed in the next section.

This review is oriented toward understanding the fluid dynamics of wave-breaking, rather than discussing the above type of experiment. The account of when and how waves break is entirely in terms of inviscid, initially irrotational flow, without concern for the air above the surface or surface tension. This idealization is proving sufficiently successful that other physical aspects of the problem are considered secondary and are discussed separately.

THE APPROACH TO WAVE-BREAKING

There are two major theoretical approaches to the problem of finding where waves break on a beach; both are only appropriate for beaches of gentle slope, and both originated well over 100 years ago.

Shallow-Water Steepening

Equations to describe finite-amplitude shallow-water waves are obtained by assuming that water-surface slopes are sufficiently gentle that water-particle accelerations are negligible compared with gravity. This assumption implies that the pressure at any point consists solely of the hydrostatic pressure due to the weight of water above that point. This theory was developed by Airy (1845) in response to Russell's (1834; see Miles 1980) observation of the existence of the solitary wave. Airy found that the equations can be put into a form showing that the front face of any wave of elevation propagating on water of uniform depth must steepen (see Lamb 1932, Sect. 187, or Lighthill 1978, Chap. 2). Airy, and many others since then (e.g. see Stoker 1957), took this steepening to imply that such shallow-water waves necessarily break; this despite the existence of the solitary wave, which is a wave of elevation that propagates unchanged on uniform water depth, i.e. without breaking.

Shallow-water steepening can be simply described in the case where a wave is propagating into uniform water conditions, e.g. still water of constant depth. Each portion of a wave with elevation ζ and horizontal water velocity u travels with the long-wave velocity corresponding to the total depth plus the water velocity, i.e. with velocity $[g(D+\zeta)]^{1/2}+u$, where D is the water depth and g the acceleration due to gravity. Thus the higher parts of a wave travel faster.

The result of wave steepening is that water accelerations increase to the point where they have a significant effect on the pressure, and this must be accounted for. The only analytically tractable case has been for waves of small amplitude; that is, a "near-linear" approximation is made and only the "first" nonlinear terms are included. The resulting equations are the Boussinesq equations (e.g. see Whitham 1974, Sect. 13.11). Among their solutions is the solitary wave in which the shallow-water steepening is exactly balanced by the effect of the water's acceleration, more commonly called a dispersive effect (see Miles 1980 for a survey of solitary waves).

The balance between shallow-water steepening and the effect of water acceleration is expressed in terms of the Ursell number (Ursell 1953)

$$\mathrm{Ur} = H/D\sigma^2, \tag{1}$$

where H is wave height and σ is a measure of maximum gradients, e.g. $D\partial/\partial x$. If, as is often convenient, σ is replaced by D/L, where L is the wavelength, the natural interpretation of large Ur corresponding only to shallow-water steepening is not correct, since long-wavelength waves frequently look like a train of solitary waves with long flat troughs between crests. However, small values of Ur reliably indicate that any shallow-water approximation is inappropriate (e.g. $HL^2/D^3 < 4\pi^2$).

When solutions of the Boussinesq equations are calculated for waves longer than solitary waves, shallow-water steepening occurs, but this is then countered by wave curvature and maximum elevation increasing so as to form a sequence of undulations without the occurrence of breaking. For example, a wave that is a smooth change in water depth between depth D and $D + \Delta D$ develops into a set of waves called an undular bore (see Peregrine 1966, or Fornberg & Whitham 1978, where equivalent solutions of the Korteweg-de Vries equation are given).

Despite the existence of nonbreaking solutions, a simple smooth "step-up" wave of height ΔD may break because of shallow-water steepening. If $\Delta D/D$ is greater than 0.7, then the wave continues to steepen and rapidly breaks, forming a turbulent bore (known as a hydraulic jump if there is sufficient current to hold it stationary). For the range $0.3 < \Delta D/D < 0.7$, undulations occur but the leading wave breaks (Binnie & Orkney 1955). Near the lower limit of this range, breaking may not occur for a relatively long time (Favre 1935).

From the above behavior of waves over a flat bed we can deduce when shallow-water steepening is the primary cause of breaking for waves on a beach. The waves must have propagated to a portion of the beach where (a) their length is much longer than the water depth, (b) their slopes are still gentle, and (c) their height is almost as great as the depth.

A periodic solution of the shallow-water equations on a plane beach is given by Carrier & Greenspan (1958). It can be matched away from the shore with the linear wave solution (Keller 1961), and for those cases where the beach has only gentle surface slopes it gives an accurate solution for the perfect reflection of incident periodic waves. Linear water-wave theory has solutions that correspond to perfect reflection for any beach slope, but the assumptions of linear theory do not hold at the shoreline unless the beach slope there is $O(1)$. The Carrier-Greenspan solution provides a local solution near the shoreline for gentle slopes.

A wave with near-total reflection satisfies the description of a surging wave. The Carrier-Greenspan solution gives a limit to such waves, since as the amplitude increases a vertical surface slope is predicted at the shoreline. Meyer & Taylor (1972) show that there is a solution corresponding to total reflection of waves if

$$a_0(\pi/2\alpha)^{1/2}\omega^2/g\alpha^2 \leqslant 1/2, \tag{2}$$

where the factor $(\pi/2\alpha)^{1/2}$ connects the amplitude offshore, a_0, with the amplitude of the Carrier-Greenspan solution. See Guza & Bowen (1976) for further details and experimental measurements agreeing with this result.

Parameters equivalent to $a\omega^2/g\alpha^2$, which appears in (2), have been found useful for correlating surf-zone properties as well as determining whether or not waves may break. For a discussion comparing several surf-zone properties, see Battjes (1974), where Iribarren & Nogales (1949) are credited with first using such a parameter to divide breaking and nonbreaking waves. As might be expected from the above, it is the properties of waves that are close to this nonbreaking condition that are well correlated by using the parameter in (2) (e.g. see Guza & Bowen 1976). Its relevance to the above discussion of equations for shallow-water waves is demonstrated by Munk & Wimbush (1969), who consider it as a ratio of $a\omega^2/\alpha$ (a measure of water acceleration up the beach slope) to αg (the component of gravity parallel to the beach). Battjes gives further physical interpretations.

Refraction and Waves of Limiting Steepness

For calculating the refraction of waves approaching a beach, the two most common assumptions are that (*a*) there is no reflection of the waves, and (*b*) the beach slope is so gentle that the waves are like plane periodic waves on water of constant depth. In the simplest case of steady waves normally incident on a beach, the amplitude variation is obtained from the constancy of wave-action flux, which is equivalent to constant energy flow in the absence of currents. The first example of this method is for linear long waves (Green 1838) and in that case it leads to Green's Law, that wave amplitude is proportional to $D^{-1/4}$.

The position of wave breaking is estimated by introducing a limit to wave steepness. The existence of a limiting wave steepness for traveling waves has been known since Stokes (1880) studied the flow near the crest of such a wave, and the limiting waves may now be calculated with considerable accuracy for any depth (Williams 1981).

In practice, coastal engineers use formulas such as

$$(Hk)_{\max} = 0.89 \tanh kD, \tag{3}$$

due to Miche (1944), where $H = 2a$ is the wave height, $k = 2\pi/L$ is the wave number, and L is the wave length. Van Dorn (1978) shows that Equation (3) is a reasonable fit to experimental data from beaches of gentle slope. Since small values of kD are most frequently encountered at breaking, both in experiments and in nature, a limiting ratio, D/H, is more often used. This ratio is called a "breaker index" and is given values in the range 1.1–1.3.

Limiting-steepness waves have provided a starting point for theoretical studies of wave breaking. Longuet-Higgins (1980a) includes an account of the part they have played in the study of breaking waves. However, development of accurate solutions for periodic waves of lesser steepness (Schwartz 1974, Longuet-Higgins 1975, Cokelet 1977b, Longuet-Higgins & Fox 1978, reviewed in Schwartz & Fenton 1982) has shown that wave phase velocity and most other integral properties of waves such as energy flow have their maximum, for given mean depth and wave number, for waves of less than maximum steepness. When accurate wave solutions are used in a refraction calculation, this results in (*a*) two possible steep-wave solutions for a restricted range of water depths, and (*b*) in solutions ceasing to exist for shallower water (e.g. Sakai & Battjes 1980, Stiassnie & Peregrine 1980, and Ryrie & Peregrine 1982). [There is a second type of double-valued solution in Ryrie & Peregrine (1982), which Peregrine (submitted for publication) discusses in more detail, showing the second solution is rarely relevant.] The first two of these papers show reasonable agreement for wave height and velocity with the detailed measurements of Hansen & Svendsen (1979), except within one wave length of breaking.

In experiments, waves on beaches do not retain the symmetry about their crests that a periodic wave train on uniform depth has. Hansen & Svendsen (1979) include measures of asymmetry and profile measurements that illustrate this point. The same authors (Svendsen & Hansen 1978) give a perturbation analysis of cnoidal waves on a sloping bottom. Their comparisons with experiment look satisfactory in the range $25 \leqslant \mathrm{Ur} \leqslant 300$, and they find a maximum asymmetry for $\mathrm{Ur} \sim 50$. Here we are using $\mathrm{Ur} = HL^2/D^3$.

Any solution depending on local plane-wave solutions, or perturbations to them, implies that wave properties can adjust to changes in depth. If the time scale of such an adjustment is long, then there must be a correspondingly slow variation of depth. An adjustment involving adjacent waves can occur more readily in deep water, where any disturbance is communicated through the half-space of fluid, than in shallow water, where disturbances propagate along a strip of fluid with velocities that cannot be much greater than $(gD)^{1/2}$. An indication of this difference is the way the ratio of group velocity to phase velocity approaches unity as $D/L \to 0$. In long waves, wave crests behave like independent entities. Thus periodic waves can be accurately represented by a train of solitary waves [e.g. see Stiassnie & Peregrine 1980, Witting (unpublished) 1981, and Williams 1981].

The "rate of adjustment" of a train of solitary waves to disturbances can be estimated from the interaction between a nearly equal pair of solitary waves for which there is an exact solution of the Korteweg-de Vries equation (Whitham 1974, Equation 17.21). (The Korteweg-de Vries equation is an appropriate approximation to the Boussinesq equations.) By using such an estimate, Stiassnie & Peregrine (1980) show that in this regime waves have time to interact with their neighbors only if

$$\alpha < 2(3)^{1/2}(H/D)^{3/2}\exp\left[-\tfrac{1}{2}(3\mathrm{Ur})^{1/2}\right]. \tag{4}$$

This is a very severe requirement on α, since Ur should be greater than $O(50)$ for the train-of-solitary-waves approximation and if Ur = 50, the exponential in expression (4) equals 0.002.

This suggests that for practical beach slopes a slowly varying periodic-wave solution is inappropriate once Ur > 50, and consideration of each wave crest as an independent entity may be better. An appropriate solution to examine is that for a solitary wave. It has often been remarked that waves on beaches resemble solitary waves (e.g. Munk 1949). Departures from a true solitary-wave profile as it propagates over varying depth have been analyzed for (*a*) wave reflection by Peregrine (1967) and (*b*) direct perturbations by Kaup & Newell (1978). Miles (1979) discusses the changes that occur, Grimshaw (1979) provides a theoretical framework for a more general study, and Ippen & Kulin (1955) and Street & Camfield (1966) report experimental results. Chan & Street (1970) give a numerical solution. To date no work allows for seaward flow between each crest.

All the perturbations of a solitary wave as it moves over differing depths are at the back of the wave. These can grow and give rise to further wave crests (Madsen & Mei 1969), and very long waves on gentle beaches show this phenomenon (Gallagher 1972). Perhaps it is more relevant that in the regime described by the Boussinesq equations the forward face of any wave of elevation tends to become like a solitary wave, unless it is interacting with other significant waves.

Freilich (1982) has made a comparison between calculations with the Boussinesq equations and observation of ocean waves arriving at a beach in the region before they break. A very satisfactory agreement was obtained between measured spectra at depths of 10 m and 3 m when the 10 m spectrum was used as input to a spectral representation of Boussinesq's equations to calculate that at 3 m.

Instabilities

When approximate methods, such as refraction methods or integration of the Boussinesq equations, are used for waves on beaches they fail or are unreliable as waves approach the steepness that limits periodic waves, or the maximum solitary-wave height. This is usually interpreted to imply that breaking soon follows, as is observed to be the case (except where the possibility of multiple-crest formation must also be considered). If waves at this point are close to symmetrical, the study of wave-train instabilities is relevant.

For symmetrical periodic traveling waves, Longuet-Higgins (1978b) has found that deep-water waves become unstable with a rapidly growing instability once their steepness, ak, exceeds 0.406. This should be compared with a maximum steepness of 0.443 and the steepness of maximum phase velocity, which is 0.436. The perturbation giving this instability involves a reduction in amplitude of alternate crests and an increase in amplitude of the remaining crests. Longuet-Higgins finds that the perturbation has no contribution from the first harmonic in its Fourier description and is stationary relative to the wave; hence he associates the instability with the maximum of the first harmonic of the basic periodic solution, which Schwartz (1974) shows to be at $ak = 0.412$.

However, Longuet-Higgins (1978a) shows that there is almost certainly an instability associated with the maximum of a wave's phase velocity. Further support for this instability comes from Cleaver's (1981) stability analysis of Longuet-Higgins & Fox's (1977) local solution for flow at and near the crest of steep waves. Cleaver finds an instability that becomes more stable if matching conditions from less-than-maximum-steepness waves are included.

Numerical computations (Longuet-Higgins & Cokelet 1978) clearly show that the first-mentioned instability rapidly leads to wave breaking. It is possible that the energy transfer between wave crests is simply sufficient to increase the local wave steepness to a point where the second instability, associated with maximum phase velocity, can grow to breaking. There is a corresponding slackening followed by a final increase in the growth rate shown in Figure 16 of Longuet-Higgins & Cokelet (1978), together with a localization of the perturbation, as shown in their Figure 14. Thus it appears that all waves close in form to the steepest possible wave suffer from one or more rapidly growing instabilities. It is appropriate to refer to them as "Longuet-Higgins instabilities."

There are other instabilities of wave trains. The modulational instability of Benjamin & Feir (1967) is investigated by Longuet-Higgins (1978b) and Cleaver (1981) and does eventually lead to wave breaking (Benjamin 1967, Longuet-Higgins & Cokelet 1978). However, (*a*) it is likely that the final breaking process can be ascribed to a Longuet-Higgins instability, (*b*) the growth rate of the instability is relatively small, and (*c*) it diminishes as the water depth is decreased, so it is unlikely to be of much importance on beaches.

An analysis by McLean et al. (1981) of three-dimensional perturbations extends Longuet-Higgins (1978a,b). The Benjamin-Feir instability is found to extend to waves of steepness $ak = 0.39$ before disappearing. The Longuet-Higgins instability that involves alternate wave crests is found to exist, with small growth rates, at all wave steepnesses and to have a maximum growth rate for oblique perturbations. Su et al. (1982a) report experiments in which steep two-dimensional deep-water waves develop a three-dimensional breaking pattern that is qualitatively similar to what one would expect for this instability. The work of McLean et al. (1981) does not extend to sufficiently steep waves to examine the phase-velocity-maximum instability. For values of $kD = O(1)$, the Benjamin-Feir instability diminishes in importance (Cleaver 1981), but experiments (Su et al. 1982b) and theory (McLean 1982) show that the alternate-crests instability becomes more important.

For shallow-water depths, the modulational and alternate-crests instabilities are likely to be unimportant because they involve transfer of energy between each crest and its neighbors. Natural breaking waves are frequently observed to be uniform along their crests for considerable distances, so it is also unlikely that three-dimensional instabilities are important in this context. This leaves the constant-phase-velocity instability as the one most likely to be relevant to wave breaking, though even that may often be of little or no significance, since waves on a beach are being subjected to a finite disturbance.

WAVE OVERTURNING

In a majority of wave-breaking events on beaches, an element of the water surface becomes vertical; a portion of the surface then overturns, projects forward, and forms a jet of water. Such overturning may be small or large compared with the wave and is well developed in plunging breakers. Observation shows it occurs both when waves are nearly symmetrical (perhaps liable to instability) and quite asymmetrical (usually due to shallow-water steepening).

Overturning looks very similar whatever its scale, suggesting that there may be some similarity solution that gives a local description of the overturning motion. The demonstration by Cleaver (1981) that the local flow at the crest of a steep progressing wave suffers from an instability supports this notion. It is possible that the full nonlinear development of the most unstable linear perturbation gives such a similarity solution. However, the disturbances that provoke breaking are not infinitesimal, nor are the waves initially symmetrical. Numerical solutions for overturning waves give only a little support to this idea.

Experimental measurements of wave profiles and of velocity distributions as waves begin to break are numerous. Recent publications are by Van Dorn (1978), Hansen & Svendsen (1979), Kjeldsen & Myrhaug (1980), Flick et al. (1981), and Hedges & Kirkgöz (1981). Reference to these papers and the surveys by Galvin (1972) and Cokelet (1977a) will reveal most earlier work. Field measurements are less comprehensive. Papers to refer to are by Iwagaki et al. (1974), Thornton et al. (1976), Suhayda & Pettigrew (1977), Weishar & Byrne (1978) and Hotta &

Mizuguchi (1980). However, since the flow is unsteady, measures of acceleration or pressure are desirable; these have become available from numerical solutions for wave overturning.

The first demonstration of a numerical solution for wave overturning is by Longuet-Higgins & Cokelet (1976) and has been followed by others—Longuet-Higgins & Cokelet (1978), Cokelet (1978), Peregrine et al. (1980), Vinje & Brevik (1980), McIver & Peregrine (1981), and Srokosz (1981). The comments here are based on these papers and further results from P. McIver and A. New of Bristol University. Except for the work of Vinje & Brevik (1981) and A. New, all results are for deep-water waves; however, the finite-depth computations appear to be similar. A variety of disturbances are used to provoke breaking, yet superficially the overturning motions resemble each other and natural waves unless there is an appreciable standing-wave component. Velocity and acceleration fields, however, show more variation. Figure 1 illustrates a wave breaking in finite water depth.

A detailed study of a few overturning-wave solutions (Peregrine et al. 1980) reveals three features of the overturning motion, all of which are apparent *before* the face of the wave has a vertical tangent (see Figure 2).

(i) Water-particle velocities exceed the wave velocity. This property has often been quoted as a criterion for wave breaking. In fact the "wave velocity" is not well determined, since its shape is unsteady and each point such as the highest point or point of maximum slope has a different, time-varying velocity. Computations indicate that it is realistic to expect velocities up to twice the phase velocity of a linear wave.

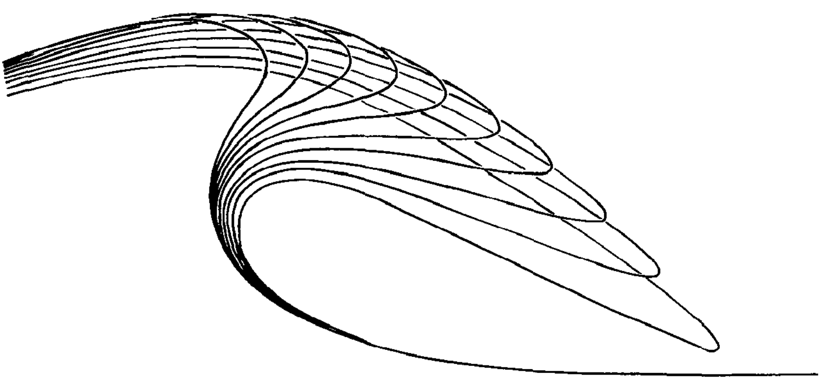

Figure 1 A sequence of computed wave profiles for a periodic wave after an "impulsive depth perturbation" from 0.20 to 0.07, where the wavelength is unity. (Figure supplied by A. New, Bristol University.) Profiles are plotted with a uniform velocity imposed to match the trough profiles.

(ii) In a thin region on the front of the wave, water accelerations exceed the acceleration of gravity. The existence of such a region was unexpected; with hindsight, it is clear that appreciable accelerations are necessary to accelerate the water near the surface that is projected forward in a jet. Computed solutions show accelerations greater than $5g$ in the subsequent development of the overturning, and it seems likely that such accelerations occur in natural waves.

(iii) An extensive, poorly defined region on and beneath the back slope of the wave has low water accelerations. This region ensures that the high pressure gradients necessary to produce the acceleration in region (ii) can exist. Hydrostatic pressure and wave asymmetry suffice to provide this "support" to region (ii). Such support can have a more precise physical form in other circumstances. For example, if a rigid vertical plate is moved horizontally "sweeping up" a layer of water, that water rises up the plate and is then projected forward. The plate will support whatever pressure is necessary to accelerate the water.

Study of the dynamics of overturning reveals nothing special about the instant when the wave first has a vertical tangent, or the emergence of the overhanging jet. The following comments provide a partial explanation. The dynamic boundary condition at the free surface is physically "weak": the pressure is constant, it is not driving the flow, all acceleration is due to the water's inertia and to gravity. The kinematic boundary condition is that the surface is a material surface moving with the water. In almost every velocity field, material surfaces become strongly distorted and folded. Thus, for motion with a free surface, one can expect the surface

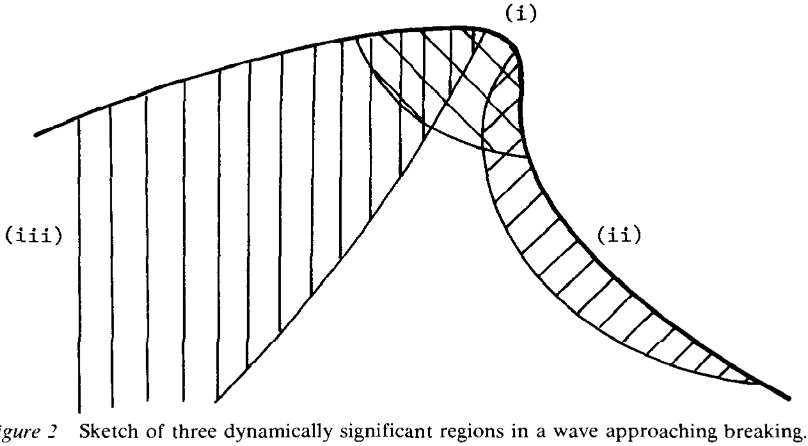

Figure 2 Sketch of three dynamically significant regions in a wave approaching breaking. (i) Particle velocity greater than phase velocity of steepest wave. (ii) Water accelerations greater than gravity. (iii) Water accelerations less than one-third gravity (after Peregrine et al. 1980).

to develop thin sheets in any region in which the water motion is so energetic that gravity is of little importance. The argument above indicates that overturning should not be interpreted as a singularity of the surface or the flow in the water. This argument is insufficient in itself to explain wave breaking. It is also relevant to the later spray-forming stages of the breaking process.

The velocity field in the water can be represented by flow singularities outside the region occupied by water. This proves to be surprisingly effective (McIver & Peregrine 1981, 1982), and only a few singularities are required. This approach was investigated when the author discovered that in certain instances most of the velocity field of the near-vertical front face of the wave could be described by a single line sink.

There are some analytic solutions that contribute to understanding aspects of an overturning wave. Longuet-Higgins (1980b) gives a solution having the shape of a rotating hyperbola with asymptotes that enclose a steadily reducing angle. McIver & Peregrine (1981) verify with numerical computation that it is a suitable model for flow near the tip of a projected jet, but in order to obtain sufficient resolution near the sharply curved tip, an atypical example with zero-gravity was examined. Longuet-Higgins (1981a) investigates the simplest combination of Stokes's (1880) solution for the crest of the steepest wave and a branch point of the velocity potential, which gives some realistic-looking profiles for the surface as it becomes vertical.

The high water accelerations on the face of the wave reveal the importance of that area. A. L. New (conference lecture, 1981) fitted ellipses to large portions of the face in some computed solutions and photographs of waves. He found that there was a good fit and the ratio of the axes of the ellipses was always close to $\sqrt{3}$. An analytic solution that has fluid round the exterior of an ellipse has been found (New, personal communication). New's results in turn stimulated Longuet-Higgins (1981b, 1982) to find further analytic solutions that satisfy the boundary conditions, and one of these, P_3, fits the underside of breaking waves remarkably well. It is a self-similar solution; in suitably oriented Cartesian coordinates the surface is given by

$$(x, y) = At^4\left(-3\mu^2 + \tfrac{1}{3}, -\mu^3 + 2\mu\right), \tag{5}$$

where A is a constant (see Figure 3). Comparisons between analytic solutions and computational results are sufficiently good (McIver & New, personal communication) to encourage further investigation.

If satisfactory analytic solutions are found, they should help to provide a measure of the "strength" of an overturning event. A quantitative measure is desirable since the established descriptive approach is entirely qualitative, and there are significant differences other than those of geometric scale relative to the rest of the wave. The emerging jet can have quite different velocities relative to the wave (e.g. see Srokosz 1981). For example, most of the face of a wave may become vertical but the jet can be very small with a low relative velocity, whereas in another case the jet can have sufficient relative velocity to plunge far ahead of the wave crest.

Another property that varies is the direction of the jet, e.g. if there is a strong reflected wave the jet can be projected almost vertically. In a field study, Weishar & Byrne (1978) found that the horizontal distance the plunging jet traveled and the time it took to fall were both greater than the simple free-fall trajectory that Galvin (1969) obtained by assuming horizontal projection at the wave-crest velocity. Their suggestion that this may in part be due to a vertical component of projection is consistent with computed solutions that show variations in the direction of projection of the jet.

The duration of the overturning flow is an aspect of the motion about which very little is known. For example, can it be defined? It is important to define some such quantity in order to be able to quantify energy and momentum loss from the wave motion.

There is likely to be some interdependence among these properties, especially for waves on beaches, where breaking most commonly occurs for the single reason that waves propagate into diminishing water depths.

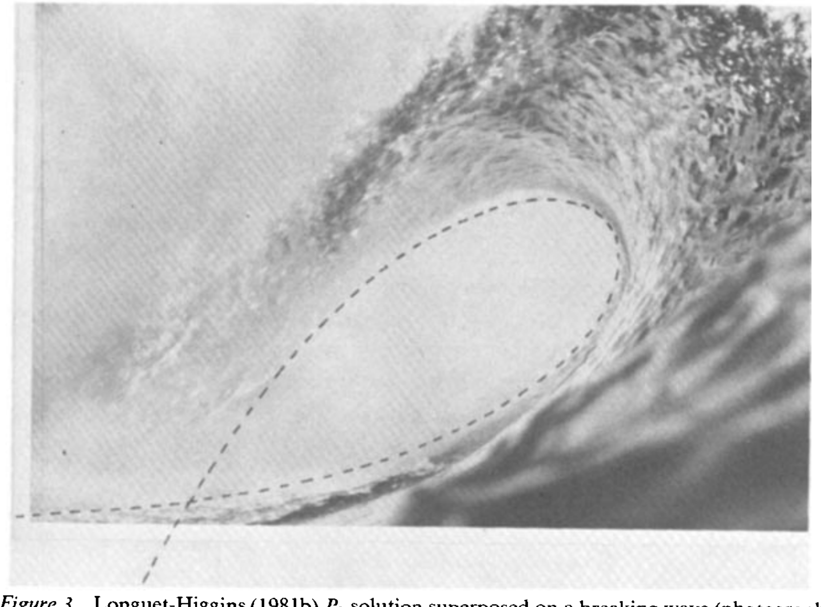

Figure 3 Longuet-Higgins (1981b) P_3 solution superposed on a breaking wave (photograph courtesy of *Surfing Magazine*).

Overturning does not necessarily occur in spilling breakers. There is reasonable doubt about the initiation of spilling breaking. In many cases it commences with a relatively small overturning. Miller (1976) illustrates this with a photograph in which overturning on the scale of 5 mm is occurring. However, in other cases white water seems to appear without surface slopes becoming near-vertical. A possible mechanism in these cases is Rayleigh-Taylor instability. This occurs if a sufficiently large portion of the surface has an acceleration component directed into the fluid greater than the component of gravity in that direction. The development of Rayleigh-Taylor instability usually leads to a highly convoluted surface that is also characteristic of a breaking wave.

There is, however, no clear evidence for the occurrence of Rayleigh-Taylor instability in water-wave motion. As mentioned above, white water appears to arise without overturning. Note that photography of wave profiles through the transparent side of a flume is an unreliable guide in this context because the wall boundary layer often "breaks" in a different manner from the main part of the waves, and bubbles in the boundary layer can obscure a small jet in mid flume (e.g. see comments in Ippen & Kulin 1955).

Steadily propagating wave solutions have no accelerations greater than $\frac{1}{2}g$, though the maximum standing wave has an acceleration of g at its crest. In computations, marginally destabilizing accelerations have been found in circumstances with an appreciable standing-wave component. The large scale of the numerical discretization relative to the size of these regions, and the presence of numerical instabilities, allow no definite conclusions to be drawn (McIver & Peregrine 1982).

Another possible "cause" of wave breaking is the steepening and breaking of short waves as they are overtaken by the crest of a longer wave. In a wind-driven sea, this occurs frequently and it is also seen on beaches. There are many factors involved in the interaction of short waves with long waves. Those interested may consult Garrett & Smith (1976) and Peregrine (1976, Sect. IIF). Dagan (1975) analyzes the case of linearized short waves on a steep free surface, which is equivalent to an instability analysis of spatial growth. For waves on beaches the simultaneous breaking of both long and short waves can occur, and it is unlikely that short waves are more than incidental to the breaking of larger waves.

THE EVOLUTION OF A PLUNGING BREAKER

Once the jet from an overturning wave hits the water, at the plunge point, water splashes up, sometimes to a height greater than the original wave. From the plunge point onward the breaking process appears to degenerate rapidly into a chaotic motion of air and water. However, close and careful inspection often reveals a surprising amount of order within the wave. A misleading impression of the flow is easily obtained, since in any one direction a single drop or bubble is sufficient to interrupt a line of sight.

The subsequent evolution of the wave depends on the position of the *plunge point*, i.e. the place and instant of time where the falling jet touches the undisturbed surface. If it is near the crest of the wave the resulting splash may be directed down the wave and it becomes a spilling breaker. At the other extreme, which is most likely on a steep beach, the jet may travel beyond the base of the wave; if it lands in water of negligible depth the jet is simply redirected up the beach and constitutes the major part of the run-up due to the wave. Such an event can lead to an excursion of the shoreline that greatly exceeds that due to a more normal wave impeded by backwash from its predecessor. The more usual intermediate case is now discussed.

The plunging jet closes over the air beneath it to form a tube around which there is considerable circulation. Air pressure usually prevents the rapid collapse of this tube, and even without air pressure the circulation around the tube implies that there is a minimum radius at which centrifugal acceleration balances an inward pressure gradient. The non-circular initial state and three-dimensional instabilities both contribute to this tube having a relatively short life. Sometimes the trapped air vents through the surface with a sudden spout of spray. Sawaragi & Iwata (1974) indicate that the tube or vortex descended toward the bed in their experiments, which included measurements of several relevant quantities.

The splash-up commences from the plunge point. At the present time only a few visual and photographic observations of the splash-up have been made and its mechanism and the origin of the water in the splash-up are not clear. The water must come partly from the jet and partly from previously undisturbed water; the division might be even or tend toward one extreme.

One view is that the jet "rebounds" (Figure 4*a*). At the other extreme, it can be considered to penetrate the surface and then, because of its forward motion and downward momentum, it acts like a solid surface and "pushes up" a jet of previously undisturbed fluid. This is illustrated by a sketch in Figure 4*b* and by a photograph in Figure 5. The photograph shows a wedge of water free from bubbles that looks as if it has been pushed upward. An intermediate possibility is sketched in Figure 4*c*.

Peregrine (1981) describes an initial attempt to analyze a simplified model of the splash-up. A thin uniform jet falls onto a thin layer of water resting on a rigid surface. A one-dimensional model is used but is inadequate because it does not consider the vorticity at the interface between the two fluid regions.

Figure 6 is a photograph of a large jet, caused by overturning, falling onto a thin fast-flowing backwash; a splash is emerging above a thin layer of air. Peregrine's (1981) analysis indicates that the relative velocity between the two bodies of water is too great for any simple splash solution to be relevant. The dynamics of this example are obscure.

The type of questions a model of the splash-up might answer could also be examined experimentally. For example, is it possible for the

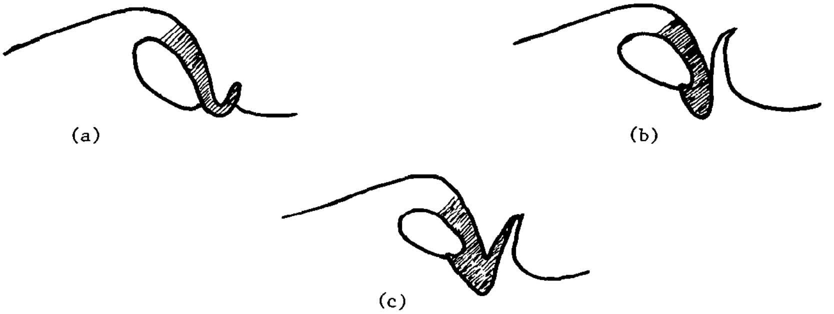

Figure 4 Sketches of possible modes of splash-up after the plunge point. Water in and from the falling jet is shaded.

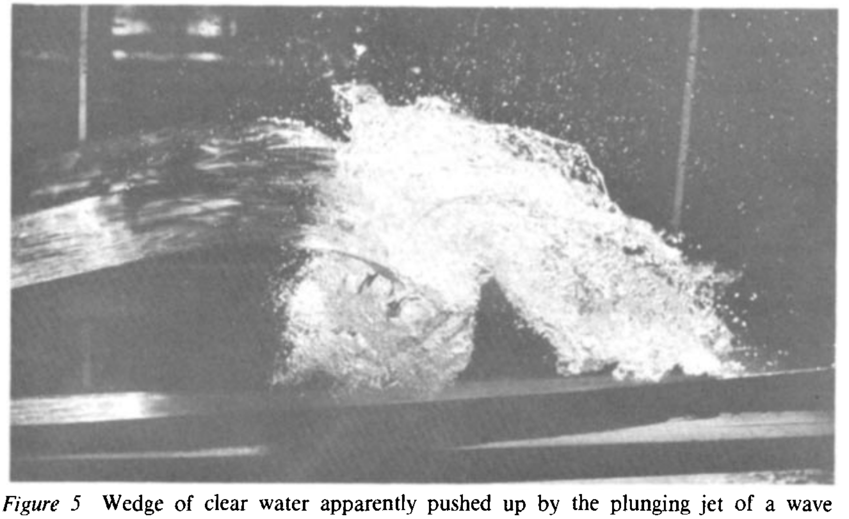

Figure 5 Wedge of clear water apparently pushed up by the plunging jet of a wave breaking from the left on a 1:35 beach. Photograph taken at ISVA, Technical University of Denmark. (The upper sloping black strip is a support for the flume.)

splash-up to fall back onto the initiating jet? Can part of the jet "return" under the tube of trapped air and augment (or disrupt) the circulation around it?

Although the range of possible motions is wide, on gently sloping beaches a plunging jet usually causes a splash to be projected forward over another tube of air to hit undisturbed water at a second plunge point, with the cycle starting again with another splash-up. Galvin (1969) calls this second plunge point the splash touchdown point and gives measurements of the distance between the first two plunge points. Miller (1976) draws attention to these cycles of plunge and splash-up that entrap tubes of air and give rise to strong vortex-like motions in each cycle.

Several splash-up cycles can occur before the turbulence, which is usually evident from the plunge point onward, destroys the organized nature of the motion and a bore results. The two photographs in Figure 7 illustrate three cycles and how the motion in the flume is sufficiently deterministic that it may be reproduced on different occasions. These cycles also occur in natural waves, as is shown in Figure 8, where the plunge point for each cycle is visible as a cleft in the surface.

The vortex-like motions from each cycle all have the same direction of rotation, and hence high rates of shear exist between them. This and the high shears arising at each plunge imply that the turbulent intensity is

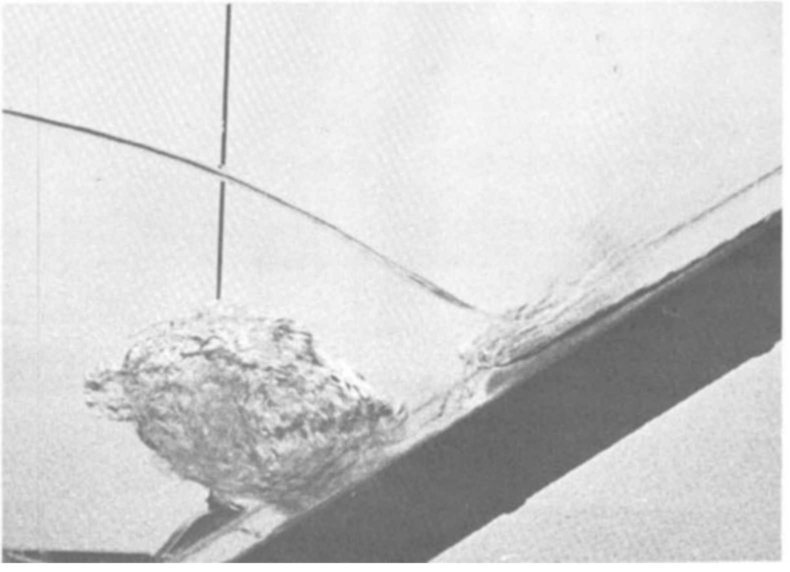

Figure 6 Splash of a large plunging jet into a thin rapid backwash.

very high. This is consistent with the rapid decrease in height and loss of energy from the wave motion in this region. Svendsen et al. (1978) call this the "outer region" of the surf zone and plot results from experiments on gently sloping beaches that indicate that many waves lose 50% of their breaking height while traveling a distance less than 10 times the water depth at breaking. Sawaragi & Iwata (1974) estimate the energy loss from the wave into the first vortex-like motion to be about 15–30% of the dissipation that occurs there.

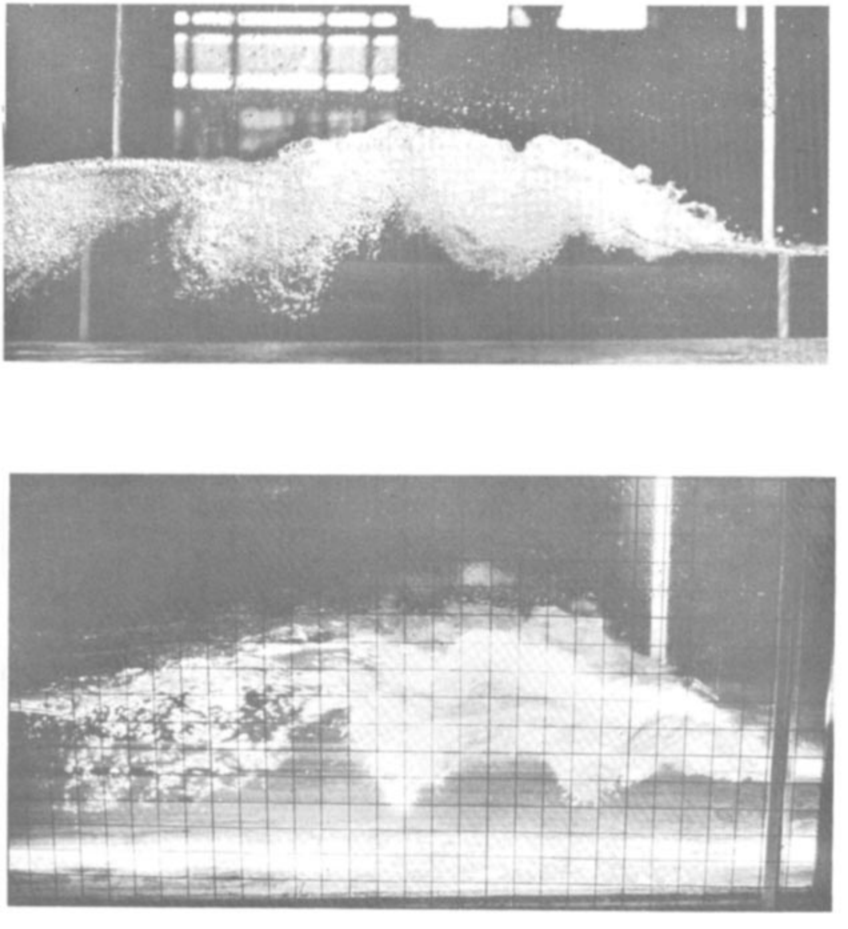

Figure 7 These two photographs are taken with the same wave conditions on different occasions and illustrate the order in the cycles of "plunge-splash-up" that occur after breaking. The exposure in the top photo is much longer than in the bottom. Bottom photograph courtesy of I. A. Svendsen. Both taken at ISVA, Technical University of Denmark.

Figure 8 A natural breaking wave illustrating the "clefts" that mark successive plunge points. Photograph taken at the Scripps Institution of Oceanography pier.

QUASI-STEADY BREAKING WAVES

Most breaking waves settle into a quasi-steady state after the plunge point and any ensuing plunge-splash-up cycles. Exceptions occur where the plunging jet or its succeeding splash penetrate shallow water and are deflected by the bed to become part of the run-up, or where irregularities of the beach cause breaking to be an intermittent process.

The quasi-steady state is one in which the wave form changes relatively slowly and has a strongly turbulent region on the face of the wave. If the turbulence is confined to a region near the crest of the wave, the wave is a "spilling breaker." On the other hand, if the whole face of the wave is turbulent it is a "bore" (or "turbulent bore" if considered in circumstances where undular bores may be occurring). There is, of course, a whole range of intermediate waves.

The mean flow in quasi-steady breaking waves includes a recirculating region, or "roller," since water can be seen "tumbling" down the front of the wave. However, the turbulent velocities in the roller exceed the mean velocities relative to the wave in the roller and so attention should be focused on the turbulence (Peregrine & Svendsen 1978). Right from the toe of the roller, turbulence can be seen to spread away from the surface. It is generated by the velocity difference between the undisturbed water and that tumbling down the wave's face. In this respect the mechanism for generation of turbulence is similar to the one occurring in the turbulent mixing layer that arises when parallel streams of differing velocities are allowed to meet. However, it is likely that active generation of turbulence is confined to those portions of the front of the wave with a significant slope of the mean surface.

A moderately detailed model of the mean flow in a steady bore has been devised (Madsen 1981, Madsen & Svendsen 1982) that gives a good account of many features, such as the roller and the bore's surface profile, for the steady case of a transition between two constant levels. Extension to the more general cases found on beaches would involve modeling the balance between (*a*) gravity causing turbulent water to fall forward, and (*b*) the wave's velocity relative to water in front, which tends to sweep the turbulent water over the wave's crest.

The classical bore model of a simple transition between uniform levels is helpful in studying the surf zone. However, the bores are often sufficiently weak, or the level behind them drops so quickly, that secondary undulations grow. On the other hand, Svendsen et al. (1978) find that their estimates of the rate of energy dissipation in bores on a beach are greater than the classical value. These properties, the dynamics of spilling breakers, and the unsteadiness of these waves are closely related and require more study.

Behind the region of active turbulence generation, the turbulence continues to spread, as is demonstrated by Banner & Phillips (1974) and Peregrine & Svendsen (1978) (see Figure 9). Detailed velocity measurements in this region by Battjes & Sakai (1981) and Duncan (1981) for a wave behind a two-dimensional hydrofoil and measurements by Stive (1980) of an ensemble of breaking waves in a laboratory beach confirm the views expressed by Banner & Phillips and Peregrine & Svendsen that the turbulence is similar to that in a two-dimensional turbulent wake. In Figure 9, the spread of turbulence is made visible by minute bubbles (rise velocity around 1 mm s^{-1}) that were originally floating on the water surface.

In considering the experiments with hydrofoils, it is tempting to extend the analogy with a wake and to use the momentum deficit associated with the wake as a parameter for breaker strength. However, if this is done a paradox appears once a turbulent bore is considered. The successful classical approach to finding the changes in height and velocity across a bore involves an assumption of conservation of momentum, and hence no momentum deficit.

This apparent paradox may be resolved by considering the dimensionless energy-momentum flux diagram introduced by Benjamin & Lighthill

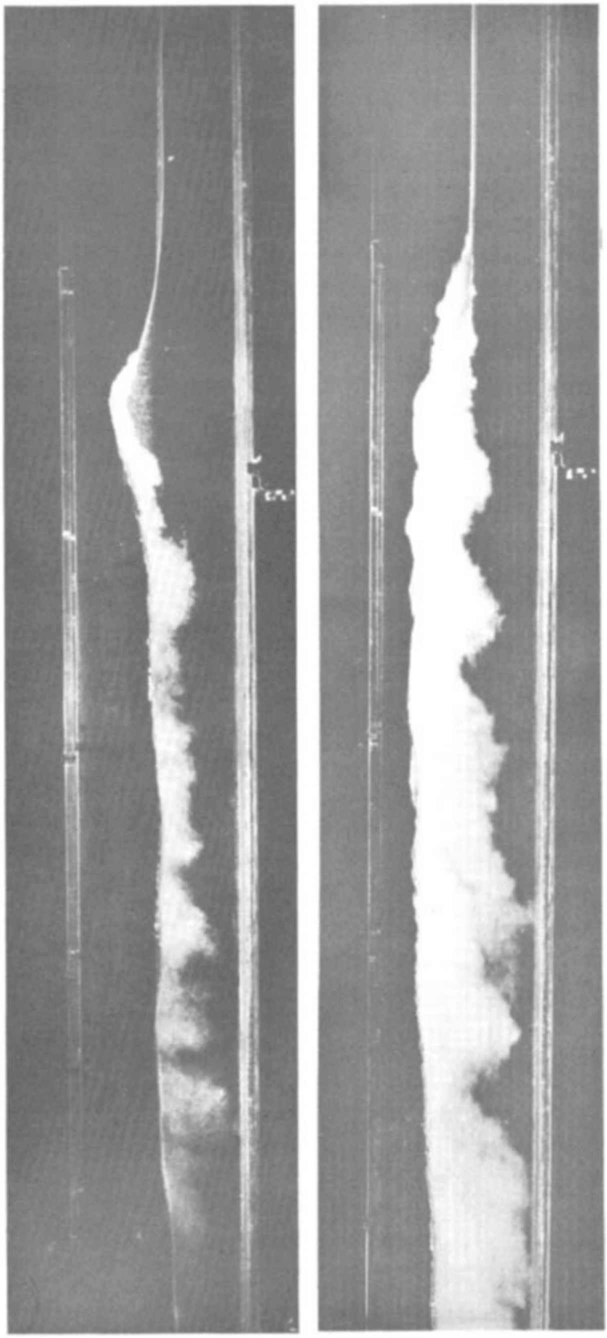

Figure 9 Flow visualization showing the approximate extent of turbulence behind (*top*) a spilling breaker and (*bottom*) a bore. Photographs courtesy of I. A. Svendsen, ISVA, Technical University of Denmark.

(1954) (see Figure 10). In the figure, the hydrofoil meets water with a subcritical velocity corresponding to a flow at A. The wave drag on the hydrofoil gives an equal and opposite force on the flow, reducing the momentum flux (or flow-force) and creating a flow at point B. If there is no flow separation, viscous forces and dissipation are negligible so that the flow's energy is unaltered. However, point B is outside the region of steady wave solutions, breaking occurs without further loss of momentum from the flow, and the resulting wave train corresponds to point C. The energy loss BC is the measure of the breaking. Any "momentum deficit" must be examined in this context; there is a change in mean depth and to analyze an experiment it is necessary to measure such changes as well as the wave train.

A bore meets water at a supercritical velocity, i.e. a flow relative to the bore corresponding to point D on the supercritical flow curve. The turbulence in the bore reduces the energy to point E on the subcritical flow curve, or else to an intermediate point F where a wave train forms behind the initial breaker. The effect of breaking in both these examples

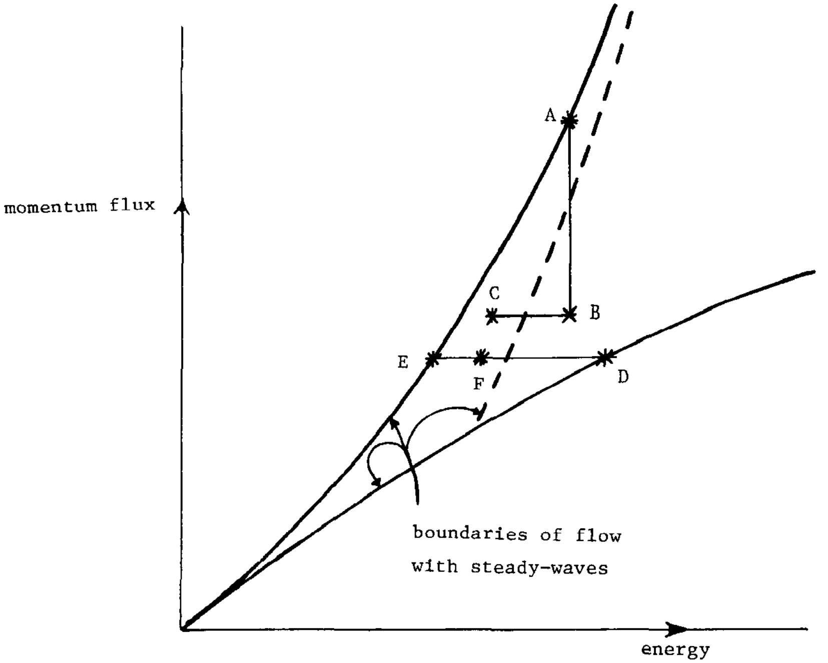

Figure 10 Energy-momentum flux diagram, after Benjamin & Lighthill (1954). The diagram is distorted for clarity. The two heavy lines represent steady uniform flows, the upper being subcritical, the lower supercritical. The broken line represents flows with stationary waves of maximum steepness.

is a loss of energy. Transfers of momentum between waves and mean flow are not considered since S is the total momentum flux.

A hydrofoil suffers no wave resistance in a two-dimensional supercritical flow since the flow force cannot be reduced; an exception occurs if it has sufficient drag to generate an upstream propagating bore that transforms the flow it meets into a subcritical one.

For breaking waves on beaches there is no reference frame in which the waves are steady, but the overall effect of wave breaking can be deduced in a similar manner. For example, in an idealized case on a horizontal bed consider a uniform wave train that suffers wave breaking (say, due to an instability) but then reforms into another uniform wave train. The conservation of mass and momentum then give, after averaging over a wave period,

$$U_1 D_1 + I_1 = U_2 D_2 + I_2 \tag{6}$$

and

$$(D_1 U_1 + I_1)(U_1 + I_1/D_1) + \tfrac{1}{2} g D_1^2 + S_{xx1} = (D_2 U_2 + I_2)(U_2 + I_2/D_2) + \tfrac{1}{2} g D_2^2 + S_{xx2}, \tag{7}$$

where U is the uniform current, D the depth, I the mass flow associated with the waves, S_{xx} the wave-momentum flux, density is taken equal to unity, and subscripts refer to conditions on each side of the breaking region. If the wave motion is defined in such a way that $I_1 = I_2 = 0$ [corresponding to Stokes's (1847) second definition of phase velocity], Equations (6) and (7) simplify and it can be readily shown that a loss of momentum from the wave motion leads to an increase of water depth if U is zero or if the flow is subcritical. Subcritical flow conditions are normal on a beach and the resulting depth increase, "wave set-up," is well known. Supercritical flows can occur, for example, on the shoreward side of shallow sand bars, and in such cases there is a decrease of depth due to wave breaking.

The above interpretation is only appropriate in shallow water. In deeper water the momentum loss from the waves is better interpreted as a surface shear stress acting on the mean flow. Duncan (1981) considers the detailed dynamics of the roller region to be best described as giving a surface shear stress, and he makes a first approximation for distinguishing between wave and mean motion. However, as McIntyre (1981) states, caution is necessary in ascribing momentum to the wave motion.

This consideration of mean flow changes and apparent surface stresses brings us to the topic of surf-zone dynamics, which is outside the scope of this review. However, the above discussion shows that a full understanding of wave breaking should include properties such as the rate-of-momentum transfer from the wave motion to the mean flow.

OTHER PHYSICAL EFFECTS

Air

The direct dynamic effects of the air on breaking are probably slight unless there is a large, say $O(20 \text{ m s}^{-1})$, relative velocity between the air and the water, in which case there can be considerable spray formation before the plunge point. For steady waves, Dore (1978) shows that surface boundary-layer effects are sometimes significant. The indirect effects of air, as in bubbles (see below) and in cushioning the collapse of the tube beneath a plunging jet, are more important.

Surface Tension

The clearest effect of surface tension is on steep waves less than about 10 cm high. The development of a plunging jet and the entrainment of air are both inhibited. Such waves, right down to an amplitude of around 2 mm, can still develop strongly turbulent regions, so that it is sensible to retain the description "breaking" even though the water surface remains continuous. Banner & Phillips (1974) note how important this type of breaking wave can be for sustaining wind stress on the sea.

Drops and Bubbles

The combination of air and surface tension in drops and bubbles fashions much of what is visible of a breaking wave. Their dynamic effects on waves are mainly such as to increase the rate at which the motion becomes more disorganized and turbulent. The buoyancy of bubbles is irrelevant in the main breaking process, but has some influence on the nature of the decaying turbulence left behind by breakers.

Perhaps the most important dynamic effect of bubbles is the way in which they, along with the air, cushion the impact of waves on any objects in the breaking zone of a beach. The velocity of sound in water can be reduced by an order of magnitude due to the presence of bubbles. It should be noted, however, that the typical size of bubbles is very different in salt and fresh water (e.g. see Scott 1975 and Monahan 1969, 1971). This is of particular importance if laboratory experiments are being used to model ocean conditions.

Vorticity

The main features of breaking waves generate ample vorticity. Here we mention the effects of preexisting vorticity, which has been generated by viscosity or turbulence in the water before the wave meets it.

The effect of a velocity shear near the surface, such as might be due to an on- or offshore wind, has been described by Banner & Phillips (1974) and Phillips & Banner (1974). An onshore surface shear tends to reduce the height of a wave at breaking; an offshore shear increases it. An extreme example is the surface-shear wave described by Peregrine (1974). It occurs in fast-flowing backwash and may reach a height several times the depth of water before collapsing.

If water is flowing toward the wave, the shear near the bed can have an overwhelming effect on the internal-flow pattern. The increase in pressure on the bed as a wave approaches can lead to flow separation from the bed. This has not been studied thoroughly, but it is clearly important for sediment transport. Matsunaga & Honji (1980) report experiments where flow separation at the base of a breaking wave is a major influence on the beach profile.

Measurements have been made of the flow pattern in hydraulic jumps, i.e. stationary bores. Resch & Leutheusser (1972) have shown how the inflow conditions affect its structure. A uniform inflow, which would correspond to conditions a bore would meet when propagating onto a weak current, leads to an internal-flow pattern as described above, with turbulence spreading downward into the incident flow. An inflow that has the fully developed profile of a steady turbulent flow gives rise to separation under the front of the wave and a substantially different flow field, though without any significant change in the surface profile. This latter case is relevant to waves that are almost brought to rest by the backwash, a common happening.

CONCLUDING REMARKS

In this review I have attempted to indicate how much, or little, is known about the dynamics of wave breaking. This means that breaking is treated as a specific event for each identifiable wave. Such an approach is desirable for understanding and accounting for natural waves. The variation from wave to wave is usually quite considerable. For example, some discussions of waves on beaches refer to a "break point." Figure 11 shows the distribution of the points at which breaking started from a 24-minute filmed sequence of waves on a North Devon beach. There is no one break point; there is instead a wide breaking zone. The beach from which this record comes was almost perfectly plane, with a slope of 1:60 at the time of filming. The majority of natural beaches have nonuniform profiles, often including bars.

Another source of variation is the response of the surf zone to the incident waves. Appreciable surface displacements and corresponding currents occur on gently sloping beaches; these are sometimes known as "surf beats" and are also ascribed to edge waves, but they may primarily be due to the envelope of amplitudes of the incident waves. The existence of these longer-scale motions means that successive waves enter different depth and current conditions.

To gain adequate understanding of wave breaking on beaches, it seems desirable to exploit the fact that most wave breaking occurs when waves are not far removed from the solitary wave in character, and to examine analytically, numerically, and experimentally the behavior of such waves on differing slopes against differing currents.

Only occasional reference has been made here to the large body of literature of measurements of wave properties on beaches. This is because properties such as the change of wave height with depth are better understood if considered in the context of surf-zone dynamics. For example, some experimenters measure the first few waves of a wave train in order to avoid the influence of reflected waves; if those waves are breaking waves, however, the chances are high that the "mean" level is steadily rising and the shoreward mass flow is significant, since those waves are establishing the set-up of mean level on the beach. In natural waves, such changes of level and current do not cease. Recent papers in this area are Guza & Thornton (1982) and Symonds et al. (1982).

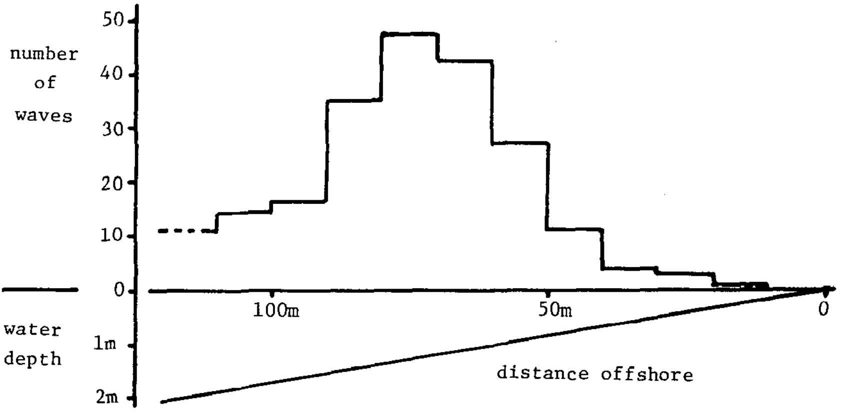

Figure 11 The spatial distribution of breaking waves in a time interval of 24 minutes on a natural beach of slope 1:60.

ACKNOWLEDGMENTS

The author acknowledges support from the La Jolla Foundation for Earth Sciences, which has permitted this review to be written close to a beach at the Institute of Geophysics and Planetary Physics, University of California, San Diego. Thanks are due for reprints, comments on a draft, and for discussions or other assistance to E. D. Cokelet, R. T. Guza, R. E. Flick, M. S. Longuet-Higgins, P. McIver, A. L. New, S. Pazan, I. A. Svendsen, and others.

Literature Cited

Airy, G. B. 1845. Tides and waves, Section 392. In *Encyc. Metropolitana*, 5:241–396

Banner, M. L., Phillips, O. M. 1974. On the incipient breaking of small scale waves. *J. Fluid Mech.* 65:647–56

Battjes, J. A. 1974. Surf similarity. *Proc. Conf. Coastal Eng., 14th*, pp. 466–79

Battjes, J. A., Sakai, T. 1981. Velocity field in a steady breaker. *J. Fluid Mech.* 111:421–37

Benjamin, T. B. 1967. Instability of periodic wave trains in non-linear dispersive systems. *Proc. R. Soc. London Ser. A* 299:59–75

Benjamin, T. B., Feir, J. E. 1967. The disintegration of wave trains on deep water. I. Theory. *J. Fluid Mech.* 27:417–30

Benjamin, T. B., Lighthill, M. J. 1954. On cnoidal waves and bores. *Proc. R. Soc. London Ser. A* 224:448–60

Binnie, A. M., Orkney, J. C. 1955. Experiments on the flow of water from a reservoir through an open channel. II. The formation of hydraulic jumps. *Proc. R. Soc. London Ser. A* 230:237–46

Carrier, G. F., Greenspan, H. P. 1958. Water waves of finite amplitude on a sloping beach. *J. Fluid Mech.* 4:97–109

Chan, R. K.-C., Street, R. L. 1970. Shoaling of finite-amplitude waves on plane beaches. *Proc. Conf. Coastal Eng., 12th,* pp. 345–61

Cleaver, R. P. 1981. *Some nonlinear properties of steep surface waves.* PhD thesis. Cambridge Univ.

Coastal Engineering Research Center, US Army. 1977. *Shore Protection Manual*. US Gov. Print. Off. 3 vols., 3rd ed.

Cokelet, E. D. 1977a. Breaking waves. *Nature* 267:769–74

Cokelet, E. D. 1977b. Steep gravity waves in water of arbitrary uniform depth. *Philos. Trans. R. Soc. London Ser. A* 286:183–230

Cokelet, E. D. 1978. Breaking waves—the plunging jet and interior flow field. In *Mechanics of Wave-Induced Forces on Cylinders*, ed. T. L. Shaw, pp. 287–301. London: Pitman. xiv + 752 pp.

Dagan, G. 1975. Taylor instability of a non-uniform free-surface flow. *J. Fluid Mech.* 67:113–23

Dore, B. D. 1978. A double boundary-layer model of mass transport in progressive interfacial waves. *J. Eng. Math.* 12:289–301

Duncan, J. H. 1981. An experimental investigation of breaking waves produced by a towed hydrofoil. *Proc. R. Soc. London Ser. A* 377:331–48

Favre, H. 1935. *Ondes de translation*. Paris: Dunod. 215 pp. (4 plates)

Flick, R. E., Guza, R. T., Inman, R. L. 1981. Elevation and velocity measurements of laboratory shoaling waves. *J. Geophys. Res.* 86:4149–60

Fornberg, B., Whitham, G. B. 1978. A numerical and theoretical study of certain nonlinear wave phenomena. *Philos. Trans. R. Soc. London Ser. A* 289:373–404

Freilich, M. H. 1982. *Resonance effects on shoaling surface gravity waves.* PhD dissertation. Univ. Calif., San Diego

Gallagher, B. 1972. Some qualitative aspects of nonlinear wave radiation in a surf zone. *Geophys. Fluid Dyn.* 3:347–54

Galvin, C. J. 1968. Breaker type classification on three laboratory beaches. *J. Geophys. Res.* 73:3651–59

Galvin, C. J. 1969. Breaker travel and choice of design wave height. *J. Waterw. Harbors Div., Proc. ASCE* 95:175–200

Galvin, C. J. 1972. Wave breaking in shallow water. In *Waves on Beaches and Resulting Sediment Transport*, ed. R. E. Meyer, pp. 413–56. New York: Academic

Garrett, C., Smith, J. 1976. Interaction between long and short surface waves. *J. Phys. Oceanogr.* 6:925–30

Green, G. 1838. On the motion of waves in a variable canal of small depth and width. *Trans. Cambridge Philos. Soc.* 6:457–62
Grimshaw, R. 1979. Slowly varying solitary waves. I. Korteweg-de Vries equation. *Proc. R. Soc. London Ser. A* 368:359–75
Guza, R. T., Bowen, A. J. 1976. Resonant interactions for waves breaking on a beach. *Proc. Conf. Coastal Eng., 15th*, pp. 560–79
Guza, R. T., Thornton, E. B. 1982. Swash oscillations on a natural beach. *J. Geophys. Res.* 87:483–91
Hansen, J. B., Svendsen, I. A. 1979. Regular waves in shoaling water, experimental data. *Inst. of Hydrodyn. Hydraul. Eng., Tech. Univ. Denmark, Ser. Pap. 21.* (Erratum Oct. 1979). 20 + 212 pp.
Hedges, T. S., Kirkgöz, M. S. 1981. An experimental study of the transformation zone of plunging breakers. *Coastal Eng.* 4:319–33
Horikawa, K. 1978. *Coastal Engineering: an Introduction to Ocean Engineering*. New York: Wiley. xii + 402 pp.
Hotta, S., Mizuguchi, M. 1980. A field study of waves in the surf zone. *Coastal Eng. Jpn.* 23:59–79
Ippen, A. T., Kulin, G. 1955. Shoaling and breaking characteristics of the solitary wave. *Tech. Rep. No. 15*, MIT Hydrodyn. Lab., Cambridge, Mass. vii + 56 pp.
Iribarren, C. R., Nogales, C. 1949. Protection des ports, Section II. *Comm. 4, 17th Int. Nav. Congr.*, 31:80. Lisbon
Iwagaki, Y., Sakai, T., Tsukioka, K., Sawai, N. 1974. Relationship between vertical distribution of water particle velocity and type of breakers on beaches. *Coastal Eng. Jpn.* 17:51–58
Kaup, D. J., Newell, A. C. 1978. Solitons as particles, oscillators, and in slowly changing media: a singular perturbation theory. *Proc. R. Soc. London Ser. A* 361:413–46
Keller, J. B. 1961. Tsunamis—water waves produced by earthquakes. *Internat. Union Geodesy. Geophys. Monogr. 24*, pp. 154–66. Tsunami Hydrodyn. Conf., Honolulu
Kjeldsen, S. P., Myrhaug, D. 1980. Wave-wave interactions, current-wave interactions and resulting extreme waves and breaking waves. *Proc. Conf. Coastal Eng., 17th*, pp. 2277–303
Lamb, H. 1932. *Hydrodynamics*. Cambridge, Engl: Cambridge Univ. Press. xv + 738 pp. 6th ed.
Lighthill, J. 1978. *Waves in Fluids*. Cambridge, Engl: Cambridge Univ. Press. xv + 504 pp.
Longuet-Higgins, M. S. 1975. Integral properties of periodic gravity waves of finite amplitude. *Proc. R. Soc. London. Ser. A* 342:157–74
Longuet-Higgins, M. S. 1978a. The instabilities of gravity waves of finite amplitude in deep water. I. Super-harmonics. *Proc. R. Soc. London Ser. A* 360:471–88
Longuet-Higgins, M. S. 1978b. The instabilities of gravity waves of finite amplitude in deep water. II. Sub-harmonics. *Proc. R. Soc. London. Ser. A* 360:489–505
Longuet-Higgins, M. S. 1980a. The unsolved problem of breaking waves. *Proc. Conf. Coastal Eng., 17th*, pp. 1–28
Longuet-Higgins, M. S. 1980b. On the forming of sharp corners at a free surface. *Proc. R. Soc. London Ser. A* 371:453–78
Longuet-Higgins, M. S. 1981a. On the overturning of gravity waves. *Proc. R. Soc. London Ser. A* 376:377–400
Longuet-Higgins, M. S. 1981b. A parametric flow for breaking waves. *Int. Symp. Hydrodyn. Ocean Eng., Trondheim, Norway*, pp. 121–35
Longuet-Higgins, M. S. 1982. Parametric solutions for breaking waves. *J. Fluid Mech.* In press
Longuet-Higgins, M. S., Cokelet, E. D. 1976. The deformation of steep surface waves on water. I. A numerical method of computation. *Proc. R. Soc. London Ser. A* 350:1–26
Longuet-Higgins, M. S., Cokelet, E. D. 1978. The deformation of steep waves on water. II. Growth of normal-mode instabilities. *Proc. R. Soc. London Ser. A* 364:1–28
Longuet-Higgins, M. S., Fox, M. J. H. 1977. Theory of the almost highest wave: the inner solution. *J. Fluid Mech.* 80:721–41
Longuet-Higgins, M. S., Fox, M. J. H. 1978. Theory of the almost highest wave. Part 2. Matching and analytic extension. *J. Fluid Mech.* 85:769–86
Madsen, O. S., Mei, C. C. 1969. The transformation of a solitary wave over an uneven bottom. *J. Fluid Mech.* 39:781–91
Madsen, P. A. 1981. A model for a turbulent bore. *Inst. Hydrodyn. Hydraul. Eng., Tech. Univ. Denmark, Ser. Pap. 28.* 149 pp.
Madsen, P. A., Svendsen, I. A. 1982. Turbulent bores and hydraulic jumps. *J. Fluid Mech.* Submitted for publication
Matsunaga, N., Honji, H. 1980. The backwash vortex. *J. Fluid Mech.* 99:813–15
McIntyre, M. E. 1981. On the "wave-momentum" myth. *J. Fluid Mech.* 106:331–47

McIver, P., Peregrine, D. H. 1981. Comparison of numerical and analytical results for waves that are starting to break. *Int. Symp. Hydrodyn. Ocean Eng., Trondheim, Norway*, pp. 203–15

McIver, P., Peregrine, D. H. 1982. Motion of a free surface and its representation by singularities. *J. Fluid Mech.* Submitted for publication

McLean, J. W. 1982. Instabilities of finite-amplitude waves on water of finite depth. *J. Fluid Mech.* 114:331–41

McLean, J. W., Ma, Y. C., Martin, D. V., Saffman, P. G., Yuen, H. C. 1981. Three-dimensional instability of finite-amplitude water waves. *Phys. Rev. Lett.* 46:817–20

Meyer, R. E., Taylor, A. D. 1972. Run-up on beaches. In *Waves on Beaches and Resulting Sediment Transport*, ed. R. E. Meyer, pp. 357–411. New York: Academic

Miche, M. 1944. Le pouvoir réfléchissant des ouvrages maritimes exposés à l'action de la houle. *Ann. Ponts Chaussées* 121:285–318

Miles, J. W. 1979. On the Korteweg-de Vries equation for a gradually varying channel. *J. Fluid Mech.* 91:181–90

Miles, J. W. 1980. Solitary waves. *Ann. Rev. Fluid Mech.* 12:11–43

Miller, R. 1976. Role of vortices in surf zone prediction: sedimentation and wave forces. *Soc. Econ. Paleontol. Mineralog., Spec. Publ. No. 24*, pp. 92–114

Monahan, E. C. 1969. Fresh water whitecaps. *J. Atmos. Sci.* 26:1026–29

Monahan, E. C. 1971. Oceanic whitecaps. *J. Phys. Oceanogr.* 1:139–44

Munk, W. H. 1949. The solitary wave and its application to surf problems. *Ann. NY Acad. Sci.* 51:376–424

Munk, W., Wimbush, M. 1969. A rule of thumb for wave breaking over sloping beaches. *Oceanology* 6:56–59

Peregrine, D. H. 1966. Calculations of the development of an undular bore. *J. Fluid Mech.* 25:321–30

Peregrine, D. H. 1967. Long waves on a beach. *J. Fluid Mech.* 27:815–27

Peregrine, D. H. 1974. Surface shear waves. *J. Hydraul. Div., Proc. ASCE* 100:1215–27

Peregrine, D. H. 1976. Interaction of water waves and currents. *Adv. Appl. Mech.* 16:9–117

Peregrine, D. H. 1981. The fascination of fluid mechanics. *J. Fluid Mech.* 106:59–80

Peregrine, D. H., Svendsen, I. A. 1978. Spilling breakers, bores and hydraulic jumps. *Proc. Conf. Coastal Eng., 16th*, pp. 540–50

Peregrine, D. H., Cokelet, E. D., McIver, P. 1980. The fluid mechanics of waves approaching breaking. *Proc. Conf. Coastal Eng., 17th*, pp. 512–28

Phillips, O. M., Banner, M. L. 1974. Wave breaking in the presence of wind drift and swell. *J. Fluid Mech.* 66:625–40

Resch, F. J., Leutheusser, H. J. 1972. Reynolds stress measurements in hydraulic jumps. *J. Hydraul. Res.* 10:409–30

Ryrie, S., Peregrine, D. H. 1982. Refraction of finite-amplitude water waves obliquely incident to a uniform beach. *J. Fluid Mech.* 115:91–104

Sakai, T., Battjes, J. A. 1980. Wave shoaling calculated from Cokelet's theory. *Coastal Eng.* 4:65–84

Sawaragi, T., Iwata, K. 1974. Turbulence effect on wave deformation after breaking. *Coastal Eng. Jpn.* 17:39–49

Schwartz, L. W. 1974. Computer extension and analytic continuation of Stokes' expansion for gravity waves. *J. Fluid Mech.* 62:553–78

Schwartz, L. W., Fenton, J. D. 1982. Strongly nonlinear waves. *Ann. Rev. Fluid Mech.* 14:39–60

Scott, J. C. 1975. The role of salt in whitecap persistence. *Deep-Sea Res.* 22:653–57

Silvester, R. 1974. *Coastal Engineering*. Vols. 1,2. Amsterdam: Elsevier. 457 pp., 338 pp.

Srokosz, M. A. 1981. Breaking effects in standing and reflected waves. *Int. Symp. Hydrodyn. Ocean Eng., Trondheim, Norway*, pp. 183–202

Stiassnie, M., Peregrine, D. H. 1980. Shoaling of finite-amplitude surface waves on water of slowly-varying depth. *J. Fluid Mech.* 97:783–805

Stive, M. J. F. 1980. Velocity and pressure field of spilling breakers. *Proc. Conf. Coastal Eng., 17th*, pp. 547–66

Stoker, J. J. 1957. *Water Waves*. New York: Interscience. 567 pp.

Stokes, G. G. 1847. On the theory of oscillatory waves. *Trans. Cambridge Philos. Soc.* 8:441–55 [*Math. Phys. Pap.* 1:197–229 (1880)]

Stokes, G. G. 1880. Considerations relative to the greatest height of oscillatory irrotational waves which can be propagated without change of form. *Math. Phys. Pap.* 1:225–28

Street, R. L., Camfield, F. E. 1966. Observations and experiments on solitary wave deformation. *Proc. Conf. Coastal Eng., 10th*, pp. 284–301

Su, M.-Y., Bergin, M., Marler, P., Myrick, R. 1982a. Experiments on nonlinear instabilities and evolution of steep gravity wave trains. Submitted for publication

Su, M.-Y., Bergin, M., Myrick, R., Roberts, J. 1982b. Experiments on shallow-water wave grouping and breaking. *Proc. 1st Int. Conf. Meteorol. Air-Sea Interaction Coastal Zone,* The Hague: K. Ned. Meteorol. Inst.

Suhayda, J. N., Pettigrew, N. R. 1977. Observations of wave height and wave celerity in the surf zone. *J. Geophys. Res.* 82:1419–24

Svendsen, I. A., Hansen, J. B. 1978. On the deformation of periodic long waves over a gently sloping bottom. *J. Fluid Mech.* 87:433–48

Svendsen, I. A., Madsen, P. A., Hansen, J. B. 1978. Wave characteristics in the surf zone. *Proc. Conf. Coastal Eng., 16th,* pp. 520–39

Symonds, G., Huntley, D. A., Bowen, A. J. 1982. Two-dimensional surf beat: long wave generation by a time-varying breakpoint. *J. Geophys. Res.* 87:492–98

Thornton, E. B., Galvin, J. J., Bub, F. L., Richardson, D. P. 1976. Kinematics of breaking waves. *Proc. Conf. Coastal Eng., 15th,* pp. 461–76

Ursell, F. 1953. The long-wave paradox in the theory of gravity waves. *Proc. Cambridge Philos. Soc.* 49:685–94

Van Dorn, W. G. 1978. Breaking invariants in shoaling waves. *J. Geophys. Res.* 83:2981–88

Vinje, T., Brevik, P. 1981. Numerical simulation of breaking waves. *Adv. Water Resour.* 4:77–82

Weishar, L. L., Byrne, R. J. 1978. Field study of breaking wave characteristics. *Proc. Conf. Coastal Eng., 16th,* pp. 487–506

Whitham, G. B. 1974. *Linear and Non-Linear Waves.* New York: Wiley-Interscience. xvii + 636 pp.

Wiegel, R. L. 1964. *Oceanographical Engineering.* Englewood Cliffs, NJ: Prentice-Hall. xi + 532 pp.

Williams, J. M. 1981. Limiting gravity waves in water of finite depth. *Philos. Trans. R. Soc. London Ser. A* 302:139–88

Annual Review of Fluid Mechanics
Volume 15, 1983

CONTENTS