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HYDROPHYSICAL AND HYDRODYNAMIC PROCESSES

The Problem of the Surface Wave Propagation in a Basin with a Rough Bottom: Berkhoff Approximation

E. N. Pelinovskii*, A. V. Razin, and E. V. Sasorova*****

** Institute of Applied Physics, Russian Academy of Sciences, ul. Ul'yanova 46, Nizhni Novgorod, 603600 Russia*
*** Radiophysical Research Institute, ul. Bol'shaya Pecherskaya 25, Nizhni Novgorod, 603600 Russia*
**** State Institute of Oceanography, Kropotkinskii per. 6, Moscow, 119034 Russia*

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Abstract—The problem of the propagation of surface gravity waves of small amplitude in a basin with unidimensional bottom roughness was solved in the context of the Berkhoff model (the original three-dimensional hydrodynamic equations were replaced by two-dimensional equations). The solution obtained was compared with that found through the exact linearized system of hydrodynamic equations for the incompressible fluid. Through a correlation between exact and approximate values of the attenuation factor for the average field, the Berkhoff approximation was established to be applicable for the shallow water case for any roughness correlation scale, whereas for a basin with moderate and great depths, this approximation is valid only where the correlation distance of roughness is several times greater than the basin depth. The limits of applicability of the Berkhoff model, which result from a low variation in the vertical structure of the wave field, were explained. The effect of the bottom roughness on the velocity of surface gravity wave propagation was investigated. The propagation of the surface wave in a basin with a rough bottom was shown to be, on average, either slower or faster than that in the basin with a level bottom, depending on the relationship between the wavelength, depth, and scale of the bottom roughness correlation.

INTRODUCTION

Propagation of surface waves in the basin with a rough bottom have been studied in many papers. The shallow water approximation yields analytical results fairly readily. Thus, the effect of small two-dimensional bottom roughness on the attenuation and on variations in the velocity of the surface wave propagation was studied [9, 12] in the context of the wave field method [2, 11, 13]. The same problem was solved in the Markovian approximation [1] in the case of unidimensional delta-correlated roughness that occupies a limited bottom area. For an arbitrary relationship between the basin depth and the wavelength, only the problem of the average field evolution was considered [7].

In the theory of surface waves, especially for calculations of the wave field within large water areas, the Berkhoff approximation, obtained essentially by the Galerkin technique and a number of assumptions concerning the vertical structure of the wave current [3–5, 8, 14], is used in addition to the exact hydrodynamic equations. The application of this model can be substantiated in two limiting cases: shallow water with gentle bottom slopes and vertical obstacles in the fluid of an arbitrary constant depth. Thus, this model is frequently referred to as the Berkhoff refraction-diffraction model. The Berkhoff model ignores the vertical coordinate, thus substantially simplifying the analysis of the wave propagation in a basin with a variable depth, which is especially advantageous in the case of the numerical methods. The model is usually applied to the deterministic depth profiles. All efforts based on asymptotic methods to validate the application of the Berkhoff model to an arbitrary relationship between the basin depth, wavelength, and scale of bottom roughness have failed. Therefore, the limits of its applicability are determined from some numerical solutions of three-dimensional models, results of laboratory experiments, and field data. We will assess the limits of applicability of the Berkhoff approximation for the bottom with random roughness, using as a test the results described above in the context of the average field method.

THE BERKHOFF APPROXIMATION OF THE MEAN FIELD

Let the plane $z = 0$ in the Cartesian coordinates coincide with the undisturbed water surface (z-axis is in opposition to the gravity); the fluid is restricted by the absolutely rigid bottom, the surface of which is given by the function $z = -h(x)$. For simplicity, we consider here a two-dimensional problem. In the Berkhoff approximation, variations in the fluid level with respect to the undisturbed surface for the harmonic wave with a small amplitude, which propagates along the x-axis, are described by the following equation (the temporal factor $\exp(-i\omega t)$ is omitted):

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$$cc_g\frac{d^2\eta}{dx^2}+\frac{d}{dx}(cc_g)\frac{d\eta}{dx}+k^2cc_g\eta = 0, \quad (1)$$

where $k = k(x)$ is the wavenumber of the surface wave related to the frequency by the exact dispersion equation

$$\omega^2 = gk\tanh kh, \quad (2)$$

where g is the gravitational acceleration,

$$c = [(g/k)\tanh kh]^{1/2}, \quad (3)$$

$$c_g = \frac{1}{2}\left(\frac{g}{k}\tanh kh\right)^{1/2}\left(1+\frac{kh}{\cosh^2 kh\tanh kh}\right), \quad (4)$$

c and c_g are the phase and group velocities of the wave, respectively.

Let us represent the basin depth as $h(x) = h_0 + \chi(x)$, where h_0 is the constant average depth, and $\chi(x)$ is its small (as compared with its mean value) random variations with $\langle\chi\rangle = 0$ (broken brackets denote the statistical averaging). We express the coefficients cc_g and k^2 from (1) in the form:

$$cc_g = c_0c_{g0}[1+\alpha(x)], \quad k^2 = k_0^2[1+\beta(x)], \quad (5)$$

where $\alpha(x)$ and $\beta(x)$ are random functions, and c_0, c_{g0}, and k_0 are values of velocities and wavenumber corresponding to the average depth h_0 and obtained from (2)–(4) for $h = h_0$.

If the bottom roughness is small, the functions $\alpha(x)$ and $\beta(x)$ can be readily found in an explicit form

$$\alpha(x) = u\varepsilon(x), \quad \beta(x) = v\varepsilon(x), \quad \varepsilon(x) = \chi(x)/h_0, \quad (6)$$

$$u = (3k_0h_0(1-k_0h_0\tanh k_0h_0)\tanh k_0h_0 + k_0^2h_0^2)/((k_0h_0 + \sinh k_0h_0\cosh k_0h_0)[(1 - k_0h_0\tanh k_0h_0)\tanh k_0h_0 + k_0h_0]), \quad (7)$$

$$v = -\frac{2k_0h_0}{k_0h_0+\sinh k_0h_0\cosh k_0h_0}. \quad (8)$$

Thus, the equation for the amplitude of the surface wave can be written in the form:

$$\frac{d^2\eta}{dx^2}(1+u\varepsilon)+u\frac{d\varepsilon}{dx}\frac{d\eta}{dx}+k_0^2\eta(1+w\varepsilon) = 0, \quad (9)$$

where $w = u + v$. We represent variations in the fluid level as a sum of the mean field η_0 and fluctuation field η' in which $\langle\eta'\rangle = 0$. Averaging (9), we obtain an equation for the coherent part of the field:

$$\frac{d^2\eta_0}{dx^2}+u\left\langle\varepsilon\frac{d^2\eta'}{dx^2}\right\rangle+u\left\langle\frac{d\varepsilon}{dx}\frac{d\eta'}{dx}\right\rangle+k_0^2\eta_0 + k_0^2w\langle\varepsilon\eta'\rangle = 0. \quad (10)$$

Subtracting (10) from (9) and neglecting, as in [2, 11–13], the terms of the second order of smallness with respect to the fluctuating values (the Bourret approximation [6]), we obtain an equation for the fluctuation field:

$$\frac{d^2\eta'}{dx^2}+k_0^2\eta'+u\varepsilon\frac{d^2\eta_0}{dx^2}+u\frac{d\varepsilon}{dx}\frac{d\eta_0}{dx}+k_0w\varepsilon\eta_0 = 0. \quad (11)$$

We will consider the case where the depth fluctuations are statistically homogeneous, i.e., the following relationship is valid:

$$\langle\varepsilon(x)\varepsilon(x')\rangle = \langle\varepsilon^2\rangle\Gamma(|x-x'|), \quad (12)$$

where Γ is the correlation coefficient, and $\langle\varepsilon^2\rangle$ is the variance of fluctuations.

We solve equation (11) by the Green function method, express the fluctuation field in terms of the mean field, and then insert this expression into the right-hand part of equation (10). This results in an integro-differential equation for the mean field η_0:

$$\frac{d^2\eta_0}{dx^2}+k_0^2\eta_0-\langle\varepsilon^2\rangle\int_{-\infty}^{+\infty}\left[u\frac{d^2\mathrm{g}(x-x')}{dx^2}+k_0^2w\mathrm{g}(x-x')\right]\times\left[u\Gamma(|x-x'|)\frac{d^2\eta_0(x')}{dx'^2}+u\frac{d\Gamma(|x-x'|)}{dx'}\frac{d\eta_0(x')}{dx'} + k_0^2w\Gamma(|x-x'|)\eta_0(x')\right]dx' - \langle\varepsilon^2\rangle u\int_{-\infty}^{+\infty}\frac{d\mathrm{g}(x-x')}{dx}\times\left[u\frac{d\Gamma(|x-x'|)}{dx}\frac{d^2\eta_0(x')}{dx'^2}+u\frac{d^2\Gamma(|x-x'|)}{dxdx'}\frac{d\eta_0(x')}{dx'} + k_0^2w\frac{d\Gamma(|x-x'|)}{dx}\eta_0(x')\right]dx' = 0, \quad (13)$$

$$\mathrm{g}(z) = -\frac{i}{2k_0}e^{ik_0|z|}, \quad (14)$$

where $\mathrm{g}(z)$ is the Green function for the unidimensional wave equation. Inserting into (13) the representation of the mean field as the Fourier integral

$$\eta_0(x) = \int_{-\infty}^{+\infty}\tilde{\eta}_0(k)e^{ikx}dk, \quad (15)$$

we obtain a dispersion equation for the surface waves with small amplitude that propagate in a basin with a rough bottom:

$$k^2 = k_0^2 - \langle\varepsilon^2\rangle \int_{-\infty}^{+\infty} \left[u \frac{d^2 g(|x-x'|)}{dx^2} + k_0^2 w g(x-x') \right]$$

$$\times \left[iku \frac{d\Gamma(|x-x'|)}{dx'} + k_0^2 w \Gamma(|x-x'|) - k^2 u \Gamma(|x-x'|) \right]$$

$$\times e^{ik(x'-x)} dx' - u\langle\varepsilon^2\rangle \int_{-\infty}^{+\infty} \frac{dg(x-x')}{dx} \left[iku \frac{d^2\Gamma(|x-x'|)}{dxdx'} \right. \qquad (16)$$

$$\left. + k_0^2 w \frac{d\Gamma(|x-x'|)}{dx} - k^2 u \frac{d\Gamma(|x-x'|)}{dx} \right] e^{ik(x'-x)} dx'.$$

Let $k \approx k_0$ in the right-hand part of equation (16) and z be replaced by $x' - x$. The dispersion equation takes the form

$$k^2 = k_0^2 - \langle\varepsilon^2\rangle k_0^2 v \int_{-\infty}^{+\infty} g(z) \left[k_0^2 v \Gamma(|z|) + ik_0 u \frac{d\Gamma(|z|)}{dz} \right]$$

$$\times e^{ik_0 z} dz - \langle\varepsilon^2\rangle u \int_{-\infty}^{+\infty} \frac{dg(z)}{dz} \left[k_0^2 v \frac{d\Gamma(|z|)}{dz} \right. \qquad (17)$$

$$\left. + ik_0 u \frac{d^2\Gamma(|z|)}{dz^2} \right] e^{ik_0 z} dz - \langle\varepsilon^2\rangle ik_0 u^2 \Gamma'(0) - \langle\varepsilon^2\rangle u v k_0^2.$$

In deriving equation (17), we took into account that $\Gamma(0) = 1$.

Transformation of integrals from equation (17) results in the following expression for the effective wave number of the surface wave:

$$k \simeq k_0 + \delta k_w + i\gamma_w, \qquad (18)$$

$$\delta k_w = \frac{1}{2}\langle\varepsilon^2\rangle k_0 u^2 - \langle\varepsilon^2\rangle k_0^2 \left(u^2 + uv + \frac{v^2}{4} \right)$$

$$\times \int_0^{\infty} \Gamma(z) \sin 2k_0 z dz, \qquad (19)$$

$$\gamma_w = \frac{\langle\varepsilon^2\rangle}{4} k_0^2 \left[v^2 \int_0^{\infty} \Gamma(z) dz + (4u^2 + 4uv + v^2) \right.$$

$$\left. \times \int_0^{\infty} \Gamma(z) \cos 2k_0 z dz \right]. \qquad (20)$$

Here, δk_w describes the variation in the real part of the effective wavelength, and γ_w describes the attenuation of the mean field. In the case $\Gamma(z) = \exp(-z/L)$ where L is the correlation radius for the bottom roughness, we obtain:

$$\delta k_w = k_0 \langle\varepsilon^2\rangle \left[\frac{1}{2}u^2 - \frac{k_0^2 L^2}{4k_0^2 L^2 + 1} \left(u^2 + uv + \frac{v^2}{4} \right) \right], \qquad (21)$$

$$\gamma_w = \frac{\langle\varepsilon^2\rangle}{4} k_0^2 L \left[v^2 + \frac{4u^2 + 4uv + v^2}{4k_0^2 L^2 + 1} \right]. \qquad (22)$$

MEAN FIELD IN THE CONTEXT OF THE POTENTIAL MODEL

In order to determine the limits of applicability of the Berkhoff model, we will consider more rigorously the solution of the problem of propagation of the surface wave with small amplitude, which requires solving the linearized system of hydrodynamic equations for the incompressible fluid. The case of two-dimensional roughness was analyzed in [7]. Here, we perform essentially the same analysis, but for the unidimensional roughness. The solution obtained will be used as a test. The linearized system for the velocity potential φ will look like the boundary value problem

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial z^2} = 0, \quad -h(x) < z < 0, \qquad (23)$$

$$\frac{\partial\varphi}{\partial z} = \frac{\omega^2}{g}\varphi, \quad z = 0, \qquad (24)$$

$$\frac{\partial\varphi}{\partial z} = -\frac{\partial h}{\partial x}\frac{\partial\varphi}{\partial x}, \quad z = -h(x). \qquad (25)$$

Using the assumed smallness of the roughness, we transfer the boundary condition (25) from the surface $z = -h(x)$ to the plane $z = -h_0$:

$$\frac{\partial\varphi}{\partial z} = -\frac{\partial}{\partial x}\left[\chi(x) \frac{\partial\varphi}{\partial x} \right], \quad z = -h_0.$$

We write the velocity potential as a sum of the mean and fluctuation fields ($\varphi = \varphi_0 + \varphi'$). The Bourret approximation describes the values φ_0 and φ' by the following system of equations:

$$\frac{\partial^2\varphi_0}{\partial x^2} + \frac{\partial^2\varphi_0}{\partial z^2} = 0, \quad -h_0 < z < 0, \qquad (26)$$

$$\frac{\partial\varphi_0}{\partial z} = \frac{\omega^2}{g}\varphi_0, \quad z = 0, \qquad (27)$$

$$\frac{\partial\varphi_0}{\partial z} = -\frac{\partial}{\partial x}\left\langle \chi \frac{\partial\varphi'}{\partial x} \right\rangle, \quad z = -h_0, \qquad (28)$$

$$\frac{\partial^2\varphi'}{\partial x^2} + \frac{\partial^2\varphi'}{\partial z^2} = 0, \quad -h_0 < z < 0, \qquad (29)$$

$$\frac{\partial\varphi'}{\partial z} = \frac{\omega^2}{g}\varphi', \quad z = 0, \tag{30}$$

$$\frac{\partial\varphi'}{\partial z} = -\frac{\partial}{\partial x}\left(\chi\frac{\partial\varphi_0}{\partial x}\right), \quad z = -h_0. \tag{31}$$

We represent the potential functions $\varphi_0(x, z)$ and $\varphi'(x, z)$ as Fourier integrals:

$$\varphi_0(x, z) = \int_{-\infty}^{+\infty}\tilde{\varphi}_0(k, z)e^{ikx}dk, \tag{32}$$

$$\varphi'(x, z) = \int_{-\infty}^{+\infty}\tilde{\varphi}'(k, z)e^{ikx}dk. \tag{33}$$

From (29)–(33) it follows that

$$\tilde{\varphi}'(k, z) = \frac{\sigma(k)(kg\cosh kz + \omega^2\sinh kz)}{k(\omega^2\cosh kh_0 - kg\sinh kh_0)}, \tag{34}$$

$$\sigma(k) = \int_{-\infty}^{+\infty}k\xi\tilde{\chi}(k-\xi)\tilde{\varphi}_0(\xi, -h_0)d\xi, \tag{35}$$

$$\tilde{\chi}(k) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\chi(x)e^{-ikx}dx, \tag{36}$$

where $\tilde{\varphi}'(k, z)$ is the spectrum of the bottom roughness. From (26)–(28), with allowance made for (34), we obtain the equation and boundary conditions for $\tilde{\varphi}_0$:

$$\frac{d^2\tilde{\varphi}_0(k, z)}{dz^2} - k^2\tilde{\varphi}_0(k, z) = 0, \tag{37}$$

$$\frac{d\tilde{\varphi}_0(k, z)}{dz} = \frac{\omega^2}{g}\tilde{\varphi}_0(k, z), \quad z = 0, \tag{38}$$

$$\frac{d\tilde{\varphi}(k, z)}{dz} = \langle\chi^2\rangle k^2\tilde{\varphi}_0(k, z)$$
$$\times\int_{-\infty}^{+\infty}T(|k-\xi|)\frac{g\xi - \omega^2\tanh\xi h_0}{\omega^2 - g\xi\tanh\xi h_0}\xi d\xi, \quad z = -h_0, \tag{39}$$

$$T(k) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\Gamma(x)e^{-ikx}dx, \tag{40}$$

where $T(k)$ is the spectrum of the correlation coefficient of the bottom roughness.

The general solution of equation (37) takes the following form:

$$\tilde{\varphi}_0(k, z) = Ae^{kz} + Be^{-kz}. \tag{41}$$

Substituting (41) in the boundary conditions (38) and (39) results in a system of homogeneous linear algebraic equations in A and B. The condition of the existence of nontrivial solutions to this system consists in the vanishing of its determinant. This condition takes the following form:

$$\frac{\omega^2 - gk\tanh kh_0}{gk - \omega^2\tanh kh_0} = I, \tag{42}$$

$$I = \langle\chi^2\rangle k\int_{-\infty}^{+\infty}T(|k-\xi|)\frac{g\xi - \omega^2\tanh\xi h_0}{\omega^2 - g\xi\tanh\xi h_0}\xi d\xi. \tag{43}$$

Equation (42) is the dispersion equation for the mean field. Its solution will be sought in the form $k = k_0 + \kappa$, where k_0 is the solution to the dispersion equation (2) for the constant mean depth h_0, and κ is a small term (as compared with the mean value of k_0).

Expanding (42) and (43) into a series in small κ, about $k = k_0$, we obtain the following approximate solution of the dispersion equation for the mean field of surface waves:

$$\kappa = \delta k + i\gamma = -\frac{2k_0}{2k_0h_0 + \sinh 2k_0h_0}I. \tag{44}$$

Integral (43) involved in (44) is complex-valued. Its real part governs the variation in the real part of the wavelength and, therefore, the velocity of propagation of the mean wave field. The imaginary part of integral (43), which varies in proportion to the sum of half-residues in the poles of the integrand, controls the attenuation of the mean field. Performing the relevant calculations, we obtain the following expression for the attenuation coefficient of the mean field:

$$\gamma = \pi\langle\chi^2\rangle\left(\frac{2k_0^2}{2k_0h_0 + \sinh 2k_0h_0}\right)^2[T(0) + T(2k_0)]$$
$$= \langle\chi^2\rangle\left(\frac{2k_0^2}{2k_0h_0 + \sinh 2k_0h_0}\right)^2 \tag{45}$$
$$\times\left[\int_0^{\infty}\Gamma(x)dx + \int_0^{\infty}\Gamma(x)\cos 2k_0x dx\right].$$

In the special case where the correlation function of the bottom roughness is exponential, the attenuation coefficient of the mean field for the surface gravity wave takes the following form:

$$\gamma = \frac{8\langle\varepsilon^2\rangle k_0^4h_0^2L(2k_0^2L^2 + 1)}{(4k_0^2L^2 + 1)(2k_0h_0 + \sinh 2k_0h_0)^2}. \tag{46}$$

ASSESSMENT OF THE LIMITS OF APPLICABILITY OF THE BERKHOFF APPROXIMATION

We will analyze expressions for the attenuation coefficients of the mean field of surface gravity waves, obtained in the context of the Berkhoff model (22) and in the context of the more complete potential model (46). For simplicity, we will consider the exponential expression for the correlation coefficient, although many of our conclusions can be drawn under more general assumptions.

We will start with the Berkhoff model. Under the shallow-water assumptions where $k_0h_0 \ll 1$, from (7) and (8), it follows that $u = -v = 1$. In this case, the attenuation coefficient is found from (22) in the form

$$\gamma_w = \frac{\langle\varepsilon^2\rangle}{2}k_0^2L\frac{1+2k_0^2L^2}{4k_0^2L^2+1} \qquad (47)$$

and coincides with that of (46), which was obtained in the context of a more exact potential problem statement. It is worth noting that expression (47) can also be obtained as a result of the solution of the main shallow-water equation by the mean field method

$$\frac{d^2\eta}{dx^2} + k_0^2\eta = -\frac{d}{dx}\left[\varepsilon(x)\frac{d\eta}{dx}\right]. \qquad (48)$$

This conclusion is obvious, since under the assumption that $k_0h_0 \ll 1$, both the Berkhoff model and the initial potential model are reduced to the shallow-water equations.

In the case of deep water, where $k_0h_0 \gg 1$, it follows from (7) and (8) that

$$u = -8k_0^2h_0^2e^{-2k_0h_0}, \quad v = -8k_0h_0e^{-2k_0h_0}, \qquad (49)$$

and in the context of the Berkhoff model,

$$\gamma_w = 64\langle\varepsilon^2\rangle k_0^4h_0^2Le^{-4k_0h_0}\frac{k_0^2h_0^2+k_0^2L^2}{4k_0^2L^2+1}. \qquad (50)$$

In the context of the potential model (46), the attenuation coefficient is determined from the following expression:

$$\gamma_w = 32\langle\varepsilon^2\rangle e^{-4k_0h_0}\frac{k_0^4Lh_0^2(2k_0^2L^2+1)}{4k_0^2L^2+1}. \qquad (51)$$

For large-scale roughness, these expressions are identical, whereas they differ greatly for small-scale roughness. Thus, the Berkhoff approximation can be applied to the wave scattering on large-scale roughness. This conclusion can be explained by the fact that the vertical structure of the wave field in the Berkhoff model is assumed to be fixed. But if the bottom roughness is large-scale, then even in the context of the more exact potential model, the vertical field structure is virtually invariable, which results in identical expressions for the attenuation coefficient. It should, however, be remembered that in deep water, the effect of bottom roughness on the propagation of the surface wave proves to be, as expected, exponentially small.

We will point out that in the limiting cases of both long and short waves, the attenuation coefficient is relatively small, with its maximum falling on the wavelengths comparable with the basin depth. For the arbitrary basin depths, the attenuation coefficient of the mean field should be studied numerically. To this purpose, we will represent γ and γ_w in the form:

$$\gamma = \frac{\langle\varepsilon^2\rangle}{L}\tilde{\gamma}, \quad \gamma_w = \frac{\langle\varepsilon^2\rangle}{L}\tilde{\gamma}_w, \qquad (52)$$

where $\tilde{\gamma}$ and $\tilde{\gamma}_w$ are numerical coefficients depending on the wave frequency, average basin depth, and correlation radius of roughness. The form of these coefficients can be readily determined from (22) and (46), which were derived assuming the exponential function of correlation.

The results of calculations for $\tilde{\gamma}$ performed in the context of the Berkhoff and potential models are shown in Fig. 1, where the mean basin depth–correlation distance ratio ($q = L/h_0$) is a parameter of the model. Figure 1 shows that, in the neighborhood of the maximum, the Berkhoff approximation can be in a good agreement with the exact results only if the bottom irregularities are relatively smooth. This fact results from the effect of the preservation of the vertical structure of the wave field, which was discussed above.

In the case of small-scale irregularities, the calculations based on the Berkhoff model result in great errors. In this case, the results differ even qualitatively: the curve becomes double-humped, whereas in the context of the exact model, the curve remains invariably single-humped.

The analysis performed showed that where the vertical structure of the wave field is virtually invariable (shallow water and arbitrary irregularities or arbitrary depth and large-scale irregularities), the Berkhoff model closely approximates the wave attenuation coefficient in the basin with a rough bottom. If the field vertical structure varies, the Berkhoff approximation, which is based on the invariability of this structure, turns out to be inefficient.

THE EFFECT OF BOTTOM IRREGULARITIES ON THE WAVE PROPAGATION VELOCITY

Along with the attenuation of the mean field, the bottom irregularities result in varying the velocity of propagation of the surface gravity waves. In the Berkhoff approximation, the variation in the real part of the effective wavelength is described by expression (21), which in the case of shallow water is reduced to the form:

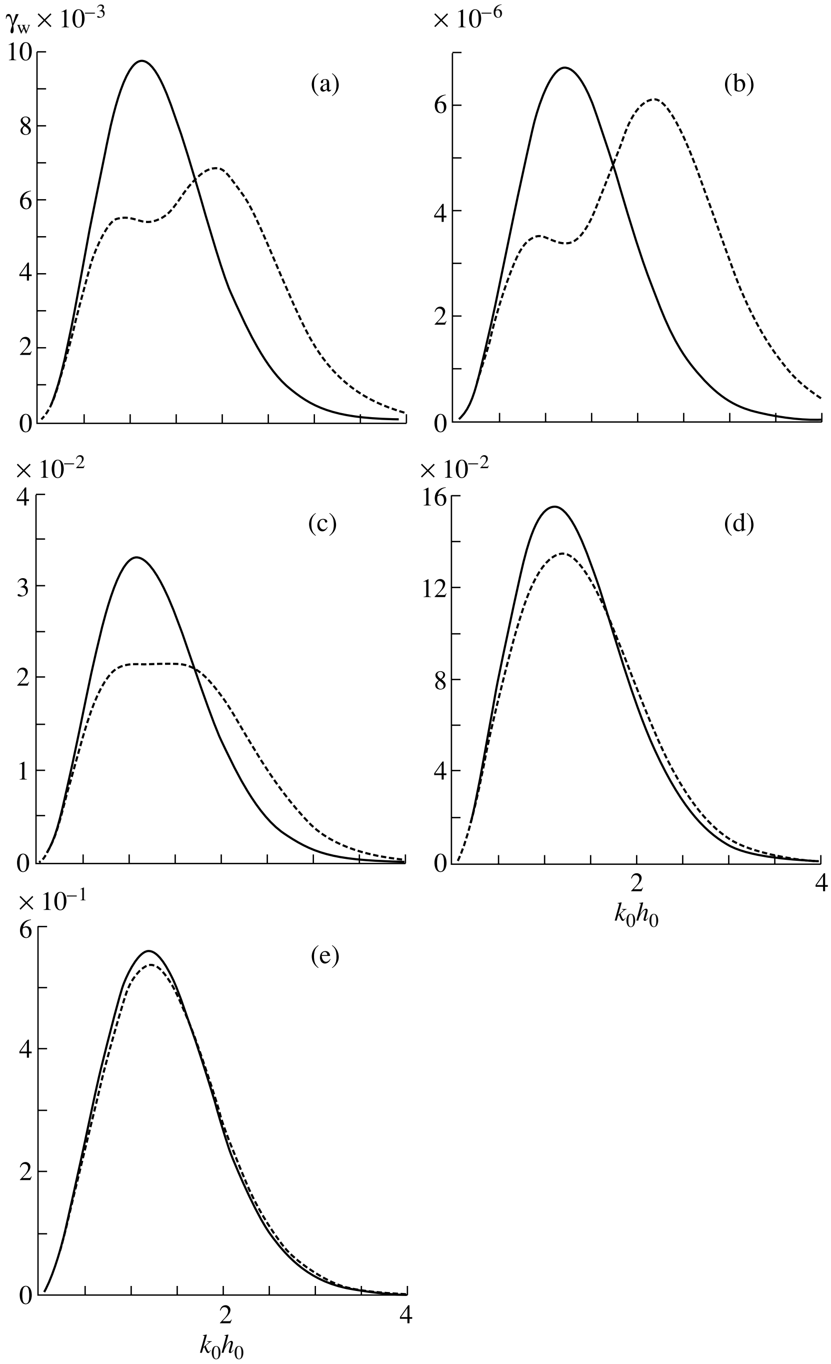

Fig. 1. the coefficient of the surface wave attenuation for $q = L/h_0 = 0.05$ (a), 0.2 (b), 0.4 (c), 2 (d), and 5 (e). The solid line denotes the potential model; the dashed line, the Berkhoff model.

$$\delta k_w = k_0 \langle \varepsilon^2 \rangle \sigma, \quad (53)$$

$$\sigma = \frac{3k_0^2L^2 + 1}{2(4k_0^2L^2 + 1)}. \quad (54)$$

The value of σ is always positive and steadily decreases from 0.5, in the case of small-scale irregularities ($k_0L \ll 1$), and to 0.375, in the case of large-scale irregularities ($k_0L \gg 1$). This means that in shallow water in the basin with a rough bottom, the surface wave propagates, on average, more slowly than that in the basin with a smooth bottom. The same result is valid without the use of the Berkhoff approximation in the context of the shallow water model and is important in explaining the time lag of the tsunami wave arrival as compared with the calculated time [10, 15].

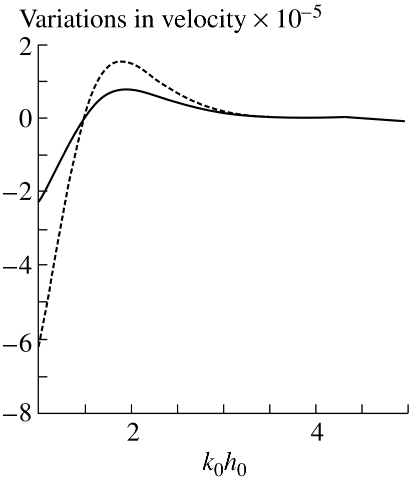

Fig. 2. Variations in the wave propagation velocity. The solid line denotes the phase velocity; the dashed line denotes the group velocity.

In the case of deep water and an exponential correlation function,

$$\sigma = 32k_0^2h_0^2e^{-4k_0h_0}\frac{k_0^2h_0^2 - 4k_0^3h_0L^2 - k_0^2L^2}{4k_0^2L^2 + 1}. \quad (55)$$

If the bottom irregularities are small-scale, then

$$\sigma \approx 32k_0^4h_0^4e^{-4k_0h_0} \quad (56)$$

and the surface wave propagates, on average, more slowly than that in the basin with a smooth bottom. If the bottom irregularities are large-scale, then $\sigma < 0$ and

$$\sigma \approx -32k_0^3h_0^3e^{-4k_0h_0}, \quad (57)$$

and the wave propagates, on average, more rapidly than that over the smooth bottom.

The results of the calculations of $\sigma_f = c/c_0 - 1$ and $\sigma_g = c_g/c_0g - 1$, which describe variations in the phase and group velocities of the surface gravity wave for $\langle\varepsilon^2\rangle = 10^{-3}$ and $q = 5$, are presented in Fig. 2. This figure shows that variations in the surface wave velocities caused by the effect of bottom irregularities are quite small. The variation in velocity is of 10^{-4} order of magnitude and, thus, the variation in the time lag of the wave arrival does not exceed 20 s even along transoceanic paths. Therefore, this effect will not be discussed in this paper.

CONCLUSION

The Berkhoff approximation describes well the coefficient of attenuation of the average field in the case of shallow water and an arbitrary scale of bottom irregularities, as well as in the case of an arbitrary depth and large-scale bottom irregularities.

Such a good correspondence results from the preservation of the vertical structure of the wave field in the cases listed above; the Berkhoff model is based precisely on this approximation.

For the small-scale irregularities and arbitrary basin depths, the Berkhoff approximation is inefficient, and calculations in the context of this model result in high errors.

In the basin with a rough bottom, the velocity of wave propagation also varies, with roughness resulting either in an increase or a decrease in the velocity. For actual cases, the velocity of propagation varies only slightly.

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