GROWTH RESPONSE OF WAVES TO THE WIND STRESS

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A re-analysis wave growth data available in the literature has been undertaken during this study. Data from the sources used by Plant (1982) have been assembled and supplemented by substantial data sets from six other sources. These data sets show growth rates compatible with Plant's curve only if Plant's uncertainty factor of 2 is retained. We have found systematic wave growth behaviour when wave growth is normalised by wave speed multiplied by total wind stress and then plotted as a function of the mean wave steepness. Wave energy input increases approximately quadratically with mean wave steepness to the point of initiation of both breaking and flow separation. Across the transition to breaking, there is no further increase in normalised wave energy input but for strong breaking, no other measurements are available at present. Net wave attenuation rates in the presence of an opposing wind are much greater than wave growth rates for comparable forcing once the mean steepness exceeds approximately 0.05.

1. Introduction

Determining the wave climate is fundamental to analysis and design in the coastal environment. Unfortunately, the wind conditions responsible for the most severe coastal wave climates can be rarely approximated by a delineated fetch exposed to a unidirectional wind field. Consequently, nomograms of peak spectral wave height and period of wave fields based on measurements (e.g. USACE, 1984; Donelan, 1990, Young, 1999) must be applied with caution and considerable uncertainty. The most severe wave states are usually associated with cyclonic storms that are characterised by a moving core and rapidly turning wind fields. The wave climate radiated by these systems is critically dependent on the region of most intense wave generation which is close to the centre of the storm.

Investigations over the last 50 years have shown that predicting the design wave conditions associated with storms requires analysis of spectral response for directional components over a selected spatial domain. Numerical models of sea state development for variable surface wind fields have developed in complexity and capability since the pioneering work of Pierson, Neumann and James, 1955. More recent assessments of operational wave model capability in the prediction of severe states are contained in Komen *et al.*, 1994 and Hasselmann *et al.*, 1998. Significant deficiencies remain and model tuning techniques have had to be use to obtain maximum agreement between observations and model predictions. Recent field measurements by Wright *et al.*, 2001 have highlighted the complexity of wave fields inside tropical cyclones and the prevalence of conditions of intense winds not aligned with the direction of propagation of peak wave energy.

Most numerical representations of the development of wind-wave fields use the wave energy balance equation as their formulation (for example, Komen *et al.*, 1994, pages 33 and 47):

$$\frac{dF(\omega,\theta)}{dt}=S_{in}(\omega,\theta)+S_{nl}(\omega,\theta)+S_{diss}(\omega,\theta)+S_{bed}(\omega,\theta) \quad (1)$$

where F is the spectral wave energy as a function of frequency ω and direction θ; S_{in} are the energy inputs from the wind; S_{nl} are non-linear energy transfers between wave frequencies due to wave-wave interactions; and, S_{diss} and S_{bed} (negative quantities) are the loss of energy from the wave field due to surface (predominantly, wave breaking) and bed interactions respectively. Note that the above equation uses the total derivative and therefore considers changes moving with the group velocity of the waves.

The primary difficulty in predicting wind-wave field development is that it is the cumulative response to at least four processes that have proved difficult to quantify. Cross couplings between the terms on the right hand side of equation (1) are also possible. S_{diss} was included in the balance equation because of the obvious role of wave breaking in the loss of energy from the wave field but recent synthetic numerical studies (e.g. Makin et al., 2002) stimulated by earlier strategic laboratory studies (Banner and Melville, 1976 and Banner, 1990) indicate that breaking mays a major role in determining the source term S_{in} as well. At present, a much wider role for wave breaking in development of wind-forced wave fields is now being contemplated.

The purpose of this contribution is to present some recent analysis of S_{in} and more detailed discussion of current theoretical approaches to quantifying the other terms on the right right hand side of Equation (1) is not possible. The interested reader is referred to more extensive discussions found in Komen *et al.* (1994) and Young (1999).

We briefly review current understanding of wind-input to the wave field, present a new approach to wave growth measurements which indicates systematic behaviour that can be understood in terms of drag development and identify current issues that remain to be resolved.

2. Wind-input to the wave field

The dominant synthetic and quantitative approach to determining the magnitude of the wind input source term was published by Plant (1982). He analysed four data sets gathered in the laboratory and field and showed normalised energy growth rates with a quadratic dependence on inverse wave age (u_a^* / c) (where c is the wave speed and u_a^* is the friction velocity in the air). This finding was consistent with a critical layer theory developed by Miles (1957) but Plant found that the magnitude of the rate of growth was, on average, approximately a factor of 2 higher than that predicted by Miles. However, with Plant's normalisation, the data exhibited an unsatisfactorily high degree of scatter (approximately a factor of 2). Since that time, there have been extensive theoretical studies to try to determine the source of the disparity between the mean rates of growth predicted and those measured (See review in Belcher and Hunt, 1998).

At this point, we must distinguish broadly four categories of growing waves:

1. *Gravity-capillary* waves (i.e. wavelengths less than 60mm) have been shown to grow by different processes to longer gravity waves. Theoretical prediction of wave growth due to shear instability at the surface of the water (van Gastel *et al.*, 1985, Merink, 2002) has been able to account for the observed rates of growth (Larson and Wright, 1975, Caulliez *et al.*, 1998).
2. *Fast* waves have speeds close to or exceeding that of the wind. Reliable measurements of fast wave growth are very difficult because of the weak coupling between wind and waves with comparable speeds. Theoretical predictions show comparable rates of growth (or attenuation) with available measurements.
3. *Slow* waves are gravity waves that propogate slowly relative to the wind (approximately $c/U_{10}<0.2$). Most of the measurements presented by Plant (1982) were of slow waves and in spite of detailed theoretical and numerical studies over the last 20 years, the disparity in measured and predicted growth rates remains.
4. Wave propagating against the wind. Recent measurements by Peirson *et al.* (2003) have shown very strong attenuation of waves propagating against the wind and at rates much higher than theoretical predictions.

3. A Review of Wave Growth and Attenuation Measurements

There are three established methods of measuring S_{in}.

1. *Radar and scattering theory*. For small wave steepnesses (ak<0.01), there is a linear relationship between wave amplitude and the power of the radar return from water surfaces (Larson and Wright, 1975, *p*. 418). Use of this technique has been limited to waves with lengths comparable with the radar microwaves themselves, limiting its applicability to the gravity-capillary range.
2. *Pressure measurement in the air*. At the gravity wave scale, air flow-induced form drag is believed to be the primary contributor to the growth of wind waves. If this is the case, correlation of the surface pressure with

the wave slope will yield the energy input to the wave. However, pressure probes cannot be located directly at the surface and measurements above separated regions must be extrapolated to obtain surface values (Snyder *et al.*, 1981).

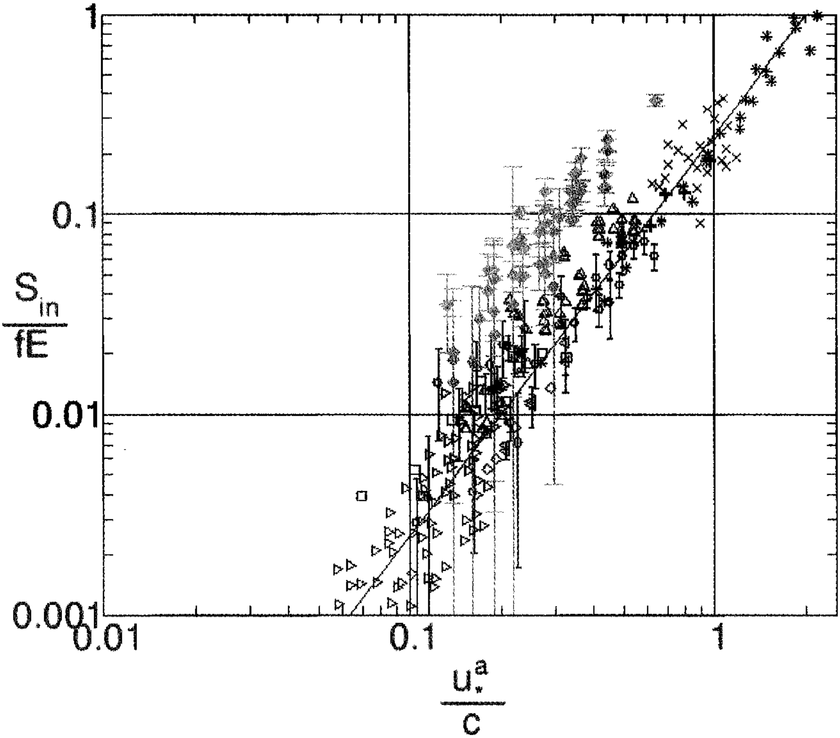

Figure 1. Wave growth data assembled by Plant (1982) with data obtained by other investigators as shown: asterices, laboratory radar measurements by Larson and Wright, 1974; x, laboratory air pressure data from Wu, Hsu and Street (1977); diamonds, laboratory air pressure measurements by Shemdin and Hsu, 1971; triangles to right, field air pressure measurements by Snyder et al., 1981; solid circles, laboratory wave energy measurements by Bole, 1967; squares, laboratory wave energy measurements by Wilson *et al.*, 1971; +, laboratory air pressure measurements by Banner, 1990; trangles to left, laboratory wave energy measurements by Bliven *et al.*, 1986; upright triangles, laboratory wave energy measurements by Mitsuyasu and Honda, 1982; downward pointing triangles, laboratory air pressure measurements by Mastenbroek *et al.*, 1996. Error bars have been included where possible. Solid line indicates best fit calculated by Plant (1982). The wave attenuation results of Peirson *et al.*, 2003 are shown as grey diamonds.

3. *Spatial changes of wave energy.* Under specific experimental conditions, it has been shown that the S_{nl} term is negligible (Bliven *et al.*, 1986) and, in the absence of surface breaking, assumptions can be invoked regarding S_{diss} and S_{bed} (Mitsuyasu and Honda, 1982). The left hand side of equation (1) can be evaluated from spatial measurements of wave growth under a steady wind. If all of these requirements are satisified, measurement of spatial variations in wave energy can be used to derive values of S_{in}. Peirson *et al.*

(2003) have shown that errors can be quantified for this technique.

We have assembled six other sets of measurements of gravity wave growth available from the literature (Bole, 1967, Wilson *et al.*, 1971, Mitsuyasu and Honda, 1982; Bliven *et al.*, 1986; Banner, 1990; Mastenbroek *et al.*, 1996) that were unused or not available to Plant (1982). From these, we have computed error bars where possible. Figure 1 shows these other data sets imposed on the Plant diagram. The Peirson *et al.*, 2003 measurements of wave attenuation have been included for comparison in this diagram as well.

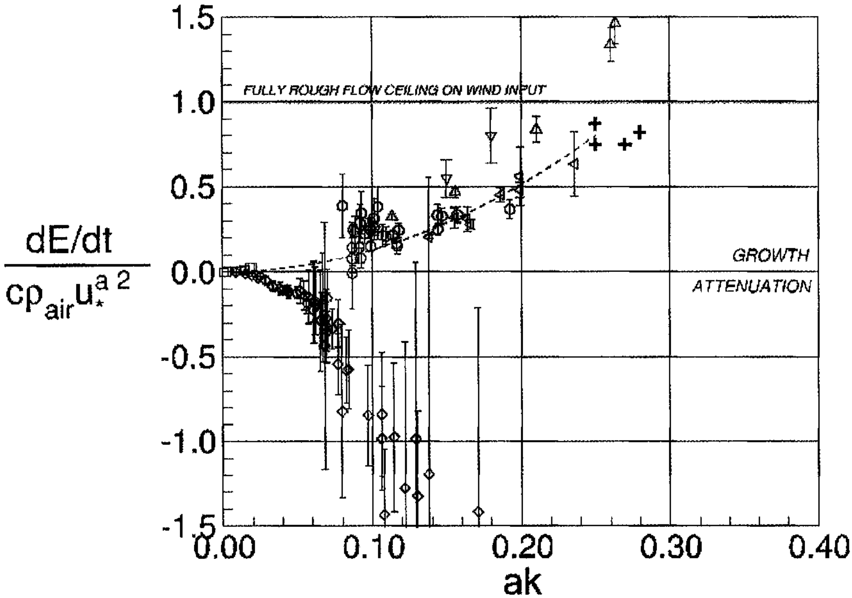

Figure 2. Rate of wave energy gain as a proportion of total wind stress multiplied by wave speed presented as a function of mean wave steepness. Symbols are as in Figure 1. The dashed line is a best fit quadratic to the available growth data.

4. Alternative Normalisations

We have applied the approach to drag over hills developed by Belcher *et al.* (1993) to the case of slow gravity waves. It could be expected that with the significant differences in wind and slow wave speed, the form drag behaviour of hills and slow waves should be similar. For hills, Belcher *et al.* (1993) showed systematic behaviour of the form drag as a proportion of the total drag in response to mean wave steepness. The increase in form drag was approximately a quadratic function of the mean hill steepness (*ak*) up to a value of *ak* = 0.25, at which point the flow separates and the form drag does not increase substantially. It is interesting to note that Banner (1990) showed that airflow separation above slow water waves was also initiated at *ak* ~ 0.26.

For water waves, wind input can be derived from the form drag via:

$$S_{in}=c\tau_{form} \tag{2}$$

where τ_{form} is the form drag which can be normalised by the total wind stress, τ_{total} to yield a wave growth rate normalised in terms of the total stress:

$$\frac{\tau_{form}}{\tau_{total}}=\frac{S_{in}}{c\rho_{air}u_*^2} \tag{3}$$

Note that the requirement of form drag strictly less than the total drag requires that both sides of equation 3 must remain less than unity if the primary input to the wave field from the wind is due to form drag.

This normalisation has been tested with data sets that have been reported with sufficient background information and is presented in Figure 2. The systematic behaviour and collapse of the wave growth data can be observed. The following observations can be made:

1. All of the data sets support a systematic increase in the wave drag proportion of the total stress as mean wave steepness increases (Equation 3).
2. The Mitsuyasu and Honda (1982) data exhibits wind-induced growth rates in excess of that which could be provided by the total stress at high values of *ak*. This appears to be a result of normalising wave growth rates of steep waves by wind stress measurements taken over waves of low slope. Banner (1990) has shown that the total wind stress can increase very rapidly at high wave steepnesses.
3. Bole (1967) used stream function techniques to quantify low-frequency wave energy in the presence of wind waves which would systematically yield growth rates that are too high at low wave steepness. If this is acknowledged, the data sets of Wilson *et al.* (1971), Bole (1967), Bliven et al., (1986) and Banner (1990) all support a single relationship between mean wave steepness and observed wave growth rates.
4. At high wave steepnesses, a ceiling value in the ratio of form drag to total drag is suggested of 0.82±0.07 in accord with the measurements of Banner (1990).
5. At low wave steepnesses, Wilson *et al.* (1971) show that as $ak \rightarrow 0$, the form drag contribution to the total stress also approaches zero.
6. Two data points are available from Mastenbroek *et al.* (1996) which indicate comparatively higher wind input at lower wave steepness than the other studies. These values were obtained from pressure measurement in the air whereas the others at comparable steepness were obtained from spatial measurements of wave energy. If these values are correct, this may be initial evidence of an effect of subsurface turbulence attenuating the wave growth (Teixeira and Belcher, 2002).
7. Comparison with the attenuation data of Peirson *et al.* (2003) support their assertion that at $ak<0.06$ form drag plays a primary role in observed wave attenuation but some other (as yet unidentified, perhaps wave-turbuence interaction) process is responsible for the strong rates of attenuation for

values of $ak > 0.06$.

8. Figure 2 shows a best fit curve to the assembled non-breaking wave data as a dashed line. If this line is transferred to the Plant diagram, it yields wave growth rates that are approximately 80% of the values given by the mean Plant curve.

There are a number of important conclusions that can be drawn from these results.

1. A collation of data from different researchers shows development of form drag over waves similar to that observed over hills by Belcher *et al.* (1993). Form drag as a proportion of the total stress increases approximately quadratically with mean wave steepness as a result of non-separated sheltering. When waves reach a mean steepness of 0.26 flow separation is initiated – a virtually identical threshold to that observed for hills (0.25).
2. By clearly linking wave growth to form drag, these results provide a stronger and more systematic relationship between momentum flux and energy flux for slow waves than has previously been available. These results confirm that non-breaking and breaking waves need to be addressed differently when developing more general representations of energy flux to naturally occurring wind-wave fields.
3. There has been little consideration of the potential significance of wave -turbulence interactions in models describing wave growth. However, it has been observed that normalised wave attenuation rates increase very rapidly and systematically with wave steepnesses above 0.06 and indicating that some other process apart from form drag is responsible for the wave attenuation. At present, there is insufficient wave growth data of suitable quality to determine whether this is an important process during the wave growth phase.

5. Conclusions

A re-analysis wave growth data available in the literature has been undertaken during this study. This investigation has focussed specifically on monochromatic wave behaviour.

Data from the sources used by Plant (1982) have been assembled and supplemented by substantial data sets from six other sources. These data sets show growth rates compatible with Plant's curve only if Plant's uncertainty factor of 2 is retained.

Total wind stress above monochromatic waves increases appreciably with mean wave steepness above a threshold value of approximately 0.15. Some investigators appear to have underestimated inverse wave age resulting in apparently very high growth rates when plotted on the Plant diagram.

Following the approach of Belcher *et al.* (1993) to drag over solid hills, we have found systematic wave growth behaviour when wave growth is normalised by wave speed multiplied by total wind stress and then plotted as a function of ak.

Reliable net wave growth rates determined by measuring the increase in monochromatic non-breaking wave energy are compatible with the wind energy input levels determined by Banner, 1990. However, there remains a disparity with the input levels measured by Mastenbroek *et al.*, 1996. This requires further investigation.

By assuming that the net growth rates of slow waves are dominated by the form drag, we have observed that slow wave growth rates show very similar behaviour to that which would be observed if they were treated as solid hills. In particular, the onset of flow separation occurs at a similar mean wave steepness. Below this threshold the development of the form drag component with mean steepness is of the same form to that observed for hills.

Reconciliation of air pressure-slope correlation measurements with net observed wave growth rates indicate that wind-induced form drag is the dominant growth mechanism for $ak > 0.12$.

Wave energy input increases approximately quadratically with mean wave steepness to the point of initiation of both breaking and flow separation. Across the transition to breaking, there is no further increase in normalised wave energy input but for strong breaking, no other measurements are available at present.

A quadratic best fit to the non-breaking data available to this investigation can be plotted on the Plant diagram yielding a curve parallel and 20% below the best fit determined by Plant (1982). Using our normalisation, wind energy input to breaking waves cannot continue to increase quadratically and therefore will not adhere to the Plant form.

A systematic form drag response also suggests that representative wave growth relationships of the form proposed by Jeffreys (1925) can been developed. This matter is currently under investigation. It also suggests that theoretical approaches are systematically over estimating the tangential stress component at the interface but the reasons for this are yet to be identified.

These results support the view of Peirson *et al.*, 2003 that net wave attenuation rates in the presence of an opposing wind are much greater than wave growth rates for comparable forcing once ak exceeds approximately 0.05. The reasons for these rapid attenuation rates are yet to be determined.

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