DRAFT

Proceedings of OMAE 2004:
23rd International Conference on Offshore Mechanics and Arctic Engineering
20-25 June 2004, Vancouver, Canada

OMAE2004-51562

IMPROVED SPECTRAL WAVE MODELLING OF WHITE-CAPPING DISSIPATION IN SWELL SEA SYSTEMS

David P. Hurdle, ***Alkyon Hydraulic Consultancy & Research***

Gerbrant Ph. Van Vledder, ***Alkyon Hydraulic Consultancy & Research***

ABSTRACT
An improved method for computing the dissipation by whitecapping in full spectral discrete wind wave models is presented. In this method the rate of dissipation of a certain spectral wave component is determined by the cumulative wave steepness of all spectral components with lower frequencies. This method is known as the cumulative wave steepness method (CSM) and was introduced by Van Vledder and Hurdle (2002). In this paper some improvements of the CSM are presented which are related to directional effects. The CSM was implemented in the latest release of the SWAN model (version 40.31 of February 2004) and a calibration against a parameteric growth curve was performed. Further, the benefits of the CSM in swell-sea systems are discussed and the method is compared to the Komen et al. (1994) method.

INTRODUCTION
Spectral wave models are an important tool for transforming of deep-water wave conditions to shallow water for deriving design loads on marine structures and coastal protection systems. In the Netherlands the SWAN model is used to derive wave boundary conditions along the water defenses. A weak point of the SWAN and similar full spectral models is an under-prediction of wave period measures in many typical shallow water situations. Detailed analysis of wave observations and wave model runs with SWAN indicate that this under-prediction is stronger in locations were relatively long offshore waves coexist with shorter locally generated waves. Analyses of spectral shapes clearly indicate an under-prediction of low-frequency energy and an over-prediction of the wind sea part. The under-prediction of low-frequency wave energy was also noted by Rogers et al. (2003). They analyzed the under-prediction of swell energy in two coastal areas and an inland lake, and they concluded that the Komen et al. (1994) method for modeling dissipation by whitecapping is responsible for the enhanced dissipation of swell waves. This is remarkable; since it seems very unlikely that whitecapping should affect swell waves.

Most present spectral wave models, including WAM, SWAN and WAVEWATCH, are third-generation wave prediction models. The aim of such models is to model each individual physical process on the basis of first principles in the form of source terms. This aim is rather ambitious in view of our lack of understanding of many these processes. One of the least understood processes is the dissipation by white-capping. A commonly used formulation for white-capping is the one proposed by Komen et al. (1994). A key element of this formulation is the use of a mean wave steepness. This steepness is used to scale the amount of white-capping. This formulation works well in idealized growth situations as long as the wave spectra are uni-modal. As indicated in Van Vledder and Hurdle (2002) this white-capping formulation has erroneous properties in double-peaked wave spectra, especially in relation with its effect on mean period measures. The presence of a small amount of swell energy lowers the mean wave steepness, resulting in a lower rate of dissipation of the whole spectrum. As a result the wind sea peak grows too fast, resulting in an over-estimation of high-frequency energy and an under-estimation of the mean wave period. In contrast, a small wind sea superimposed on a swell system increases the mean wave steepness, resulting in a higher dissipation rate of the whole spectrum. As a result the unrealistic dissipation of the swell takes place, also leading to an under-prediction of wave period.

These problems can be avoided by the cumulative wave steepness method, introduced by Van Vledder and Hurdle (2002). In this method the dissipation rate at a certain spectral component is determined by the cumulative wave steepness of

Copyright © #### by ASME

all spectral components with lower frequencies. This method is known as the cumulative wave steepness method (CSM).

The main advantage of the CSM is that is does not use integral wave parameters. Use of integral parameters has several unwanted properties. One of them is an unrealistic coupling between the dissipation of various wave systems, as described in the introduction. Another property is the dependence of the parameter values on the upper integration limit. Especially the higher frequency moments are very sensitive on the upper integration limit. In the CSM these problems are eliminated because the dissipation of a certain wave component only depends on the wave energy at lower frequencies.

In this paper new features and results of the CSM method are presented. The CSM has been extended in a natural way to includ directional effects, it has been implemented in the SWAN model (Booij et al., 1999) and calibrated against established parametric growth curves. Finally, the results of the CSM are demonstrated in a swell-sea system.

NOMENCLATURE

A_m	normalisation coefficient in CSM
C_{csm}	scaling coefficient of CSM
C_{nl4}	scaling coefficient of source term for quadruplet wave-wave interactions
d	water depth (m)
$E(\omega)$	energy density spectrum (m^2/rad)
F_*	dimensionless fetch (-)
g	acceleration due to gravity (m/s^2)
H_{m0}	significant wave height (m)
m	parameter controlling directional dependence
M_{wc}	magnitude of whitecapping source function
$S_{st}(\omega,\theta)$	steepness spectrum
$S_{wc}(\omega,\theta)$	source function for whitecapping
T_{m01}	mean wave period (s)
T_p	peak wave period (s)
u_*	friction velocity (m/s)
U_{10}	wind speed at 10 m height (m/s)
ε_*	dimensionless wave energy
λ	shape parameter of wave number configuration
θ	direction (rad)
σ	directional spreading
ν_*	dimensionless peak frequency
ω	radian frequency
ω_{min}	minimum discrete frequency of SWAN
ω_{max}	maximum discrete frequency of SWAN

THE CUMULATIVE WAVE STEEPNESS METHOD FOR WHITE-CAPPING

Based on ideas of Donelan (1999) Van Vledder and Hurdle (2002) proposed a possible solution for this behavior of the standard white-capping dissipation. They argue that the dissipation of a spectral component should not be based on the average steepness of the whole spectrum. Instead they proposed to link this dissipation rate to the cumulative wave steepness of all spectral components up to the frequency considered. This method is known as the Cumulative Steepness Method (CSM).

In the CSM the cumulative squared wave steepness is computed as:

$$S_{st}\left(\omega\right)=\int_0^{\omega}k^2E\left(\omega'\right)d\omega' \qquad (1)$$

In which the wave number k and the radian frequency ω are related via the linear dispersion relation $\omega^2=gk\tanh(kd)$ with d the water depth. This is the integral of the steepness spectrum up to the frequency ω.

Using the the cumulative steepness S_{st}, the source term for whitecapping is given by:

$$S_{wc}\left(\omega,\theta\right)=-C_{csm}S_{st}\left(\omega\right)E\left(\omega,\theta\right) \qquad (2)$$

with C_{csm} a tunable coefficient.

Expression (2) does not include explicit directional effects, but directional effects are included in this method by considering the physical process of the straining mechanism. In this mechanism short waves propagating on top of larger waves are compressed on the forward face of the longer wave and stretched at the backward face. The compressed waves become steeper, resulting in enhanced dissipation. It is assumed that this process does not act when waves propagate at an angle of 90° with respect to one another. It acts similarly when the waves have opposing direction components of propagation and weaker when the waves are travelling at an oblique angle. A simple way to account for this affect is to introduce a cosine-type dependence of the angular difference in combination with an absolute operation. This leads to the following extended formulation for computing the cumulative steepness at a certain frequency- direction bin:

$$S_{st}\left(\omega,\theta\right)=\int_0^{\omega}\int_0^{2\pi}k^2\left|\cos\left(\theta-\theta'\right)\right|^m E\left(\omega',\theta'\right)d\theta' d\omega' \qquad (3)$$

In this expression m is a tunable coefficient, which controls the directional dependence. It is expected that this coefficient is order 1 if the straining mechanism is dominant. If directional effects do not play a role, the coefficient m takes the value 0 and expression (3) reduces to expression (1). On the other hand, if the whitecapping can be considered to be directionally decoupled (i.e. the dissipation is not influenced at all by waves in other direction sectors) then m approaches infinity.

Equation (3) has the property that for a given vale of C_{csm} the amount of steepness depends on the coefficient m. For practical reasons it is desirable to isolate directional effects, as controlled by the coefficient m, from the total amount of dissipation, as controlled by the coefficient C_{csm}. This is achieved by normalizing the right-hand side of expression (3) by assuming

Copyright © #### by ASME

that the $\cos^m(\theta)$ function is a distribution function. The normalization coefficient A_m is such that:

$$\int_0^{2\pi} A_m \cos^m(\theta) = 1 \quad (4)$$

This leads to the following expression for A_m:

$$A_m = \frac{1}{\sqrt{\pi}} \frac{\Gamma(m/2+1)}{\Gamma(m/2+0.5)} \quad (5)$$

The following expression for the steepness spectrum then becomes:

$$S_{st}(\omega,\theta) = A_m \int_0^{\omega} \int_0^{2\pi} k^2 \left|\cos(\theta-\theta')\right|^m E(\omega',\theta')\, d\theta' d\omega' \quad (6)$$

The source term for the whitecapping is given by:

$$S_{wc}(\omega,\theta) = -C_{csm} S_{st}(\omega,\theta) E(\omega,\theta) \quad (7)$$

Expression (7) has been implemented in SWAN 40.31 with the parameters C_{csm} and m as tunable parameters. In this paper this SWAN model version has been used to calibrate and validate the CSM method.

PROPERTIES OF THE CSM METHOD FOR WHITECAPPING

Some basic properties of the CSM are illustrated using a uni-modal and a bi-modal spectrum. The directional integrated cumulative wave steepnesses for these spectra are shown in Figure 1. As can be seen, the cumulative steepness is practically zero for the lowest frequencies of the swell system. It increases quite quickly for frequencies above the peak of the wind sea spectrum. Especially for the higher frequencies the extra contribution of the swell system is visible.

The corresponding directionally integrated source functions of the CSM are shown in Figure 2, together with a comparison with the Komen et al. (1994) source function. Inspection of this figure clearly shows that the Komen formulation reduces the dissipation of the wind sea in the presence of a swell system, whereas the CSM slightly increases this dissipation. Further, the Komen method increases the dissipation of the swell system in the presence of a wind sea system.

The applicability of the normalization coefficient A_m was tested for a JONSWAP type spectrum with a cos-shaped directional distribution, and various values of the coefficient m. To that end the CSM source function was computed using a range of values for m.. The total dissipation rate was then computed by integration of the source function for white-capping according to:

$$M_{wc} = \int_0^{2\pi} \int_{\omega_{\min}}^{\omega_{\max}} \left|S_{wc}(\omega,\theta)\right| d\omega d\theta \quad (8)$$

in which ω_{min} and ω_{max} are the lowest and highest frequencies in the discrete SWAN spectrum. The normalized variation of M_{wc} as a function of m is shown in Figure 3. It can be seen that M_{wc} is practically constant with varying power m. This implies that the coefficients C_{csm} and m can practically be treated as independent variables.

The dependence of the directional distribution of the dissipation rate on the coefficient m is illustrated in Figure 4 for a mean JONSWAP spectrum with a $\cos^4(\theta)$ directional distribution. The directional distribution of the whitecapping dissipation is computed by integration over the frequencies as:

$$S_{wc}(\theta) = \int_{\omega_{\min}}^{\omega_{\max}} S_{wc}(\omega,\theta) d\omega \quad (9)$$

Figures 4 contains three curves. The solid curve is for m=0, the dashed curve is for m=2 and the dotted curve is for m=1000. The directional resolution of the discrete spectrum is 10°. The results in this figure show that with increasing power m the directional distribution of the white-capping dissipation becomes narrower. This behaviour is intriguing since the only other source term affecting the directional distribution is the one for the quadruplet wave-wave interactions. This implies that the parameter m should be used in combination with the parameter C_{nl4} and λ of the Discrete Interaction Approximation for the quadruplet source term to calibrate SWAN 40.31.

CALIBRATION OF CSM

The two coefficients of the CSM have been determined by means of a calibration against the growth curve of Kahma and Calkoen (1992). To that end the SWAN 4031 model was applied in one-dimensional mode to generate deep water growth curves for a fetch of 25 km and two wind speeds of 10 m/s and 20 m/s. The dimensionless fetch laws were taken from the composite data set of Kahma and Calkoen (1992). They are given by:

$$\begin{aligned} \varepsilon_* &= 6.5\times10^{-5} \times F_*^{0.9} \\ \nu_* &= 3.08 \times F_*^{-0.27} \end{aligned} \quad (10)$$

with ε_* and ν_* the dimensionless wave energy and peak frequency, respectively, as normalized by the friction velocity u_*. In the calibration of SWAN 4031 also the coefficients C_{nl4} and λ of the source term for quadruplet wave-wave interactions were optimized. This was necessary since the source terms for whitecapping and quadruplet interactions act together during active wave growth.

The results of the calibration yielded the following parameter values: C_{csm} = 0.4 , m = 2, C_{nl4} =4e7, and λ=0.28.

Copyright © #### by ASME

An example of a computed growth curve and the comparison with the parametric growth curve of Kahma and Calkoen is shown in Figure 5. The agreement is very satisfactory. The development of the frequency spectra along the fetch is shown in Figure 6. The spectra are very realistic and they clearly show the overshoot at higher frequencies.

WAVE GROWTH IN A SWELL SEA SYSTEM

The benefits of the of the CSM over the Komen method are demonstrated in a situation of wave growth with and without the presence of background swell system. To that end two wave model runs with the SWAN model were made. The first run comprises an ideal deep water wave growth situation starting with zero waves and a wind speed of 20 m/s. In the second run a background swell with a peak period T_p=10 s, a significant wave height of H_{m0}=0.5 m and a $\cos^{10}(\theta)$ directional spreading was specified as initial condition. The growth curves of the significant wave height H_{m0} and mean wave period T_{m01} are presented in Figure 7. The frequency spectra at fetches of 1, 5 10, 15, 20 and 25 km are shown in Figure 8.

The results in these figures show that the presence of a background swell slightly increases the overall level of the significant wave height H_{m0} and mean wave period T_{m01}. Comparison of the frequency spectra shown in the Figures 6 and 8 indicate that the wind-sea spectra of swell situation are slightly lower than the pure wave growth situation. This is due to the increased level of dissipation by the presence of the background swell. As the wave become more developed the wind-sea and swell start to interact via the CSM and the quadruplet wave-wave interactions. The precise characteristics of these interactions need to be investigated in more detail.

DISCUSSION AND CONCLUSION

In the present paper an improved version (with respect to Van Vledder and Hurdle, 2002) of the CSM for white-capping dissipation is presented. The results show that the CSM was succesfully implemented in the SWAN 40.31 model and could be calibrated against established growth curves. The main benefit of the CSM is its ability to predict realistic wave growth in a situation with a background swell.

The CSM deviates considerably from the well-known Komen et al. (1994) or the extended Komen formulation, proposed by Holthuijsen et al. (2001). The extended Komen formulation reduces to the standard Komen formulation in the case of uni-modal spectra. Both of these methods use mean properties of the wave spectrum, which are not used in the CSM. Moreover, the CSM uses a completely different method for estimating the dissipation by white-capping, which may lead to a further questions and understanding of the source term balance in wave growth situations.

Further work is in progress to study the directional behaviour of the CSM in two-dimensional situations. Also, detailed studies in the field situations are needed to assess the ability to predict wave conditions in the Dutch coastal waters, especially in situations with double peaked spectra. Another point of interest is the nature of the interactions between the wind-sea and swell systems by non-linear quadruplet wave-wave interactions.

REFERENCES

Booij, N., R.C. Ris, and L.H. Holthuijsen, 1999: A third generation wave model for coastal regions. 1. Model description and validation, J. Geophys. Res., 104, 7649-7666.

Donelan, M.A., 1999. Presentation at WISE meeting, Annapolis, USA.

Holthuijsen, L.H., R.C. Ris, N. Booij and E. Cecchi, 2000. Swell and Whitecapping, A Numerical Experiment. Proc. 27th Int. Conf. on Coastal Eng. Sydney, Australia.. Pp. 346-354.

Kahma, K.K., and C.J. Calkoen, 1992: Reconciling discrepancies in the observed growth rates of wind waves. J. Phys. Oceanogr. Vol. 22, 1389-1405.

Komen, et al., 1994: Dynamics and modelling of ocean waves. Cambridge Univ. Press.

Rogers, W.E., P.A. Hwang, and D.W. Wang, 2003: Investigation of wave growth and decay in the SWAN model: Three Regional-Scale Applications. Journal of Phys. Oceanogr., Vol. 33, 366-389.

Van Vledder, G.Ph., and D.P. Hurdle, 2002: Performance of formulations for whitecapping in wave prediction models. Proc. OMAE 2002.

Copyright © #### by ASME

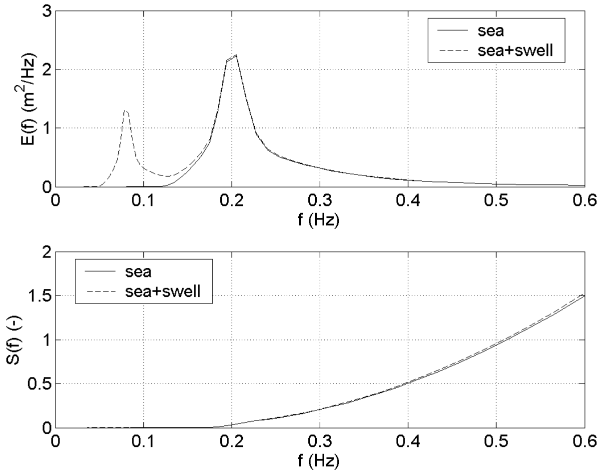

Figure 1: Frequency spectra (upper panel) and cumulative wave steepness (lower panel) for a uni-modal spectrum and a bi-modal spectrum.

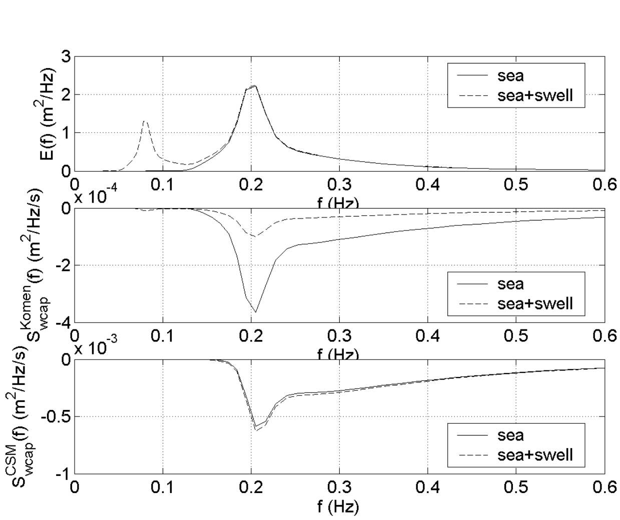

Figure 2: Frequency spectra (upper panel), Komen white-capping source terms (middle panel) and CSM white-capping source terms (lower panel) for a uni-modal and a bi-modal spectrum.

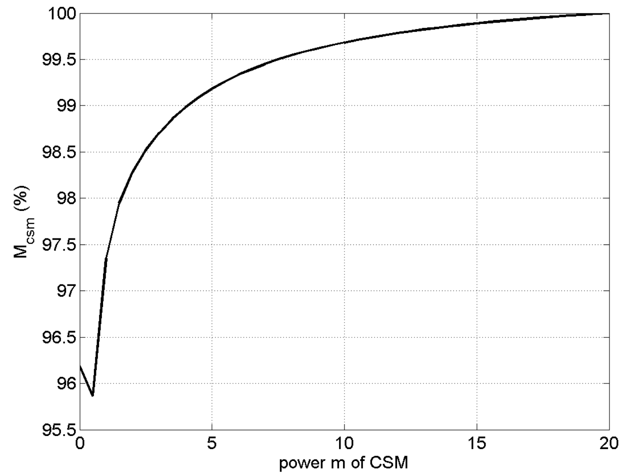

Figure 3: Normalized variation of white-capping source term magnitude with the parameter m for a mean JONSWAP with a $\cos^2(\theta)$ directional distribution.

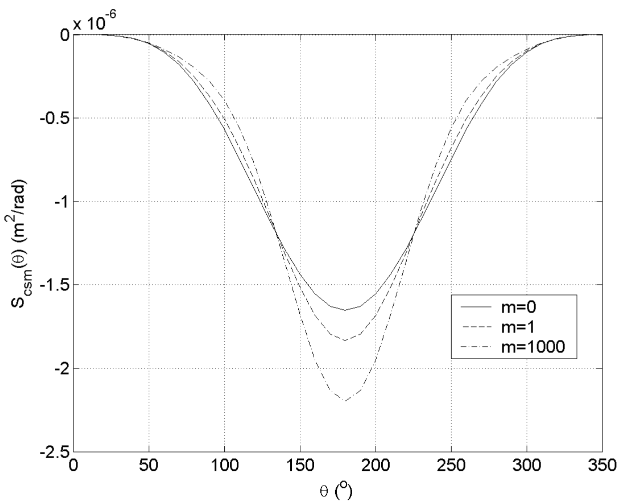

Figure 4: Directional distribution of CSM white-capping for a mean JONSWAP spectrum with a a $\cos^2(\theta)$ directional distribution for m=0, 1 and 1000.

Copyright © #### by ASME

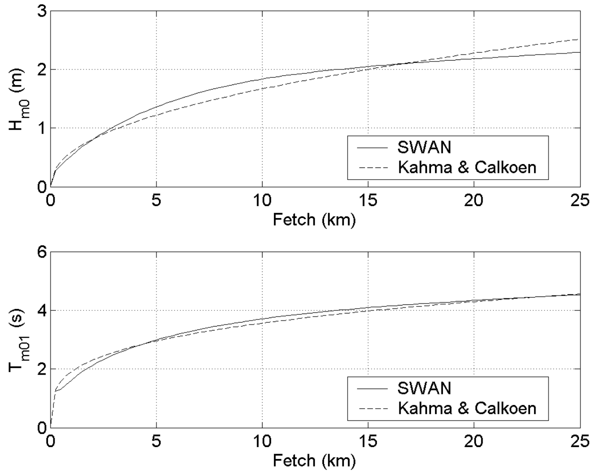

Figure 5: Growth curve of significant wave height H_{m0} and mean wave period T_{m01} of fetch-limited deep water wave growth for a wind speed of 20 m/s computed with the calibrated SWAN 40.31 and compared with the parametric growth curve of Kahma and Calkoen (composite data set, scaled with friction velocity u_*).

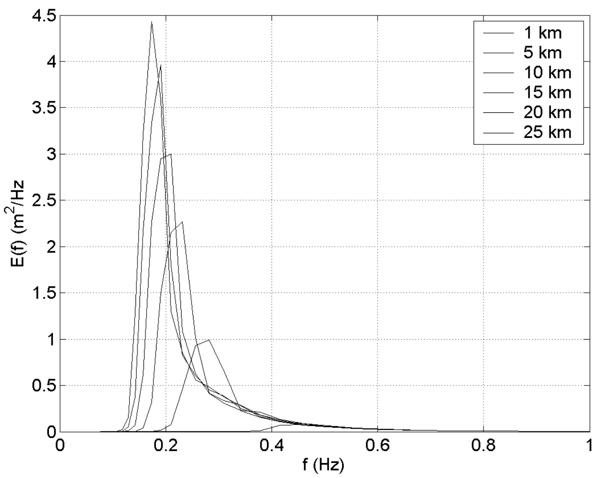

Figure 6: Frequency spectra at fetches of 1, 5, 10, 15, 20 and 25 km for the situation described in legend of Figure 5.

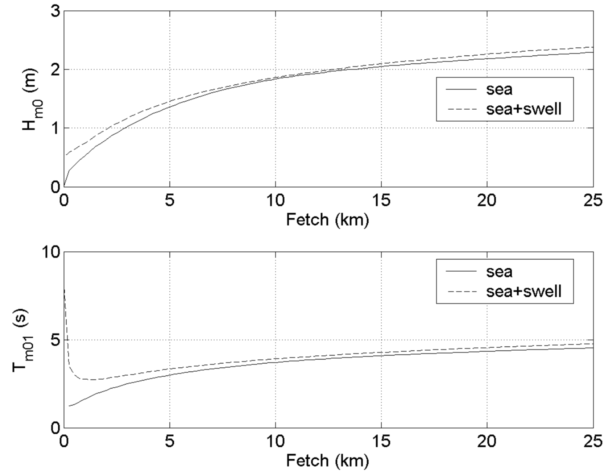

Figure 7: Growth curve of significant wave height H_{m0} and mean wave period T_{m01} of fetch-limited deep water wave growth for a wind speed of 20 m/s and an incoming swell with a significant wave height H_{m0}=0.5 m and a peak period T_p=10 s, computed with the calibrated SWAN 40.

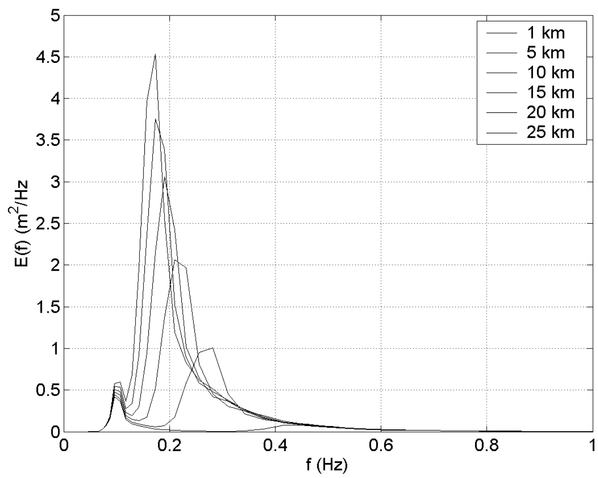

Figure 8: Frequency spectra at fetches of 1, 5, 10, 15, 20 and 25 km for the situation described in legend of Figure 6.

Copyright © #### by ASME