Pergamon

Phys. Chem. Earth (B), Vol. 24, No. 5, pp. 437–441, 1999

1464-1909/99/$ - see front matter

PII: S1464-1909(99)00025-8

Tsunami Generation in Compressible Ocean

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Received 10 August 1998; accepted 10 November 1998

Abstract. Most of the tsunami modelling studies have been carried out within the framework of incompressible fluid theory. Estimations we made show that this approach quite appropriately describes the stages of tsunami propagation and run-up, whereas incompressible fluid theory fails to describe the process of tsunami generation properly. Comparative studies of wave generation by piston bottom displacements in compressible and incompressible fluids have been carried out within the framework of linear potential theory. The analysis of exact analytical solutions to the problem has shown that a substantial difference exists between the behaviour of compressible and incompressible fluids under typical tsunami source conditions. It was also shown that bottom displacements of seismic origin must give rise to standing acoustic waves in the source region. As a result each tsunami should have its own "voice", the characteristics of which depend on bottom topography, sediment features and on the time history and spatial structure of the bottom displacement. This "voice" may serve as an additional important tool for tsunami forecasting.

1 Introduction

In order to predict with reasonable accuracy the possible height a tsunami will reach at a given point on the shore, and thus to limit loss of life and property, detailed knowledge of the following three processes is required: 1. The generation of the wave; 2. The propagation of the wave in the deep ocean; 3. The propagation of the wave in the continental shelf zone and its run-up on the ocean shore.

This paper focuses on the first of the three stages mentioned above, i.e. on the mechanism that leads to the generation of a tsunami by bottom displacements. For reliable tsunami prediction, this is the most important stage, since it depends on it, whether a given underwater earthquake leads to the formation of a tsunami or not, and if it does, of what intensity the tsunami will be in a given direction.

In spite of the fact that the tsunami problem was under scientists investigation for many years an exact and reliable forecast of the tsunami heights at shore still remains a task to be solved (Pelinovsky, 1996). One of the possible reasons for that is shortage of knowledge about physical mechanisms of processes in the ocean above the submarine earthquake epicentre region. The overwhelming majority of the tsunami models uses oversimplified approximations and assumptions (incompressible fluid, shallow water equation, impulse bottom displacements, the initial water surface elevation is assumed to be equal to the residual bottom displacements). Thus generally speaking, one can not expect an exact and reliable tsunami forecast if the numerical model is based on inaccurate physical statements. In particular the water compressibility is neglected, or the sound velocity is assumed to be infinite. It should be noted that some aspects of the problem of tsunami generation in consideration of water compressibility certainly was under investigation earlier (Sells, 1965; Kajiura, 1970; Pelinovsky, 1996; Nosov, 1998; Nosov and Sommer, 1998), but generally the problem still remains a task to be solved. In the present paper it will be shown that the water compressibility should be taken into consideration if one pretends to describe the tsunami generation accurately.

2 Parameters and estimations

From the very formal physical point of view (Landau and Lifshits, 1987) there are two conditions to be satisfied for the tsunami generation process one can consider as a process running in the incompressible fluid: 1. $v \ll c$; 2. $\tau \gg \{H c^{-1}, S^{1/2} c^{-1}\}$, where v is the fluid velocity (~1m/s), c is the sound velocity in water (1500m/s), τ is the duration of bottom displacement (10^0-10^2s), H is the total

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ocean depth (10^3m), S is the area of the tsunami source (10^8-10^{11}m^2). Thus the first condition is always satisfied, whereas the second one is not satisfied in most cases.

It should be noted that these two conditions are completely satisfied for the stages of tsunami propagation and run-up. The second condition for these cases has the following form: $T >> \{H c^{-1}, \lambda c^{-1}\}$, where λ is the length of tsunami wave (10^4-10^5 m), T is the tsunami period which can be estimated as $T = \lambda (g H)^{-1/2}$.

Also there is an additional reason to take into account the water compressibility for the tsunami generation process. For the piston bottom displacement of the finite duration (in terms of the model which is described in details in section 3 and on Fig. 1.) it is easy to estimate the energy associated with the gravitational tsunami wave W_1 (potential energy of initial elevation) and the acoustic waves generated by moving bottom W_2: $W_1 = 0.5 \rho g S (v\tau)^2$, $W_2 = c \rho S v^2 \tau$, where g is the acceleration due to gravity, ρ is the density of water, and v is the vertical velocity of moving bottom. The result of the estimation is as follows: $W_2 / W_1 = 2c / (g\tau) >> 1$. It means that the considerable part of the energy transfers from moving bottom to the ocean certainly exists not as the gravitational but as the acoustic waves.

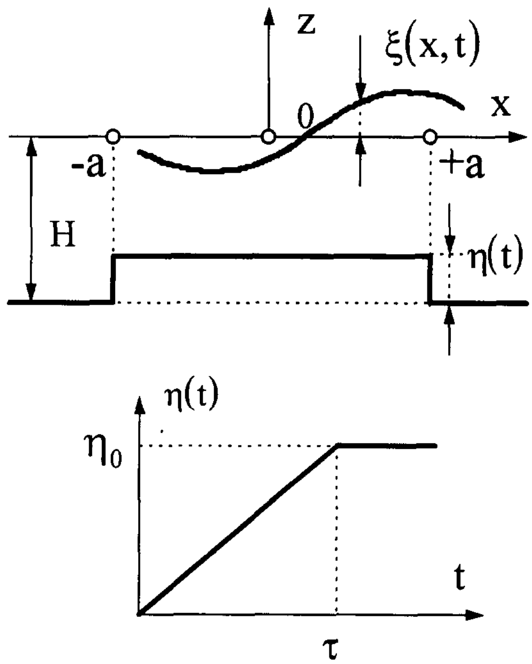

Fig. 1. Mathematical statement of the problem.

3 Mathematical model

Let us consider a layer of an ideal compressible homogeneous fluid of constant depth H in the field of gravity and assume that it is unbounded along the OX-axis (Fig. 1). The origin of the Cartesian coordinate system OXZ finds itself at the unperturbed free surface, and the OZ-axis is oriented vertically upward. To find the wave disturbance $\xi(x,t)$ that is excited at the fluid surface by bottom motions of the form $\eta(x,t)$ it is necessary to solve the following problem for the flow velocity potential $\varphi(x,z,t)$ (Landau and Lifshits, 1987):

$$\varphi_{xx} + \varphi_{zz} = \frac{1}{c^2}\varphi_{tt}; \tag{1}$$

$$\varphi_{tt} = -g\varphi_z, \quad z = 0; \tag{2}$$

$$\varphi_z = \eta_t, \quad z = -H, \tag{3}$$

where c is the sound wave velocity, g is the acceleration due to gravity. For the incompressible fluid the equation (1) degenerates into the Laplace equation:

$$\varphi_{xx} + \varphi_{zz} = 0,$$

whereas the boundary conditions (2), (3) remain the same. In what follows the obtaining of solution will be given in details only for the case of compressible fluid.

Let us search for a solution to problem (1)-(3) in the form of Laplace and Fourier transforms with respect to the temporal and spatial coordinates respectively:

$$\varphi(x,z,t) = \int_{s-i\infty}^{s+i\infty} dp \int_{-\infty}^{+\infty} dk\, \Phi(z,p,k) \exp(pt - ikx). \tag{4}$$

The disturbance of the fluid surface is expressed in terms of the flow velocity potential as follows:

$$\xi(x,t) = -g^{-1}\varphi_t(x,0,t). \tag{5}$$

Omitting the necessary manipulations, I give the resulting expressions for the disturbance of the fluid surface excited by arbitrary bottom motions $\eta(x,t)$:

$$\xi(x,t) = g^{-1} \int_{s-i\infty}^{s+i\infty} dp \int_{-\infty}^{\infty} dk \frac{p^2 \Psi(p,k)}{\alpha\, sh(\alpha H) + p^2 g^{-1} ch(\alpha H)} \exp(pt - ikx), \tag{6}$$

where

$$\alpha^2 = k^2 + p^2 c^{-2},$$

$$\Psi(p,k) = \frac{1}{4\pi^2 i} \int_0^{\infty} dt \int_{-\infty}^{+\infty} dx\, \eta(x,t) \exp(-pt + ikx).$$

I introduce nondimensional variables (the superscript "*" will be omitted in what follows)

$$k^* = kH, \quad a^* = \frac{a}{H}, \quad x^* = \frac{x}{H}, \quad t^* = \frac{tc}{H}, \quad \tau^* = \frac{\tau c}{H}, \quad p^* = \frac{pH}{c}, \quad c^* = \frac{c}{(gH)^{1/2}} \tag{7}$$

I assume the following model law to hold for bottom motion (Fig. 1.):

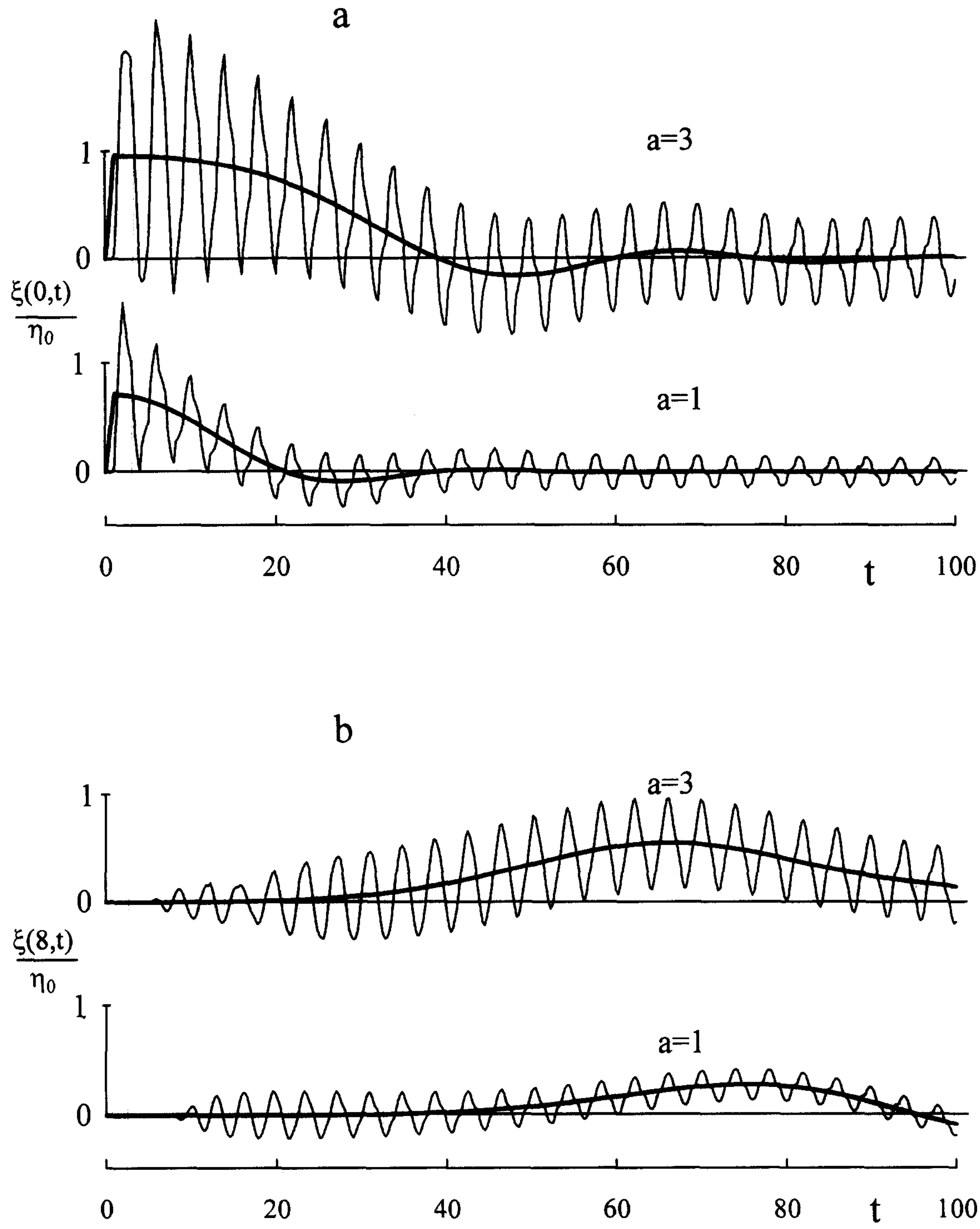

Fig. 2. Time-history of surface disturbance generated by bottom displacement of duration $\tau=1$ at the points $x=0$ (a) and $x=8$ (b) in incompressible (thick curves) and compressible (thin curves) fluid for two different sizes of the source.

$$\eta(x,t)=\eta_0[\theta(x+a)-\theta(x-a)]\left[\frac{\theta(t)t-\theta(t-\tau)(t-\tau)}{\tau}\right] \quad (8)$$

where $\theta(z)$ is the Heaviside step function, η_0 is the amplitude of the motion, 2a is the horizontal size of the excitation area, and τ is the duration of the motion. For this particular case the final expression is as follows:

$$\xi(x,t)=\theta(t)\zeta(x,t)-\theta(t-\tau)\zeta(x,t-\tau), \quad (9)$$

where

$$\zeta_{com}(x,t)=\frac{\eta_0 c^2}{2\pi^2 i\tau}\int_{s-i\infty}^{s+i\infty}dp\int_{-\infty}^{+\infty}dk\frac{\sin(ka)\exp(pt-ikx)}{k(\alpha\,\mathrm{sh}(\alpha)+p^2c^2\,\mathrm{ch}(\alpha))}, \quad (10)$$

$\alpha^2=k^2+p^2$.

Following the way that was shown above, it is quite easy to obtain the solution of the same problem for incompressible fluid. The final expression (9) has the same structure but instead of the expression (10) the expression (11) to be used.

$$\zeta_{inc}(x,t)=\frac{\eta_0 c}{\pi\tau}\int_{-\infty}^{+\infty}dk\frac{\exp(-ikx)\sin(ka)\sin\left[c^{-1}(k\,\mathrm{th}(k))^{1/2}t\right]}{k\,(k\,\mathrm{th}(k))^{1/2}} \quad (11)$$

In contrast to the case of compressible fluid for the incompressible model it was easy to evaluate the integral with respect to p analytically. This is why the expression (11) contains only single integral.

The imaginary parts of (10) and (11) vanish, because the respective parts of the integrands are odd functions. The integrals in the real parts of (10) and (11) are computed numerically.

In what follows, the source length will be taken as 2a=2, 6, 10 and the speed of sound as c=8. With the scaling relations (7) for an ocean depth of 4 km, these values correspond to an active length 8, 24, 40 km and a speed of sound of 1585 m/s, which is approximately the speed of sound in water.

4 Discussion of results

The results of the calculations for both compressible and incompressible fluid are given in Fig. 2. as time-history of water surface disturbance at the points x=0 (in the centre of the generation region, Fig.2a.) and x=8 (out of the generation region, Fig. 2b). The bottom displacement duration is 1 (~2.7 seconds). The time steps between two calculated points are 0.25.

First of all it should be emphasised that the compressible fluid model describes the process more adequate than the incompressible one, in terms of proper time the disturbance reaches the given point.

Before establishing the conditions under which the compressibility of water has to be taken into account in order to describe the surface disturbance adequately, the main difference between the compressible and incompressible case shall be discussed. From Fig. 2 it can be seen that the most striking feature of the surface disturbance of compressible water is the appearance of fast vertical oscillations of the surface level. "Fast" here means that these oscillations are of considerably higher frequency than the oscillations due to the purely gravitational waves. These fast surface oscillations are a consequence of compressive and rarefactive waves travelling back and forth between the bottom and the surface of the water layer. The period of the oscillations of roughly T=4 (~10.7 seconds) can be explained as follows: the upward motion of the bottom of the water layer creates a compressive wave which causes an upward motion of the water surface. In compressible water this gravito-acoustical upward motion overshoots the level of the purely gravitational surface disturbance created in incompressible water and leads to the formation of a rarefactive wave travelling downwards. The rarefactive wave reaches the bottom after $\Delta t_1 = 1$, is reflected there, and returns to the surface $\Delta t_2 = 1$ later. There it causes the surface level to fall beneath its purely gravitational equilibrium level, thus leading to the formation of a compressive wave travelling downward. The compressive wave reaches the bottom after $\Delta t_3 = 1$, is reflected there, and returns to the surface $\Delta t_4 = 1$ later. It causes the surface level to rise above its purely gravitational "equilibrium" level, and so on. So the time that passes between two subsequent maximums of the surface level in compressible water is given by $\Delta t=\Delta t_1+\Delta t_2+\Delta t_3+\Delta t_4=4$.

In other words the compressible water layer bounded above by free surface, and bounded below by hard bottom, is a natural resonator. The resonant frequencies are given by the following expression:

$$\nu_k=\frac{c(1+2k)}{4H}, \quad (12)$$

where k=0,1,2,3,... Thus the lowest frequency is equal to c/4H. With the scaling relations (7) this value corresponds to the oscillation period of 4.

Fig. 3 shows the maximum amplitude of the surface oscillations due to the fluid compressibility at the centre of generation region (x=0) as a function of the duration of bottom displacement τ for three different sizes of source. The maximum amplitude was calculated as a half of maximum difference between surface disturbances for compressible and incompressible cases:

$$A_{max}=0.5\,\eta_0^{-1}\,\mathrm{Max}\,(\xi_{com}(t)-\xi_{inc}(t)). \quad (13)$$

The maximum amplitude decreases as the duration of the bottom displacement goes up. Because of the resonant features of compressible water layer the dependence is nonmonotone. It should be also noted that the maximum amplitude does not increase indefinitely as either the duration of the bottom displacement τ decreases or the source size 2a increases. The Fig. 3 let me to make the very important conclusion: up to values of the bottom displacement duration of 10, the amplitude of the oscillations due to water compressibility can reach more than 10% of the maximum vertical bottom displacement η_0. With the scaling relations (7) for an ocean depth of 4 km, this value approximately corresponds to τ~30 s.

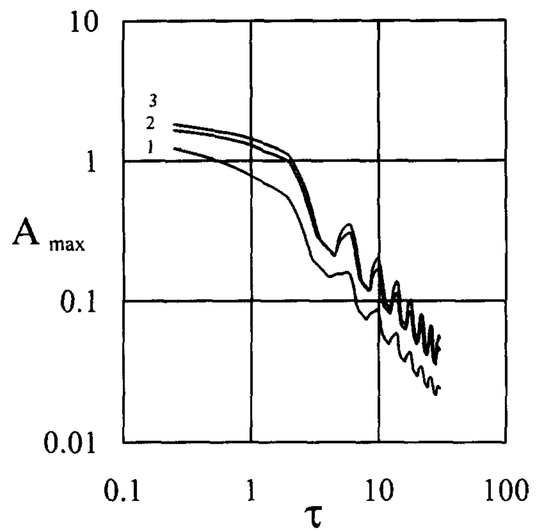

Fig. 3. The dependence of the maximum amplitude of surface oscillations due to fluid compressibility on the duration of the bottom displacement for a=1, 3, and 5 (curves 1-3, respectively).

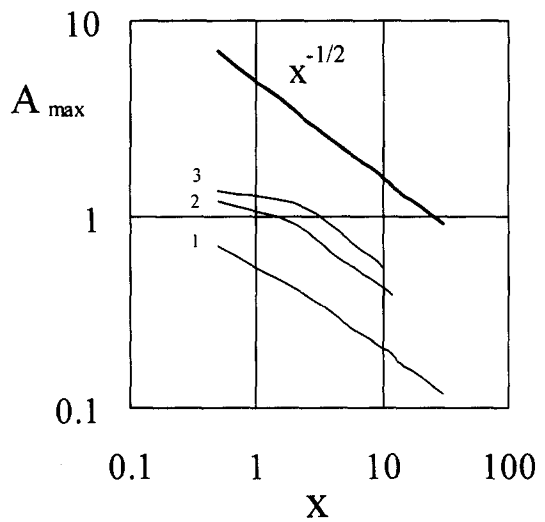

Fig. 4. The dependence of the maximum amplitude of surface oscillation due to compressibility on the distance given point x from x=0 for t=1 and a=1. 3, and 5 (curves 1-3, respectively).

In terms of the tsunami prediction it is very interesting to explore whether this surface oscillations are a pure local phenomenon or they can propagate for a long distance and serves as tsunami precursor. Fig. 4 demonstrates the relationship between the maximum amplitude and distance the given point x from the centre of the source (x=0) for $\tau=1$ and a=1, 3, and 5. As the distance increases the maximum amplitude decreases approximately as $x^{-1/2}$. Thus the surface fast oscillations can be observed at a distance of hundreds kilometres from the tsunami source.

5 Conclusion

The results of this study indicate that the compressibility of water can not be neglected, if one pretends to give an adequate description of the tsunami generation by bottom displacements of a duration less than 30 s (generally if $\tau<10H/c$). Tsunami source radiates both gravitational and acoustic waves, or more correctly a system of gravito-acoustical waves and the energy of the acoustic waves can significantly exceed the gravitational one. Due to the resonant features of the water layer, the bottom displacements of seismic origin must give rise to standing acoustic waves in the source region, thus the acoustic mode has a certain frequency spectrum which depends on bottom topography, sediment features and on the time history and spatial structure of the bottom displacement. The further study of the features of the gravito-acoustical waves radiated from tsunami source could be of interest for tsunami forecasting in terms of tsunami forerunners.

Acknowledgements. The research was supported by Russian Foundation for Basic Research under grant98-05-64522.

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