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Statistics of Ocean Wave Groups

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ABSTRACT

Statistical parameters are derived that describe the occurrence probability for the number and height of consecutive waves in a group, which are large compared to the average wave height. Such wave groups can create extreme forces in mooring lines of large vessels. All of the parameters that describe the statistics of wave groups can be derived from the energy spectrum representing the sea state, if the energy spectrum is assumed to contain only a narrow band of wave frequencies. Good agreement was found between the theoretical expressions derived in this paper and actual wave data.

INTRODUCTION

It has been shown by Hsu and Blenkarn[1] that the long period sway motion and peak mooring line force observed in a model study of a barge resulted from the combined action of a sequence of consecutive high waves in the random wave train. Such a sequence of waves will be referred to as a wave group. Independent model-tank tests conducted by Remery and Hermans[2] substantiated the findings presented in Ref. 1. It is anticipated that the action of wave groups may also be important in other types of ocean engineering systems that are subjected to horizontal motions with periods much longer than the period of individual waves. Such systems could include dynamically positioned vessels. In the analysis and design of such systems it would be prudent to check whether the action of wave groups could subject the systems to motions beyond those calculated on the basis of individual waves. One can either use recorded wave groups measured in the sea, or wave groups generated in a model tank as a basis to perform such a check. There are also available electronic and numerical methods to simulate ocean waves. However, in order to make proper engineering decisions, one would need to know whether the occurrence of such wave groups is a common or rare event. More specifically, one would require a probabilistic description of the basic characteristics of wave groups in order to assess the likelihood of a system to survive.

References and illustrations at end of paper.

The purpose of this paper is to develop tools with which one can characterize the statistics of wave groups in terms of occurrence probabilities. No attempt is made in this paper to illustrate how such statistical tools could be used in practical engineering. It is believed that development of rational procedures to utilize such tools for specific problems can best be developed by individual users.

THEORETICAL DERIVATIONS

In this paper a wave group will be defined in terms of the envelopes of wave crests and troughs as shown in Fig. 1. The distance between the crest envelope and the trough envelope will be called the height envelope, whereas the distance between the crest envelope and the mean water surface will be called the amplitude envelope. The height envelope indicates the change of wave heights. Similarly, the amplitude envelope indicates the change of wave amplitude. In reality, ocean waves are not symmetrical as is implied by the term, amplitude. Measured from mean water level, the distance to the crest of a wave is generally larger than the distance to the trough. However, in order to utilize the theoretical methods developed by S. O. Rice,[3] we shall assume that the wave amplitude is half of the wave height.

As one can see from Fig. 1, the height envelope has the characteristics of a wave with a much longer period than the individual waves. Therefore, a logical way to characterize a height envelope, or ultimately a wave group, is by its height and period. Since our primary interest here is in groups of large waves, we will be interested in the portion of the envelope with heights larger than a specified value. In such a case, the time duration for which the height envelope exceeds a specified level will be used to characterize the group's period as shown in Fig. 1. Because the term "period" is generally associated with the description of individual waves, we shall use the term "time duration" for the time variable associated with wave groups. The analysis given here is based on the continuous envelope curves. It should be pointed out, however, that one can derive wave group statistics based on the discrete waves. It is the authors' opinion that the continuous envelope model is more straightforward and rigorous than the discrete wave model. In this paper it will be assumed that the energy spectrum that characterizes the individual waves is concentrated in a relatively narrow band of frequencies. The narrow band assumption is powerful and permits useful statistical information to be derived. In general, energy spectra calculated for ocean waves have a high frequency tail. However, for practical purposes the narrow band assumption has been used extensively for ocean waves and has provided a beneficial insight and rationale for solving problems.

The probability of the height envelope crossing the level, h_o, during an interval of time will be assumed to be described by a Poisson process. The assumptions associated with using the Poisson model[4] are as follows.

1. The probability of the envelope crossing the level, h_o, in any nonoverlapping time interval is independent and time invariant.

2. The probability of crossing the level, h_o, in a small time interval is proportional to the size of the interval.

3. The probability of multiple crossings in a small time interval is negligible relative to the probability of a single crossing.

The time duration for a group of Level h_o will be defined as the elapsed time beginning when the envelope crosses above the level, h_o, and ending when the envelope crosses below the level, h_o. For the Poisson model, the probability that the time duration is greater than t can be expressed[4] as

$$P(h_o,t) = \exp[-t/\tau(h_o)] \quad , \ldots\ldots (1)$$

where $\tau(h_o)$ represents the average time duration for groups of Level h_o. Therefore, for a given sea state and a given Level h_o, $P(h_o,t)$ will be known when $\tau(h_o)$ is determined.

Let $p(a)$ denote the probability density function of the amplitude envelope and $p(a,\dot{a})$ the joint probability density function for the amplitude envelope, a, and the time derivative of the amplitude envelope, $\dot{a}$. Then, the fraction of time that the amplitude envelope exceeds the level, a_o, is given by

$$P(a_o) = \int_{a_o}^{\infty} p(a)\,da \, . \ldots\ldots\ldots (2)$$

The average frequency with which the amplitude envelope crosses the level, a_o, with a positive slope (i.e., an upcrossing) is found by extending the result given in Ref. 3 for the average frequency of zero crossings and may be written as

$$G(a_o) = \int_0^{\infty} \dot{a}\, p(a_o,\dot{a})\,d\dot{a} \ldots\ldots (3)$$

If L represents the total time duration of a wave record, the time that the amplitude envelope exceeds a_o during the record is $P(a_o)L$. The average number of upcrossings, i.e., the average number of groups, during the record is $G(a_o)L$. The average time duration that the envelope stays above the level, a_o, during the record is the amount of time the envelope exceeds the level, a_o, divided by the expected number of groups. Thus, the average time duration for groups of Level a_o is

$$\tau(a_o) = \frac{P(a_o)L}{G(a_o)L} = \frac{\int_{a_o}^{\infty} p(a)\,da}{\int_0^{\infty} \dot{a}\, p(a_o,\dot{a})\,d\dot{a}} \ldots (4)$$

Evaluation of the integrals in the above equation is straightforward, but somewhat lengthy.

Appendix A outlines the basic steps involved in the evaluation without the detailed mathematics. From the explanations given below and in Appendix A, the reader will be able to fill in the details.

By assuming ocean-wave energy spectra to be narrow banded, the probability density function, $p(a)$, for the envelope amplitude can be shown to be a Rayleigh distribution. From this assumption, the envelope amplitude and the slope of the envelope curve can also be shown to be independent random variables.

$$p(a,\mathring{a}) = p(a)\,p(\mathring{a}) \quad , \quad \ldots \ldots \ldots \quad (5)$$

where $p(\mathring{a})$ has a normal distribution. By substituting the expressions, for $p(a)$ and $p(a,\mathring{a})$ given in Appendix A, into Eq. 4, and letting $h_o = 2a_o/H_s$, for which H_s is the significant wave height, one has

$$\tau(h_o) = \frac{1}{2\sqrt{2\pi}\,h_o}\left[\frac{1}{f_e^{\,2} - \bar{f}^{\,2}}\right]^{1/2} \quad , \quad . \quad (6)$$

in which h_o is the level of the envelope height divided by the significant wave height. f_e is the average wave frequency[3] defined by

$$f_e^2 = \frac{\int_0^\infty S(f)f^2 df}{\int_0^\infty S(f)df} \quad \ldots \ldots \ldots \quad (7)$$

$S(f)$ is the wave energy spectrum, and $\bar{f}$ is the centroid of the wave energy spectrum defined as

$$\bar{f} = \frac{\int_0^\infty S(f)f\,df}{\int_0^\infty S(f)df} \quad . \quad \ldots \ldots \ldots \ldots \quad (8)$$

Eq. 6 can be rewritten as

$$h_o\,\tau(h_o) = \frac{K}{2\sqrt{2\pi}} \quad , \quad \ldots \ldots \ldots \ldots \quad (9)$$

where $K = \left[\frac{1}{f_e^{\,2} - \bar{f}^{\,2}}\right]^{1/2}$.

Since f_e and $\bar{f}$ will be constants for any given wave energy spectrum, K is a constant for any sea state specified by its energy spectrum. The inverse of K can be shown to equal the root-mean-square width of the energy spectrum about its centroid. Therefore, based on Eq. 9, the average time duration, $\tau(h_o)$, is inversely proportional to both the root-mean-square width of the spectrum and the level, h_o.

With $\tau(h_o)$ known, one can derive the following parameters which characterize the statistics of wave groups.

1. The average frequency of occurrence for wave groups of the level h_o will be represented by $\nu(h_o)$ and is equal to the average frequency of upcrossings of the level h_o. Hence, $\nu(h_o)$ is equal to $G(a_o)$ of Eq. 3 and can be derived from Eqs. 2 and 4 as

$$\nu(h_o) = \frac{P(h_o)}{\tau(h_o)} = \frac{\exp(-2\,h_o^2)}{\tau(h_o)} \quad . \; . \; . \; . \; . \quad (10)$$

The numerator of Eq. 10, in terms of a Rayleigh distribution for the height envelope, is equal to Eq. 2.

2. Assuming that the average wave period is T, then the average number of waves, N, in the time interval, t, is $N = t/T$. Thus the probability that a wave group contains more than N waves, higher than the level h_o, can be found from Eq. 1 as

$$P(h_o,N) = \exp\,[-NT/\tau(h_o)] \quad . \; . \; . \; . \; . \; . \quad (11)$$

3. Since $\nu(h_o)$ is the average frequency of occurrence for wave groups of Level h_o, the probability for the number, M, of such groups that would be encountered in a length of time, L, can be predicted in terms of the Poisson model[4] as

$$P_M(h_o) = \frac{[\nu(h_o)L]^M}{M!}\exp\,[-\,\nu(h_o)\,L] \quad . \quad (12)$$

4. In a wave group of N waves exceeding the level, h_o, the probability that the highest wave in the group is less than h can be expressed by the following expression, which is derived in Appendix B.

$$P(h|h_o,N) = \{1 - \exp[-2(h^2-h_o^2)]\}^N \quad . \; . \quad (13)$$

The energy spectra for the envelope and the square of the envelope can be developed from information given in Part IV of Ref. 3. This reference gives the energy spectrum for the square of a wave trace, and the spectrum for the output which would result if the wave trace were passed through a linear rectifier. In addition, it is shown that the envelope of the wave trace is equal to the low frequency portion of the rectified wave trace and that the square of the envelope is equal to the low frequency portion of the square of the wave trace. Using this information, one can derive the energy spectra for the envelope and the square of the envelope. By comparing the expressions for the two spectra, it is found that the two spectra differ only by a constant. Thus, in normalized form, the spectra for the envelope and the square of the envelope are equal and can be expressed as

$$E(f) = \frac{\int_0^\infty S(x)S(f+x)dx}{\int_0^\infty S^2(x)dx} \quad , \; . \; . \; . \; . \; . \; . \; . \; (14)$$

where S(x) is the energy spectrum for the wave trace and x is a dummy variable representing frequency.

COMPARISON OF THEORETICAL RESULTS WITH DATA

The joint industry Ocean Data Gathering Program recorded wave data in the Gulf of Mexico during a winter storm. These data will be compared with the theoretical results obtained in the previous section. The wave record was 90 minutes long and consisted of about 750 waves. The wave crest envelope was constructed by connecting adjacent wave crests with straight lines. This procedure was also used to construct the trough envelope. The envelope height was taken as the distance between the crest envelope and the trough envelope.

The energy spectrum for the waves is shown in Fig. 2. Eqs. 7 and 8, which are in terms of the energy spectrum, were evaluated by numerical integration. The range of integration was from 0 to 0.15 cp. It is to be pointed out that the high frequency cutoff at 0.15 cp is somewhat arbitrary. Such a cutoff was necessary to comply with the narrow band assumption employed in the derivation of the theoretical expressions.

The products of the average time duration, $\tau(h_o)$, and the level, h_o, for various h_o's found by analysis of the data are shown in Fig. 3. According to the theoretical results, these products should have a constant value. The data points in Fig. 3 show that these products have essentially a constant value up to h_o = 1.2, and then decrease to zero at h_o = 1.62. It is to be pointed out that the data is from a record of finite length in which the maximum envelope height in the record is h_o = 1.62. Thus for the data, the probability is zero for the occurrence of an envelope height greater than h_o = 1.62. On the other hand, the theoretical analysis assumes that the envelope height is described by the Rayleigh distribution for which the probability of having heights greater than h_o = 1.62 has a nonzero value. Because of the finite wave population of the record, the distribution for the envelope height of the data near the maximum value could not possibly be described accurately by the Rayleigh distribution. As shown in Appendix A, Eq. A-11, the value of $\tau(h_o)$ can be expressed as the following proportion:

$$\tau(h_o) \propto \frac{\int_{h_o}^\infty p(h)dh}{p(h_o)} \quad , \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (15)$$

where p(h) is the probability density function for the envelope height. For the Rayleigh distribution, the above expression equals $1/(4h_o)$. Fig. 4 shows a comparison of the cumulative probability of the envelope height determined from the data and the Rayleigh distribution. This plot gives the value of the integral in Eq. 15. The density function, $p(h_o)$, based on the data, can be obtained from the cumulative probability by numerical differentiation. Then by substituting the cumulative probability and $p(h_o)$ into Eq. 15, one obtains $\tau(h_o)$ values, which are independent of the assumption of a Rayleigh distribution. The values of $h_o\,\tau(h_o)$ evaluated by Eq. 15 are shown as a curve in Fig. 3 and agree very well with the data points.

The probability distribution for the time duration of the wave groups, $p(h_o,t)$, was calculated from Eq. 1 using the $\tau(h_o)$ value calculated by Eq. 15. The results are compared with data points in Figs. 5 through 7 for h_o = 0.75, 1.0 and 1.3. It is seen that the agreement is excellent and implies that the Poisson model, assumed in the previous section, is appropriate for the wave-group problem.

Figs. 5 and 6 present comparisons between Eq. 11 and data for the probability, $P(h_o,N)$, of the number of waves in a wave group with h_o 0.75 and 1.0. There were not enough wave group for h_o = 1.3 to allow a meaningful evaluation of $P(h_o,N)$ for the data. The data points shown in Figs. 5 and 6 were obtained by finding all wave groups with levels higher than h_o, tabulating the number of waves in each of the groups, and calculating the percentage occurrence of groups containing more than N waves. The $\tau(h_o)$ used in Eq. 11 was calculated from Eq. 15 and the average wave period, T, for high waves had been determined from the wave record to be abou 10 seconds. The correlations between the analytical results and data points are very good.

The average frequency of occurrence of wave groups for the level of h_o = 1, was predicted from Eq. 10 to be 0.0097 cp. In other words, on the average, one such wave group would occur every 103 seconds. The 90-minute wave record contained 59 wave groups with a level of h_o = 1. Therefore, for the data, $\nu(h_o = 1)$ = 0.011 cp, or one such wave group occurred every 91 seconds compared with the predicted value of 103 seconds.

There were 16 wave groups in the record that contained exactly two waves (i.e., N = 2) for the level of $h_o = 1$. From Eq. 13, one can calculate the probability of the maximum wave height in wave groups with N = 2 and $h_o = 1$. For this case, the predicted probability distribution $P(h|h_o,N)$ as a function of h is shown by the curve in Fig. 8, along with the statistical distribution derived from the 16 data values. The agreement between the predicted distribution and the data is good.

The time trace of the height envelope for the wave record was digitized at a 2-second interval and processed through a high-pass filter to eliminate the DC component of the time trace. The energy spectrum for the height envelope was then calculated from the processed time trace. The spectrum computed from the data, and the spectrum calculated from Eq. 14 are shown in Fig. 9. As one can see, both spectra show that the energy content decreases with increasing frequency. Eq. 14 predicts that the maximum energy will always be contained infinitely close to zero frequency. The spectrum for the data was not plotted for frequency less than 0.008 cp because the high-pass filter affected the low frequency components.

CONCLUSIONS

The good agreement observed between the theoretical predictions and the data substantiates the potential usefulness of the theoretical results. The use of the Poisson model produced results correlating very well with the data. The narrow band wave spectrum or the Rayleigh distribution assumption appears to be applicable. However, there will always be a difference between theoretical results, which assume an infinite wave population, and the data derived from a wave record of finite length. The authors believe that the average of a large number of finite records would yield even better correlations with the theoretical results.

The analysis presented in this paper is aimed at developing tools for characterizing the statistical behavior of ocean wave groups. The practical significance and the engineering application of the results are left to the individual users.

NOMENCLATURE

a, a_o = wave envelope amplitude
$E(f)$ = wave envelope energy spectrum
h, h_o = normalized wave envelope height
H_s = significant wave height
L = length of wave record
M = number of wave groups in Length L
N = number of waves in a wave group
$p(X)$ = probability density function for the variable X
$P(X)$ = cumulative probability function for the variable X
$S(f)$ = wave energy spectrum
T = average period of larger waves
t = time duration of wave group
$\tau(h_o)$ = average time duration that the wave height envelope stays above the level, h_o
$\nu(h_o)$ = average frequency of occurrence of wave groups of Level h_o

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APPENDIX A

In this appendix, the expression for $\tau(h_o)$ as given in Eq. 6 of this paper will be derived. The basic methods used are from the work by Rice.[3] The fundamental assumptions are that the wave record is Gaussian and has a narrow band energy spectrum. The wave record can be represented by the time series given below.

$$Y(t) = \sum_{n=1}^{N} C_n \cos(\omega_n t - Q_n)$$

or

$$Y(t) = Y_c \cos \omega_m t - Y_s \sin \omega_m t \quad , \ . \ . \ (A\text{-}1)$$

where

$$Y_c = \sum_{n=1}^{N} C_n \cos(\omega_m t - \omega_n t - Q_n)$$

$$Y_s = \sum_{n=1}^{N} C_n \sin(\omega_m t - \omega_n t - \dot{Q}_n)$$

ω_m = average circular frequency of wave spectrum

Q_n = random phase angle with uniform distribution between 0 and 2π

$C_n = \sqrt{2\, S(f)\, \Delta f}$

S(f) = wave energy spectrum

f = frequency

Through lengthy but straightforward calculations, the four-dimensional normal distribution can be derived for Y_c, Y_s, $\dot{Y}_c$ and $\dot{Y}_s$. The dot above the variable denotes the time derivative. The amplitude envelope of the waves is

$$a = \sqrt{Y_c^2 + Y_s^2} \quad \ldots\ldots\ldots\ldots (A\text{-}2)$$

By making the transformation

$$Y_c = a \cos \sigma$$
$$Y_s = a \sin \sigma$$

one can express the four-dimensional distribution for the Y's in terms of $(a, \dot{a}, \sigma, \dot{\sigma})$. After the change of variables is made the joint probability distribution for the envelope, $\dot{a}$, and its time derivative, $\dot{a}$, is found.

$$p(a, \dot{a}) = \int_{-\infty}^{+\infty} d\dot{\sigma} \int_{0}^{2\pi} p(a, \dot{a}, \sigma, \dot{\sigma})\, d\sigma$$

$$= \frac{a}{b_o} \exp\left[-a^2/2b_o\right]$$

$$\cdot \left(\frac{1}{2V}\right)^{1/2} \exp\left[-\dot{a}^2/aV\right], \quad \ldots (A\text{-}3)$$

where

$$V = b_2 - (b_1/b_o)^2$$

$$b_n = (2\pi)^n \int_0^\infty S(f)\,(f - f_m)^n\, df$$

$$n = 0, 1, 2$$

$$f_m = \frac{\omega_m}{2\pi} = \text{average frequency of wave spectrum}$$

The function p(a) is given by

$$p(a) = \int_{-\infty}^{\infty} p(a, \dot{a})\, d\dot{a} = \frac{a}{b_o} \exp\left[-a^2/b_o\right]. (A\text{-}4$$

From Eqs. A-3 and A-4, one can conclude that th wave envelope amplitude has a Rayleigh distribu tion and is independent of the envelope slope which has a normal distribution. Also from Eqs A-3, A-4 and Eq. 4, one can show that the average time duration for which the wave amplitude envelope stays above the level, a_o, is

$$\tau(a_o) = \left[(2\pi b_o/a_o^2)\,(b_o/V)\right]^{1/2} \quad \ldots (A\text{-}5$$

By defining

$$\bar{f} = \frac{\int_0^\infty S(f)\, f\, df}{\int_0^\infty S(f)\, df}$$

and

$$f_e^2 = \frac{\int_0^\infty S(f)\, f^2\, df}{\int_0^\infty S(f)\, df}$$

one can show that

$$b_o = \int_0^\infty S(f)\, df$$

$$b_1/b_o = 2\pi(\bar{f} - f_m)$$

$$b_2/b_o = (2\pi)^2(f_e^2 - 2f_m \bar{f} + f_m^2)$$

and

$$V/b_o = (2\pi)^2(f_e^2 - \bar{f}^2) \quad \ldots\ldots (A\text{-}6$$

As defined in Eq. A-1, the wave spectrum S(f) i the half-amplitude square spectrum. Therefore, the significant wave height

$$H_s = 4\sqrt{b_o} = 2\, a_s \quad \ldots\ldots\ldots (A\text{-}7$$

Substituting Eqs. A-6 and A-7 into Eq. A-5 one has

$$\tau(a_o) = \frac{1}{2\sqrt{2\pi}} \frac{a_s}{a_o} \left[\frac{1}{f_e^2 - \bar{f}^2}\right]^{1/2}$$

Letting $h_o = a_o/a_s$ yields the results given in Eq. 9, i.e.,

$$h_o\, \tau(h_o) = \frac{K}{2\sqrt{2\pi}}, \quad \ldots\ldots\ldots (A\text{-}8$$

where $K = \left[\frac{1}{f_e^2 - \bar{f}^2}\right]^{1/2}$

Although the result given in Eq. A-8 was derive from the assumption of a narrow band wave spectrum that resulted in a Rayleigh distribution for the amplitude envelope, the relationship

$$\tau(a_o) = \frac{\int_o^\infty p(a)\,da}{\int_o^\infty \dot{a}\,p(a_o,\,\dot{a})d\dot{a}} \quad \ldots\ldots\ldots \text{(A-9)}$$

does not depend on the assumptions of a narrow band or a Rayleigh distribution. Eq. A-9 can be directly evaluated using the wave data. The only assumption made is that $\dot{a}$ and a are independent random variables, which implies $p(a_o, \dot{a}) = p(a_o)\,p(\dot{a})$. Therefore, by substituting $h = 2a/H_s$, Eq. A-9 can be expressed as

$$\tau(h_o) = \frac{\int_{h_o}^\infty p(h)\,dh}{p(h_o)}\,[\bar{\dot{h}}]^{-1}, \quad \ldots\ldots \text{(A-10)}$$

where $\bar{\dot{h}} = \int_o^\infty \dot{h}\,p(\dot{h})\,d\dot{h}$.

Therefore, Eq. A-10 may be expressed as

$$\tau(h_o) \propto [\int_{h_o}^\infty p(h)\,dh]/p(h_o)\,, \quad \ldots\ldots \text{(A-11)}$$

where p(h) can be any distribution.

APPENDIX B

In a wave group of Level h_o containing N waves, all N waves will have a height greater than h_o. One may wish to determine the height of the highest wave among the N waves. This question can be answered in statistical terms. In this appendix a method is developed for calculating the occurrence probability of the highest waves in a group of N waves with a level of h_o.

The wave-height distribution, p(h), for all waves represented by a narrow band energy spectrum is Rayleigh, i.e.,

$$p(h) = 4h\,\exp\,[-2h^2] \quad \ldots\ldots\ldots \text{(B-1)}$$

The probability that any one of these waves is higher than h_o is

$$P(h_o) = \int_{h_o}^\infty p(h)\,dh = \exp\,[-2h_o^2] \quad \ldots \text{(B-2)}$$

By dividing Eq. B-1 by Eq. B-2, one has the conditional probability distribution, q(h), for wave heights that are greater than h_o, i.e.,

$$q(h) = \frac{p(h)}{P(h_o)} = 4h\,\exp\,[-2(h^2-h_o^2)] \quad \ldots \text{(B-3)}$$

The probability that a wave is less than h, given that the wave is greater than h_o (i.e., $h > h_o$), is found from the integral

$$p(h|h_o,\,1) = \int_{h_o}^{h} q(h)\,dh = 1-\exp\,[-2(h^2-h_o^2)]. \quad \ldots\ldots \text{(B-4)}$$

For N waves, in a wave group containing waves higher than h_o, the probability that all N waves are less than h is

$$p(h|h_o,N) = \{1-\exp\,[-2(h^2-h_o^2)]\}^N \quad \ldots \text{(B-5)}$$

The condition that all N waves are less than h is equivalent to the condition that the maximum wave height, for the N waves, is less than h. Thus, Eq. B-5 gives the probability that the maximum wave height, in a group of N waves with a level, h_o, is less than h.

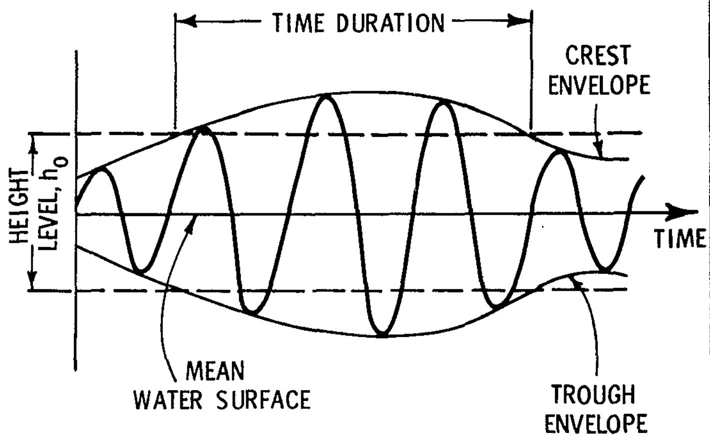

Fig. 1 - Wave group containing waves higher than h_o.

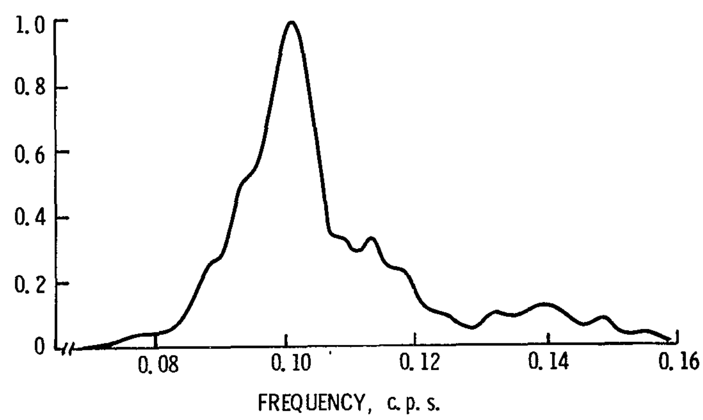

Fig. 2 - Normalized wave energy spectrum.

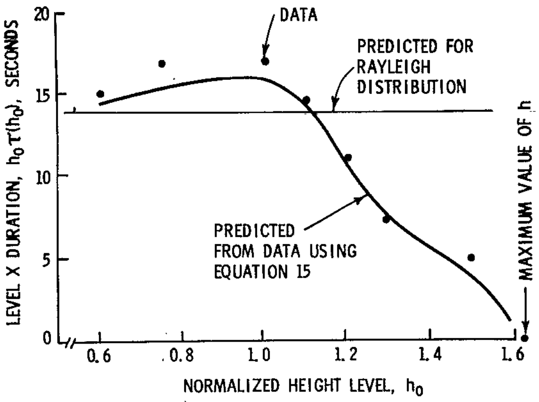

Fig. 3 - Average time duration of groups.

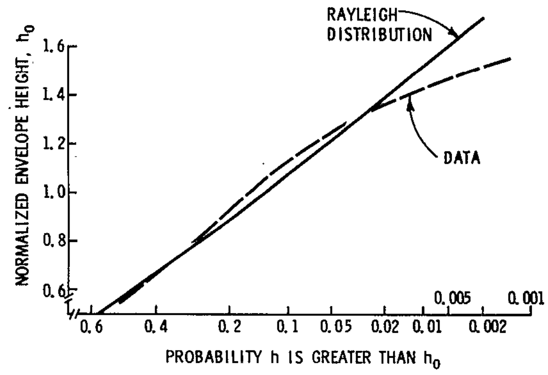

Fig. 4 - Comparison of probability distribution of data to Rayleigh.

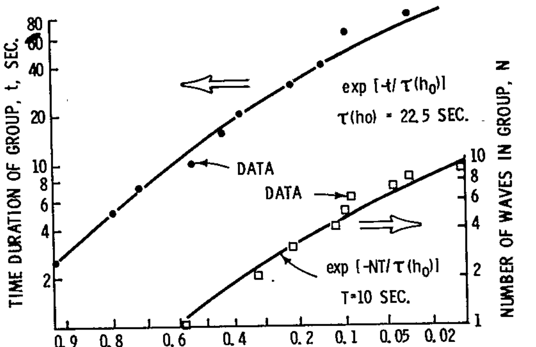

Fig. 5 - Probability distributions for groups of level h_o = 0.75.

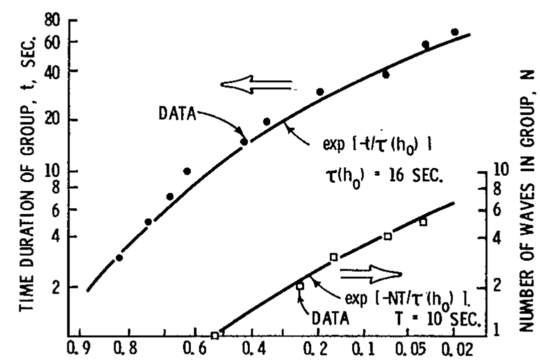

Fig. 6 - Probability distributions for groups of level h_o = 1.0.

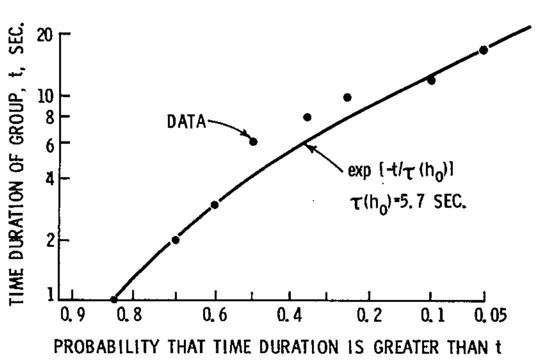

Fig. 7 - Probability distribution for groups of level h_o = 1.3.

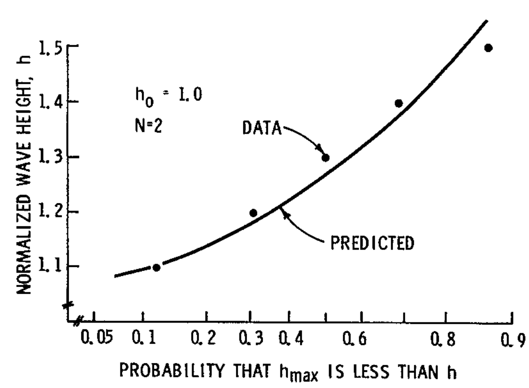

Fig. 8 - Probability distribution for maximum wave height, h_{max} in group.

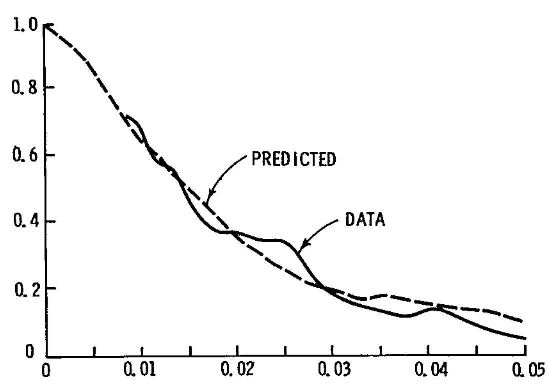

Fig. 9 - Normalized energy spectrum for wave envelope.