Sensitivity to wave breaking and the Prandtl number in the ocean mixed layer model and its dependence on latitude

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[1] Investigation of the sensitivity to wave breaking and the Prandtl number in the ocean mixed layer model under surface heating reveals an interesting latitude-dependent feature in the dynamical process of the ocean mixed layer. The effect of Prandtl number variation with stratification plays an important role in the equatorial ocean, whereas, in the extratropical ocean, the effect of strong near-surface turbulence caused by wave breaking is dominant. The contrasting responses to parameterization is attributed to the fact that turbulence production in the ocean mixed layer is dominated by turbulent kinetic energy flux from the sea surface in the extratropical ocean, and by shear production in the equatorial ocean. Comparisons were also made between the sensitivity tests from different ocean mixed layer models. *INDEX TERMS:* 4572 Oceanography: Physical: Upper ocean processes; 4568 Oceanography: Physical: Turbulence, diffusion, and mixing processes; 4255 Oceanography: General: Numerical modeling; 4508 Oceanography: Physical: Coriolis effects; 4504 Oceanography: Physical: Air/sea interactions (0312). **Citation:** Noh, Y. (2004), Sensitivity to wave breaking and the Prandtl number in the ocean mixed layer model and its dependence on latitude, *Geophys. Res. Lett.*, *31*, L23305, doi:10.1029/2004GL021289.

1. Introduction

[2] The ocean mixed layer (OML) is of primary importance in controlling the air-sea interaction and in determining the structure of the upper ocean, and numerous models have been developed for its prediction [see, e.g., *Garrett*, 1996]. The ocean mixed layer models (OMLM) can be classified into three groups; turbulence closure models [*Mellor and Yamada*, 1982; *Gaspar et al.*, 1990; *Large et al.*, 1994; *Noh and Kim*, 1999], bulk type models [*Kraus and Turner*, 1967; *Price et al.*, 1986; *Chen et al.*, 1994], and the parameterization of mixing coefficients in terms of local variables [*Pacanowski and Philander*, 1981].

[3] The models differ mainly in the parameterizations of various processes present in the OML; e.g., the effects of stratification, convection, the Prandtl number, wave breaking, and Langmuir circulation. Although various intercomparison experiments have been carried out with an intention to evaluate the model performance [*Martin*, 1985; *Chen et al.*, 1994; *Halpern et al.*, 1995; *Reason*, 1996; *Li et al.*, 2001; *Noh et al.*, 2002], it is not yet clear how model results are affected by the parameterizations used in the models.

[4] Recent measurements of the turbulent structure of the OML have revealed that wave breaking significantly increases turbulent kinetic energy (TKE) near the surface, contrary to the atmospheric boundary layer (ABL) [*Agrawal et al.*, 1992; *Drennan et al.*, 1996]. A high level of TKE near the surface helps to maintain the well-mixed layer, and it makes TKE flux dominant in the TKE budget of the upper mixed layer, while rendering shear production relatively unimportant [*Noh*, 1996; *Craig and Banner*, 1994]. This effect also has been corroborated recently by large eddy simulation (LES) [*Noh et al.*, 2004] and direct numerical simulation [*Sullivan et al.*, 2004].

[5] Some efforts have been made to incorporate the effect of wave breaking into the OMLM [*Noh and Kim*, 1999; *D'Alessio et al.*, 1998; *Burchard*, 2001]. Although the strong mixing by wave breaking is implied in bulk models that assume uniform density within the OML [*Kraus and Turner*, 1967; *Chen et al.*, 1994], it is not taken into account in many turbulence closure models [*Mellor and Yamada*, 1982; *Gaspar et al.*, 1990; *Large et al.*, 1994].

[6] Meanwhile, it has been found from the observation of the equatorial turbulence [*Peters et al.*, 1988] that the eddy diffusivity K_h decreases faster than the eddy viscosity K_m with increasing stratification, thus increasing the turbulent Prandtl number Pr ($=K_m/K_h$). This effect is taken into consideration in some models [*Blanke and Delecluse*, 1993; *Pacanowski and Philander*, 1981], but it is neglected in a large number of models including all bulk models [e.g., *Mellor and Yamada*, 1982; *Large et al.*, 1994; *Noh and Kim*, 1999].

[7] This apparent inconsistency raises the necessity to clarify the importance of these parameterizations in the OMLM; i.e., the effects of strong near-surface turbulence by wave breaking and the increase of Pr with increasing stratification.

[8] In addition, the dependence of the dynamical processes of the OML on the latitude has not been considered properly yet in view of the OMLM. Note, however, that the downward transports of momentum and heat are restricted to the Ekman layer depth D_E such as

$$D_E = \pi(2K_m/f)^{1/2} \tag{1}$$

if K_m is assumed to be constant, where f is the Coriolis parameter. In the equatorial ocean, however, the downward transports of momentum and heat continue indefinitely. Hence, it is possible that the response of the OMLM reveals substantial difference depending on the latitude.

[9] Therefore, in this paper, we attempted to investigate how wave breaking and the Prandtl number affect the vertical mixing process in the upper ocean, and how they differ depending on the latitude. For this purpose, we examined the OMLM by *Noh and Kim* [1999] (NMIX

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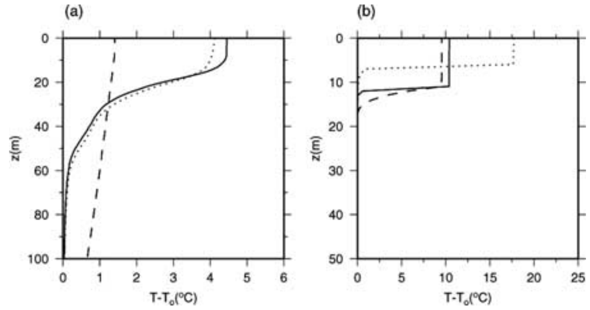

Figure 1. Temperature profiles evolved from constant temperature after 30 days of surface heating (T_0 = 29°C): (a) f = 0 s^{-1}, (b) f = 10^{-4} s^{-1} (solid: NMIX_O, dashed: NMIX_B, dotted: NMIX_A).

hereafter) and by *Pacanowski and Philander* [1981] (hereinafter referred to as PPMIX).

2. Ocean Mixed Layer Model

[10] In NMIX [*Noh and Kim*, 1999; *Noh et al.*, 2002], K_m and K_h are calculated by

$$K_m = S_m q l, \tag{2}$$

$$K_h = S_h q l, \tag{3}$$

where $q^2/2$ is TKE, l is the length scale of turbulence, and S_m and S_h are empirical constants. Here q is calculated by TKE equation, similarly to other turbulence closure models [e.g., *Gaspar et al.*, 1990; *Mellor and Yamada*, 1982].

[11] The effect of wave breaking is incorporated by modifying the surface boundary conditions for the TKE flux F and the length scale z_0. They are given by $F = mu_*^3$ with m = 100, and z_0 = 1 m, following *Craig and Banner*'s [1994] analysis of the observation data. Here u_* is the frictional velocity due to wind stress.

[12] Since TKE production is dominated by the TKE flux with very small mean velocity shear in the OML under the influence of wave breaking [*Craig and Banner*, 1994; *Noh et al.*, 2004], the effect of stratification is parameterized in terms of Rt (=$(Nl/q)^2$), instead of Ri (=$(N/S)^2$); e.g.,

$$S_m/S_{m0} = (1 + \alpha \mathrm{Rt})^{-1/2}. \tag{4}$$

Here N is the Brund-Väisälä frequency, S is the mean velocity shear, α is an empirical constant, and S_{m0} is the value in the absence of stratification. For the detailed description of the model, one can refer to *Noh and Kim* [1999] and *Noh et al.* [2002].

[13] In the present paper, we modified NMIX by parameterizing the influence of stratification on the Prandtl number. The new parameterization is given in the form of

$$\mathrm{Pr}/\mathrm{Pr}_0 = (1 + \beta \mathrm{Rt})^{1/2} \tag{5}$$

where Pr_0 is the value in the absence of stratification. For the values of α and β, α = 5 and β = 0.5 were used, based on the analysis of LES data of the OML [e.g., *Noh et al.*, 2004].

[14] For the investigation of the sensitivity, we examined three different versions of NMIX; one in which the effects of both wave breaking and the Prandtl number are present (m = 100, z_0 = 1 m, β = 0.5) (NMIX_O), one without the effect of wave breaking (m = 0, z_0 = 0 m, β = 0.5) (NMIX_A), and one without the effect of the Prandtl number (m = 100, z_0 = 1 m, β = 0) (NMIX_B).

[15] In PPMIX, K_m and K_h are given as follows.

$$K_m = K_0/(1 + aRi)^2 + K_{m0}, \tag{6}$$

$$K_h = K_m/(1 + bRi) + K_{h0}, \tag{7}$$

where the empirical constants are given by $a = b = 5$, K_0 = 5 × 10^{-3} m^{2}s^{-1}, $K_{m0} = K_{h0}$ = 1.34 × 10^{-6} m^{2}s^{-1}.

[16] Here we can regard that the intensity of mixing near the surface and Pr are represented by K_0, and 1 + bRi, respectively. Therefore, we performed the sensitivity tests with different values of K_0 and b; the original PPMIX (PPMIX_O), the one with K_0 = 5 × 10^{-1} m^{2}s^{-1} (PPMIX_A), and the one with b = 0 (PPMIX_B). Here we examined the case with a larger value of K_0 in PPMIX_A, representing a stronger near-surface mixing, contrary to the case of NMIX_A, since it has been suggested that the vertical mixing in PPMIX might be too weak [*Blanke et al.*, 1993; *Schneider and Müller*, 1994; *Li et al.*, 2001; *Noh et al.*, 2002].

[17] In the present paper, only the response of the OML under surface heating was presented. Difference in the model results was much smaller in the response to surface cooling, as the OML is dominated by convective eddies. The contribution of salinity was neglected, and the vertical resolution was 1 m in all simulations.

3. Sensitivities of Parameterizations in NMIX

[18] Figure 1 shows the temperature profiles obtained from NMIX_O, NMIX_A, and NMIX_B after 30 days under the surface heating of Q_0 = 200 Wm^{-2} and the wind stress of u_* = 0.01 ms^{-1}, both cases of the equatorial (f = 0 s^{-1}) and extratropical (f = 10^{-4} s^{-1}) oceans, starting from the homogeneous OML.

[19] Note that, in the extratropical ocean, the downward transports of momentum and heat are restricted by D_E, leading to a well-defined thermocline at a relatively shallow depth. On the other hand, in the equatorial ocean, the downward transports of momentum and heat penetrate much deeper.

[20] The comparison between NMIX_O, NMIX_A, and NMIX_B reveals a very interesting pattern. Including wave breaking effect leads to a substantial increase of the mixed layer depth (MLD) and a decrease of the sea surface temperature (SST) in the extratropical ocean, but its effect is insignificant in the equatorial region. On the contrary, the inclusion of the Prandtl number effect suppresses the downward transport of heat, thus decreasing the MLD and increasing the SST remarkably in the equatorial ocean, while its effect is insignificant in the extratropical ocean.

4. Difference in the TKE Budget in the Equatorial and Extratropical Oceans

[21] Different nature of the mixed layer dynamics depending on the latitude is substantiated from the profiles

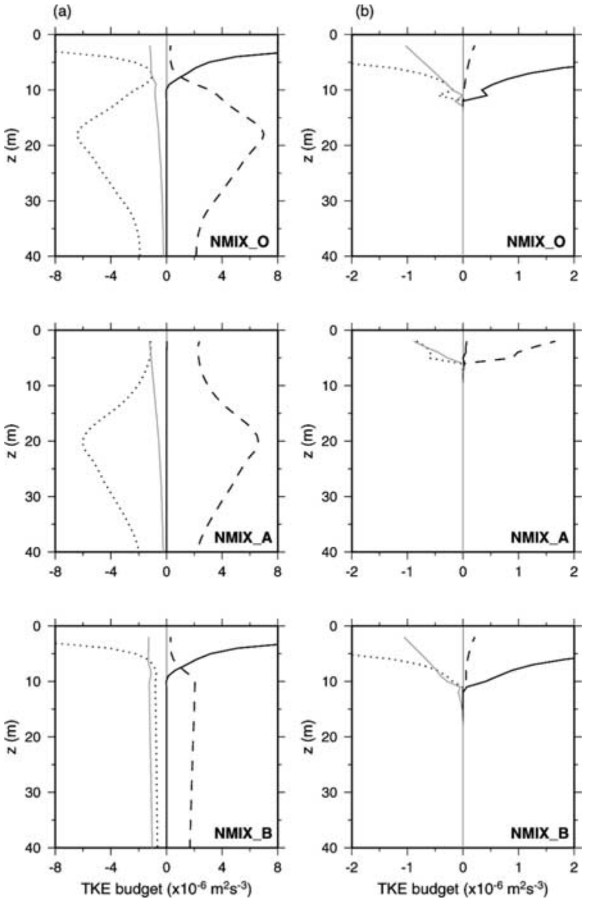

Figure 2. Profiles of the terms of the TKE budget from NMIX: (a) f = 0 s^{-1}, (b) f = 10^{-4} s^{-1} (solid: TKE flux, dashed: shear production, dotted: dissipation, gray: buoyancy).

of the TKE budget corresponding to the cases of Figure 1 (Figure 2). In the case of NMIX_O, TKE production is dominated by TKE flux near the surface (z < 10 m), but by shear production below it [*Craig and Banner*, 1994; *Noh*, 1996; *Noh et al.*, 2004; *Sullivan et al.*, 2004]. Meanwhile, shear production dominates throughout the depth in NMIX_A, similarly to the case of the ABL.

[22] In the extratropical ocean, TKE flux dominates within the whole OML that is restricted by D_E. Here the MLD is estimated by (1), in which K_m is determined by the TKE level near the surface, whereas the effect of the Prandtl number is relatively unimportant, because of very weak stratification within the OML.

[23] On the other hand, below the near-surface zone in the equatorial ocean, the TKE flux becomes negligible, and the TKE budget is balanced by $K_mS^2 - K_hN^2 - \varepsilon = 0$. In this case, the decreases of K_m and K_h by stratification are followed by the increase of S, resulting in a new balance [*Noh*, 1996]. When Pr increases with stratification, however, K_h decreases further, and it induces stronger stratification, which suppresses the downward transports of heat and momentum.

[24] The contrasting pattern of the mixed layer dynamics depending on the latitude can be also identified from shear profiles (Figure 3). Note that the velocity shear remains strong over the extensive depth in the equatorial ocean, but it is negligible except near the thermocline in the extratropical ocean.

5. Comparison With PPMIX

[25] Figure 4 shows the response from PPMIX in the corresponding sensitivity tests. In general, PPMIX shows much stronger stratification near the surface than NMIX, suggesting the insufficient near-surface mixing. The MLD in the extratropical ocean is much shallower, as it is known in ocean general circulation model (OGCM) experiments [*Li et al.*, 2001; *Noh et al.*, 2002].

[26] Compared with the case of PPMIX_O, vertical mixing is substantially enhanced in both equatorial and extratropical oceans in PPMIX_B, but no significant modification appears in PPMIX_A, in spite of the large difference in K_0. The presence of strong stratification within the OML causes the effects of the Prandtl number to be important in the extratropical ocean as well as in the equatorial ocean. Meanwhile, the stronger near-surface mixing makes the vertical shear smaller. Consequently, much larger values of Ri appear, and it offsets the larger K_0 in (6) and (7).

[27] Note that the parameterization in terms of Ri presumes that shear production dominates in the TKE budget of the OML, which is contrary to the recent evidences [*Craig and Banner*, 1994; *Noh*, 1996; *Noh et al.*, 2004; *Sullivan et al.*, 2004]. The presence of strong shear-free

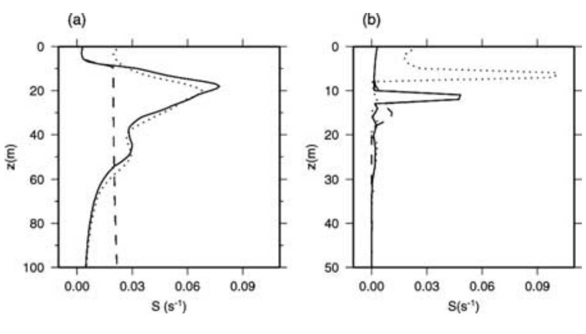

Figure 3. Shear profiles after 30 days of surface heating: (a) f = 0 s^{-1}, (b) f = 10^{-4} s^{-1}, (NMIX_O and NMIX_B coincide at the peak in (b)). (solid: NMIX_O, dashed: NMIX_B, dotted: NMIX_A).

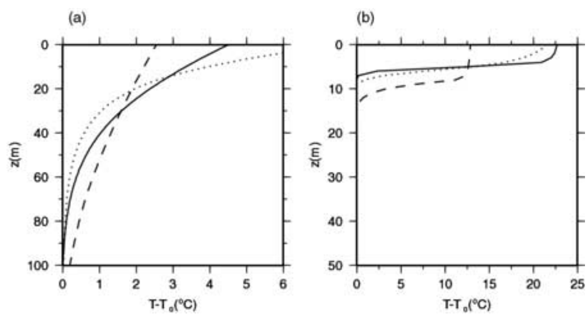

Figure 4. Temperature profiles evolved from constant temperature after 30 days of surface heating (T_0 = 29°C): (a) f = 0 s^{-1}, (b) f = 10^{-4} s^{-1} (solid: PPMIX_O, dashed: PPMIX_B, dotted: PPMIX_A).

turbulence generated by wave breaking makes Rt smaller, but Ri much larger.

6. Concluding Remarks

[28] Analysis of the OMLM revealed that the dynamical process of the OML under surface heating is radically different, depending on whether it is in the extratropical or equatorial ocean. TKE flux plays a dominant role in the former, whereas shear production plays a dominant role in the latter. As a result, the response of the OMLM is sensitive to the effect of wave breaking in the extratropical ocean and the effect of the Prandtl number in the equatorial ocean in the case of NMIX.

[29] The layer, in which shear production dominates, may appear as D_E becomes sufficiently deeper than the depth h proportional to the Monin-Obukhov length scale $L(\equiv u_*^3/B_0$, where B_0 is the buoyancy flux) at which the TKE flux and the buoyancy flux become balanced [*Noh*, 1996]. If we estimate typically as $h \sim 10$ m and $K_m \sim 10^{-3}$ ms^{-2}, we can expect that $D_E \gg h$ can be applied within the latitudinal band up to $\sim 5°$ from the equator. Further, we expect that shear production be even more important in the real equatorial ocean, because of the presence of the Equatorial Undercurrent.

[30] The present results suggest that the excessive vertical mixing in the equatorial ocean from the OGCM with NMIX_B [*Noh et al.*, 2002] can be improved while maintaining a proper MLD in the extratropical ocean, by introducing the effect of the Prandtl number, i.e., NMIX_O.

[31] In the case of PPMIX, the parameterization in terms of Ri and the simulated OML with too strong stratification and too shallow MLD may not reflect the dynamics of the OML accurately. It also leads to somewhat different responses to the sensitivity tests.

[32] Finally, it is important to mention that, if the parameterizations of stratification are made in terms of Ri rather than Rt, for (4) and (5) in NMIX, it leads to unrealistic and unstable results due to the very small values of S in the OML under the influence of wave breaking.

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