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PRECISE NEARSHORE CURRENTS MODEL USING SIGMA COORDINATE SYSTEM

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The nearshore currents have a 3-dimensional structure, and the driving force of the currents can be explained by the vertical distribution of radiation stress. Based on this principle, a 3-dimensional nearshore current model was already proposed. Using the horizontal level method on the Cartesian coordinates, however, this model generates only rough vertical distribution of the current in a shallow water region. In this study, a nearshore current model based on the vertical distribution of radiation stress was developed using the sigma coordinate system. This new model can reproduce the vertical current patterns and water levels measured in experiments.

Keywords: radiation stress, vertical distribution, nearshore currents, undertow, Sigma coordinate system, Cartesian coordinate system

1. Backgrounds and Purpose

Predicting 3-dimensional nearshore current in a wide area is important to the management of sand transport and water-quality in the coastal area. Although the nearshore currents are calculated by the direct simulation of Navier-Stokes equation, the object area covered by the Navier-Stokes equation model is narrow even today because of the limited capacity of computer. Therefore, we need to develop an efficient model, which can predict the 3-dimensional nearshore currents preciously enough in a wide area ranging from shallow to deep water region.

Nobuoka et al. (1998) have shown that the driving force of 3-dimensional nearshore currents is the vertical distribution of radiation stress, and have developed a time-mean current model based on this principle. However, this model is not practical as it uses the horizontal level method on the Cartesian coordinates. The vertical current distribution cannot be expressed sufficiently in detail in the shallow water, while in the deep water region the number of layers increases to cause a long CPU time. Introduction of the sigma coordinate can overcome such problems, as the number of the calculation point is constant for all the water depth.

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The purpose of this study is to develop a nearshore current model defined on the sigma coordinate system, which takes the vertical distribution of radiation stress for the driving force. First, the governing equations of the nearshore current on the sigma coordinate are derived from the Navier-Stokes equation. Second, the calculation method of the radiation stress in this coordinate system is verified. Finally, the capacity of this model is examined by comparing the calculation with the experimental results in a wave flume. Comparisons are also made for the results of the undertow model.

2. Governing Equations Using Sigma Coordinate System

The governing equations of the previous 3-dimensional nearshore current were derived from the Navier-Stokes equations and the primitive continuity equation, adding three assumptions and a time averaging during one wave period, as shown by Nobuoka (1998). The assumptions were that a velocity can be separated to the wave component, the time-averaged component and the turbulent component, no interactions exist between the different components, and vertical accelerations of time-averaged flow are very small. In this study, the same methods are taken to derive the governing equations expressed by sigma coordinate system.

The one of the sigma coordinate systems as illustrated in Figure 1 is non-dimensional scale of total local water depth so that vertical coordinate z is divided by that depth.

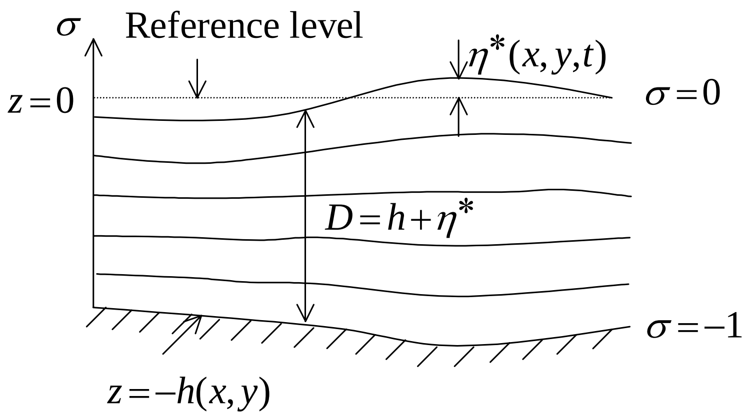

Fig. 1. Definition sketch of the sigma coordinate system

The horizontal axes *x* and *y* and the vertical axis σ by the sigma coordinate system are shown as Equation 1, against the axis x^*, y^* and z^* by Cartesian coordinate system,

$$x = x^*, \quad y = y^*, \qquad \sigma = \frac{z - \eta^*}{D} \tag{1}$$

where η^* is water surface elevation above static water level (z=0), *D* is the total water depth

(D=h+ η*) and h is the static water depth. The definition of Equation 1 indicates that the origin of sigma coordinate is water surface, where σ=0, and that value at all the sea bottom is σ=-1. The velocity u, v and ω in x, y and σ direction respectively by sigma coordinate system are also expressed as Equation 2 using u*, v* and w* of Cartesian velocities.

$$
\begin{aligned}
&u = u^*, \quad v = v^* \\
&\omega = w^* + D\left(\frac{\partial \sigma}{\partial t} + u\frac{\partial \sigma}{\partial x} + v\frac{\partial \sigma}{\partial y}\right)
\end{aligned}
\tag{2}
$$

The governing equations of nearshore currents, Equation 3 and 4, are derived from the basic equations, using Equation 1 and 2 and the above mention's method.

$$
\frac{1}{D}\left\{\frac{\partial u_f}{\partial x} + \frac{\partial v_f}{\partial y} + \frac{\partial \omega_f}{\partial \sigma_f}\right\} = 0 \tag{3}
$$

$$
\begin{aligned}
&\frac{1}{D_f}\left\{\frac{\partial u_f^2 D_f}{\partial x} + \frac{\partial u_f v_f D_f}{\partial y} + \frac{\partial u_f \omega_f}{\partial \sigma_f}\right\} \\
&= -\frac{\partial}{\partial x}\left(-gD_f\sigma_f\right) \\
&\quad \overline{-\frac{\partial}{\partial x}\left[-gD_w\sigma_w - \frac{\partial}{\partial x}\int_0^{\sigma w} w_w u_w D_w d\sigma - \frac{\partial}{\partial y}\int_0^{\sigma w} w_w v_w D_w d\sigma - w_w\omega_w\right]} \\
&\quad \overline{-\left(\frac{\partial \sigma}{\partial x}\right)\left(-gD_w\sigma_w - \frac{\partial}{\partial x}w_w u_w D_w - \frac{\partial}{\partial y}w_w v_w D_w - \frac{\partial}{\partial \sigma_w}w_w\omega_w\right)} \\
&\quad \overline{-\frac{1}{D_w}\left\{\frac{\partial u_w^2 D_w}{\partial x} + \frac{\partial u_w v_w D_w}{\partial y} + \frac{\partial u_w \omega_w}{\partial \sigma_w}\right\}} + R_x
\end{aligned}
\tag{4}
$$

where subscript w and f, and symbol of Rx, are expressed as the wave, the time-averaged flow and the turbulent component (Reynolds stress) respectively. The acceleration-terms, which are the left-hand side of Equation 4, are the same as those of a tidal-current equation because those equations are similarly approximated as a long wave. Accordingly, the new proposal in this study is the radiation stress components, the second to fourth terms on the right-hand side of Equation 4, which is the driving force of wave-induced currents.

If we integrate these radiation stress components over the whole water column, the derived terms should agree with the conventional radiation stress regardless of the coordinate. The integrated result of these components is shown in Equation 5. The terms in the parenthesis [ ] of Equation 5 are in agreement with the conventional radiation stress shown by Mei (1989).

$$
\begin{aligned}
S_{xx} = & -\overline{\frac{\partial}{\partial x}\left[-gD_w^2\int_{-1}^{0}\sigma_w d\sigma - D_w\int_{-1}^{0} w_w w_w d\sigma\right.} \\
& \overline{\left. -\frac{\partial}{\partial x}\left\{D_w\int_{-1}^{0}\int_{0}^{\sigma_w} w_w u_w D_w d\sigma\right\} + \frac{\partial h}{\partial x}\int_{0}^{-1} w_w u_w D_w d\sigma\right]} \\
& \overline{-\frac{\partial h}{\partial x}\frac{\partial}{\partial x}\left\{\int_{0}^{-1} w_w u_w D_w d\sigma\right\} - \frac{\partial}{\partial x}\left[\int_{-1}^{0} u_w^2 D_w d\sigma\right]}
\end{aligned}
\tag{5}
$$

This agreement confirms that the radiation stress components of Equation 4 are appropriate.

The finite difference method is adapted for the numerical solution of these governing equations. In order to keep the conservation law against this method, the Equation 3 and 4 are integrated in each vertical layer divided arbitrarily in the water depth. The integrated equations are governing equations of nearshore current simulation model proposed by this study. The detail of the numerical solution of these equations is the same as that of the previous model (Nobuoka, 1998). The variables to be obtained in this model are the vertical distribution of time-mean flux and time-mean water level in all the area.

3. Calculation of Vertical Distribution of Radiation Stress

Even if a sea bottom is very mild, the effect of that inclination needs to be taken into consideration on the vertical distribution of radiation stress (Nobuoka, 2002). Therefore, the vertical distribution of radiation stress was calculated using the Biesel wave theory (1952) which can take the effect of inclination.

This calculation method of the radiation stress was examined by the two methods. One is that the vertical profile of the stress by sigma coordinate system should be the same as that by Cartesian coordinate system. The other is that the vertical sum of the calculated stresses in each layer should be the same as the conventional radiation stress. The condition of calculation is the wave flume of which the water depth in offshore area and the angle of bottom inclination are 0.35m and 1/20. The height and period of incident waves are 2.5cm and 1.0s, respectively. The thickness of layer in the case of the sigma and Cartesian coordinate system are 1/10 of the water depth and 1cm respectively.

The result of the vertical profile at the point where the water depth is 5cm in the surf zone is shown in Figure 2. The horizontal axis is the horizontal gradient of the radiation stress. The black circles and white squares are the result by the sigma and Cartesian coordinate system, respectively. The result based on sigma coordinate system has ten points in vertical direction. On the other hand, the result based on a Cartesian coordinate system has only five points. If the water depth in this line is partitioned into more than five levels in Cartesian coordinate systems, the numerical simulation method becomes extremely difficult because the maximum wave set-down level reaches second layer. In both results, vertical profile of radiation stress is almost the same. In the case of sigma coordinate system, the momentum gradient generated by the breaking wave on the water surface can be expressed more correctly. Figure 3 shows the results of the sum of the radiation stresses of all the layers in each local depth to compare with the conventional radiation stress introduced by Longuet-Higgins and Stewart (1964), at each point from the offshore side to the shore line. Both results coincide well with each other in all area. These two results also

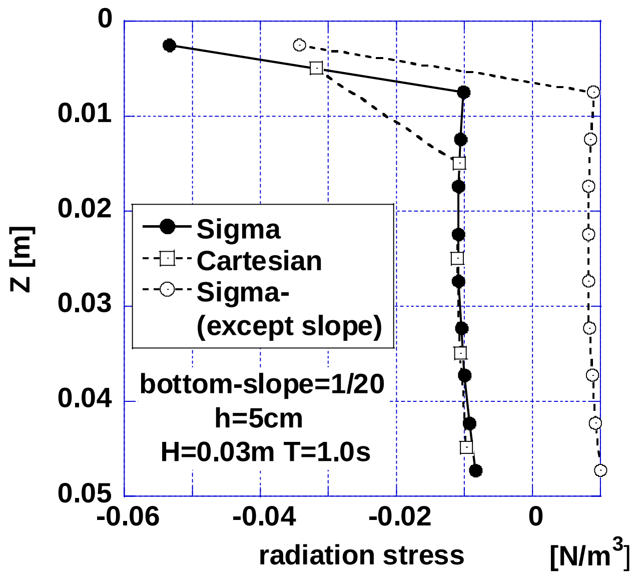

Fig 2 Vertical profile of horizontal gradient of radiation stress

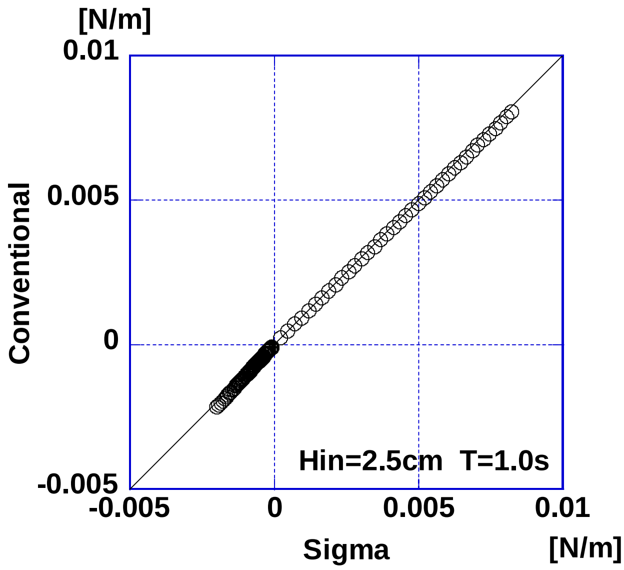

Fig. 3 Comparing the two calculation methods of conventional radiation stresses

indicate that the radiation stress of sigma coordinate system calculated by proposed method is valid and accuracy improves.

The effect of inclination of sigma plane is also examined. The white circles shown in Figure 2 are the calculated results except the inclination term which is the third-term in the right hand side of Equation 4. This result indicates that even if the gradient of sigma plane is very mild less than 1/20, the effect of the gradient acts on radiation stress greatly. It means that we should include the effectiveness of the inclination to calculate the vertical distribution of radiation stresses.

4. Verification of Nearshore Current Model Using Sigma Coordinate System

The present model was applied to predict time-mean water level and vertical distribution of time-mean currents in cross-shore section. The two experiments are chosen for the comparison. The length and height of mesh size is 1cm and 1/10 of total water depth at each point. The breaking waves are modeled by the use of the relative wave height, *H/h*, where *H* and *h* are wave height and water depth respectively.

4.1 Time-mean water level

The gradient of time-mean water level is one of the driving forces of time-mean currents. On/off-shore currents including undertow occur by the local imbalance of radiation stress gradient and static pressure caused by time-mean water level gradient (Nobuoka, 1998). A part of alongshore currents, which is not examined in this paper, also is generated by the water level gradient. Therefore, it is important to predict the time-mean water level correctly. The condition of experiment for the comparison is that the offshore water depth and the slope of sea bottom is 35cm and 1/20 respectively. The different wave conditions are used; spilling wave breaker (Case-1-a, H=10cm, T=1.0s) and plunging wave breaker (Case-2-a, H=12cm, T=2.0s).

Figure 4 (a) and (b) show the both results of time-mean water level. The horizontal and vertical axis is non-dimensional distance and time-mean water level respectively. The line and symbols show the result of calculation and experiment respectively. As the wave flume is finite length, the time-mean water level in offshore area is lower than the static water level. The calculation results are in good agreement with experimental results of mild wave set-down and steep wave set up.

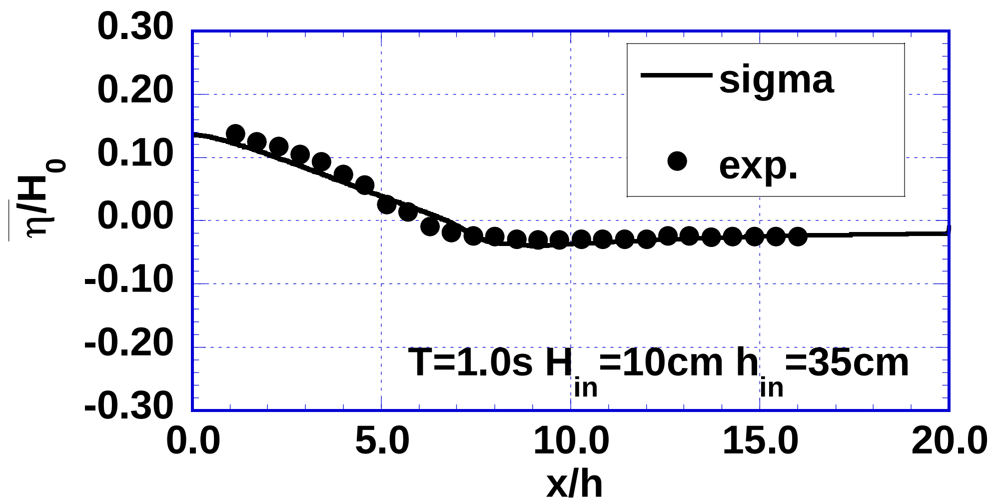

Fig. 4 (a) Cross shore distribution of time-mean water level (Spilling)

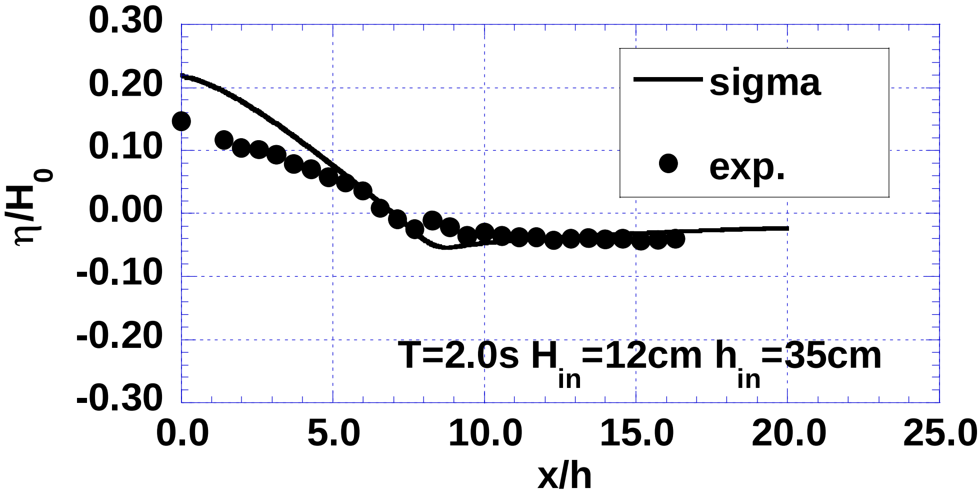

Fig. 4 (b) Cross shore distribution of time-mean water level (plunging)

4.2 Vertical distribution of on and off-shore current

To verify simulated results of currents, those were compared with experimental results measured by Okayasu (1988). In these experiments, the vertical distribution of cross-shore current was measured in detail by the use of laser-Doppler velocimeter. The wave flume was 23m long and 0.4m wide with 1/20 slope. Two incident wave conditions were chosen; plunging breaker (case-2-a, H=5.63cm, T=2.0s) and spilling breaker (case-2-b, H=9.87cm, T=1.17s). The line-3 is the location where surface rollers fully developed, and the line-6 is on the still water shoreline.

Figure 5 shows the results of several measuring line. The white squares and black triangles are calculated result by the present model and experimental results, respectively. The white circles are shown the results by the old Cartesian model for the comparison. The time-mean current calculated by using the sigma coordinate system is in good agreement with that of experiment. On the other hand, the result using the Cartesian coordinate system cannot express the vertical distribution near the shoreline well because of the above mention about the limit of number of layers. These results confirm us that the sigma coordinate system improves the prediction accuracy of a vertical distribution profile.

4.3 Comparing with the undertow model

Although vertical distribution of long shore currents and offshore-zone currents are not predicted, the undertow model is one of the good models to predict vertical distribution of time-mean cross-shore current in a surf zone. The original undertow model was developed by Svendsen (1984) and there are several improved models these days. In this study, the capacity of present model was compared with Okayasu's undertow model (1988). The depth-mean velocity, which needs to calculate in Okayasu's model, was gotten from the results predicted by him (figure 8 of Okayasu, 1988). The comparison results are shown in Figure 6. The vertical axis is the root-mean-square of error of time-mean currents of all layers, which are non-dimensionless value divided by depth-averaged currents. These results indicate that the present model using sigma coordinate system is the most same for the capacity of prediction as the undertow model.

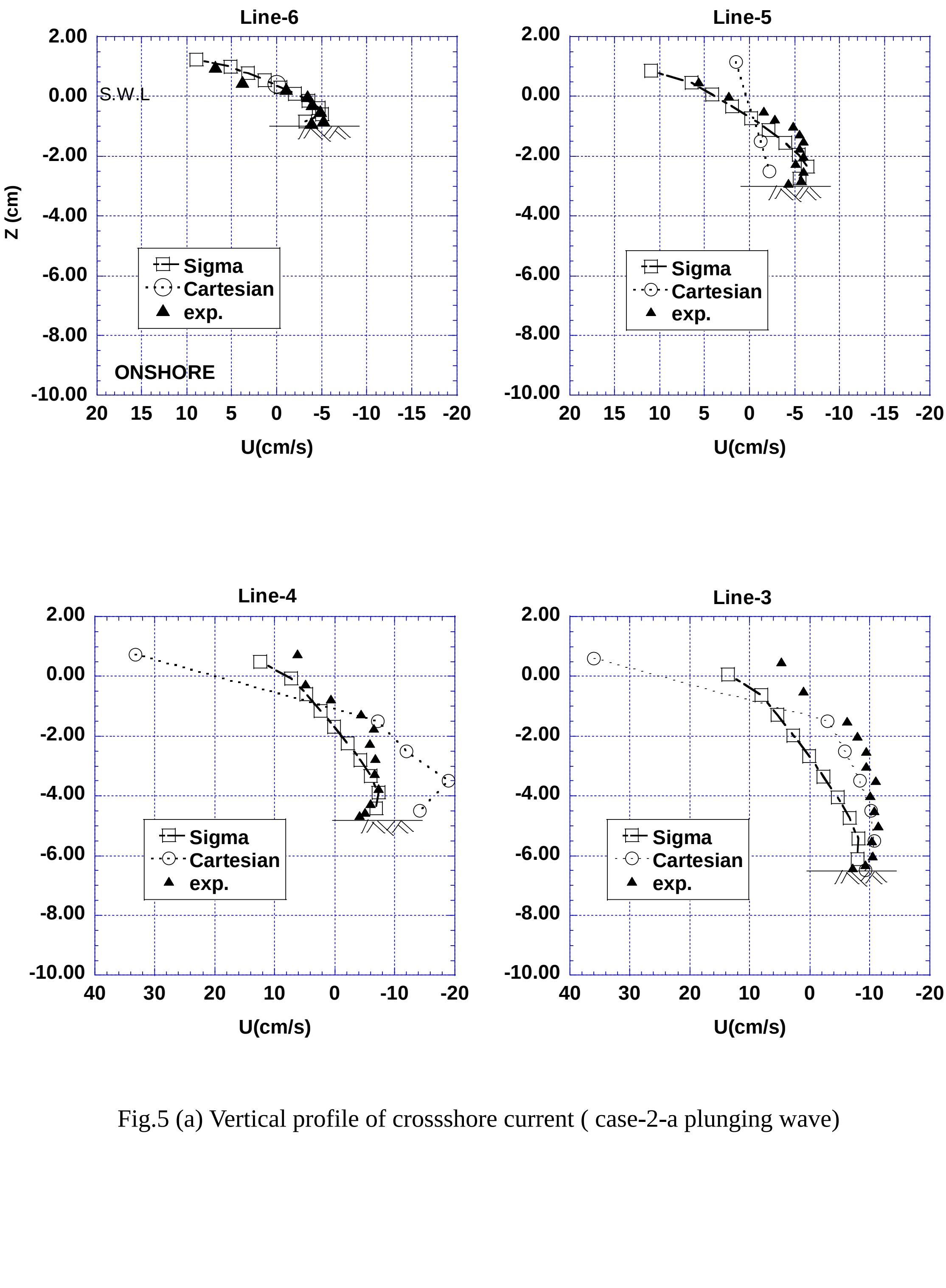

Fig.5 (a) Vertical profile of crossshore current (case-2-a plunging wave)

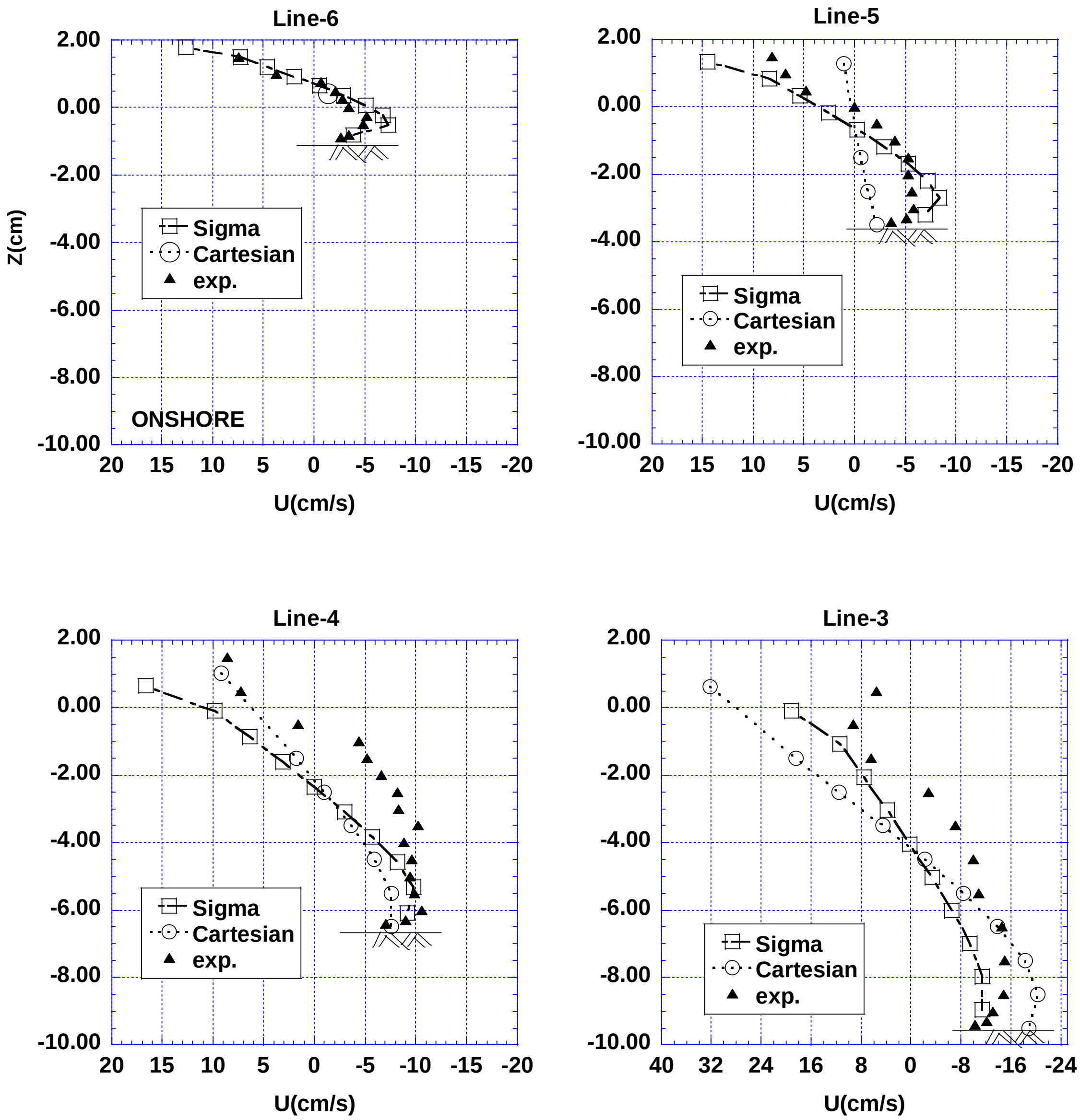

Fig.5 (b) Vertical profile of crossshore current (case-2-b spilling wave)

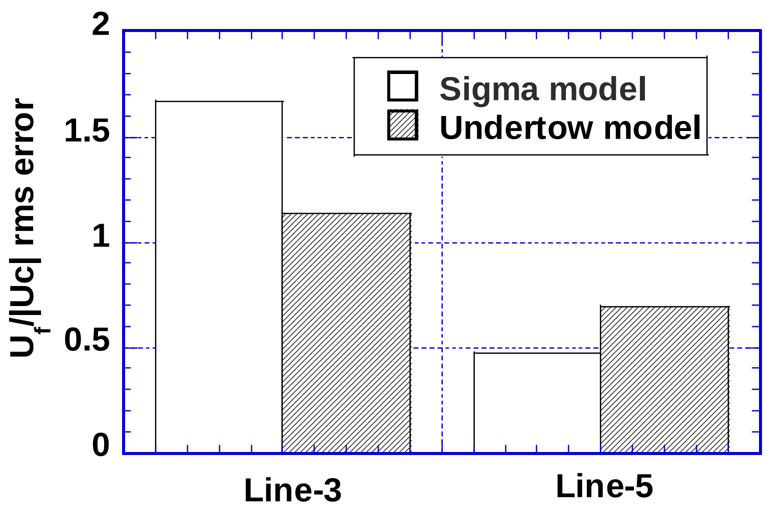

Fig 6 Comparison of the proposed model and undertow model

The old model using Cartesian coordinate system can predict 3-dimensional currents passably well (Nobuoka 1998). Therefore, the advantage of the proposed model based on the vertical distribution of radiation stresses, having a good capacity of prediction as the undertow model in cross-shore section, is to be able to expand to 3-dimensional area comparatively easily.

4.4 Reduce the calculation CPU time using sigma coordinate system

We need the nearshore currents model that can calculate in as short CPU time as possible. As the proposed model has to solve the simultaneous equations, of which variables to be obtained are time-mean currents in each layer and water level all of the area, the number of these variables is much related to the CPU time. The CPU time using sigma coordinate system is about 1/5 times shorter than that using Cartesian coordinate system in same calculation condition. The defect using Cartesian system is to increase the number of variable as for offshore area, where the vertical profile of currents becomes also uniform. On the other hand, that number by using sigma coordinate system is constant from onshore side to offshore side. Another merit by using sigma coordinate system is that the handling of sea-bottom condition becomes easy.

5. Conclusion

In this study, the governing equations of nearshore current model using sigma coordinate system were proposed, the model for these equations was developed and the following points were clarified.

1) The inclination of each sigma surface contributes greatly to the gradient of the vertical distribution of the radiation stresses. We must take account of the inclination of the sigma plane in order to calculate the radiation stress correctly.

2) The results of time-mean on/off-shore currents and time-mean water level in a surf zone, predicted by the proposed model, are in good agreement with experimental data. The proposed model can predict vertical profile of time-mean current more correctly than that of the old Cartesian coordinate system model, in shallow water region.
3) The capacity of prediction by proposed model is the almost same as that by an undertow model in a surf zone.
4) The proposed model can save the calculation CPU time considerably, compared with the old Cartesian coordinate system model.

The above results confirm us that the nearshore current model using sigma coordinate system is highly practical.

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