International Journal of Modern Physics B,

A SIMPLE MODEL FOR THE VARIOUS PATTERN DYNAMICS OF DUNES

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Received (2 June)

A simple computational model is proposed that reproduces various aspects of the complex dynamics of dunes such as the barchan dunes formation process, the evolution process from a barchan dune to a seif dune, the network-dunes formation under time-dependent directional winds, etc. Although this model may be oversimplified in several respects, there is a hope that it helps us to sift relevant factors out the vast sea of numerous factors influencing the rich dynamics of desert dunes.

1. Introduction

In spite of the accumulation of field observational data and the development of the satellite camera utilization, the comprehensive understanding of the formation and the evolution process of large scale sand dunes has not satisfactorily been attained.[1,2,3,4,5] The difficulty seems to come mainly from the fact that the characteristic time scales of large dunes are huge; longer than tens of thousand of years for dunes of the height over 100m.[3] Consequently, although many qualitative models have been proposed, their results have not seriously been confronted with observations with only a few exceptions.[6]

Recent remarkable development of computers has enabled us to simulate, e.g., complicated hydrodynamic phenomena. Therefore, computational approaches with the synergistic use of analytical theories is a hopeful means to facilitate quantitative understanding of the entire process of dune formation. Some computational models have already been studied to reproduce the formation process of dunes, typically a barchan dune. For example, Howard and Walmsley [7] have tried to make a barchan from an initially conic sand hill, but they have failed to realize the shape of a barchan because of the numerical instability. Wipermann and Gross's model [8] is more refined and successfully forms an isolated barchan dune. Fisher and Galdies [9] have simulated the pattern deformation from an initial barchan with symmetric wings to an asymmetric barchan after a sudden change of the wind direction.

In above models they simulate rather short time scales compared to the *whole life of a dune*: from the initiation of a sand hill to its dispersal or its arrival at a stationary state. Furthermore, the spatial scales these models can cover are restricted to the area several times as large as that of individual dunes. In consequence, more remarkable aspects of dunes dynamics have not been studied such as the merger of two isolated dunes, the cooperative evolution of numerous isolated dunes into complex network dunes, the formation of various types of dunes depending on several factors, etc.[1,10,11,12] This current incomplete situation of computational studies (and also of theoretical approaches) is due to the following intrinsic difficulties of the systems ranging over various scales.

Firstly, at the macroscopic level, where the interesting length scale is much larger than that of individual sand grains, the main problem for numerical and theoretical approaches has been the lack of a simple closed set of equations to describe the dynamics of the whole system (the dynamics of wind and that of the sediment transport). In fact, a large amount of effort has been made to describe the specific parts of the whole system under particular conditions. For example, 1+1-dimensional (one horizontal direction plus one vertical direction) air flow over a fixed dune has long been investigated.[13] Also the relation of sediment transport rate and the shear stress exerted by the wind has been studied by many researchers.[14]

However, many important subjects remain to be studied such as the estimation of the slope effect on the sediment transport [15,16], and the unsteadiness of the flow profile over a dune. Such incompleteness of the macroscopic approaches is partially caused by the inherent instability within the macroscopic scale. However, another crucial difficulty seems to originate from the inseparability of the macroscopic dynamics from the microscopic one. That is, the macroscopic equation does not close itself. As a symbolic example, the profile of dunes, i.e., the boundary of the domain for the air flow, often has sharp crests which are, not rarely, cusped with the scale much smaller than the size of dunes. Consequently, the irregular dynamics of air flow inevitably results. Also the macroscopic movement of sand through the 'avalanche' along the slip face is caused by the microscopic fluctuation at or around the brink of the slip face. Therefore, the 'microscopic' degrees of freedom cannot easily be 'renormalized' into the macroscopic dynamics.

In the other extreme, we could start with the microscopic treatment in which the dynamics of individual sand grains is taken into account. The main macroscopic difficulty, the lack of fundamental equation, almost disappears. Here, the system would, at least in principle, be described by the combination of established equations and laws; the Navier-Stokes equation (for the air flow) and hard core interaction between grains. The interaction between a grain and the air flow could be solved as a boundary problem for the air flow and the consequent drag force on the grains would be obtained. However, needless to say, huge amount of computation is needed to make a typical dune.

The characteristic time for the elementary dynamics (=saltation (see Fig.2)) of a sand grain is around 0.2sec [17], and the characteristic size, its diameter, is around 1mm.[17] Hence, considering a typical barchan, 100m $\times$ 100m in horizontal size and at least 100 years in 'age', the number of the iteration steps required is $\frac{100m \times 100m}{1mm \times 1mm} \times \frac{100years}{0.2sec} \approx$ (the number of sand grains on the surface of a barchan) $\times$ (the number of time steps to make the barchan) $\approx 1.6 \times 10^{19}$. These iterations occupy a 100GFLOPS computer around 1.6×10^{8} seconds, more than one year! Note that the above estimation is still for the 'ideal sand gas' with no interaction among individual grains, and that the heaviest task for a computer in simulating the granular dynamics is, without doubt, to treat the direct and indirect interactions between grains.[18] Reproduction of the various dynamics of dunes with the aid of a microscopic model is, at present, hopelessly unrealistic. Besides, even if we could describe a dune simulating grains, we must not forget that most computational resources are wasted in this case to compute the motions we are not at all interested in. What is of utmost interest to us is not a faithful reproduction of the dynamics of numerous grains but the extraction of the essential factors that decisively rule the evolution process of dunes.

As we have seen, the large scale dynamics of dunes remains to be investigated systematically. The difficulty we encounter in our problem is more or less inherent to any strongly nonlinear systems. Therefore, to study dunes may serve as a paradigm of studying complex nonlinear systems. We must pay attention to the fact that the large scale morphology of dunes and many aspects of their dynamics observed in various deserts have some common features.

For example, barchan dunes are formed generally in the area where a unidirectional wind prevails and the amount of available sand is limited, whereas transverse dunes are formed in desert areas with rich amount of sand.[12] As another example, in the migration process on the desert floor individual barchan dunes keep their shapes like solitons in fluid dynamics. Moreover, if their profile are assumed to be similar to each other, dunes move with constant velocities almost inversely proportional to their heights as shown by Rubin and Hunter.[19] A similar analysis has been made for sub-aqueous dunes by Fredsøe.[20] These facts indicate the existence of certain universal mechanisms that are insensitive to the details of individual dunes. Therefore, we expect that it is possible to invent a computationally efficient minimum model that can describe the large scale dynamics of dunes.[21]

In section 2, we will introduce the effective numerical model to describe the essential morphodynamics of dunes. In section 3, the simulation results for the 1D and 2D systems are shown. The summary and the discussion are in section 4.

2. Models

Since our problem is highly nonlinear, inevitable use of computers urges us to make a model directly adapted to discrete digital means. The system consists of horizontally extended lattice. Depending on the aspect of dynamics, 1-dimensional (1D) lattice or 2-dimensional (2D) square lattice is alternatively used.[22] Hereafter, we mainly

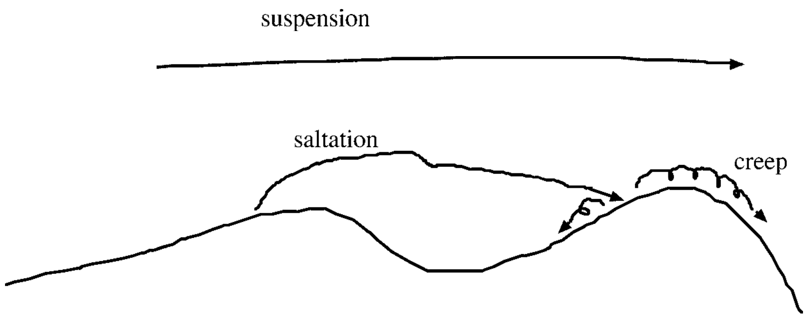

Fig. 1. Typical elementary dynamics of wind-blown sand grains.

explain the 2D model whereas it is evident that 1D model is a special case of the 2D model.

In the 2D model, each cell of the lattice corresponds to the area of the ground sufficiently larger than individual sand grains. At each cell (i, j), at each coarse-grained time step n, a continuous field variable $h_n(i, j)$ is allocated to denote the average height of the sand surface within the cell. Therefore, the evolution of $h_n(i, j)$ at one time step does not express the movement of individual sand grains. It rather describes the resulting surface height change after the collective motion of many grains during the unit time period sufficiently shorter than the characteristic time of a dune formation but much larger than the time scale of individual sand grain dynamics.

According to Bagnold the elementary dynamics of individual sand grains consists of creep and saltation (Fig.1).[1] Creep is the process where sand grains move along sand surface sliding or rolling. Saltation is the process in which sand grains make short jumps typically in the order of 10 cm. However, in the real process of dune formation the distinction between creep and saltation is not clear.[23] In the ripple formation model by Anderson et al., they stressed the important role of small jumps called reptation with the intermediate scale between saltation and creep.[23] Apart from the above processes, suspension process (Fig.1) plays an important role for sub-aqueous dunes,[24] but is not a decisive factor for the desert dunes because of the large difference of the mass density between the air and sand grains.

Here, we construct a phenomenological model at the space-time scale much larger than these elementary processes. We wish to describe effectively the outcome of the accumulation of the elementary moves such as creep and saltation. Our purpose is to grasp the essential feature of the dynamics of dunes and to investigate what kind of description should be effective from the macroscopic viewpoint. We divide the dynamical process into two:

i) The *inertial* or *advection* process; This describes the average transport effect of wind over the time step. This process may look like saltation in Bagnold's sense, but the similarity is only superficial as can be seen from the difference of involved space-time scale.[25]

ii) The *frictional* (or *diffusion*) process; There must be fluctuation effects around the average motion due to erratic wind directions, irregularities of the dune surface, etc.[26] This erratic 'Brownian' motion is modified by the slope effect [28] as the chemical potential does in the microscopic Brownian motion. The reader might be tempted to conclude that this process covers creep and small jumps mentioned above, but again the space-time scales are completely different.

The actual dynamics of our model are as follows;
In the frictional process, we may assume a local conservation law of the amount of sand in the coarse grained space mesh. Therefore, the conservation equation

$$h_{n+1}(i,j) = h_n(i,j) + [\sum_{(i',j')\in NN} j_n^{NN}(i',j':i,j) + \sum_{(i',j')\in NNN} j_n^{NNN}(i',j':i,j)] \quad (1)$$

holds. Here, NN are nearest cells of (i,j) and NNN are 2nd nearest cells of (i,j), the quantity $j_n^{NN}(i',j':i,j)$ or $j_n^{NNN}(i',j':i,j)$ is the net flux of sand from (i',j') to (i,j). This flux is the horizontal component of the flow which is assumed to be proportional to the gravitational force along the slope.[27] Hence, if the local slope is sufficiently gentle, the relations

$$\begin{aligned} j_n^{NN}(i',j':i,j) &= a(h_n(i',j') - h_n(i,j)) + b\delta_{j',j}(i > i') \\ &= a(h_n(i',j') - h_n(i,j)) - b\delta_{j',j}(i' > i) \quad (2) \\ j_n^{NNN}(i',j':i,j) &= \frac{a}{2}(h_n(i',j') - h_n(i,j)) \quad (3) \end{aligned}$$

hold, where a and b are positive constants. The symbol δ in the r.h.s.of (2) means Kronecker's delta, and the terms proportional to b in these equations describe the constant drift in the wind direction. However, they are ineffective in this model because of the conservation relation (1).

The advection process is modeled as follows.

$$h_{n+1}(i,j) = h_n(i,j) + \sum_{(i',j)} q_n(i',j)(\delta_{i'+L_n(i',j),i} - \delta_{i',i}) \quad (4)$$

Here $L_n(i,j)$ is the average transport length (note that this is not the saltation length) of sand grains which take off from (i,j), whereas $q_n(i,j)$ is the 'height transfer' associated with the grains transfer from (i,j) to $(i + L_n(i,j), j)$ by the inertial process (Fig.2). The symbol δ in the r.h.s. of (4) means Kronecker's delta, and the 2nd and the 3rd terms express the incoming advection flux to (i,j) and the outgoing one from (i,j), respectively.

The transport length $L_n(i,j)$ depends on many factors in real deserts. These factors are mutually coupled in a complicated manner. For example, the flow field of the wind directly affects the saltation length of individual sand grains, whose accumulation results in the transport length. On the other hand, the flow field sensitively depends on the present profile of the sand surface. At the same time the surface profile varies with time through the transport of sand grains. In the

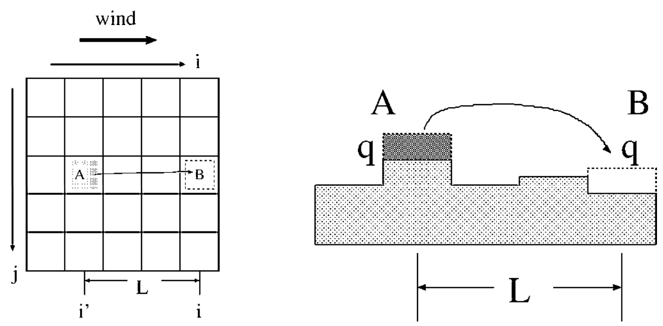

Fig. 2. Saltation process for the present model; At a saltation step sand grains at cell A will jump into cell B. In this process the surface height at A decreases by q and the height at B increases by q. The left figure is the view from the above while the right is the corresponding profile.

same way, the specification of the amount of the height transfer $q_n(i,j)$ in terms of possible relevant factors, also, is not an easy task.

Here, to make a minimal model for the large scale dynamics of dunes, we ignore the details of actual systems and assume a set of rather simple rules for the transport length $L_n(i,j)$ and the height transfer $q_n(i,j)$ as

$$L_n(i,j) = \alpha(\tanh(\nabla_i h(i,j)) + 1) \tag{5}$$

$$q_n(i,j) = \beta(-tanh(\nabla_i h(i,j)) + 1 + \epsilon). \tag{6}$$

Here, $\nabla_i h_n(i,j)$ means $h_n(i+1,j) - h_n(i,j)$, i.e., the local slope in i-th (wind) direction, where α, β and ϵ are positive constants. The above rules reflect the following observations:[29]

(i) In the windward side of a sand hill, the wind velocity is higher than the flat area.

(ii) Particularly around the crest of the real sand hill, a sharp peak of the surface wind velocity is observed.

(iii) At the lee side of the hill a drastic decrease of the wind velocity is observed.

Our simulation (see Appendix) for the flow field around a fixed sand hill also demonstrates the existence of a sharp peak of shear velocity at the top of a hill and its drastic decrease behind the hill. The advection flux $f = L \times q$ for our model is set to be consistent to the above facts as shown in Fig.3.

Now, the remaining problem is the non-uniqueness of the combination of L and q after determining the product $f = L \times q$. We have confirmed, by surveying the 'model space,' [30] that rough profiles of the dunes does not sensitively depend on the details of the combination of L and q. However, it should also be noted that the detailed shapes of dunes (e.g., the existence of a straight and sharp slope at the lee side of a barchan dune and a gentle slope at the windward of it, etc.) inevitably depends on the combination of L and q.

Apart from the above dynamical rules, as mentioned in the introductory section, seasonal change of wind directions and other geological conditions play very important roles in dunes evolution. Here, we introduce two parameters controlling these

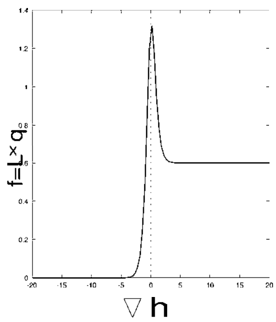

Fig. 3. Saltation flux $f = L \times q$ for the present model as the function of local slope ∇h. Here, α and β are 1 and ϵ is 0.3. (see equations (5)and(6).)

factors; i) the amount of available sand in a desert area, ii) the directional variability of wind in the desert area. The available sand in a desert area is limited because the existence of the unerodible hard ground beneath a sand layer. To incorporate these factors in the model we set the surface of the hard ground at a certain depth H below the average height of sand surface. To realize the above factor ii, that is, the directional variability of wind, we set the control parameter V as the index of wind directional variability. Specifically, the direction of wind, i.e., the direction of the advection, is periodically shuffled into one of the previously selected directions which are perpendicular to one another.

3. Simulations

Two kinds of simulations are performed. One is the 1D simulation in which the dynamics of the isolated dunes are discussed. The other is the 2D simulation in which the morphodynamics of the top view of dunes is discussed. Both simulations are performed with periodic boundary conditions.

3.1. *1D simulation of isolated dunes*

The dynamical rules for the 1D model is the same as (1)-(6) except that diffusion flow occurs only between nearest cells, that is, the last term of the r.h.s. of (1) is eliminated. If the initial condition is set as an almost flat surface with small fluctuations and if only a small amount of available sand is prepared, isolated dunes are spontaneously formed in the system. Here, by the term 'isolated dunes' we mean sand hills surrounded by the unerodible hard ground.

Individual isolated dunes migrate with the velocity depending on their height. The relation between the height of a dune and its migration velocity is known as the 'inverse relation', that is, the higher dune migrate with the lower velocity. Especially if the profile of several dunes in a desert area are similar to one another, migration velocities of them are supposed to be inversely proportional to their height.[19,20] More concretely, if it is assumed that the volume q_C per width per unit time of sand crossing the crest is deposited on the lee side of the dune, then the velocity of

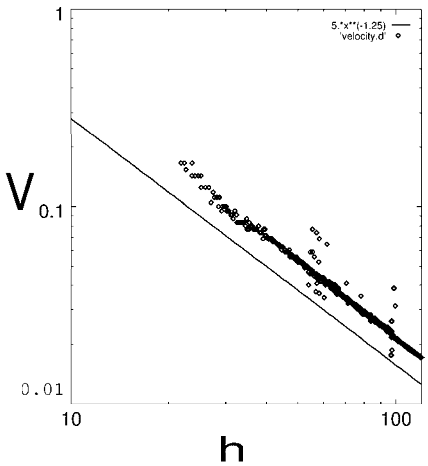

Fig. 4. The relation between the velocity v and the height h of mobile dunes obtained by 1D simulations. The inclination of solid line is -1.25.

the dune is given by:

$$v = \frac{q_C}{h}, \tag{7}$$

where h is the height of the dune. Actually, the velocity v is not just inversely proportional to the height, because q_C is a function of the height as well. In our simulation, the relation between the velocity and the height of dunes is measured and the above 'inverse relation' is reproduced. Moreover a power low relation $v = h^{-b}$ between the height h and the velocity v holds for a wide range of dune heights except for small dunes (Fig.3), where the value of b varies depending on details of the model.

If a small and isolated dune catches up with a larger and slower dune, two types of collisions occur (Fig.4). One is the perfect absorption of the smaller dune by the larger one. Through this process the average size of isolated dunes in the system increases. The other type of collision causes the tunneling of a smaller dune through a larger dune. More precisely, the smaller dune climbs up the larger dune and, before being completely absorbed by the larger, reaches the crest of the latter, is pushed forwarded, and eventually escapes from the lee side. This process still remains to be confirmed in real dunes. Such a 'tunneling process' is known for two solitons solution of e.g., the KdV equation. However, unlike the behavior of the KdV equation, through the collision, mass transfer between two waves (dunes) takes place. Besides, the higher wave migrates with the lower velocity. This is the opposite case to the soliton solution for the KdV equation.

3.2. *2D simulation for morphodynamics of dunes*

In the 2D simulation, we investigate the morphodynamics of dunes. As explained in the previous section, there are two control parameters in the simulation. One is H,

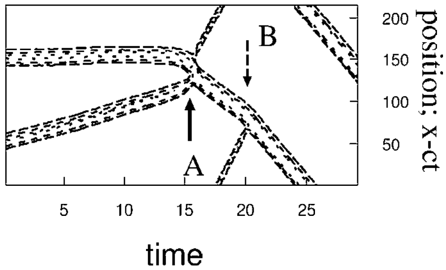

Fig. 5. The trajectories of initially two isolated dunes. Dotted lines are contours to denote the foots of individual dunes. Here it is noted the vertical axis is the position in a frame which moves with the initial velocity of the largest dune. At point A the tunneling process of a smaller dune through the larger dunes takes place while at point B the perfect absorptions of the smaller dune by the larger dune occurs.

the average depth of available sand. The other is V, the directional variability of wind. Depending on the number V of possible directions, we call the wind regime, the mono-directional regime ($V = 1$), the bi-directional regime ($V = 2$), or the tri-directional regime ($V = 3$).

We start simulations with such initial conditions that the sand surface is set almost flat except small fluctuations. As time proceeds, in the system, various types of dunes spontaneously emerge, depending on the values of the control parameters.

Firstly, dunes formation under the mono-directional wind regime is simulated. When the amount of available sand is sufficient, that is, if the layer of sand is so deep that hard ground surface is never exposed to the atmosphere, transverse dunes whose crests align almost perpendicular to the wind direction are spontaneously formed in the system (Fig.6(a)). When the amount of sand is poor, that is, if the surface of hard ground is easily exposed to the atmosphere, isolated barchan dunes are formed in the system(Fig.6(b)). Individual barchan dunes almost maintain their shapes and sizes in the migration process. However, if a small barchan catches up with a larger and, consequently, slower one, several types of collisions take place. As a typical case the perfect absorption of the smaller dune by the larger occurs. Eventually, networks which consist of large amount of small barchans are formed(Fig.6(c)).

Secondly, dunes formation under the bi-directional wind regime is simulated, where the wind directions time-dependently alternate stochastically between two preselected directions perpendicular to each other. Like the case of the mono-directional wind regime, two types of dunes patterns are formed depending on the amount of available sand. If sand is sufficiently supplied in the system, transverse dunes are formed. In this case, the crests of these dunes extend in the direction perpendicular to the average wind directions. In contrast, if the amount of sand is poor, linear dunes(or seif dunes) are formed in the system. The direction of their extension is, mainly, the intermediate angle between two wind directions. As shown in Fig.6(d), the crests of these linear dunes extend from the area with the shape of barchan dunes to the leeward. It seems to correspond to the linear dune formation scenario proposed by previous researchers.[1]

Finally, the simulations of the tri-directional wind regime are performed. In this wind regime, when the available sand is sufficient, sand hills similar to star dunes are formed as shown in Fig.6(e).

In Fig.7 the above mentioned results are summarized. The horizontal axis and the vertical axis denote two control parameters; the directional variability V and the average depth of sand layer (i.e.the amount of available sand per unit area) H, respectively. In the diagram, the types of dunes spontaneously formed at several points in the parameter space are described. Except that we failed to obtain realistically-shaped star dunes because of the anisotropy of our lattice, the diagram has a nice correspondence to the empirical facts.[12] Further simulations to make a more refined diagram are now in progress. For example, the extended calculation of the above $V = 2$ simulation is made, in which the relative angle θ_{w-w} between the two direction of the wind is not fixed as $\frac{\pi}{2}$ but continuously varied from 0 to π. In the calculation the relative angle θ_{w-c} between the time averaged wind direction and the dune crests' extension suddenly changes from 0 to $\frac{\pi}{2}$ at $\theta_{w-w} = \frac{\pi}{2}$. This result is consistent with a field observational report for the ripple formation [31] and the result of Werner's numerical approach.[21]

What is the most remarkable with our simulation is that it does not incorporate the detailed dynamics of the air flow field. Specifically, we determine the sand transport rate not explicitly as a function of wind force along the sand bed, but as a function of the local bed slope. In the latter function, the influence of the wind force on the sand transport is effectively incorporated as explained in section 3 and shown in Fig.3. By means of this manipulation, the serious difficulty to treat the feedback loop between the dynamics of the air flow and the evolution of the sand bed profile is avoided. This is the crucial point which enables our model to simulate the long time and large spatial scale evolution of dunes. The correspondence between the morphology obtained in our simulations and that of real systems seems fairly good. This means that the mechanism of dunes formation is, at least at the large scale of dunes, not so seriously affected by the details of the complex profile of the wind in contrast to the previous discussions. Of course, we must admit that certain factors should be added to reproduce a more accurate phase diagram. However, our model is certainly one of minimal computational models that can describe various pattern dynamics of dunes formation [21], and may be considered as an effective step toward the comprehensive understanding of large scale and long time evolution processes of dunes.

4. Summary

We have proposed a minimal model that successfully captures macro-phenomenology of dunes dynamics. The predictive and explanatory power we have achieved here may be far below the level that satisfies the ordinary physicists. However, we must

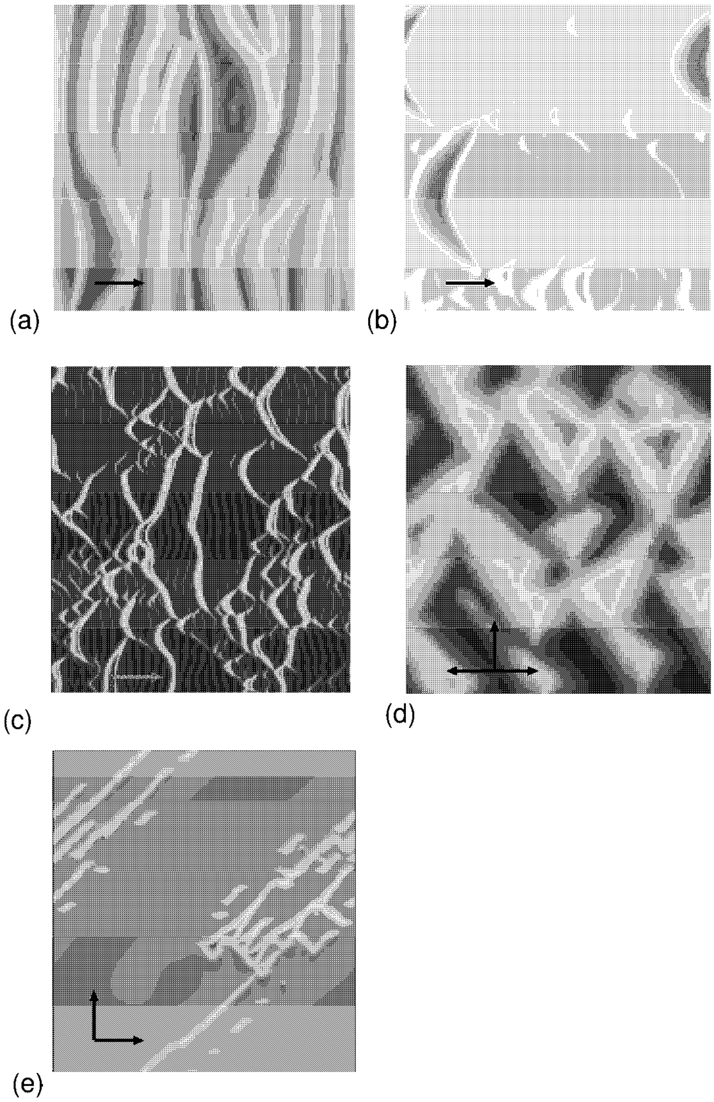

Fig. 6. Various shapes of simulated dunes; (a)transverse dunes, (b)barchan dunes, (c)network of small barchans, (d)seif dunes, and, (e)star dunes.

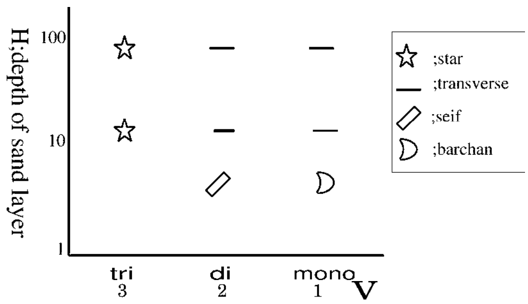

Fig. 7. Types of dunes formed at several points in the control parameters space. The control parameter V is the wind directional variability and H is the average depth of sand layer.

not forget a plain fact that modern physics has almost no effective recipe to deal with various phenomena surrounding us every day, e.g., long term weather dynamics, dynamics of land slides, and so on. Extreme examples of such phenomena may be those observed in living systems, e.g., evolution.

The dynamics of dunes may be considered as another such example. The dynamics of dunes can be reduced to hydrodynamics of air and the dynamics of granular materials both of which are in turn reducible to particle dynamics. This is also true for living systems. However, such descriptions do not allow us satisfactory analyses because of the intrinsic difficulties mentioned in the introduction. We may expect that there are phenomenological laws universal to various phenomena largely independent of the detailed microscopic descriptions. Needless to say, dunes are much simpler than, e.g., ecosystems. It is, however, our hope that studies of dunes may be used as a test ground of phenomenological approaches that are not a mere crude approximation to microscopic descriptions, but are precise descriptions of phenomena correctly capturing the mathematical structures we see behind various phenomena.

Acknowledgments

One of authors (H.N.) appreciates P. Civitanovic,T. Bohr, C. Ellegaard, M. H. Jensen, S. Watanabe and other members of the Center for Chaos And Turbulence Studies (CATS) at Niels Bohr Institute for valuable discussions and great hospitality extended to him when he was staying in the institute, where a part of this study was performed. H. J. Herrmann, M Sorensen, H. J. Hunt, and R. B. Hoyle are also acknowledged for their stimulating comments on this work. He also thanks Nishina memorial foundation for the help of the stay at Niels Bohr Institute. Inamori foundation, Kurata foundation, Nissan foundation, and Japan Ministry of Culture, Sports and Education, are appreciated for their financial support of this study. Finally we thank for Y.Oono for useful suggestions on this manuscript.

Appendix; Simulation of 2D flow over 1D dunes

The flow is calculated by use of a finite-volume solver on a orthogonal curvilinear grid.[36] The boundary conditions on the top of the domain is symmetric (i.e. zero gradient of all quantities), so that the simulations are actually made in a channel-flow with the height of the channel being two times the height of the domain. The turbulent closure is accomplished using the k–ω closure model by Wilcox. [38,39] The k-ω model has been successfully used on sub-aqueous dunes by Yoon & Patel.[40]

Sand transport

The saltation flux is described by the formula of Lettau & Lettau :[35]

$$q_0 = C'' \frac{\rho_{air}}{g} u_f^2 (u_f - u_{fc}), \tag{8}$$

where g is the acceleration of gravity, u_f is the friction velocity, C'' is a constant and u_{fc} is the threshold friction velocity. If $|u_f|$ is less than u_{fc} then q_0 is set to zero. The use of $q_0 \propto u_f^3$ corresponds well the widely used Meyer-Peter formula for bed load transport in water (i.e. [20]). The constants used in this calculation are:

$$C'' = 5.5, \qquad u_f = 0.22\text{m/s}. \tag{9}$$

With the aid of the saltation flux, the change in height of the dune, $h(x)$ can be calculated using a simple continuity equation:

$$\frac{\partial h}{\partial t} = -\frac{1}{\rho_{sand}} \frac{\partial q}{\partial x}. \tag{10}$$

Simulations

The simulations have been made over a triangular dune, with a 10° slope of the windward side, a 33° angle of the slip face and a height of 6 m. The height of the domain is $D = 30$ m, the velocity in the middle of the domain is $U_0' = 12$ m/s, the Nikuradse roughness is $k_N = 3$ mm and the density of the sediment is 1350 kg/m^3. In figure 8 is seen the profile on the flat part of the simulation, before the dune, compared to the law of the wall for a rough wall:

$$\frac{u}{u_f} = \frac{1}{\kappa} \ln \left(\frac{30y}{k_N} \right). \tag{11}$$

There seems to be a well developed logarithmic boundary layer, extending over the most part of the calculation domain.

Shown in figure 9 are the results from the flow simulation and sediment transport calculation over the triangular dune. In front of the foot of the dune, there is a marked decrease in the shear stress, caused by the flow "preparing" itself to climb the dune. This effect is the same as is seen in a flow approaching a rectangular box, where a separation bubble is formed in front of the box.[37] The same decrease is seen in the sediment transport. On the ridge of the dune there is an increase in the stress and the sediment transport, followed by a sudden peak just at the crest. This peak is even more marked in the sediment transport because of the nonlinearity of the transport function (8). Beyond the crest there is a separation zone, and then a relaxation towards the normal flat-bed conditions. In the separation zone, the shear stress is below the critical stress, so that no sediment transport is taking place here. It is also seen, that the effect of the gravity correction is negligible.

The fall in the saltation flux just before the foot of the dune, causes a deposition of sand here (Eq. 10), and this is a mechanism for the initiation of a new dune. On the wind-ward there is an erosion, which causes the dune to move forward. In the lee of the dune (around $x/D = 2$ to 3) there is an erosion zone, where a hole is formed. This again triggers the formation of a new dune.

The flux the crest is $q_C = 2.6 \times 10^{-4}$ m^2/s, which gives a velocity of the dune of $v = 0.16$ m/hour(Refs.19 and 20 to one-over-h law).

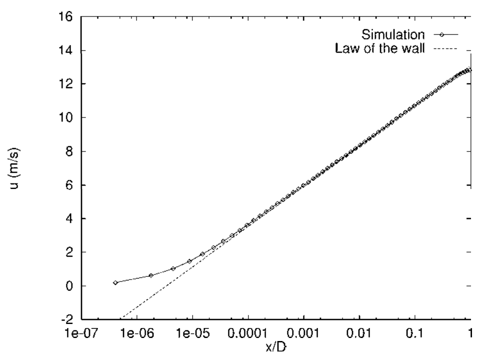

Fig. 8. The velocity profile on the flat bed. The straight line is the law of the wall.

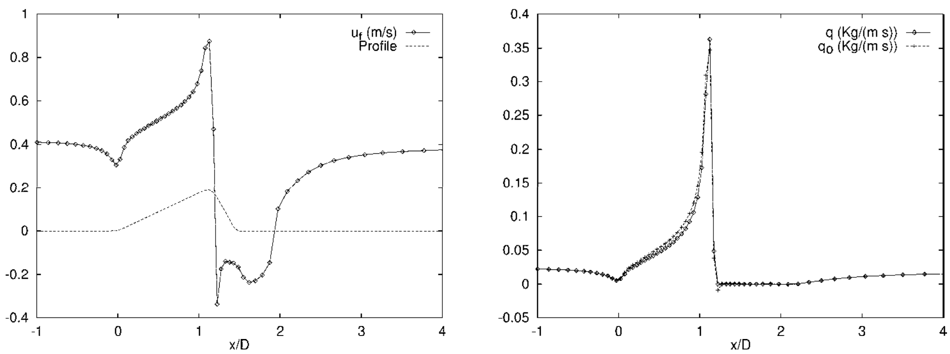

Fig. 9. The shear stress over the triangular dune (left), and the corresponding saltation flux (right). The shown profile is not to scale. The saltation flux is shown both with q and without gravity correction q_0.

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