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Comparison of laboratory data with a viscous two-layer model of wave propagation in grease ice

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Abstract. We compare laboratory measurements of wave propagation and attenuation in grease ice with two viscous fluid models. The first is the classic infinite depth, constant viscosity one-layer model. The second is a new finite depth two-layer model developed by Keller in which the upper layer (grease ice) is a constant viscosity, immiscible fluid overlying a denser but inviscid lower layer (seawater). In our comparison of these models with the laboratory results, we use the viscosity ν as a free parameter and search for that ν which gives a best fit to the data. The results of this calculation show that the two-layer model, with a grease ice viscosity $2-3\times10^4$ that of seawater, provides the best agreement of data with theory. Because the 0.1 m thickness of our laboratory grease ice is comparable to thicknesses observed in the polar oceans, the Keller model with this viscosity value should be directly applicable to the field interaction of waves and grease ice.

1. Introduction

Grease ice consists of thin slurry of frazil ice at the ocean surface, and occurs in a variety of locations under cold and windy conditions. The name "grease ice" comes from early whalers' observations that frazil ice forms into slicks which damp out short waves, giving the surface a smooth or matte appearance, as shown in surface and aerial photographs by *Martin and Kauffman* [1981] and *Pease* [1987].

Grease ice occurs in the following situations and locations. For 100 m scale leads and cold, windy conditions, *Bauer and Martin* [1983] show that frazil ice is herded downwind to accumulate into a 0.1-0.3 m thick layer of grease ice. For kilometer-scale polynyas in the Bering and Weddell Seas, *Pease* [1987] and *Eicken and Lange* [1989] show that a wind-induced Langmuir circulation herds the frazil ice into long plumes of grease ice oriented parallel to the wind. Aerial photographs show that wind waves incident on the plumes are damped out over short length scales relative to the plume widths, which contributes to the plume maintenance through the wave radiation stress [*Martin and Kauffman*, 1981]. Coring investigations of large frozen polynyas in the Bering and Chukchi Seas by one of the authors (SM) show that the frazil ice accumulates to depths of 0.1 – 0.2 m. These observations suggest that the combination of the Langmuir circulation and the wave radiation stress allows the frazil ice to accumulate to these thicknesses. *Wadhams et al.* [1996] observe grease ice formation under stormy conditions in the transient Odden ice feature in the Greenland Sea, and although they do not directly measure ice thickness, their Figure 13, combined with our laboratory value that grease ice consists of 50% ice by volume, implies a grease ice thickness of about 0.1 m.

Grease ice also reduces the surface turbulence associated with wave breaking by attenuating the waves, thereby providing favorable conditions for the formation of pancake ice, which makes up a large fraction of the ice cover in regions such as the Weddell Sea [*Eicken and Lange*, 1989]. Further, grease ice indirectly suppresses surface roughness by damping capillary waves and reducing wind stress. For example, in their investigation of the properties of many different ice types and open water under different wind conditions, *Guest and Davidson* [1991] found that grease ice had the smallest roughness lengths and drag coefficients. Finally, because grease ice attenuates the short surface waves corresponding to the radar Bragg-scattering wavelength, it is a poor radar reflector and appears dark in synthetic aperture radar (SAR) images [*Wadhams and Holt*, 1991, Figure 6; *Wadhams et al.*, 1996, Figure 4].

In previous work, from laboratory experiments on wave attenuation by grease ice, *Martin and Kauffman* [1981] suggest that this ice behaves as a shear-thinning fluid in which the viscosity increases as shear decreases. In more recent work and using better instrumentation, *Newyear and Martin* [1997] compare laboratory results on wave propagation through grease ice with the infinite-depth constant viscosity model of *Stokes* [1851] and *Lamb* [1932]. They find good agreement for $kh \approx 1$, where k is the wavenumber and h is the laboratory ice depth. In other work, *Weber* [1987] models the limiting case of a very thin, very viscous upper layer over an inviscid lower layer, and *Keller* [1998] solves for waves in a two-layer fluid where each layer has an arbitrary finite depth and a constant density, while the upper layer has a constant viscosity and the lower layer is inviscid. We next compare our laboratory results with the Stokes and Keller models and show that our data are in better agreement with *Keller*'s [1998] two-layer model.

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2. The Experiment

Newyear and Martin [1997] give a detailed description of the laboratory experiments; they take place in a wave tank located in a laboratory cold room with the air temperature regulated to within ±1°C. The tank is a flat-bottomed rectangular box, approximately 3.5 m long, 1 m wide, and 1 m deep, where one of the long tank walls is made of transparent Plexiglas. We fill the tank to a depth $H = 0.5$ m with a NaCl solution of 33 psu, which is comparable to that of Arctic surface waters. The waves are generated with a flap-type paddle, hinged at the bottom, and damped by a 0.2 m thick vertical beach made of horsehair packing material. We measure the wave properties with five independent strain gauge pressure probes, which are positioned in a line along the wave propagation direction.

To grow ice, we decrease the air temperature to about -10°C and use the paddle to produce the turbulence necessary for grease ice formation. Under these conditions, the ice layer thickness increases at rates of up to 1.5 cm hr^{-1}. When the ice reaches the desired thickness, we reset the air temperature to near freezing and measure ice properties and wave propagation. The observable wave frequencies are limited at the high and low ends by excessive damping and reflections from the beach, respectively. We measure the grease ice thickness h with a ruler by looking through the transparent tank wall; the lower layer seawater depth is H-h. Because the ice thickness increases away from the paddle due to the wave radiation stress, we approximate h as a spatially averaged constant. We calculate the complex wavenumber $K = k + iq$ using the procedures described by *Newyear and Martin* [1997]: a fast Fourier transform technique uses the relative phase shift between the pressure time series of different probes to calculate the wavenumber k, and an exponential curve fitting to the wave amplitudes determines the wave decay coefficient q.

3. Viscous Two-Layer Model

Keller [1998] presents a general solution for wave propagation in a two-layer fluid in which both layers have arbitrary but finite thicknesses, and the upper layer has a constant viscosity ν, while the lower layer is inviscid. This is an improvement over the previous models, because it describes the more realistic case of a less dense slick of viscous grease ice overlying a more dense inviscid interior. In the comparison of our experimental data with *Keller*'s [1998] model, we use the Newton-Raphson method [*Press et al.*, 1992] to solve the 4x4 matrix in his equation (21) for K as a function of the wave frequency f, the upper layer thickness h, and the viscosity ν. We directly measure or indirectly evaluate all variables in the matrix solution except for ν, which we treat as a free parameter to obtain a good match between the theory and the observations. We also nondimensionalize K at each frequency on the real wavenumber k_0 for waves in an identically stratified but inviscid fluid, so that

$$\hat{k} = \frac{k}{k_0}, \quad \hat{q} = \frac{q}{k_0} \tag{1}$$

By definition, $\hat{k} = 1$ for inviscid open water waves while $\hat{k} < 1$ indicates relative wave lengthening at a given wave frequency. Thus the effects of density stratification on waves have been removed in the normalization, leaving only those changes due to viscosity. Similarly, we nondimensionalize the effective viscosity of the grease ice by the nominal seawater viscosity of $\nu_0 = 1\times10^{-6}$ m^2 s^{-1}, so $\hat{\nu} = \nu/\nu_0$.

4. Results

Figures 1 and 2 compare the data from experiments 1 and 2 of *Newyear and Martin* [1997] with grease ice depths of $h = 0.12$ and 0.15 m. For each case, the frazil ice concentra-

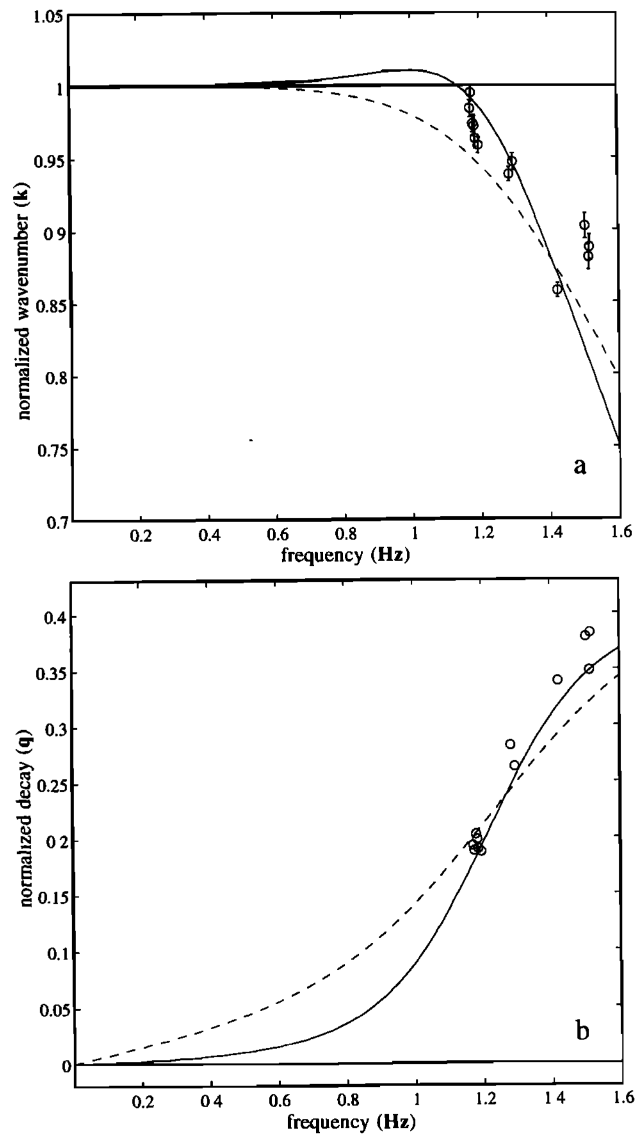

Figure 1. Comparison of laboratory to model predictions for experiment 1; see Table 1 for parameter description. On each panel the dashed curve shows the Stokes/Lamb one-layer model results for $\hat{\nu}_1 = 1.5\times10^4$; the solid curve shows the Keller two-layer results for $\hat{\nu}_2 = 2.5\times10^4$. The circles give our laboratory data; the vertical bars show the 95% confidence limits. (a) Normalized wavenumber $\hat{k}$ versus f, (b) normalized wave decay coefficient $\hat{q}$ versus f.

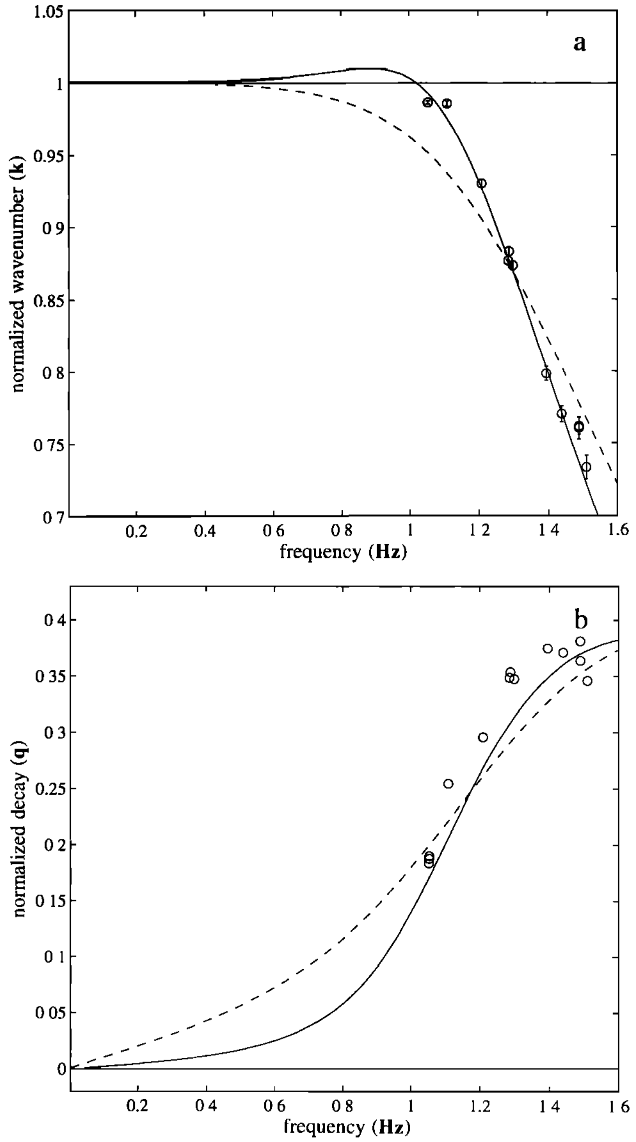

Figure 2. Comparison of laboratory data with model predictions for experiment 2; see Table 1 for parameter description. The dashed curve gives the Stokes/Lamb one-layer model results with $\hat{\nu}_1 = 2\times10^4$; the solid curve gives the Keller two-layer results with $\hat{\nu}_2 = 3\times10^4$. (a) Normalized wavenumber $\hat{k}$ versus f; (b) normalized wave decay coefficient $\hat{q}$ versus f.

tions by volume are identical at 0.48. On the figures, the top panel shows the normalized wavenumber $\hat{k}$; the bottom panel shows the wave decay coefficient $\hat{q}$, each plotted versus f. The solid horizontal lines at $\hat{k}=1$ in the top panels and at $\hat{q}=0$ in the bottom panels show the inviscid solution. The solid curve in each panel is the best fit to the data of the Keller two-layer model; the dashed line shows the best fit of the Stokes/Lamb solution. These fits were constructed by running each model at a series of different viscosities, then choosing the best fit by eye. Because of our short length of our tank, the data are concentrated at large f in the region of strong damping.

In both figures, the data show that $\hat{k}$ generally decreases, and that $\hat{q}$ increases, with f. In physical terms the figures show that the wavelength increases by as much as 25% in the observed frequency range, and the waves are damped out after propagating only a few wavelengths. For the shallower grease ice case in Figure 1, the $\hat{k}$ data exhibit such scatter, especially at the highest frequency where the experimental points lie well above both curves, that it is difficult to recommend one solution curve over the other. The wave damping comparison in Figure 1b yields a better fit to the Keller solution. In the thicker grease ice case shown in Figure 2, the Keller solution gives a fair fit to $\hat{q}$ and a much better fit to the observed $\hat{k}$ than the one-layer solution.

For each experiment, Table 1 lists h, H-h, and the viscosities used in the figures, where $\hat{\nu}_1$ is the one-layer viscosity and $\hat{\nu}_2$ is the two-layer viscosity. The uncertainties listed for $\hat{\nu}_2$ give the approximate viscosity range that provides the best fit to the data, so for the two experiments, the grease ice viscosity $\hat{\nu}$ lies in the approximate range $2-3\times10^4$, which is comparable to that of glycerin at 0°C. Because especially for Figure 2 the slopes of the one-layer curves is insufficiently steep to fit the observed $\hat{k}$ and $\hat{q}$ as well as the two-fluid curve, we omit the uncertainties for the one-layer case in the table.

Comparison of the model results with our data in Figures 1 and 2 suggests that the Keller model provides a better match to our observations than the one-layer model. Table 1 also shows that in each case, the two-layer viscosities are greater than the one-layer values. This occurs because the one-layer model allows viscous dissipation of wave energy over an infinite depth, while the *Keller* [1998] model concentrates the viscous dissipation within a relatively thin grease ice layer compared to the wavelengths being damped, thus requiring a larger absolute viscosity.

Examination of the theoretical curves show that both the one-layer and the two-layer models give the same qualitative trends for wave behavior in grease ice. At low f the effects of viscosity on wave propagation are minimal, as both models yield $\hat{k}=1$, indicating no deviation from the open water wavelength. At higher frequencies the models yield wave lengthening relative to ice-free conditions, with $\hat{k}$ decreasing as f increases. However, at midfrequencies the Keller model shows a slight overshoot in $\hat{k}$ for which the waves are slightly shortened, while the Stokes/Lamb model predicts that

Table 1. Layer Depths and Estimated Viscosities for the Two Experiments.

Experiment	H, m	H-h, m	$\hat{\nu}_1$ ($\times10^4$)	$\hat{\nu}_2$ ($\times10^4$)
1	0.11	0.39	1.5	2.5±0.25
2	0.15	0.35	2	3±0.25

The $\hat{\nu}_1$ value is the best fit nondimensional viscosity derived from the one-layer model; $\hat{\nu}_2$ is the viscosity derived from the two-layer model. See text for discussion of uncertainties.

$\hat{k} \leq 1$ for all f. Because of the practical difficulties of measuring long, low-frequency waves in our relatively short tank, we could not verify this effect in the laboratory. Each model also predicts that $\hat{q}$ is small at low f and increases with wave frequency, with the most rapid increase occurring at about the same values of f where $\hat{k}$ turns sharply downward.

For the higher frequencies shown in Figures 1b and 2b, the Keller model suggests that the $\hat{q}$ curve levels off or even decreases with a further increase in f. This is an artifact of our nondimensional variables, since the dimensional wave decay coefficient continues to increase with f but not so quickly as the open water wavenumber k_0. *Liu et al.* [1991] report field observations of such a "rollover" in wave damping at high f, but we were also unable to measure the wave properties in this very large damping regime.

5. Discussion

The success of the Keller model depends on the hypothesis that the grease ice layer can be described by a constant viscosity. One reason that the viscosity model works is that the 0.1 m vertical scale of the grease ice thickness and the 1 m horizontal scale of the incident waves are much greater than the 1 mm scale of the individual frazil crystals. This scale difference suggests that grease ice is a sufficiently continuous media that we can speak of its bulk viscosity. Such parameterization of flow in a two-phase fluid is common in the chemical engineering literature.

Martin and Kauffman [1981] suggest that the viscous nature of grease ice arises from interactions between frazil crystals. They present a thermodynamic argument for rapid sintering between crystals which come into contact. This gives rise to two processes, which are sinks for wave energy. First, mechanical agitation breaks the newly formed bonds between crystals. *Martin and Kauffman* [1981] argue that grease ice acts as a shear-thinning fluid in which viscosity increases for decreased shear rates when intercrystalline bonds form more quickly than they can be broken. Second, flocculation of frazil crystals into larger clumps increases the average roughness of the ice units, providing an increased resistance to the relative motion of the particles in wave motion. This occurs despite the rounding of individual ice crystals through wet metamorphism as they age.

These effects may also explain why the viscosity is slightly larger in our second experiment, for the ice layer was thicker and also a day older. A thicker grease ice layer leads to vertical compaction of the crystals, so sintering can occur to a greater extent. Similarly, older ice crystals have formed larger, rougher grains and flocs. Both of these effects would tend to increase the effective viscosity of grease ice and might explain why we obtain different values for our two experiments.

6. Conclusions

On the basis of our comparison of laboratory data with two theoretical models for wave propagation and attenuation, we find that grease ice can be best described by the *Keller* [1998] model. This model, which describes the grease ice-water system as a two-layer fluid, with a viscous upper layer and an inviscid lower layer, gives the best match to our data and allows us to estimate the grease ice viscosity as $2-3\times10^2$ m^2 s^{-1}, or about 4 orders of magnitude greater than seawater. This large viscosity causes wave damping, which increases with wave frequency, so grease ice becomes a low-pass filter for waves, and at high frequencies, an increased wavelength for waves in grease ice relative to open water. Because of mechanisms such as wave radiation stress, wind herding, and Langmuir circulation, grease ice in the field can accumulate to depths similar to our laboratory thicknesses. This means that the Keller model should be directly applicable to such phenomena as the interaction of wind waves with grease ice in leads and to wave propagation in grease-ice-laden regions such as the North Atlantic Odden.

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