Probability Distributions of Breaking Wave Heights Emphasizing the Utilization of the JONSWAP Spectrum

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ABSTRACT

An approximate determination has been made of the probability distribution function for breaking wave heights in the deep ocean. It was necessary to make simplifying assumptions of the joint distribution of wave height and wave period so that a semi-closed mathematical solution could be obtained as an illustration of the total processes that actually occur. These assumptions lead one to a calculation which predicts more and larger breaking waves than those which actually occur. Thus, for the design of structures in the deep ocean which are sensitive to breaking waves, a conservative determination of the probability distribution function is obtained. The distributions can be obtained for any location in the deep ocean given a sufficient history of surface meteorological data on wind speed and fetch. The joint distribution of wind speed and fetch for station PAPA in the North Pacific Ocean proved to be independent of storm duration. Thus, 6 h unit storms were considered to be independent in the computations.

1. Introduction

The motions of hydrofoils and surface effect ships as well as other floating objects, such as oceanographic research buoys, can be severely influenced by large breaking waves. A sailing vessel may pitch-pull, or capsize in other modes, if it is subjected to a breaking wave that has a height approximately equal to the length of the craft. Large breaking waves in deep water occur in severe sea states; therefore they happen more frequently in some areas of the ocean than in others due to the global distribution of prevailing malevolent climates.

If the probability distributions of breaking wave heights can be determined, then some engineering design problems can be at least approximately solved in terms of the reliability, safety or probability of survival of the object. It is known from laboratory testing and occurrences at sea that a floating object may capsize when it is subjected to breaking waves (Nagai and Kakuno, 1974). Given that the conditional probability function is known for capsizing, the unconditional probability distribution for capsizing can be calculated, if the probability distribution function for breaking waves is also known. The purpose of this research is to determine the distribution of breaking wave heights for the eastern North Pacific Ocean, with engineering applications in mind.

This paper is a re-working of Nath and Ramsey (1974), but added computations were made which show the greater utility of the JONSWAP spectrum (Hasselman, 1973) over the Liu (1971) spectrum. For clarity of presentation, the analytical developments in the previous paper have been reproduced and expanded.

Within the scope of this work is the development of a design tool consisting of a parameter κ (defined later)[3] which numerically characterizes the higher percent of large breaking waves in some oceanic areas and the lower percent in others. These procedures are illustrated with data from the eastern North Pacific Ocean, but can be used in other oceanic areas as well. The technique has been applied to one spot in the Pacific—Station PAPA at 145°W, 50°N—where "hindcasts" of wave parameters from meteorological data compared favorably with measurements from the Tucker shipborne wave recorders on the Canadian weather ships on that station.

Given a point location in the ocean, a single record of water surface fluctuations as a function of time is insensitive to wave direction. However, given that wave steepness can be defined in terms of H_s/T_s^2 (defined later) such a record does provide a good indication of large wave steepness. A single predominant direction usually exists for the larger waves. Therefore, this investigation considers the waves to be traveling in one direction, at least as a first approximation for an engineering work model.

The time trace of water surface fluctuations may look like that shown in Fig. 1. The zero-upcrossing wave definition is used so that individual wave heights

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[3] See Appendix for list of symbols.

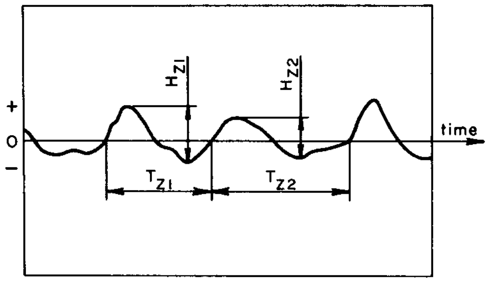

FIG. 1. Time record of water surface elevations.

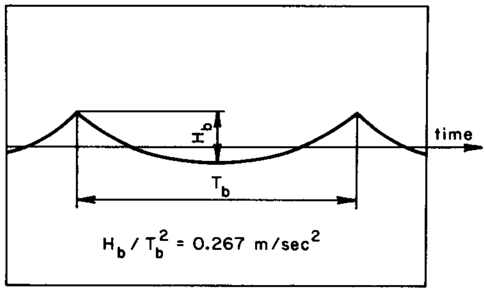

FIG. 2. Periodic wave, incipiently breaking.

and periods are H_{z1}, H_{z2}, ... and T_{z1}, T_{z2}, ... From such a record the breaking condition can only be a defined quantity since we have no hope of deriving an instability condition without knowing the wave propagation directions and the influence they have on wave instability. In addition, strong winds can blow the tops off the waves and short steep waves can break while superimposed on longer waves making the breaking condition difficult to define. Thus, the following arbitrary definition (which seems to have some engineering applicability) was adopted.

Fig. 2 represents a periodic wave that has just reached the point of breaking. For this condition the relationship between wave height and period can be considered to be

$$H_b = \nu T_b^2, \tag{1}$$

where ν is a dimensional constant and is determined by a chosen wave theory. For this work Dean's stream-function theory was adapted so that ν is 0.267 m s^{-2} (0.875 ft s^{-2}) for deep water waves (Dean, 1965). If desired, other theories or empiricisms can be used which may change the value of ν. Thus, the ith wave is defined as a breaking wave if

$$H_i \geqslant 0.267 T_i^2. \tag{2}$$

An interesting laboratory investigation on breaking waves is reported in Van Dorn and Pazan (1975), but no help is given on defining a breaking wave in an irregular sea. The report clearly shows that a breaking periodic wave is not symmetrical. However, the work of Dean (1965) is utilized here for definition purposes. How this asymmetry influences the results herein is not known now.

The following development will begin with fundamental questions about wave height and period distributions and will then methodically progress to an expression for the probability distribution function for the maximum breaking wave height at one location for Y years "on station." The result is essentially obtained by hindcasting from several years of meteorological charts of wind and fetch conditions for a large area of ocean around the location. Certain simplifying assumptions were made for the sake of expediency and they are considered as topics of continuing research.

2. The probability that a wave will break

Wave heights are generally assumed to have a Rayleigh distribution (Longuet-Higgins, 1952, 1957). The distribution of wave period is much less known. Bretschneider (1964, 1966) assumed the square of the wave periods to have a Rayleigh distribution. Longuet-Higgins (1972) derived a different distribution of wave period, assuming a normally distributed water surface elevation and a very narrow sea surface spectrum. The result is the only theoretical development on wave period distribution known to the authors. In addition, most past developments have assumed the wave period and height to be independent. However, from data obtained with Wave Rider buoys off the East coast of Canada, and from Station PAPA data, this investigation shows that the rms wave height for the distribution within a particular band of wave periods depends on the mean of the wave periods within that band. The wave heights (within the band of wave periods) have a Rayleigh distribution, but the rms wave height varies with wave period as indicated in Fig. 3.

Longuet-Higgins (1975) recently presented the joint distribution of wave periods and amplitudes for a narrow banded wave spectrum. For a sufficiently narrow banded condition one can substitute wave heights for the amplitudes, or maxima, of the waves, as is done for the correspondence between wave heights and amplitudes with respect to the Rayleigh distribu-

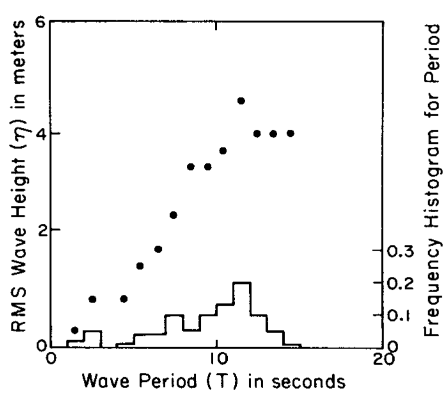

FIG. 3. RMS wave height as a function of wave period and a histogram of wave periods.

tion. The results of the new work require numerical integrations at each step of the calculations that follow in this paper. However, it is desired to avoid this for the sake of illustration of the processes involved, and to expedite a conservative engineering solution for the problem. This was achieved by making the assumption that wave height and period are independent. The dependency condition will be included in a later development.

Some data for this work were taken with Wave Rider buoys. They consisted of twenty records of wave height and period, provided by the Department of Public Works, Wave Climate Study, Ottawa, Canada. The location was around Western Head, Nova Scotia, in a water depth of about 40 m. Tucker wave gage records of water surface elevations at Station PAPA were also provided by the same department to the Fleet Numerical Weather Central, Monterey, Calif., where they were made available to us. They covered recordings from 1968 through 1970. In addition, deck charts of observed and recorded sea states were provided from 1965 through 1972 for PAPA. The meteorological information of the Pacific Ocean was obtained from the U. S. Weather Bureau National Meteorological Service and it consisted of 40 microfilm reels of Northern Hemisphere surface charts for the periods of April 1962 to September 1972.

The data show the conservative trend of the independence assumption between wave heights and periods. That is, if it is assumed that wave height and wave period are independent random variables, then a derivation for the percent of the waves which are breaking will show a larger value than if the dependency relation is maintained. Moreover, we will assume that wave heights and the square of the wave periods are Rayleigh distributed.

For wave heights and periods we now have

$$P\{H_z>h\}=\exp[-(h/\zeta)^2], \tag{3}$$

$$P\{T_z>t\}=\exp[-(t/\tau)^4], \tag{4}$$

where ζ is the rms H_z and τ^2 is rms T_z^2. A specific zero-crossing wave height is designated by h and a specific zero-crossing wave period by t.

Next, the probability that a wave is breaking (or the percent of breaking waves in a given record) will be determined. Let the probability that a particular wave is breaking be represented by

$$P_{bw}=P\{H_z\geqslant \nu T_z^2\} \tag{5}$$

so that the domain of realization is $h\geqslant 0$, $t\geqslant 0$. Because of the assumption of independence between H_z and T_z, H_z can assume any positive value, for a given T_z. [Later for the development of (26) we will impose the restriction that $H_b=\nu T_b^2$, where the subscript b stands for the breaking condition.] Thus,

$$P_{bw}=\int_0^\infty P\{H_z\geqslant \nu t^2|T_z=t\}dF_{T_z}(t), \tag{6}$$

where

$$F_{T_z}(t)=P\{T_z\leqslant t\}. \tag{7}$$

Then

$$P_{bw}=\int_0^\infty \exp[-(\nu t^2/\zeta)^2]\frac{4t^3}{\tau^4}\exp[-(t/\tau)^4]dt \tag{8}$$

$$=\int_0^\infty \exp\{-u[1+(\nu\tau^2/\zeta)^2]\}du, \tag{9}$$

where

$$u=(t/\tau)^4. \tag{10}$$

Integration then gives

$$P_{bw}=\kappa^2/(\kappa^2+1), \tag{11}$$

where

$$\kappa=\zeta/\nu\tau^2. \tag{12}$$

The parameter κ can be considered as a measure of the "steepness of the sea." Even without the simplifying assumptions leading to (12) one can consider the ratio of rms H_z to rms T_z^2 to be a good empirical indicator of the general steepness of the waves in a given record. Kappa is dimensionless, so that the total severity of the sea state is not indicated by it alone. For an empirical "feel" for total sea severity, a height parameter needs to be considered in conjunction with κ.

It is also convenient to express kappa in terms of the sea surface spectrum $S(f)$. In terms of the spectrum moments M_n Cramer and Leadbetter (1967) and Longuet-Higgins (1957) show that the average zero-crossing wave period is

$$\bar{T}_z=\left(\frac{M_0}{M_2}\right)^{\frac{1}{2}}. \tag{13}$$

In addition, from (4) we have

$$\bar{T}_z=\int_0^\infty t dF_{T_z}(t) \tag{14}$$

$$=\int_0^\infty 4\left(\frac{t}{\tau}\right)^4\exp[-(t/\tau)^4]dt \tag{15}$$

$$=\tau\Gamma\left(\frac{5}{4}\right), \tag{16}$$

where Γ represents the gamma function. It is shown by Cartwright and Longuet-Higgins (1956) that

$$M_0=\frac{\zeta^2}{8}. \tag{17}$$

Therefore, from (12), (13), (16) and (17),

$$\kappa=8.76\frac{M_2}{M_0^{\frac{1}{2}}}, \tag{18}$$

where $S(f)$ is expressed in m^2 s.

The Pierson-Moskowitz, Liu and JONSWAP spectra formulations are well known and are clearly represented in the references. Therefore, they will not be reproduced here. The useful moments of the Pierson-Moskowitz and Liu spectra can be expressed in closed form, but the JONSWAP spectrum requires a numerical integration.

It is of interest to compare κ for three different empirical sea states: 1) the "fully developed sea" spectrum as expressed by Pierson and Moskowitz (1964), 2) the simple fetch and wind-speed dependent spectrum specified by Liu (1971) [which produces some simple closed solution relationships for this work], and 3) the more complicated fetch and wind-speed dependent JONSWAP spectrum as described by Hasselman (1973).

The terminology "fully developed seas" is controversial. A fully developed sea with regard to fetch does not occur according to the Liu and JONSWAP spectral representations. They are fully developed with regard to duration only, and therefore are not functions of duration. The Pierson-Moskowitz spectrum supposedly represents sea conditions that are fully developed with regard to both duration and fetch.

Eq. (18) was utilized to determine κ for each of the three spectra. A numerical integration was necessary for the JONSWAP spectrum. For the Pierson-Moskowitz spectrum, κ has a constant value of 0.172. The Liu spectrum shows κ to be a function of wind speed and fetch, i.e.,

$$\kappa=0.12\frac{U^{0.417}}{X^{0.208}} \quad \text{(Liu spectrum)}, \qquad (19)$$

where U is the wind speed (kt) and X is fetch (n mi).

Fortuitously, when integrating numerically for κ with the JONSWAP spectrum we found that κ vs wind speed plotted as straight lines on log-log paper for different values of fetch. Thus, an empirical derivation therefrom for κ yielded

$$\kappa=0.1588\frac{U^{0.1117}}{X^{0.1087}} \quad \text{(JONSWAP spectrum)}. \qquad (20)$$

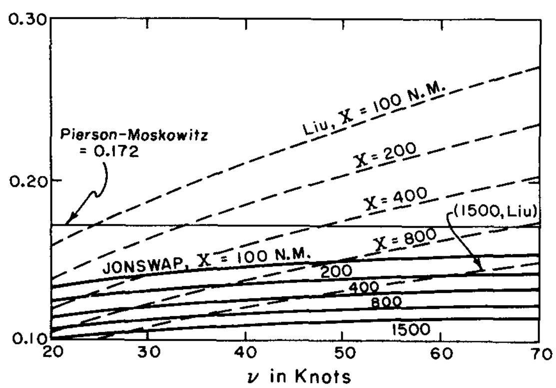

FIG. 4. Sea steepness in terms of κ for Pierson-Moskowitz, Liu and JONSWAP spectra.

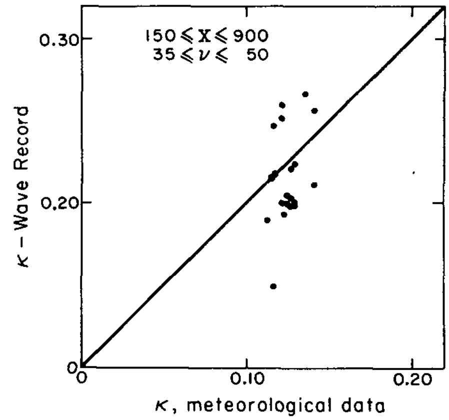

FIG. 5. Kappa correlation at Station PAPA, utilizing the JONSWAP spectrum.

The three results are compared in Fig. 4. From (11) it is evident that the JONSWAP spectrum will predict a lower percent of breaking waves than the Pierson-Moskowitz spectrum even though the JONSWAP spectrum has a higher peak value. In addition, for the usual engineering design storm conditions (high wind speed and fetch) the JONSWAP spectrum predicts lower percentages than the Liu spectrum.

Fig. 5 shows theoretical values of κ as determined from meteorological information, utilizing (20), compared with values from the actual shipboard wave recordings of the sea conditions during the same periods at station PAPA. These values of κ are from particularly severe events that were selected from the data. Thus, κ seems to be a reasonable parameter to use to describe the general sea steepness. The JONSWAP spectrum (with simplifying assumptions) predict good values at station PAPA using meteorological data for U and X. The correlation coefficient for Fig. 5 is 0.97 and the values of κ are conservative. A similar diagram of κ using (19) yields a correlation coefficient of 0.25.

3. The height distribution of waves that break

We next develop the probability distribution of the heights of waves for a particular storm record, given that they break (in other words, conditioned on the event that they are breaking). For any specific wave period t Eq. (3) gives

$$P\{H_z > \nu t^2\} = \exp[-(\nu t^2/\zeta)^2]. \qquad (21)$$

The relative frequency of the event the wave breaks and $t < T_z < t+dt$ is

$$Q(t)dt = \exp[-(\nu t^2/\zeta)^2]\frac{4t^3}{\tau^4}\exp[-(t/\tau)^4]dt, \qquad (22)$$

similar to (8). The event "$H_z > h$ and the wave breaks" is the same as "$T_z > t(h)$ and the wave breaks," where $h = \nu t^2$, because the height of a breaking wave, as defined, is determined by its period. In other words, we

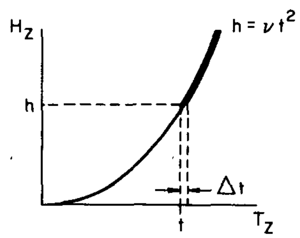

FIG. 6. Domain of realization for Eq. (25)—the heavy line.

are now seeking the probability that H_z is greater than h and that $h = \nu t^2$, which is the very restricted region shown by the heavy line in Fig. 6. Thus,

$$P\{H_z>h \text{ and the wave breaks}\}$$

$$=\int_{t(h)}^{\infty} Q(t)dt \tag{23}$$

$$=\int_{a}^{\infty} \exp[-u(1+\kappa^{-2})]du \tag{24}$$

$$=P_{bw}\exp[-(h/\zeta)^2(1+\kappa^2)], \tag{25}$$

where $a=[t(h)/\tau]^4$. Eq. (25) represents the percent of the entire sample of waves that are breaking and that are higher than the specific value h. It follows that the height distribution of breaking waves is

$$P\{H_z>h|H_z>\nu T_z^2\}$$

$$=\frac{P\{H_z>h \text{ and the wave breaks}\}}{P\{H_z>\nu T_z^2\}} \tag{26}$$

$$=\exp[-(h/\zeta_b)^2], \tag{27}$$

where

$$\zeta_b=\zeta/(1+\kappa^2)^{\frac{1}{2}}. \tag{28}$$

For a wave record, if one considers only the breaking waves, (27) represents the percent of these breaking waves that are greater than h. Thus, within the very broad set of assumptions made thus far, the heights of breaking waves have a Rayleigh distribution with an rms breaking wave of ζ_b. The rms value ζ_b is always slightly smaller than ζ because most of the breaking waves in a record are small breaking waves.

Now consider a statistically stationary storm that contains N waves. Given that successive waves are independent, the probability that a wave breaks is a binomial trial. In N waves, the probability of k breaking waves would be

$$P\{k \text{ breaking waves}|N \text{ waves}\}$$

$$=\binom{N}{k}P_{bw}k(1-P_{bw})^{n-k}. \tag{29}$$

Then the distribution for the largest of k breaking waves would be

$$F_{H_{b\max}}(h)=\sum_{k=0}^{N}\{1-\exp[-(h/\zeta_b)^2]\}^k\binom{N}{k}$$

$$\times P_{bw}^k(1-P_{bw})^{N-k} \tag{30}$$

$$=\{1-P_{bw}+P_{bw}[1-\exp(-(h/\zeta_b)^2)]\}^N \tag{31}$$

or

$$P\{H_{b\max}\leqslant h|N \text{ waves}\}$$

$$=\{1-P_{bw}\exp[-(h/\zeta_b)^2]\}^N. \tag{32}$$

4. Storm intensity and duration relationships

Storm intensity for this work refers to the sea conditions, which can be defined in terms of κ and ζ, which in turn are determined from the wind speed U and storm fetch $\boldsymbol{X}$. At a point on the ocean the sea state is also influenced by waves produced by a distant disturbance. A study was made for station PAPA to determine how we can best hindcast the sea state from the surface meteorological charts. Three hindcast methods were used: 1) all the storms within about 1000 n mi were considered that could possibly influence the conditions at PAPA [the computerized technique was developed by Enfield (1973) and included wave dispersion, superposition and attenuation]; 2) only single large storms were considered which passed anywhere near PAPA. This was done in the early stages of this investigation and utilized only the Liu spectrum, which we now know should give somewhat conservative results. For the first and second methods the spectrum is modified, due to dispersion, superposition and attenuation. The third calculates the spectrum directly, using the Liu spectrum. The significant wave height from hind cases, H_{m_0}, was calculated from the zeroth spectral moment from the predicted spectrum. Thus the result for the third method using the Liu spectrum is

$$H_{m_0}=0.0217U^{1.53}X^{0.24} \quad \text{(Liu spectrum).} \tag{33}$$

The significant wave height was also determined from numerical integration of the JONSWAP spectrum for various values of U and X. It was then found empirically that the results can be closely matched with the formulation

$$H_{m_0}=0.05042U^{0.7705}X^{0.5522}$$

$$\text{(JONSWAP spectrum),} \tag{34}$$

where H_{m_0} is in feet for (33) and (34).

Plots of the three methods, using the Liu spectrum, compared to the measured significant wave height $H_{\frac{1}{3}}$ from the shipborne Tucker wave gage, appear in Fig. 7. The results for the simple third method are surprisingly good, considering the human error involved in the construction of the meteorological data, the interpretation of fetch and wind speed therefrom, the approximations

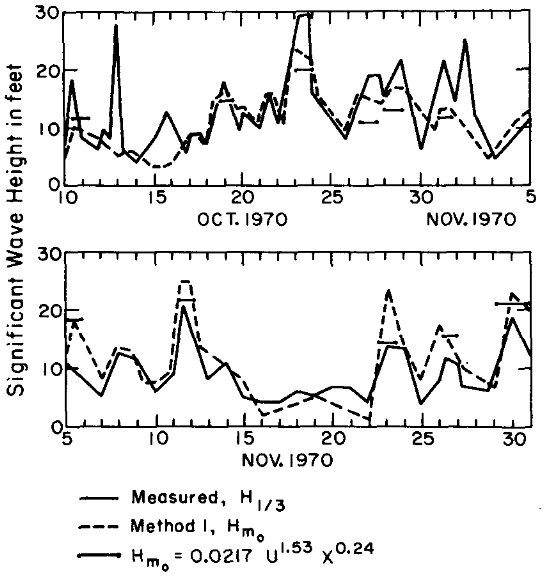

FIG. 7. Comparison of measured and hindcast significant wave heights at PAPA.

in the hindcasting, and the errors in the shipborne wave gage. Fetch estimations from surface charts are somewhat arbitrary and they were obviously underestimated for the first part of Fig. 7, and overestimated for the second part. Since Fig. 7 is for illustrative purposes only, we have not taken time to re-do it using (34). However, using the third method with (34) will bring the agreement between $H_{\frac{1}{3}}$ and H_{m_0} quite close. Of more importance in comparing measurements vs hindcasts, for this work, is Fig. 5, which has a correlation coefficient of 0.97. The simple third method of sea intensity computation was adopted for the remainder of this study.

5. The distribution of wind speed and fetch

Five years of meteorological data were analyzed. Average wind speeds over the fetch were selected by averaging ship reports, heavily weighted by the report from the weather ship at station PAPA. The data consist of observation pairs (U,X) recorded for 6 h intervals. Only those observations for which $H_{m_0} > 22$ ft (determined from the Liu spectral calculations) were recorded. Fig. 8 was used for a guide appraisal of H_{m_0}. Thus the record consisted of observation pairs grouped in time according to storms.

FIG. 8 also shows H_{m_0} hindcasts from (34). It is interesting to note the broad differences between the two computations. In addition, the Liu curves are concave upward, whereas the JONSWAP curves are concave downward, which indicates that, given a fetch, a fully developed sea with respect to wind speed would be approached asymtotically for the JONSWAP spectrum, which *should* occur due to wave breaking. Thus the JONSWAP spectrum gives much more reasonable results.

With no theoretical guidelines available, a broad modeling approach was used to determine a plausible model for the joint distribution of U and X. The data were grouped according to storm duration D and according to fetch. In each group, the empirical distribution function of wind speed

$$P = P\{U > u \,|\, X, D\} \tag{35}$$

was determined and a variety of functions of P were plotted against a variety of functions of u. In all cases, the plot of $(\log P)^{\frac{1}{2}}$ vs $\log u$ was nearly linear, suggesting the conditional distribution

$$P = \exp\{-B[\ln(u/u_0)]^2\} \tag{36}$$

for $u > u_0(\sim 20$ kt), where the parameter B depends upon fetch and duration. An analysis of the variation in the parameter estimates revealed that 1) B changed significantly with fetch, resulting in a joint distribution where extreme wind speeds are associated with relatively low fetches and vice versa; and that 2) B did not vary significantly with storm duration. The latter result is important because it allows one to regard the occurrences of severe sea states as being excursions to a high level by an approximately stationary intensity series.

To complete the picture, the distribution of fetches was modeled in the same way. The marginal distribution of fetch was also well represented by an expression similar to (36). That is,

$$\text{Prob}\{X > x \,|\, D\} = \exp\{-C[\ln(x/x_0)]^2\} \tag{37}$$

for $x \geqslant x_0(\sim 200$ n mi). It was again found that the parameter C showed no significant variation with dura-

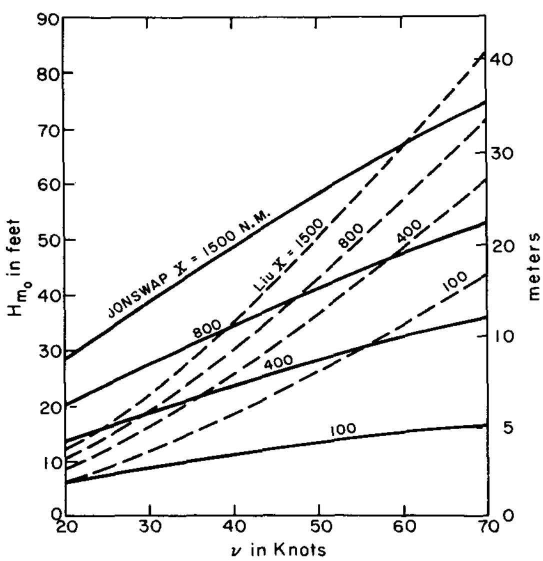

FIG. 8. Significant wave heights for wind speed and fetch from the Liu and JONSWAP spectra.

tion D. The coefficients were evaluated as

$$B=3.55+1.23(x/x_0),\quad X_0=200 \text{ n mi}, \tag{38}$$

$$C=1.75,\quad U_0=20 \text{ kt}. \tag{39}$$

The joint probability density distribution of U and X can be derived by differentiation of (36) and (37) as

$$f_{UX}(u,x)=\frac{4BC}{ux}\ln\frac{u}{u_0}\ln\frac{x}{x_0}\times\exp-\left[B\left(\ln\frac{u}{u_0}\right)^2+C\left(\ln\frac{x}{x_0}\right)^2\right]. \tag{40}$$

6. The distribution for the maximum breaking wave

For some engineering design purposes the highest breaking wave to be experienced in a given period of time needs to be considered. Thus, if an engineering structure of some sort is involved, the probability distribution function of the maximum breaking wave to be experienced during the life of the installation should be determined. An approximate solution was obtained for station PAPA. The development follows.

We now designate the probability distribution of breaking wave heights in a 6 h period as P_6. Then the conditional probability for the *maximum* breaking wave height, given the storm intensity ζ_b, is, from (32)

$$P_6^{N_6}=[1-P_{bw}\exp[-(h/\zeta_b)^2]]^{N_6}, \tag{41}$$

where N_6 is the number of waves in a 6 h record. The condition on ζ_b, which is expressed in terms of κ and ζ, is removed as follows:

$$P\{H_{b\max}\leqslant h|6 \text{ h storm}\}=\int\int P_6^{N_6}f_{\kappa,\zeta}(k,z)dkdz, \tag{42}$$

where f is the probability density distribution of κ and ζ, and 6 h storm increments can be assumed to be independent. In (42) the number of waves are actually dependent on the storm intensity because

$$N_6=21\,600/\bar{T}_z. \tag{43}$$

We now let there be L 6 h storm periods per year. Then the conditional probability for maximum breaking waves is

$$P\{H_{zb,\max}\leqslant h|L\}=\left[\int\int P_6^{N_6}f_{\kappa,\zeta}(k,z)dkdz\right]^L. \tag{44}$$

Because L appears as an exponent in (44), the condition on L can be removed with the aid of the probability generating function concept. Thus, if

$$P\{A|B\}=[g(A)]^B, \tag{45}$$

then

$$P\{A\}=G_B[g(A)], \tag{46}$$

where

$$G_B(s)=\sum_{\text{all } b}s^bP\{B=b\}. \tag{47}$$

The right-hand side of (46) reads "the probability generating function of B, evaluated at g, the function of A."

For this work, the discrete probability function for L was assumed to have a Poisson distribution, where the mean value was obtained from counting the number of 6 h storm periods used to determine the probability density distribution of wind speed and fetch [Eq. (40)]. This point needs some refinement. The *numbers* of excursions over an extreme level per year have, theoretically, an approximate Poisson distribution. But the distribution of excursion *lengths*—which is more closely related to that of L—is unknown.

For Y years on station, the expression for the probability distribution function of $H_{b\max}$ is

$$P\{H_{b\max}\leqslant h\}=\left[G_L\left(\int\int P_6^{N_6}f_{\kappa,\zeta}(k,z)dkdz\right)\right]^Y. \tag{48}$$

Since κ, ζ and N_6 can all be evaluated in terms of U and X through the spectral moments, the joint density distribution of κ and ζ can be supplanted with the joint density distribution of U and X in (48). In addition, the probability generating function of the Poisson distribution has a simple, known form. Letting $Y=1$ for one year on station, the final estimate for the probability distribution function for the height of the maximum breaking wave becomes

$$P\{H_{b\max}\leqslant h\}=\exp[-\bar{L}(1-V)], \tag{49}$$

where $\bar{L}$ is the average number of 6 h storms per year on station which have $u\geqslant u_0$ and $x\geqslant x_0$, and V is represented by the volume integral in (42).

Eq. (49) was evaluated at station PAPA, using ten years of meteorological data, wherein $H_{m_0}\geqslant 22$ ft approximately. For the following results, $\bar{L}=83.9$, $u_0=20$ kt, $x_0=200$ n mi, and the upper limits of integration were taken as 200 kt and 2000 n mi. Eq. (49) is quite sensitive to V, so that considerable care needs to be exercised in the evaluation of the double integral.

The results are on the conservative side. That is, it is expected that the h values for a given probability level should be lower than the values calculated. The results for both the Liu and JONSWAP spectra are shown in Fig. 9. According to the JONSWAP spectrum, which we feel gives the most credible results, the maximum breaking wave height in a one-year period at PAPA is nearly certain to be greater than 10 m and less than 21 m. For 5% of the years the maximum breaking wave height is predicted to be greater than 18.6 m. Once again, these results are conservatively large because the simplifying assumptions made, such as allowing wave height and period to be independent, will predict a percent of large breaking waves that will be higher than what will actually occur.

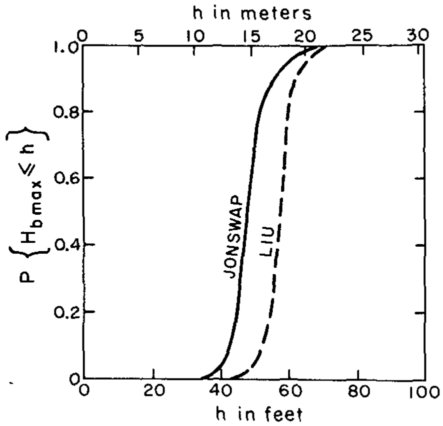

FIG. 9. Distribution function for the maximum breaking wave in one year at Station PAPA.

7. Conclusions

The probability distributions of breaking wave heights in the deep ocean can be determined, at least approximately, from a long history of surface meteorological data that describe wind speed and fetch.

The usual assumption that zero-upcrossing wave heights and periods are independent random variables leads to higher percentages of a given wave steepness than actually occur.

The joint distribution of wind speed and fetch was determined for station PAPA and both variables proved to be independent of storm duration. Thus, the 6 h unit storms were considered to be independent, which considerably simplified the computations.

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APPENDIX

List of Symbols

D	storm duration (h)
$F(\ )$	probability distribution for the argument
H_z	zero up-crossing wave height
H_b	height of wave which is incipiently breaking
H_{m_0}	significant wave height from hindcasts and spectral moments
$H_{\frac{1}{3}}$	significant wave heights from wave measurements
h	wave height index
L	number of 6 h storm periods per year
M_n	nth moment of the variance spectrum
N	number of waves in a record
N_6	number of waves in a 6 h record
P_{bw}	percent of breaking waves in a record
$Q(t)$	relative frequency of the breaking wave event
$S(f)$	variance spectrum of the sea surface fluctuations
T_z	zero up-crossing wave period
T_b	period of wave which is incipiently breaking
$\bar{T}_z$	average zero-crossing wave period in a record of N waves
t	wave period index
U	wind speed, usually in knots
V	volume integral represented by (42)
X	fetch, usually in nautical miles
Y	number of years "on station"
κ	sea steepness parameter
ν	proportionality factor between H_b and T_b^2
τ^2	rms value of T_z^2
ζ	rms value of H_z

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