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A Refinement of the Previous Hypothesis of Turbulent Energy Spectrum*

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Abstract: To refine the previous hypothesis of turbulent energy spectrum (Nan-niti, 1970), further theoretical consideration and a simple experiment using electric fans was carried out. From the hypothesis of $q=\partial\varepsilon/\partial k$ and $\varepsilon q=$const., the following relations are derived for the spectrum F, and the autocorrelation function R: $F(k)\sim k^{-(5-2\alpha)/3}$ and $R(t)\sim 1-(t/\tau)^{2(1-\alpha)/3}$, where q is the local energy supply, ε the energy dissipation rate, and k the wavenumber, t the time, τ the lifetime of the largest eddy. The following further relations are also obtained: $\varepsilon\sim l^{-\alpha}$, $v\sim l^{\beta}$, $v\sim\tau^{\gamma}$ and $\gamma=(1-\alpha)/(2+\alpha)=\beta/(1-\beta)$, where l is the length scale of the phenomenon, v turbulent velocity, $\alpha=1/2$, $\beta=1/6$, and $\gamma=1/5$. The experimental results support these relations.

1. Introduction

Ozmidov (1960) suggests that the energy dissipation rate may vary with the structure of turbulence under consideration. Kolmogorov (1962) and Obukhov (1962) conclude independently that the variation of the dissipation rate should increase with decrease of the length scale. Obukhov approximates the empirical energy spectrum by

$$F(k)=c\varepsilon^{2/3}k^{-5/3} \qquad (1)$$

where c is a constant. Obukhov shows that the intensity of turbulence varies in successive measurements in the atmosphere indicating that the dissipation rate ε is variable and he considers that $\bar{\varepsilon}$, the mean of ε, is statistically constant and that $\overline{\varepsilon^{1/3}}$ is proportional to $l^{-\alpha}$ with $1\gg\alpha>0$.

Nan-niti (1964a, 1964b) showed independently to them that

$$\varepsilon\sim l^{-\alpha} \qquad (2)$$

from field measurements in a large number of bays and seas with different sizes and that α was approximately 0.4 to 0.5. This relation suggests that the rate of energy dissipation may be constant in a given inertia subrange for a steady eddy field, but in the natural field, eddies of different size may exist at the same time that are generated by different disturbances. This suggests that there may be a local energy supply into the spectral bands and that the energy dissipation rate may be governed by the eddy of largest size in the field. The size of the largest eddy must be determined by the size of the field and turbulent velocity, and this explains why the energy dissipation rate is related to length scale l as in Eq. (2).

In Section 2 of this paper, the above hypothesis is refined and some relation among the parameters of the eddy field are presented. In Section 3, the relations are examined for experimental results obtained using two electric fans.

2. Theoretical considerations

Nan-niti (1970) shows that for energy dissipation that is dependent on wavenumber, there must be an energy supply or sink in the inertia subrange under consideration. If we assume that the local energy supply $q(k)$ is described by

$$q=\partial\varepsilon/\partial k\,, \qquad (3)$$

and that

$$\varepsilon q=\text{const.}, \qquad (4)$$

we have the relation

$$\varepsilon\sim k^{1/2}\sim l^{-1/2}. \qquad (5)$$

Equation (5) shows that α is 0.5. The energy dissipation rate can also be described in terms of turbulent velocity v and the lifetime τ of the largest eddy in the field (Nan-niti, 1964b, 1970):

$$\varepsilon\sim v^2/\tau\,. \qquad (6)$$

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When τ is proportional to L/v, where L is the length scale of the largest eddy, we have

$$\varepsilon \sim v^3/L \sim v^3/l\,. \qquad (7)$$

From Eqs. (2) and (7), we have

$$v \sim l^{(1-\alpha)/3} \sim l^{\beta}, \qquad (8)$$

$$\alpha = 1-3\beta\,. \qquad (9)$$

The energy spectrum $F(k)$, and the autocorrelation function $R(t)$ will be described by

$$F(k) \sim k^{(2\alpha-5)/3} \sim k^{-(1+2\beta)} \qquad (10)$$

$$R(t) \sim 1-(t/\tau)^{2(1-\alpha)/3} \sim 1-(t/\tau)^{2\beta} \qquad (11)$$

from the definition, namely, $F(k)=\int v^2\,dl$ and $R(t)=\int F(k)\cos kt\,dk$, (Taylor, 1938), provided τ is taken to the time $R(t=\tau)=0$. These relations show that α or β can be determined from the functional forms of $F(k)$ and $R(t)$.

On a dimensional and physical basis, the local energy supply may be given by $q \sim V.\ v^2 \sim v^3$, where V is the mean velocity. Then from Eq. (4), we have $(v^2/\tau)\cdot v^3=\text{const}$. From Eqs. (2) (7) and (9), for $v \sim \tau^{\gamma}$, we have

$$\gamma = 1/5 = (1-\alpha)/(2+\alpha) = \beta/(1-\beta)\,. \qquad (12)$$

3. Experiment

A simple experiment was carried out in a large room by setting two electric fans (Mitsubishi Electric, R35-NK) 118 cm above the floor as shown in Fig. 1, in order to examine the relations derived from the hypothesis of local energy supply. A free and fully developed turbulent wind field could be expected to be generated. The wind velocity was measured with a hot-wire anemometer (Kanomax, Model 6141), and recorded on magnetic tape. Wind velocity was

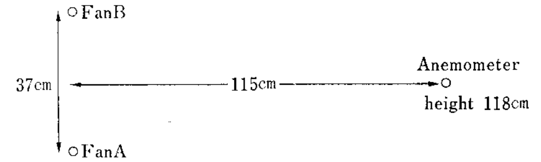

Fig. 1. Schematic figure of the experimental set-up.

Table 1. Experimental run number according to the voltage supplied to Fans A and B.

Fan B / Fan A	Supplying voltage 0	70	80	90
Supplying voltage				
0		4	8	12
70	1	5	9	13
80	2	6	10	14
90	3	7	11	15

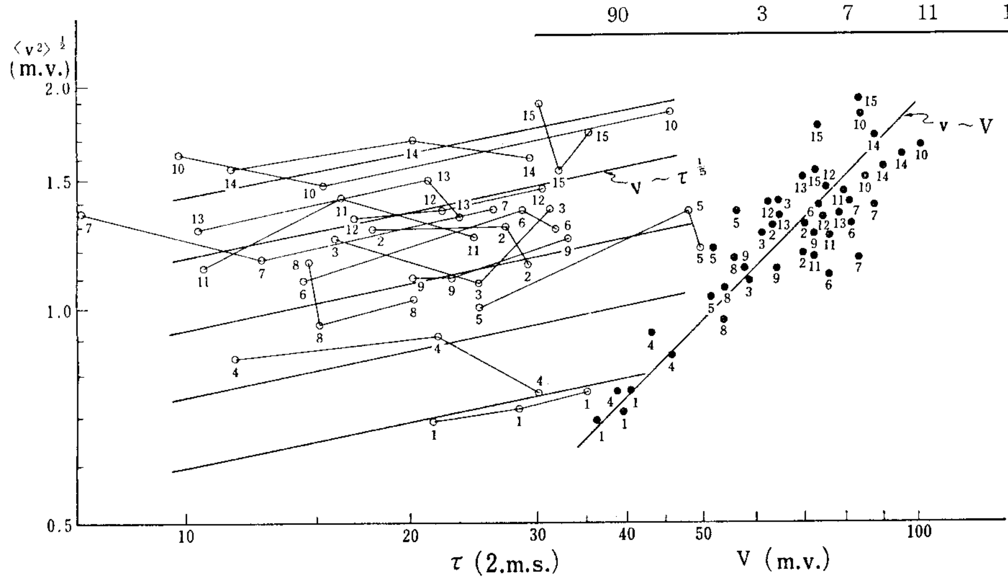

Fig. 2. Relationship between turbulent velocity, $\langle v\rangle^{1/2}$, and lifetime of the largest eddy, τ, and mean velocity, V. Numbers in the figure indicate the number of the experimental runs (as in Table 1).

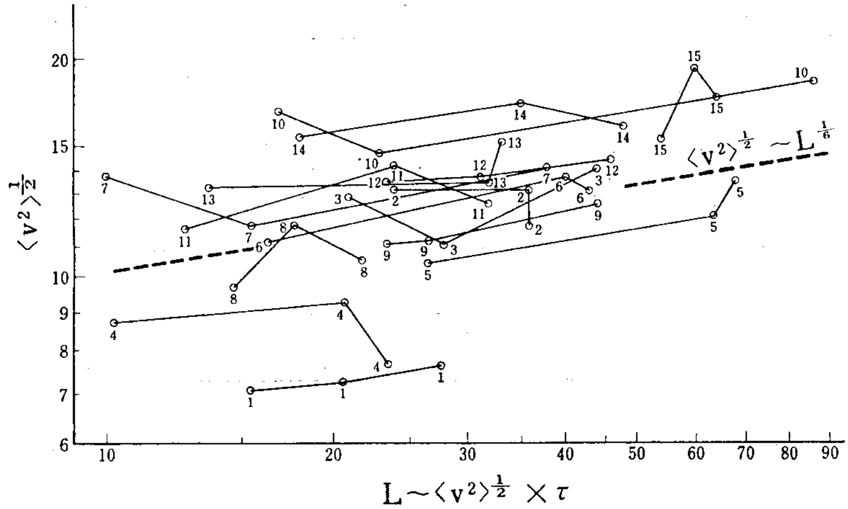

Fig. 3. Relationship between turbulent velocity, $\langle v^2 \rangle^{1/2}$, and scale of turbulence, $L \sim \langle v^2 \rangle^{1/2} \times \tau$. Numbers in the figure indicate the number of the experimental runs (as in Table 1).

controlled by the supply voltage to the fans, namely, 70, 80 and 90 volts. The sequential run numbers are given in Table 1.

Each data length of the analysis is 1,200 millisec at an interval of 2 millisec, so the number of data is 600. The data sets were taken from three portions of each 3-min long run; at the beginning, the middle and the end. The total number of the analyses was 45. Turbulent velocity $v(=\langle v^2 \rangle^{1/2})$, functional forms of spectrum F, and autocorrelation function R were estimated for each analysis.

The relation between v and V is shown in the right portion of Fig. 2. A linear curve shows $v \sim V$. In the left portion of the figure, v is plotted against τ. Triplet for each run is connected by a line. The value of v varies as a function of τ within each experimental run. The relation is described by $v \sim \tau^{1/5}$ as shown by a thick line, this means that $\gamma = 1/5$ by Eq. (12).

Figure 3 shows the relation of v to $L(=v\tau)$. Three data sets taken from each run obey the relation $v \sim L^{1/6}$ which indicates β is 1/6. Figure 4 shows the relation of $\varepsilon\ (\sim v^2/\tau)$ to L. The energy dissipation rate ε is found to be proportional to $L^{-1/2}$, *i.e.*, α is 1/2. The exponents of functional forms of the spectra and autocorrelation functions are tabulated in Table 2. They are determined for the data sets taken from the beginning of each run. Examples of the spectra are shown in Fig. 5, and examples of the autocorrelation function are shown in Fig. 6.

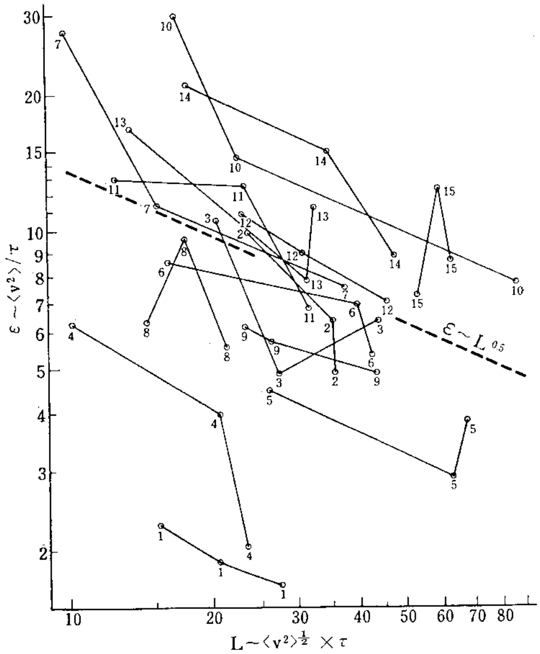

Fig. 4. Relationship between energy dissipation rate, $\tau \sim \langle v^2 \rangle / \varepsilon$, and scale of turbulence, $L \sim \langle v^2 \rangle^{1/2} \times \tau$. Numbers in the figure indicate the number of experimental runs (as in Table 1).

Table 2. Exponent of power spectra and autocorrelation, and calculated α.

number of run	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
exponent of spectra	−5/3	−5/3	−4/3	−5/3	−5/3	−4/3	−5/3	−4/3	−4/3	−5/3	−4/3	−4/3	−5/3	−4/3	−5/3
α calculated from spectra	0	0	0.5	0	0	0.5	0	0.5	0.5	0	0.5	0.5	0	0.5	0
exponent of autocorrelation	2/3	1.6/3	1.4/3	2/3	2/3	1/3	1.2/3	1.6/3	1.6/3	1.5/3	1.6/3	1/3	2/3	1.6/3	2/3
α calculated from autocorrelation	0	0.2	0.3	0	0	0.5	0.4	0.2	0.2	0.25	0.2	0.5	0	0.2	0

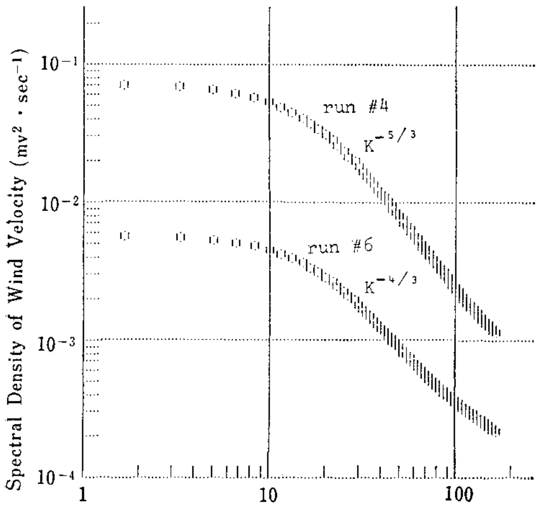

Fig. 5. Spectra of wind velocity. The energy density scale is shown for the upper spectrum. The other spectrum has been displaced downwards by 10 units.

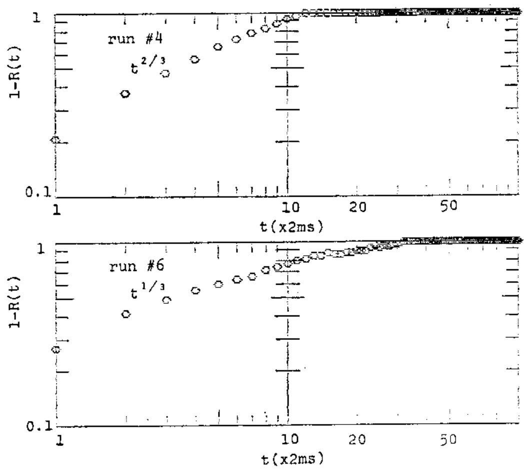

Fig. 6. Relationship between $1-R(t)$ and t.

4. Discussion

Nan-niti (1964b, 1970) and Nan-niti and Konishi (1978) showed that $\alpha=1/2$ and $\beta=1/6$, or the diffusion coefficient $\sim vL \sim L^{7/6}$, in field experiments. The values of α and β are determined from the present experiments by the method of least squares through the relations:

$$\varepsilon \sim 0.566\, L^{-0.578\pm0.053},$$

$$v \sim 0.589\, L^{0.234\pm0.243}.$$

Although the data show high scatter, they still support the conclusion that $\alpha=1/2$ and $\beta=1/6$.

The field experiments (Nan-niti and Konishi, 1978) showed that the exponent of the power spectra was $-4/3$, and $-5/3$. Table 2 of the present experiment shows that the exponent of $-4/3$ are observed in 47% of the runs and that an exponent of $-5/3$ was observed in the remaining 53%. The latter result corresponds to $\alpha=0$. The value of $\alpha=1/2$ is supported by the exponent of the autocorrelation functions in 20% of the runs, while a value of $\alpha=0$ is supported in 33% of the runs.

It should be noted that $\alpha=0$ ($\varepsilon=$const.), $\beta=1/3$ ($v\sim l^{1/3}$) and $\gamma=1/2$ ($v\sim\tau^{1/2}$) for the special case of $q=\partial\varepsilon/\partial k=0$. In conclusion, the fundamental conception of local energy supply is supported both by the field and room experiments.

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乱流エネルギースペクトル仮説の精緻化

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要旨: 先に提出した $q=\partial\varepsilon/\partial k$, $\varepsilon q=$const. なる二つの基本式による乱流エネルギースペクトル仮説に，更に理論的考察を加え，$F(k)\sim k^{-(5-2\alpha)/3}$, $R(t)\sim 1-(t/\tau)^{2(1-\alpha)/3}$, $\varepsilon\sim l^{-\alpha}$, $v\sim l^{\beta}$, $v\sim\tau^{\gamma}$, $\gamma=(1-\alpha)/(2+\alpha)=\beta/(1-\beta)$, $\alpha=1/2$, $\beta=1/6$, $\gamma=1/5$ 等の関係式を導き出し，これらが室内および野外実験で成立していることを立証した．なお local energy supply がないときは $\alpha=0$, $\beta=1/3$, $\gamma=1/2$ となり，従来の理論と一致する．

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