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Technical note

Erosion and deposition of mud beneath random waves

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Abstract

A systematic approach for predicting the mean erosion and mean deposition rates of mud beneath random waves is derived. This has been accomplished by applying formulas valid for regular waves and by describing the waves as a stationary Gaussian narrow-band random process. The present approach covers flow in the laminar, smooth turbulent and rough turbulent flow regimes. Examples are given, using data typical for field conditions representing laminar and smooth turbulent flow conditions, which are the most common flow regimes over mud beds.

Keywords: Cohesive sediments; Mud; Erosion; Deposition; Shear stress; Random waves; Stochastic approach

1. Introduction

Clays and silt are referred to as mud, where fine to medium clay has a median grain diameter d_{50} of 0.001 mm and coarse silt has d_{50} up to 0.06 mm (Soulsby, 1997). The movement of mud within coastal and estuarine waters might have large economical and ecological impact in the development of new engineering works and maintenance of existing installations, e.g., related to necessary routine dredging required for ports' accessibility to shipping. The capability to predict the movement of mud is also crucial to understand the distribution of certain pollutants adsorbed to mud as cohesive sediments are often contaminated nowadays. It appears that organic (pcb's, etc.) and inorganic (heavy metals, etc.) pollutants adhere easily to the clay particles and organic material of the sediments. Further details on the background and complexity as well as reviews of the problems are given in Whitehouse et al. (2000) (hereafter referred to as WSRM) and Winterwerp and van Kesteren (2004).

The purpose of this paper is to provide a method which accounts for the stochastic features of erosion and deposition rates of mud beneath random waves. This is made by assuming the free surface elevation to be a stationary Gaussian narrow-band random process, and the erosion and deposition rate formulas valid for regular waves are valid for individual random waves as well. Examples using data typical for field conditions are given, demonstrating how this approach can be used to make engineering assessments of erosion and deposition rates of mud beneath random waves. To our knowledge, no studies are available in the open literature dealing with erosion and deposition of mud by random waves.

2. Erosion and deposition by regular waves

2.1. Erosion

Following WSRM (Eq. SC (4.4)), the mean erosion rate during a wave-cycle of mud by regular waves is given as

$$\begin{aligned} \dot{m}_E &= m_e(\tau_w - \tau_e) \quad && \text{for } \tau_w > \tau_e \\ \dot{m}_E &= 0 && \text{for } \tau_w \le \tau_e \end{aligned} \tag{1}$$

where the dot represents the time derivative, m_e is the erosion constant, τ_w is the maximum seabed shear stress during a wave-cycle and τ_e is the critical bed shear stress for erosion of a cohesive sediment surface. The result of Eq. (1) is expressed as dry mass of mud eroded per unit area per unit time (kg/m^2 s). For cohesive bed types $\tau_e = E1 C_M^{E2}$ (based on laboratory studies), where $E1$ and $E2$ are site-specific dimensional coefficients, and C_M is the dry density of mud. More details on m_e and τ_e are given in WSRM, and more details on the calculation of τ_w are given in Section 2.3.

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2.2. Deposition

Following WSRM (Eq. (8.4)), the mean deposition rate of mud by regular waves is given as

$$\begin{aligned} \dot{m}_D &= -\left(1-\frac{\tau_w}{\tau_d}\right)C_b w_{50} \quad \text{for } \tau_w < \tau_d \\ \dot{m}_D &= 0 \quad\quad\quad\quad\quad\quad\quad\ \text{for } \tau_w \geq \tau_d \end{aligned} \tag{2}$$

where C_b is the near-bed concentration of suspended mud, w_{50} is the median settling velocity given in WSRM (Eq. SC (5.12)) and τ_d is the critical bed shear stress for deposition above which there is no deposition of suspended mud. Values of τ_d as estimated from laboratory tests are in the range 0.06 N/m^2 to 0.10 N/m^2. τ_d is typically $\tau_e/2$, although not directly related to τ_e. More details on τ_d, C_b and w_{50} are given in WSRM.

2.3. Seabed shear stress

The maximum bed shear stress during a wave-cycle is given as

$$\frac{\tau_w}{\rho} = \frac{1}{2} f_w U^2 \tag{3}$$

where U is the near-bed orbital velocity amplitude, ρ is the density of the fluid and f_w is the wave friction factor given as for laminar, smooth turbulent and rough turbulent flow.

For laminar flow, the wave friction factor is given as that for Stokes' second problem (Schlichting, 1979)

$$f_w = 2Re^{-0.5} \text{ for } Re \lesssim 3 \cdot 10^5 \tag{4}$$

where

$$Re = \frac{UA}{v} \tag{5}$$

is the Reynolds number associated with the wave motion, $A = U/\omega$ is the near-bed orbital displacement amplitude, ω is the wave frequency and v is the kinematic viscosity of the fluid.

For smooth turbulent flow, the Myrhaug (1995) smooth bed wave friction factor is adopted

$$f_w = rRe^{-s} \text{ for } Re \gtrsim 3 \cdot 10^5 \tag{6}$$

with the coefficients

$$(r, s) = (0.0450, 0.175) \tag{7}$$

Alternative coefficients (r, s) for smooth turbulent flow are given in Soulsby (1997).

For rough turbulent flow, the friction factor proposed by Myrhaug et al. (2001) is used

$$f_w = c\left(\frac{A}{z_0}\right)^{-d} \tag{8}$$

$$(c, d) = (18, 1) \text{ for } 20 \lesssim A/z_0 \lesssim 200 \tag{9}$$

$$(c, d) = (1.39, 0.52) \text{ for } 200 \lesssim A/z_0 \lesssim 11,000 \tag{10}$$

$$(c, d) = (0.112, 0.25) \text{ for } 11,000 \lesssim A/z_0 \tag{11}$$

where $z_0 = 2.5d_{50}/30$ is the bed roughness based on the median grain roughness. Note that Eq. (10) corresponds to the coefficients given by Soulsby (1997) obtained as best fit to data for $10 \lesssim A/z_0 \lesssim 10^5$. The advantage of using this friction factor for rough turbulent flow is that it is possible to derive the stochastic approach analytically.

Physically erosion and deposition take place simultaneously (similar as for sand); more discussion is found in Winterwerp and van Kesteren (2004). However, the present approach based on treating the erosion and deposition to take place independently is the one most frequently used. One should note that, although the flow conditions over mud beds most likely are in the laminar or smooth turbulent flow regime (WSRM, p. 52), rough turbulent flow is included in the formulation in order to make it complete.

3. Erosion and deposition by random waves

3.1. General

The present approach is based on the following assumptions:

(1) the free surface elevation $\zeta(t)$ is a stationary Gaussian narrow-band random process with zero expectation described by the single-sided spectral density $S_{\zeta\zeta}(\omega)$;
(2) the bottom friction formulas as well as the erosion and deposition rate formulas for regular waves given in the previous section are valid for irregular waves as well.

The second assumption implies that each wave is treated individually and that memory effects are neglected. The accuracy of this assumption has been justified by Samad (2000) for laminar and smooth turbulent boundary layer flow, for which the bottom friction is given by $\tau_w/\rho = 0.5 f_w U^2$. Here f_w is given in Eq. (4) for laminar flow and by Eq. (6) using (r, s)= (0.041, 0.16) for smooth turbulent flow. Samad (2000) found good agreement between his measured bed shear stresses (laminar and smooth turbulent) under irregular waves and simulations of bed shear stresses based on individual wave formulas. For rough turbulent flow, the validity of this approach was confirmed for seabed shear stresses by Holmedal et al. (2003) for high values of A/z_0 ($\approx$30,000). Time series of seabed shear stresses were obtained by using a standard high-Reynolds number ($k-\varepsilon$) model; the waves were generated using a JONSWAP frequency spectrum with a peakedness factor of 3 and linear potential theory was applied to evaluate the near-bed time series of wave orbital displacement and velocity. Characteristic statistical values of the resulting seabed shear stress amplitude deviated less than 20% from those obtained by the Monte Carlo simulation method by Holmedal et al. (2000). The Holmedal et al. (2000) method is essentially based on the same two assumptions upon which the present approach is based. Regarding the second assumption that each wave is treated individually, Holmedal et al. (2003) concluded for large values of A/z_0 that the main reason for the fair agreement obtained between the Monte Carlo simulations and the ($k-\varepsilon$) model predictions is the good description of the wave friction factor for individual waves. This appears to be much more important than violating the assumption of independent individual waves. Since the erosion and deposition formulas for mud are essentially based on the bed friction factors for laminar, smooth turbulent and rough turbulent flow, the assumption of treating each wave individually seems reasonable.

The narrow-band assumption is generally considered to be a better approximation at the seabed than at the surface, since most of the higher frequencies near the seabed are filtered away because of the linear wave dispersion relationship. It should also be noted that the Holmedal et al. (2003) results included both memory effects and the effect of finite bandwidth of the wave process. Thus, based on the results referred to above, this also justifies that the narrow-band assumption is reasonable for practical application. More details are given in Holmedal et al. (2003). More discussion of the accuracy of the narrow-band assumption will also be given below.

Based on the present assumptions, the time-dependent near-bed orbital displacement $a(t)$ and velocity $u(t)$ are both stationary Gaussian narrow-band processes with zero expectations and with single-sided spectral densities

$$S_{aa}(\omega) = \frac{S_{\zeta\zeta}(\omega)}{\sin \mathrm{h}^2 kh} \tag{12}$$

$$S_{uu}(\omega) = \omega^2 S_{aa}(\omega) = \frac{\omega^2 S_{\zeta\zeta}(\omega)}{\sin \mathrm{h}^2 kh}. \tag{13}$$

For a narrow-band process, the waves are specified as a "harmonic" wave with cyclic frequency ω and with slowly varying amplitude and phase. Then, for the first order, U is related to A by $U=\omega A$ (see, e.g., Sveshnikov, 1966).

It follows from the narrow-band assumption that the near-bed orbital displacement amplitude, A, and the near-bed orbital velocity amplitude, U, are Rayleigh-distributed with the cumulative distribution function given by

$$P(\hat{x}) = 1 - \exp(-\hat{x}^2), \quad \hat{x} = x/x_{\mathrm{rms}} \geq 0 \tag{14}$$

where x represents A or U, and x_{rms} is the rms value of x representing A_{rms} or U_{rms}. Now A_{rms} and U_{rms} are related to the zeroth moments m_{0aa} and m_{0uu} of the amplitude and velocity spectral densities respectively (corresponding to the variances of the amplitude (σ_{aa}^2) and the velocity (σ_{uu}^2)), given by

$$A_{\mathrm{rms}}^2 = 2m_{0aa} = 2\sigma_{aa}^2 = 2\int_0^\infty S_{aa}(\omega)\mathrm{d}\omega \tag{15}$$

$$U_{\mathrm{rms}}^2 = 2m_{0uu} = 2\sigma_{uu}^2 = 2\int_0^\infty S_{uu}(\omega)\mathrm{d}\omega \tag{16}$$

From Eqs. (16) and (13), it also appears that $m_{0uu} = m_{2aa}$, where $m_{2aa} = \int_0^\infty \omega^2 S_{aa}(\omega)\mathrm{d}\omega$ is the second moment of the amplitude spectral density. Thus, the wave frequency ω is taken as the mean zero-crossing frequency for the near-bed orbital displacement ω_z, which gives

$$\omega = \omega_z = \left(\frac{m_{2aa}}{m_{0aa}}\right)^{1/2} = \left(\frac{m_{0uu}}{m_{0aa}}\right)^{1/2} = \frac{U_{\mathrm{rms}}}{A_{\mathrm{rms}}} \tag{17}$$

where Eqs. (15) and (16) have been used. This result is valid for a stationary Gaussian random process. Note that, for a finite-band process, this zero-crossing frequency of $a(t)$ at the bed generally will be smaller than the zero-crossing frequency of $\zeta(t)$ at the surface due to greater attenuation of high frequencies; this means that the high wave frequency components will not reach the bottom. However, for a narrow-band process, these zero-crossing frequencies will be equal, since there is only one frequency present.

3.2. Erosion

Now Eq. (1) is valid for individual random waves. The quantity of interest for random waves is the mean erosion rate during the time series for the irregular wave train in a sea state, given by

$$E[\dot{m}_E] = m_{\mathrm{e}} E[\tau_{\mathrm{w}} | \tau_{\mathrm{w}} > \tau_{\mathrm{e}}] \tag{18}$$

Now τ_{w} is Rayleigh-distributed for laminar flow and Weibull-distributed for both smooth and rough turbulent flow (Myrhaug, 1995), given by the following cumulative distribution function in terms of the non-dimensional shear stress $\hat{\tau} = \tau_{\mathrm{w}}/\tau_{\mathrm{wrms}}$

$$P(\hat{\tau}) = 1 - \exp(-\hat{\tau}^\beta), \quad \hat{\tau} \geq 0 \tag{19}$$

where the Weibull parameter β and τ_{wrms} for the three flow regimes are given in Table 1. Now the results are obtained from Eq. (18) as

$$E[\dot{m}_E] = m_{\mathrm{e}} \tau_{\mathrm{wrms}} E[\hat{\tau} | \hat{\tau} > \hat{\tau}_e] \tag{20}$$

where $\hat{\tau}_{\mathrm{e}} = \tau_{\mathrm{e}}/\tau_{\mathrm{wrms}}$. By using that $\hat{\tau}$ is Rayleigh- and Weibull-distributed, the mean erosion rates for the three flow regimes are given by (see Appendix A)

$$E[\dot{m}_E] = m_{\mathrm{e}} \tau_{\mathrm{wrms}} \Gamma(1.5, \hat{\tau}_e^2)\exp(\hat{\tau}_e^2), \text{ laminar} \tag{21}$$

$$E[\dot{m}_E] = m_{\mathrm{e}} \tau_{\mathrm{wrms}} \Gamma(2-s, \hat{\tau}_e^{1/(1-s)})\exp(\hat{\tau}_e^{1/(1-s)}), \text{ smooth} \tag{22}$$

$$E[\dot{m}_E] = m_{\mathrm{e}} \tau_{wrms} \Gamma\left(2-\frac{d}{2}, \hat{\tau}_e^{2/(2-d)}\right)\exp\left(\hat{\tau}_e^{2/(2-d)}\right), \text{ rough} \tag{23}$$

where $\Gamma(\bullet,\bullet)$ is the incomplete gamma function.

Table 1
The Weibull parameter β and $\tau_{\mathrm{wrms}}/\rho$ for the three flow regimes; $\beta=2$ corresponds to the Rayleigh distribution and $Re_{\mathrm{rms}} = U_{\mathrm{rms}} A_{\mathrm{rms}}/v$

Flow regime	β	$\frac{\tau_{\mathrm{wrms}}}{\rho}$
Laminar, $Re_{\mathrm{rms}} \lesssim 3 \cdot 10^5$	2	$Re_{\mathrm{rms}}^{-0.5} U_{\mathrm{rms}}^2$
Smooth turbulent, $Re_{\mathrm{rms}} \gtrsim 3 \cdot 10^5$	$\frac{1}{1-s}$	$\frac{1}{2} r\, Re_{\mathrm{rms}}^{-s} U_{\mathrm{rms}}^2$
Rough turbulent	$\frac{2}{2-d}$	$\frac{1}{2} c \left(\frac{A_{\mathrm{rms}}}{z_0}\right)^{-d} U_{\mathrm{rms}}^2$

For rough turbulent flow, the (c, d) values are given in Eqs. (9) to (11) when A is replaced by A_{rms}.

A conventional calculation method of the mean erosion rate for random waves is to replace the wave-related quantities by their rms values in an otherwise deterministic approach, i.e., by an equivalent sinusoidal wave, see, e.g., WSRM. In this case, the deterministic result using Eq. (1) is as follows

$$\dot{m}_{E,det} = m_e(\tau_{wrms} - \tau_e) \text{ for } \tau_{wrms} > \tau_e \tag{24}$$

and zero elsewhere, where τ_{wrms} is given in Table 1 for the three flow regimes.

The stochastic to deterministic method ratio is given by

$$R_E = \frac{E[\dot{m}_E]}{\dot{m}_{E,det}}. \tag{25}$$

3.3. Deposition

Similarly, Eq. (2) is valid for individual random waves and the mean deposition rate for the irregular wave train in a sea state is obtained as

$$E[\dot{m}_D] = -\frac{C_b w_{50}}{\tau_d} E[\tau_w | \tau_w < \tau_d] \tag{26}$$

where the stochastic features of τ_w are described after Eq. (18). By using that $\hat{\tau} = \tau_w / \tau_{wrms}$, Eq. (26) takes the form

$$E[\dot{m}_D] = -\frac{C_b w_{50}}{\tau_d} \tau_{wrms} E[\hat{\tau} | \hat{\tau} < \hat{\tau}_d] \tag{27}$$

where $\hat{\tau}_d = \tau_d / \tau_{wrms}$. By using that $\hat{\tau}$ is Rayleigh- and Weibull-distributed, the mean deposition rates for the three flow regimes are given by (see Appendix A)

$$E[\dot{m}_D] = -\frac{C_b w_{50}}{\hat{\tau}_d} \left[\Gamma(1.5) - \Gamma(1.5, \hat{\tau}_d^2)\right] \left[1 - \exp(-\hat{\tau}_d^2)\right]^{-1}, \text{ laminar} \tag{28}$$

$$E[\dot{m}_D] = -\frac{C_b w_{50}}{\hat{\tau}_d} \left[\Gamma(2-s) - \Gamma\left(2-s, \hat{\tau}_d^{1/(1-s)}\right)\right] \times \left[1 - \exp\left(-\hat{\tau}_d^{1/(1-s)}\right)\right]^{-1}, \text{ smooth} \tag{29}$$

$$E[\dot{m}_D] = -\frac{C_b w_{50}}{\hat{\tau}_d} \left[\Gamma\left(2-\frac{d}{2}\right) - \Gamma\left(2-\frac{d}{2}, \hat{\tau}_d^{2/(2-d)}\right)\right] \times \left[1 - \exp\left(-\hat{\tau}_d^{2/(2-d)}\right)\right]^{-1}, \text{ rough} \tag{30}$$

where Γ is the gamma function.

Table 2
Examples of results for erosion and deposition

	Erosion	Deposition
$H_{rms}(m)$	2.83	0.28
$\bar{k}$ (rad/m)	0.0667	0.12
$U_{rms}(m/s)$	0.85	0.050
$A_{rms}(m)$	1.20	0.047
Re_{rms}	$7.5 \cdot 10^5$	1730
Flow regime	Smooth turbulent	Laminar
τ_{wrms} (N/m^2)	1.56	0.062
$E[\dot{m}_E]$ (kg/m^2 s)	0.00158	–
$E[m_E]$ (kg/m^2)	5.69	–
d_E (mm)	81	–
$\dot{m}_{E,det}$ (kg/m^2 s)	0.00136	–
R_E	1.16	–
$E[\dot{m}_D]$ (kg/m^2 s)	–	0.00403
$E[m_D]$ (kg/m^2)	–	14.5
$\dot{m}_{D,det}$ (kg/m^2 s)	–	0.00163
R_D	–	2.47

The given flow conditions for erosion and deposition are given in Sections 3.4.1 and 3.4.2, respectively.

The deterministic result using Eq. (2) is

$$\dot{m}_{D,det} = -\frac{C_b w_{50}}{\tau_d}(\tau_d - \tau_{wrms}) \text{ for } \tau_{wrms} < \tau_d \tag{31}$$

and zero elsewhere.

The stochastic to deterministic method ratio is given by

$$R_D = \frac{E[\dot{m}_D]}{\dot{m}_{D,det}}. \tag{32}$$

3.4. Examples of results

These examples are included to show some results for laminar and smooth turbulent flow, and it follows partly examples in WSRM: Example 4.2 for erosion and Example 8.1 for deposition.

3.4.1. Erosion

The given flow conditions are:

Water depth, h=15 m
Significant wave height, H_s=4 m
Mean wave period, T_z=8.9 s
Median grain diameter (medium silt according to Soulsby, 1997, Fig. 4), d_{50}=0.03 mm
Erosion constant, m_e=0.001 kg/N s
Critical bed shear stress for erosion, τ_e=0.197 N/m^2
Dry density of mud, C_M=70 kg/m^3
Kinematic viscosity of water at temperature 10 °C and salinity 35‰, $\nu = 1.36 \cdot 10^{-6}$ m^2/s
Density of water, ρ=1027 kg/m^3

The calculated quantities are given in Table 2. Here the rms wave height is $H_{rms} = H_s/\sqrt{2}$, $U_{rms} = \omega_z H_{rms}/(2\sinh \bar{k}h)$, $\bar{k}$ is the wave number corresponding to $\omega_z = 2\pi/T_z$ determined from $\omega_z^2 = g\bar{k} \tanh \bar{k}h$, $E[m_E]$ is the mean eroded mass per unit area during 1 h and $d_E = E[m_E]/C_M$ is the mean depth of eroded bed during 1 h.

It appears that the stochastic to deterministic method ratio for erosion R_E is 1.16, suggesting that the deterministic method is an adequate approximation to the stochastic method.

3.4.2. Deposition

The given flow conditions are the same as in Section 3.4.1 except for H_s=0.4 m, T_z=5.9 s and:

Near-bed concentration of suspended mud, C_b=3 kg/m^3
Median settling velocity, w_{50}=2.42 $\cdot$ 10^{-3} m/s
Critical bed shear stress for deposition, τ_d=0.08 N/m^2

The calculated quantities are given in Table 2. Here $E[m_D]$ is the mean deposited mass per unit area during 1 h.

It is noted that the flow is laminar in order to obtain a bed shear stress below the critical value for deposition (i.e., τ_d=0.08 N/m^2). It appears that the stochastic to deterministic method ratio for deposition R_D is 2.47, suggesting that a stochastic approach should be used.

4. Summary

A method for predicting the mean erosion and mean deposition rates of mud beneath random waves has been derived. The method applies formulas valid for regular waves based on treating the erosion and deposition independently, which is the procedure most frequently used. This is combined with describing the waves as a stationary Gaussian narrow-band random process. The method is valid for laminar, smooth turbulent and rough turbulent flow, although the most common flow regimes over muds are laminar and smooth turbulent. Examples using data typical for field conditions representing laminar and smooth turbulent flow conditions are given. From the results exemplified here, it appears that a stochastic approach should be used for deposition, while the deterministic method is an adequate approximation to the stochastic method for erosion. However, other examples might give other results regarding whether a deterministic or a stochastic method should be used. Comparisons with data are required before a conclusion regarding the validity of this approach can be given. In the meantime, the method should serve the purpose of assessing erosion and deposition of mud by random waves.

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Appendix A

Let x be Weibull-distributed with the probability density function

$$p(x) = \beta x^{\beta-1}\exp(-x^{\beta}), \quad x \geq 0 \tag{A1}$$

By using the results in Abramowitz and Stegun (1972, Chapters 6.5 and 26.4)

$$E[x|x > x_1] = \frac{\int_{x_1}^{\infty} xp(x)\mathrm{d}x}{\int_{x_1}^{\infty} p(x)\mathrm{d}x} = \Gamma\left(1+\frac{1}{\beta}, x_1^{\beta}\right)\exp\left(x_1^{\beta}\right) \tag{A2}$$

$$E[x|x < x_1] = \frac{\int_{0}^{x_1} xp(x)\mathrm{d}x}{\int_{0}^{x_1} p(x)\mathrm{d}x} = \left[\Gamma\left(1+\frac{1}{\beta}\right) - \Gamma\left(1+\frac{1}{\beta}, x_1^{\beta}\right)\right]\left[1-\exp\left(-x_1^{\beta}\right)\right]^{-1} \tag{A3}$$

where Γ is the gamma function and $\Gamma(\bullet,\bullet)$ is the incomplete gamma function. The results for the Rayleigh distribution are obtained for β=2.

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